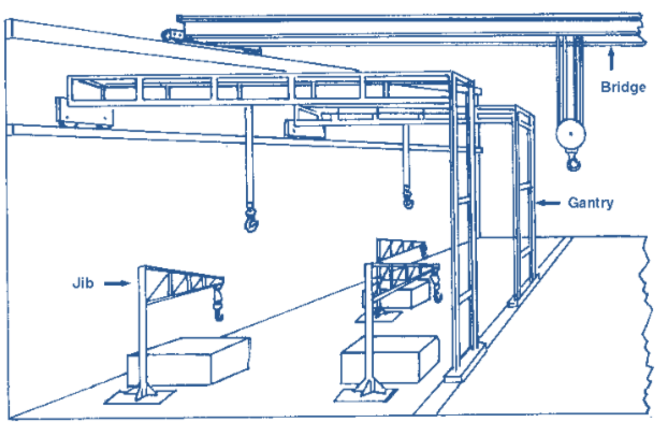
Metric Distances

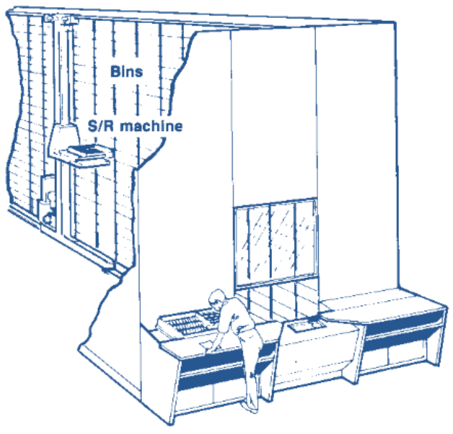
General l_p: $d_p(P_1,P_2)=\left[|x_1-x_2|^p+|y_1-y_2|^p\right]^{\frac{1}{p}}, \quad p\geq 1$

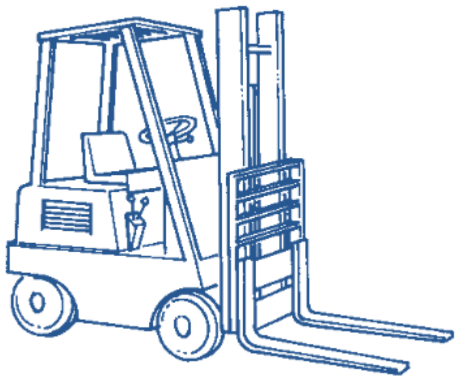
Rectilinear $(p=1)$: $d_1(P_1,P_2)=|x_1-x_2|+|y_1-y_2|$

Euclidean $(p=2)$: $d_2(P_1,P_2)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

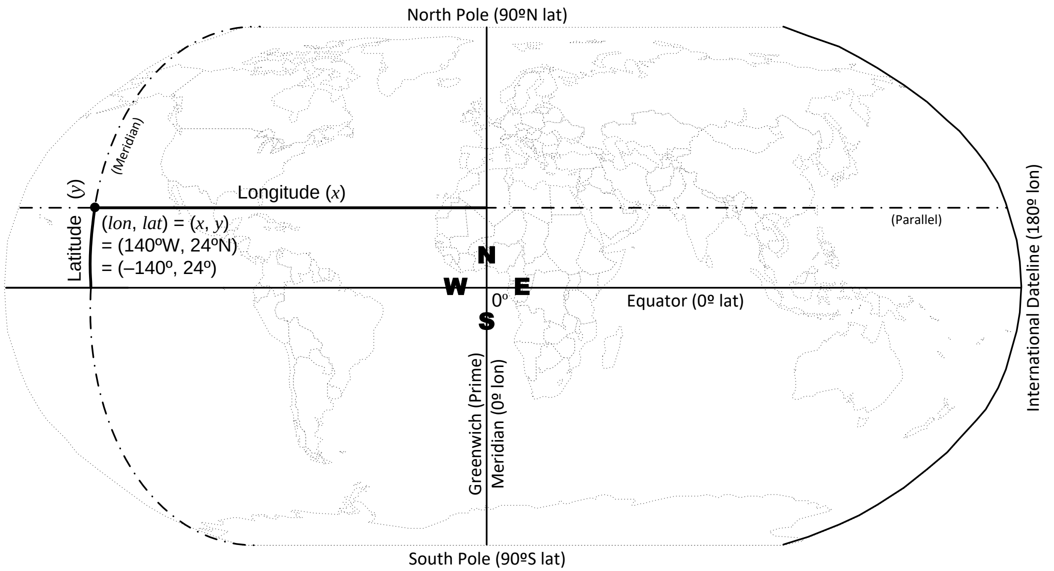
Chebychev $(p\to\infty)$: $d_\infty(P_1,P_2)=\max\{|x_1-x_2|,|y_1-y_2|\}$

