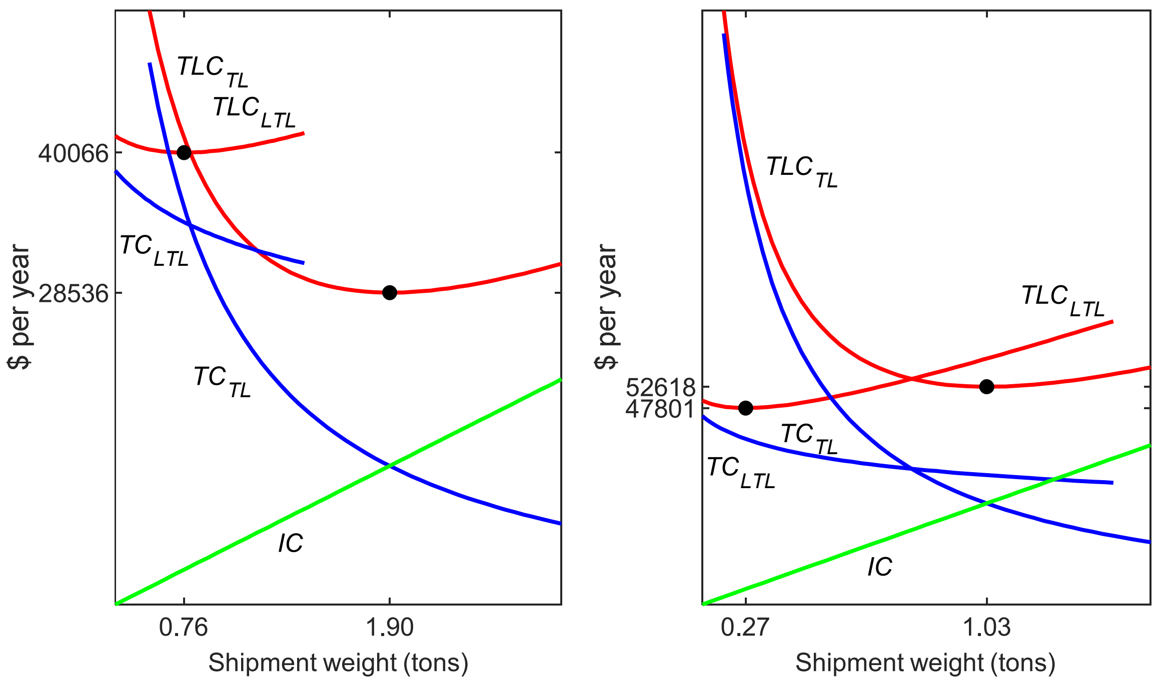
Truck Shipment Example: Periodic

19. If the value of the product increased to $85,000 per ton, should the product be shipped TL or LTL?

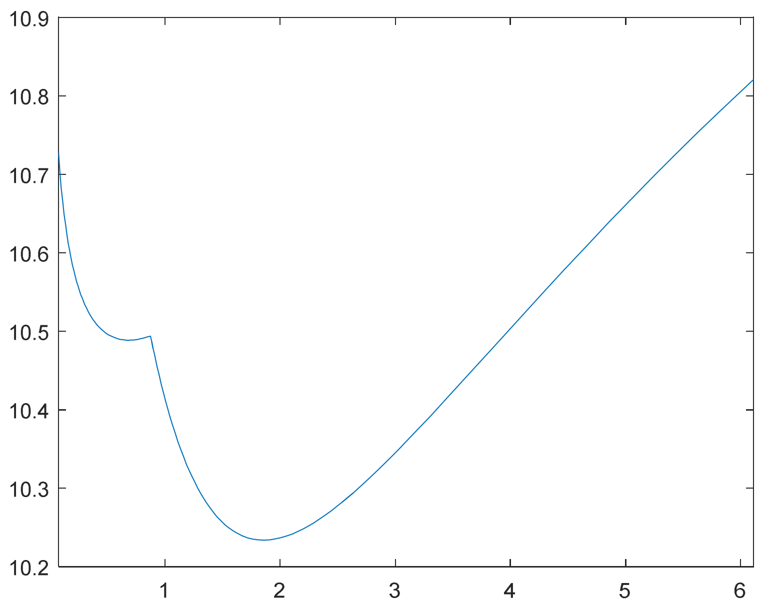


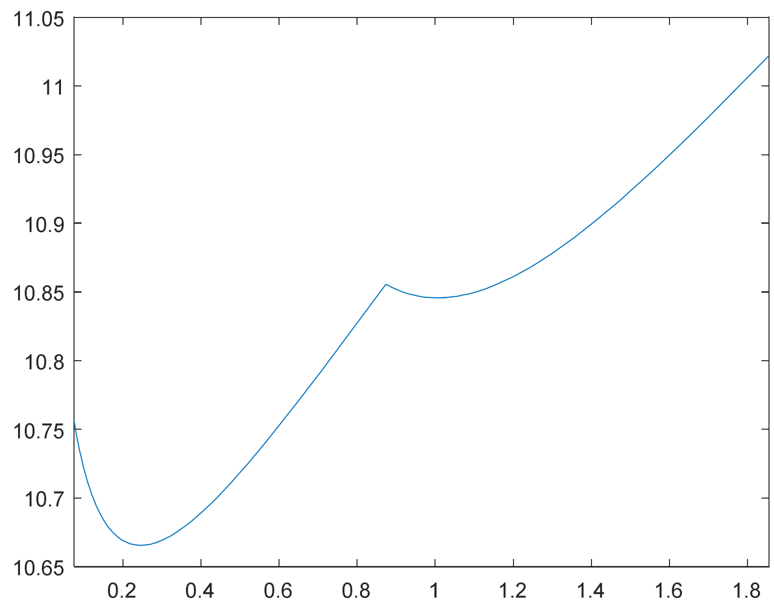
(a) $25000 value per ton (b) $85000 value per ton

Truck Shipment Example: Periodic

- Better to pick from separate optimal TL and LTL because independent charge has two local minima:

$$q_0^* = \arg\min_q \{TLC_{TL}(q), TLC_{LTL}(q)\} \qquad q_0^* \stackrel{!}{=} \arg\min_q \left\{ \frac{f}{q} c_0(q) + \alpha v h q \right\}$$





Truck Shipment Example: Periodic

20. What is optimal independent shipment size to ship 80 tons per year of a Class 60 product valued at $5000 per ton, with the same inventory fraction and carrying rate, between Raleigh and Gainesville?

$$s = 32.16 \text{ lb/ft}^3$$

$$q_0^* = \arg\min_q \left\{ TLC_{TL}(q), TLC_{LTL}(q) \right\} = 8.5079 \text{ ton}$$

$$TLC_{TL}(q_0^*) = \$25{,}523.60 / \text{yr} < TLC_{LTL}(q_0^*)$$

Truck Shipment Example: Periodic

