ICA 1: Fermi Problems

ISE 453: Design of PLS Systems — Spring 2020

"Fermi Problems," named after the physicist Enrico Fermi, are inspired guesses about quantities that seem almost impossible to determine given the limited data that you have available. Solving a Fermi Problem involves "guesstimation" of the input parameters needed and back-of-the-envelope type approximations. The goal is to have an answer that is within an order of magnitude of the correct answer (or what is termed a *zeroth-order approximation*). It works because the over- and under-estimations of each parameter tend to cancel each other out as long as there is no consistent bias.

Gusstimation is the first of several levels of modeling used in PLS system design, where each increasing level requires more data/time, while resulting in better accuracy:

0. Guesstimation (order of magnitude)
1. Mean value analysis (linear, ±20%)
2. Nonlinear models (incl. variance, ±5%)
3. Simulation models (complex interactions)
4. Prototypes/pilot studies
5. Build/do and then tweak it

Geometric Mean

In many cases, it is difficult to directly estimate an input parameter *X*, but is easy to estimate reasonable lower and upper bounds (*LB* and *UB*) for the parameter. Since the guessed LB and UB are usually orders of magnitude apart, use of the arithmetic mean, $X = (LB + UB)/2$, would give too much weight to *UB*. As a result, the geometric mean,

$$\text{Geometric Mean: } X = \sqrt{LB \times UB},$$

usually gives a more reasonable estimate because it is a logarithmic average. For example:

> How many people can be crammed into a car? Certainly more than one and less than 100. The average (50) seems to be too high, but the geometric mean (10) is reasonable.[1]

System Performance Estimation

Often easy to estimate performance of a new system if can assume either perfect (LB) or no control (practical UB), as opposed trying to develop a more detailed model of performance.

Example: Estimate the waiting time for a bus, assuming

- 8 min. avg. time (aka "headway") between buses
- Customers arrive at random (assuming no real-time bus tracking)
- Perfect control (LB): wait time = half of headway
- No control (practical UB): wait time = headway (assuming buses arrive at random, aka Poisson process)

[1] Stephan Mertens, "On the back of an envelope," Science 29 August 2008: Vol. 321 no. 5893 p. 1160

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{8}{2} \times 8} = 5.67 \text{ min}$$

- Bad control can result in higher values than no control

Typical Fermi Problem

Without looking on the web, estimate how many McDonald's restaurants there are in the U.S.?

Parameter	LB		UB	Estimate	
Annual per capita demand	1	1 order/person-day x 350 day/yr =	350	18.71	(order/person-yr)
U.S. population				300,000,000	(person)
Operating hours per day				16	(hr/day)
Orders per store per minute (in-store + drive-thru)				1	(order/store-min)
Analysis					
Annual U.S. demand		(person) x (order/person-yr) =		5,612,486,080	(order/yr)
Daily U.S. demand		(order/yr)/365 day/yr =		15,376,674	(order/day)
Daily demand per store		(hrs/day) x 60 min/hr x (order/store-min) =		960	(order/store-day)
Est. number of U.S. stores		(order/day) / (order/store-day) =		16,017	(store)

"Reasonable" (i.e., +/– 10%) guesstimates can be made for all of the parameters needed to make the estimation except for customer demand; as a result, the geometric mean of the estimated lower and upper bounds on demand is used as the estimate. The actual number of McDonald's restaurants in the U.S. as of 2013 is 14,267,[2] which is around 10% below the estimate. A key assumption in the analysis is that the number of McDonald's restaurants in the U.S. has reached market saturation, allowing the entire U.S. population to be used as the customer base.

Questions

1. How can the geometric mean be determined if the LB = 0?
2. If, during the morning rush, there are three buses operating on Wolfline Route 13 and it takes them 45 minutes, on average, to complete one circuit of the route, what is the estimated waiting time for a student who does not use TransLoc for real-time bus tracking?
3. A big-box home improvement store (like a Lowes or Home Depot) receives shipments from its distribution center (DC), where each shipment consists of the entire contents of a tractor trailer. Estimate how many truckloads of product a typical store receives from its DC during an average week.

[2] "Number of McDonald's restaurants in North America in 2012 and 2013, by country," Statista, www.statista.com/statistics/256040/mcdonalds-restaurants-in-north-america/