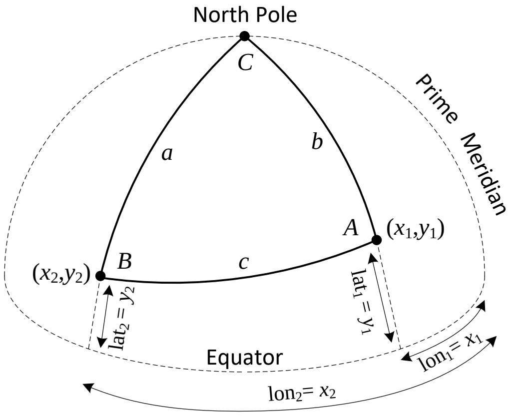


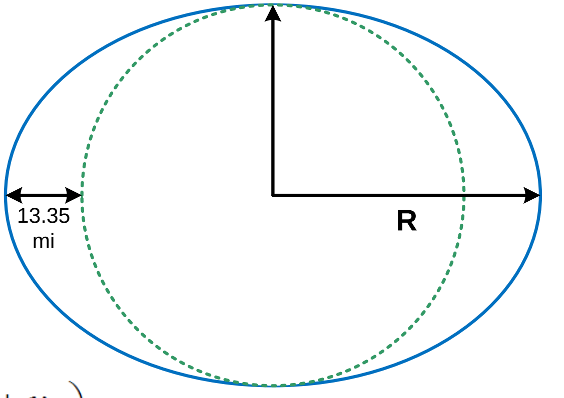




Great Circle Distances







$$(lon_1, lat_1) = (x_1, y_1), \quad (lon_2, lat_2) = (x_2, y_2)$$

$$d_{rad} = (\text{great circle distance in radians of a sphere})$$

$$= \cos^{-1}\left[\sin y_2 \sin y_1 + \cos y_2 \cos y_1 \cos(x_1 - x_2)\right]$$

$$R = (\text{radius of earth at equator}) - (\text{bulge from north pole to equator})$$

$$= 3{,}963.34 - 13.35 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ mi}, \quad = 6{,}378.388 - 21.476 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ km}$$

$$d_{\mathrm{GC}} = \text{distance } (x_1, y_1) \text{ to } (x_2, y_2) = \boxed{d_{rad} \cdot R}$$