21. What is the optimal shipment size if both shipments will always be shipped together on the same truck (with same shipment interval)?

$$d_1 = d_2, \quad h_{\text{agg}} = h_1 = h_2, \quad \alpha_{\text{agg}} = \alpha_1 = \alpha_2$$

$$f_{\text{agg}} = f_1 + f_2 = 20 + 80 = 100 \text{ ton}$$

$$s_{\text{agg}} = \frac{(\text{aggregate weight, in lb})}{(\text{aggregate cube, in ft}^3)} = \frac{f_{\text{agg}}}{\dfrac{f_1}{s_1} + \dfrac{f_2}{s_2}} = \frac{100}{\dfrac{20}{4.44} + \dfrac{80}{32.16}} = 14.31 \text{ lb/ft}^3$$

$$v_{\text{agg}} = \frac{f_1}{f_{\text{agg}}} v_1 + \frac{f_2}{f_{\text{agg}}} v_2 = \frac{20}{100} 85{,}000 + \frac{80}{100} 5000 = \$21{,}000 / \text{ton}$$

$$q_{TL}^* = \sqrt{\frac{f_{\text{agg}} r_{TL} d}{\alpha_{\text{agg}} v_{\text{agg}} h_{\text{agg}}}} = \sqrt{\frac{100(2.5511)532}{(1)21000(0.3)}} = 4.6414 \text{ ton}$$

Truck Shipment Example: Periodic

- Summary of results:

	f	s	v	qmax	TLC	q	t
1:	20	4.44	85,000	6.11	47,801.01	0.27	5.00
2:	80	32.16	5,000	25.00	25,523.60	8.51	38.84
1+2:					73,324.60		
Aggregate:	100	14.31	21,000	19.68	58,481.90	4.64	16.95