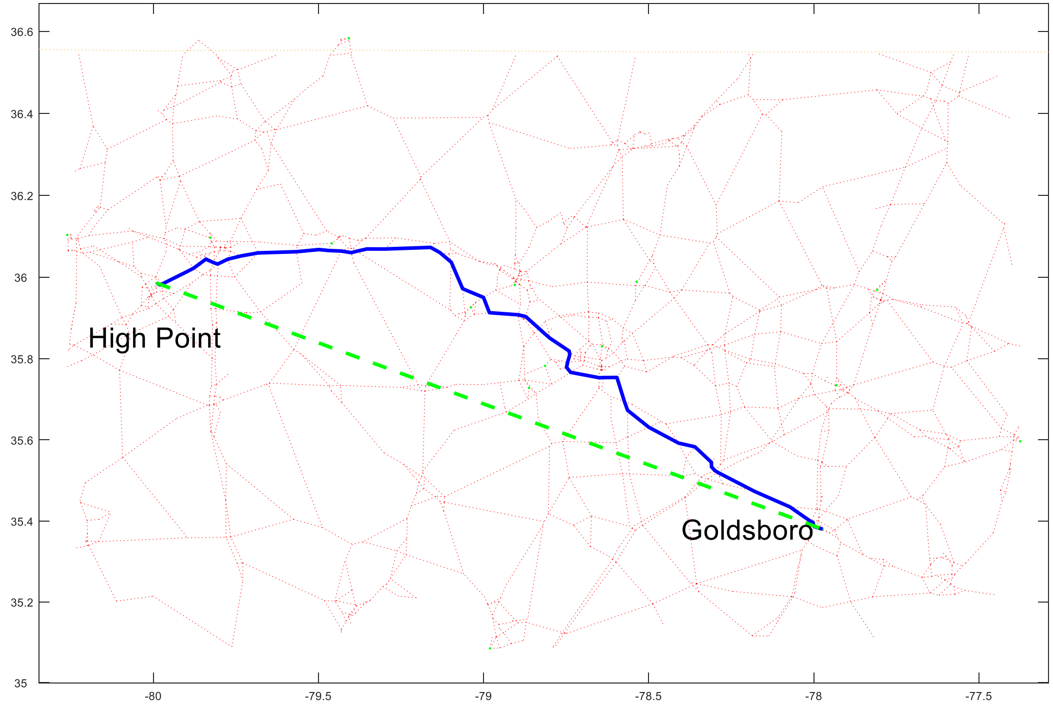
$$x_{\deg} = \begin{cases} DD + \dfrac{MM}{60} + \dfrac{SS}{3{,}600}, & \text{if } E \text{ or } N \\ -DD - \dfrac{MM}{60} - \dfrac{SS}{3{,}600}, & \text{if } W \text{ or } S \end{cases}$$