Ex 6: FTL vs Interval Constraint

- On average, 200 tons of components are shipped 750 miles from your fabrication plant to your assembly plant each year. The components are produced and consumed at a constant rate throughout the year. Currently, full truckloads of the material are shipped. What would be the impact on total annual logistics costs if TL shipments were made every two weeks? The revenue per loaded truck-mile is \$2.00; a truck's cubic and weight capacities are 3,000 ft^3 and 24 tons, respectively; each ton of the material is valued at \$5,000 and has a density of 10 lb per ft^3; the material loses 30% of its value after 18 months; and in-transit inventory costs can be ignored.

$$f = 200, \quad d = 750, \quad \alpha = \frac{1}{2} + \frac{1}{2} = 1, \quad r_{TL} = 2, \quad K_{cu} = 3000, \quad K_{wt} = 24, \quad v = 5000, \quad s = 10$$

$$h_{\text{obs}} = \frac{x_h}{t_h} = \frac{0.3}{1.5} = 0.2 \Rightarrow h = 0.05 + 0.06 + 0.2 = 0.31, \quad q_{FTL} = q_{\max} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\} = 15$$

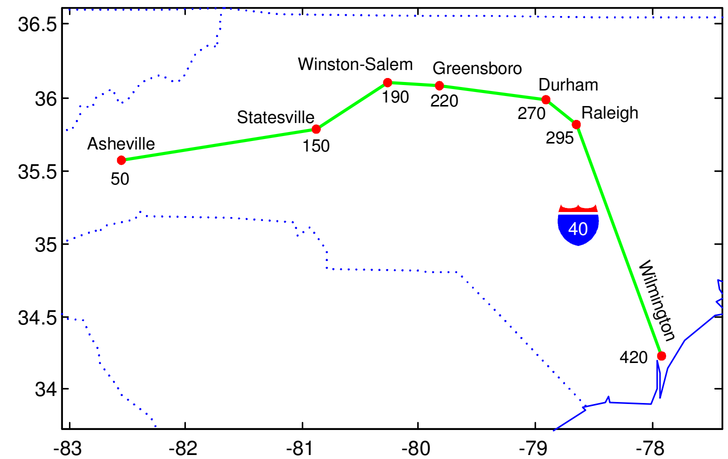
$$n_{FTL} = \frac{f}{q_{FTL}} = 13.33, TLC_{FTL} = n_{FTL} r_{TL} d + \alpha v h q_{FTL} = 43,250, \quad \text{2-wk TL} \Rightarrow \text{LTL not considered}$$

$$t_{\max} = \frac{2 \cdot 7}{365.25} \Rightarrow n_{\min} = 26.09, \quad q_{2\text{wk}} = \frac{f}{n_{\min}} = 7.67, \quad TLC_{2\text{wk}} = n_{\min} r_{TL} d + \alpha v h q_{2\text{wk}} = 51,016$$

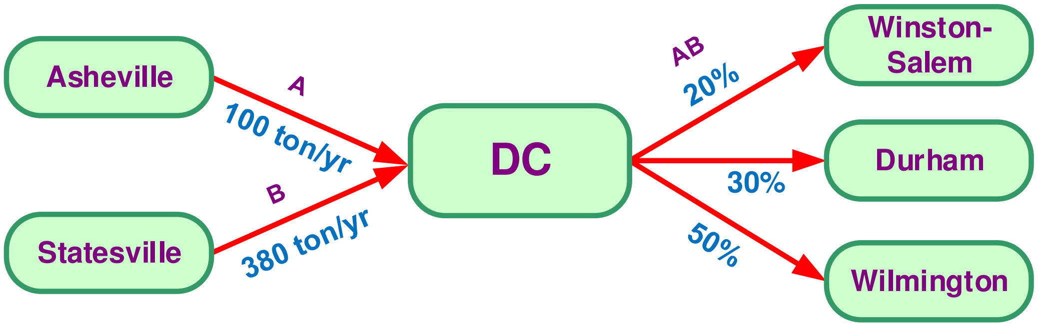
$$\Delta TLC = TLC_{2\text{wk}} - TLC_{FTL} = \$7,766 \text{ per year increase with two-week interval constraint}$$

Ex 7: FTL Location

- Where should a DC be located in order to minimize transportation costs, given:
  1. FTLs containing mix of products A and B shipped P2P from DC to customers in Winston-Salem, Durham, and Wilmington
  2. Each customer receives 20, 30, and 50% of total demand
  3. 100 tons/yr of A shipped FTL P2P to DC from supplier in Asheville
  4. 380 tons/yr of B shipped FTL P2P to DC from Statesville
  5. Each carton of A weighs 30 lb, and occupies 10 ft^3
  6. Each carton of B weighs 120 lb, and occupies 4 ft^3
  7. Revenue per loaded truck-mile is \$2
  8. Each truck's cubic and weight capacity is 2,750 ft^3 and 25 tons, respectively



Ex 7: FTL Location



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

$$\underset{(\$/\text{mi-yr})}{w_i} = \underset{(\text{ton/yr})}{f_i} \times \underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \underset{(\text{TL/yr})}{n_i} \times \underset{(\$/\text{TL-mi})}{r_i}$$

$$\underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \frac{r}{q_{\max}}, \quad n = \frac{f}{q_{\max}}$$

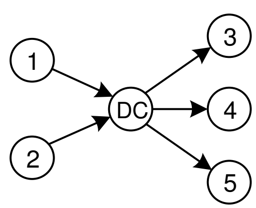
$$r = \$2\,/\,\text{TL-mi}, \quad f_{agg} = f_A + f_B = 100 + 380 = 480 \text{ ton/yr}, \quad s_{agg} = \frac{f_{agg}}{\frac{f_A}{s_A} + \frac{f_B}{s_B}} = \frac{480}{\frac{100}{3} + \frac{380}{30}} = 10.4348 \text{ lb/ft}^3, \quad q_{\max} = \left\{25, \frac{10.4348(2750}{2000}\right\} = 14.3478$$

$$s_1 = \frac{30}{10} = 3 \text{ lb/ft}^3, \; q_{\max} = \min\left\{25, \frac{3(2750)}{2000}\right\} = 4.125 \text{ ton}$$

$$f_1 = 100, \; n_1 = \frac{100}{4.125} = 24.24, \; w_1 = 24.24(2) = 48.48$$

$$s_2 = \frac{120}{4} = 30 \text{ lb/ft}^3, \; q_{\max} = \min\left\{25, \frac{30(2750)}{2000}\right\} = 25 \text{ ton}$$

$$f_2 = 380, \; n_2 = \frac{380}{25} = 15.2, \; w_2 = 15.2(2) = 30.4$$

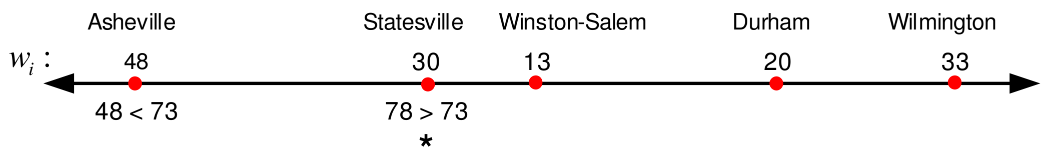


$$f_3 = 0.20 f_{agg} = 96, \; n_3 = \frac{96}{14.3478} = 6.69, \; w_3 = 6.69(2) = 13.38$$

$$f_4 = 0.30 f_{agg} = 144, \; n_4 = \frac{144}{14.3478} = 10.04, \; w_4 = 10.04(2) = 20.07$$

$$f_5 = 0.50 f_{agg} = 240, \; n_5 = \frac{240}{14.3478} = 16.73, \; w_5 = 16.73(2) = 33.45$$

$$W = 146, \quad \frac{W}{2} = 73$$



$$\text{(Montetary) Weight Losing:} \quad \Sigma w_{\text{in}} = 79 > \Sigma w_{\text{out}} = 67 \quad \left(\Sigma n_{\text{in}} = 39 > \Sigma n_{\text{out}} = 33\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$$

Ex 7: FTL Location

- Include monthly outbound frequency constraint:
  - Outbound shipments must occur at least once each month
  - Implicit means of including inventory costs in location decision

$$t_{\max} = \frac{1}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 12 \text{ TL/yr}$$

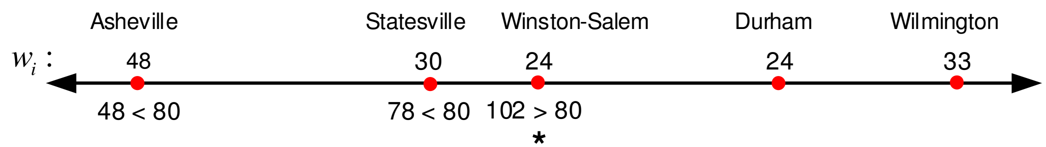
$$TC'_{FTL} = \max\left\{n, n_{\min}\right\} rd$$

$$n_3 = \max\left\{6.69, 12\right\} = 12, w_3 = 12(2) = 24$$

$$n_4 = \max\left\{10.04, 12\right\} = 12, w_4 = 12(2) = 24$$

$$n_5 = \max\left\{16.73, 12\right\} = 16.73, w_5 = 16.73(2) = 33.45$$

$$W = 160, \quad \frac{W}{2} = 80$$



$$\text{(Montetary) Weight Gaining:} \quad \Sigma w_{\text{in}} = 79 < \Sigma w_{\text{out}} = 81 \quad \left(\Sigma n_{\text{in}} = 39 < \Sigma n_{\text{out}} = 41\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$$