Circuity Factor

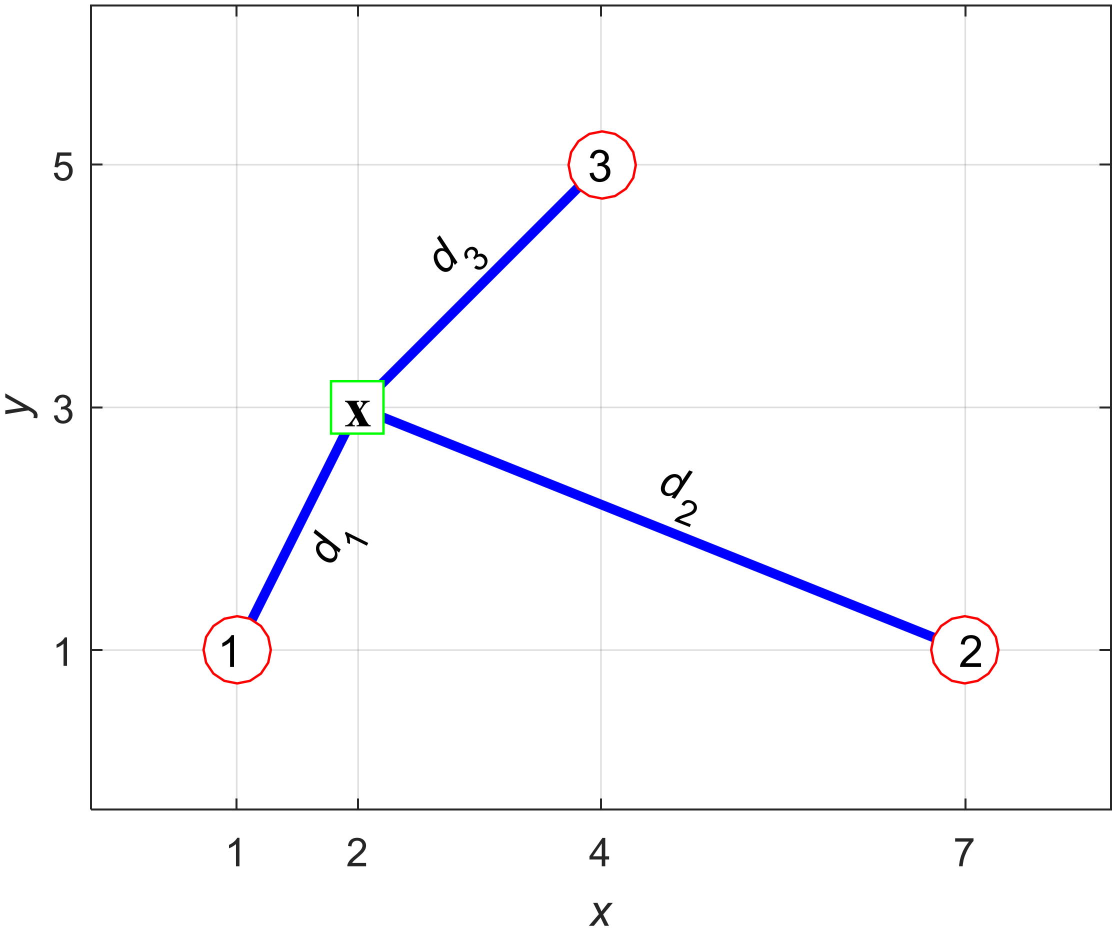
$$\textit{Circuity Factor}: \quad g = \sum \frac{d_{\text{road}_i}}{d_{GC_i}}, \quad \text{where usually } 1.15 \le g \le 1.5$$