Location and Transport Costs

- Monetary weights *w* used for location are, in general, a function of the location of a NF
  - Distance *d* appears in optimal TL size formula
  - TC & IC functions of location $\Rightarrow$ Need to minimize TLC instead of TC
  - FTL (since size is fixed at max payload) results in only constant weights for location $\Rightarrow$ Need to only minimize TC since IC is constant in TLC

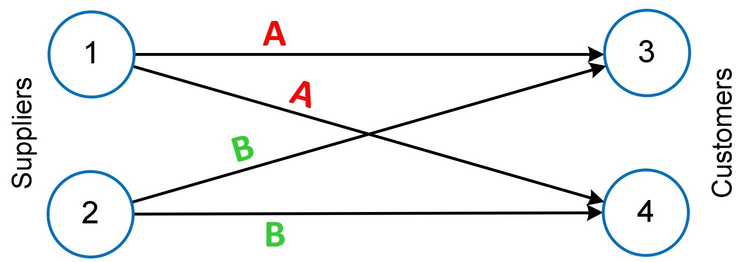
$$TLC_{TL}(\mathbf{x}) = \sum_{i=1}^{m} w_i(\mathbf{x}) d_i(\mathbf{x}) + \alpha v h q_i(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_i(\mathbf{x})} r d_i(\mathbf{x}) + \alpha v h\, q_i(\mathbf{x})$$

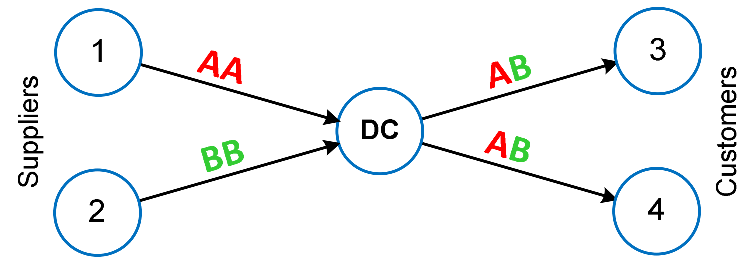
$$= \sum_{i=1}^{m} \frac{f_i}{\sqrt{\dfrac{f_i r d_i(\mathbf{x})}{\alpha v h}}} r d_i(\mathbf{x}) + \alpha v h \sqrt{\frac{f_i r d_i(\mathbf{x})}{\alpha v h}} = \sum_{i=1}^{m} \sqrt{f_i r d_i(\mathbf{x})} \left(\frac{1}{\sqrt{\alpha v h}} + \sqrt{\alpha v h} \right)$$

$$TLC_{FTL}(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_{\max}} r d_i(\mathbf{x}) + \alpha v h\, q_{\max} = \sum_{i=1}^{m} w_i\, d_i(\mathbf{x}) + \alpha v h\, q_{\max} = TC_{FTL}(\mathbf{x}) + \text{constant}$$

Transshipment

- *Direct*: P2P shipments from Suppliers to Customers

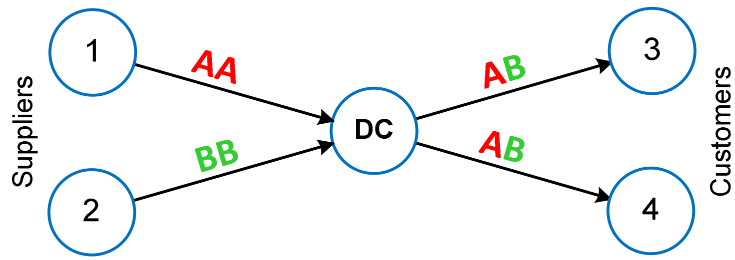




- *Transshipment*: use DC to consolidate outbound shipments
  - *Uncoordinated*: determine separately each optimal inbound and outbound shipment ⇒ hold inventory at DC
  - *(Perfect) Cross-dock*: use single shipment interval for all inbound and outbound shipments ⇒ no inventory at DC (usually only cross-dock a selected subset of shipments)

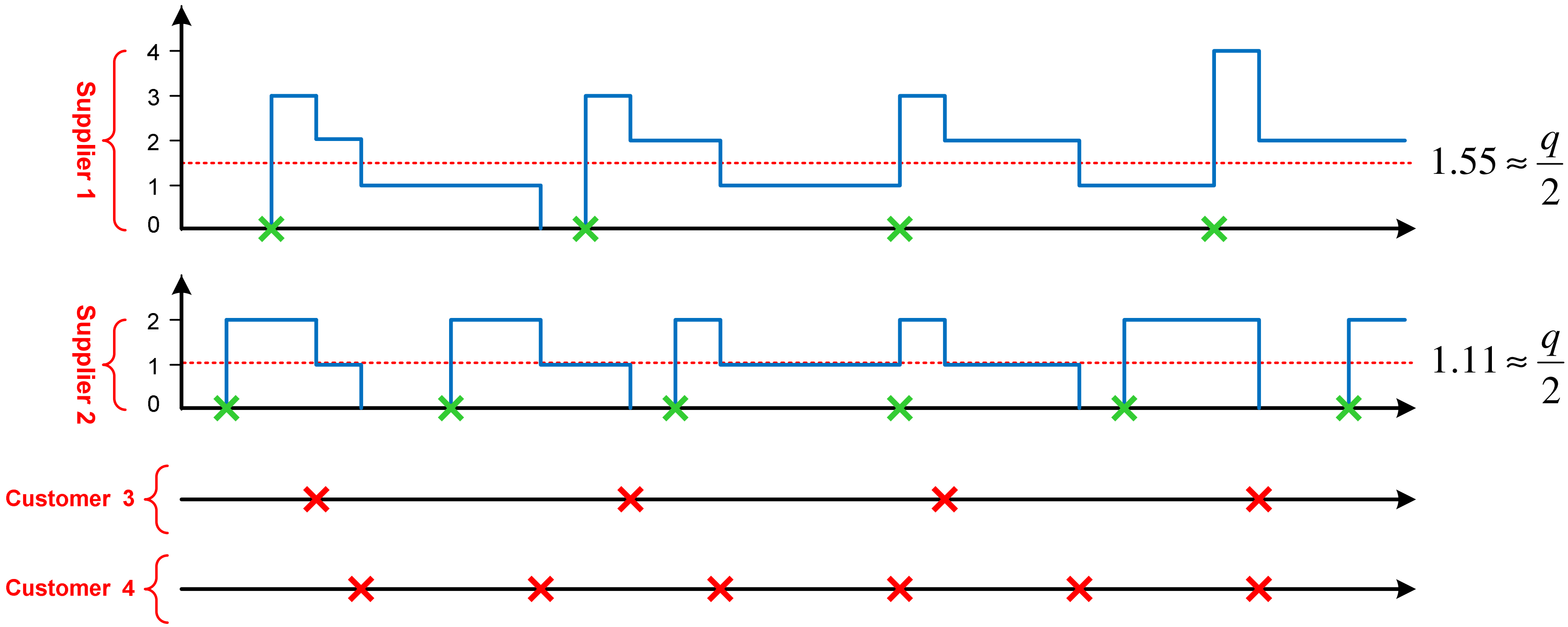
Uncoordinated Inventory

- Average pipeline inventory level at DC:



$$\alpha = \alpha_O + \alpha_D$$

$$= \begin{cases} \alpha_O + \frac{1}{2}, & \text{inbound} \\ 0 + \alpha_D, & \text{outbound} \end{cases}$$



TLC with Transshipment

- Uncoordinated: $TLC_i = TLC$ of supplier/customer i

$$q_i^* = \arg\min_q TLC_i(q)$$

$$TLC^* = \sum TLC_i\left(q_i^*\right)$$

- Cross-docking: $t = \dfrac{q}{f}$, shipment interval

$$TLC_i(t) = \frac{c_0(t)}{t} + \alpha v h f t \quad \left(\text{cf. } TLC_i(q) = \frac{f}{q} c_0(q) + \alpha v h q\right)$$

$$c_0(t) = \text{independent transport charge as function of } t$$

$$\alpha = \begin{cases} \alpha_O + 0, & \text{inbound} \\ 0 + \alpha_D, & \text{outbound} \end{cases}$$

$$t^* = \arg\min_t \sum TLC_i(t)$$

$$TLC^* = \sum TLC_i\left(t^*\right)$$