$$d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2), \quad \text{estimated road distance from } P_1 \text{ to } P_2$$

From High Point to Goldsboro: Road = 143 mi, Great Circle = 121 mi, Circuity = 1.19



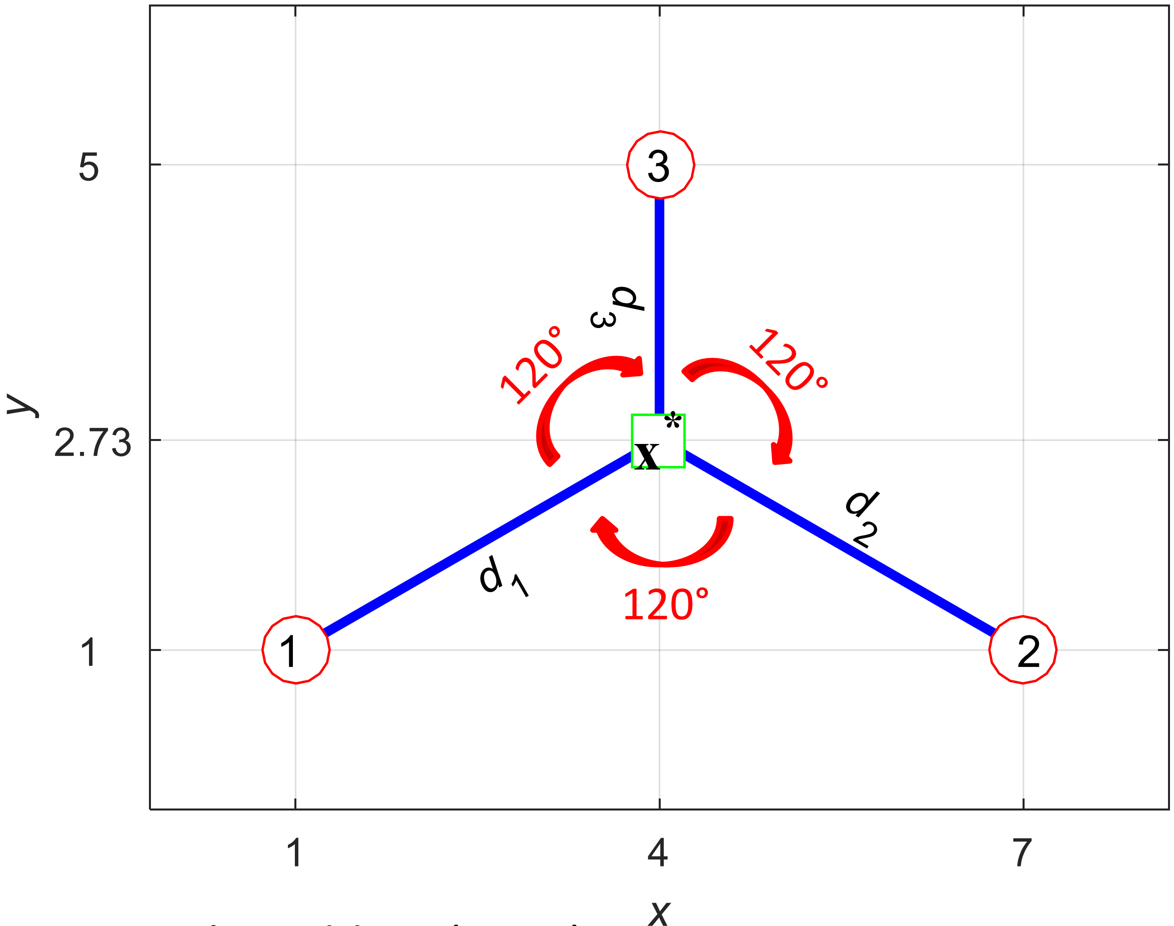
2-D Euclidean Distance



$$\mathbf{x} = \begin{bmatrix} 2 & 3 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_1 - p_{1,1}\right)^2 + \left(x_2 - p_{1,2}\right)^2} \\ \sqrt{\left(x_1 - p_{2,1}\right)^2 + \left(x_2 - p_{2,2}\right)^2} \\ \sqrt{\left(x_1 - p_{3,1}\right)^2 + \left(x_2 - p_{3,2}\right)^2} \end{bmatrix}$$

Minisum Distance Location



$$\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$d_i(\mathbf{x}) = \sqrt{\left(x_1 - p_{i,1}\right)^2 + \left(x_2 - p_{i,2}\right)^2}$$

$$TD(\mathbf{x}) = \sum_{i=1}^{3} d_i(\mathbf{x})$$

$$\mathrm{x}^* = \arg\min_{\mathbf{x}} TD(\mathbf{x})$$

$$TD^* = TD(\mathbf{x}^*)$$

Fermat's Problem (1629):

Given three points, find fourth (Steiner point) such that sum to others is minimized

(Solution: Optimal location corresponds to all angles = 120°)

Minisum Weighted-Distance Location

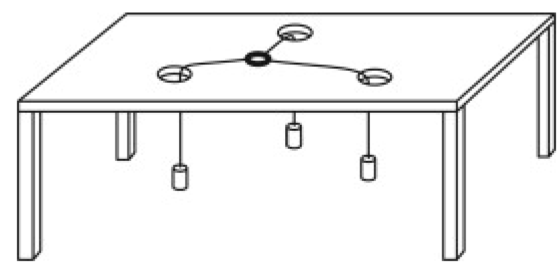
- Solution for 2-D+ and non-rectangular distances:
  - *Majority Theorem:* Locate NF at EF*j* if $w_j \geq \frac{W}{2}, \quad \text{where } W = \sum_{i=1}^{m} w_i$
  - Mechanical (Varigon frame)
  - 2-D rectangular approximation
  - Numerical: nonlinear unconstrained optimization
    - Analytical/estimated derivative (quasi-Newton, `fminunc`)
    - Direct, derivative-free (Nelder-Mead, `fminsearch`)

$$m = \text{number of EFs}$$

$$TC(\mathbf{x}) = \sum_{i=1}^{m} w_i d_i(\mathbf{x})$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

$$TC^* = TC(\mathbf{x}^*)$$



Varignon Frame

Convex vs Nonconvex Optimization