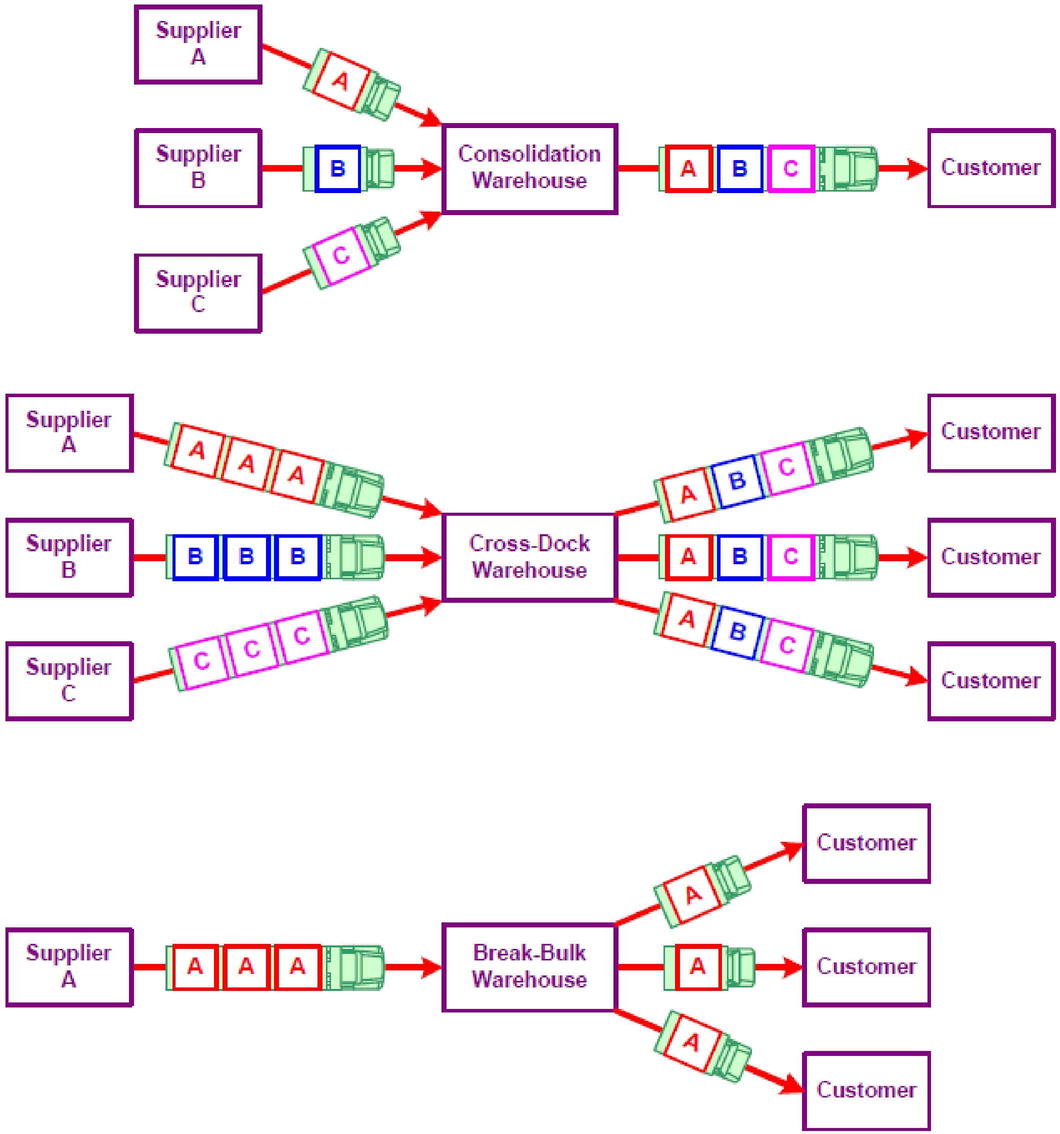
# Warehousing

- *Warehousing* are the activities involved in the design and operation of warehouses
- A *warehouse* is the point in the supply chain where raw materials, work-in-process (WIP), or finished goods are stored for varying lengths of time.
- Warehouses can be used to add value to a supply chain in two basic ways:
  1. Storage. Allows product to be available where and when its needed.
  2. Transport Economies. Allows product to be collected, sorted, and distributed efficiently.
- A *public warehouse* is a business that rents storage space to other firms on a month-to-month basis. They are often used by firms to supplement their own *private warehouses*.

# Types of Warehouses



# Warehouse Design Process

- The objectives for warehouse design can include:
  - maximizing cube utilization
  - minimizing total storage costs (including building, equipment, and labor costs)
  - achieving the required storage throughput
  - enabling efficient order picking
- In planning a storage layout: either a storage layout is required to fit into an existing facility, or the facility will be designed to accommodate the storage layout.

# Warehouse Design Elements

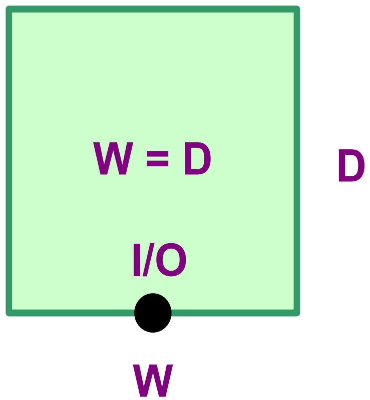
- The design of a new warehouse includes the following elements:
  1. Determining the layout of the storage locations (i.e., the warehouse layout).
  2. Determining the number and location of the input/output (I/O) ports (e.g., the shipping/receiving docks).
  3. Assigning items (stock-keeping units or *SKUs*) to storage locations (*slots*).
- A typical objective in warehouse design is to minimize the overall storage cost while providing the required levels of service.

# Design Trade-Off

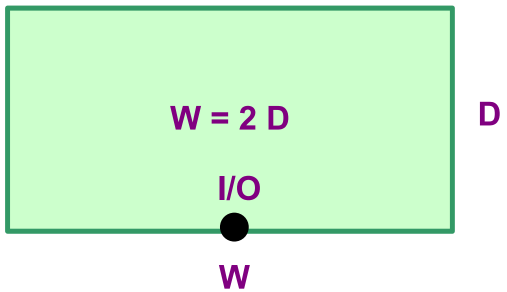
- Warehouse design involves the trade-off between building and handling costs:

| | | |
|---|---|---|
| min Building Costs | vs. | min Handling Costs |
| $\Updownarrow$ | | $\Updownarrow$ |
| max Cube Utilization | vs. | max Material Accessibility |

# Shape Trade-Off



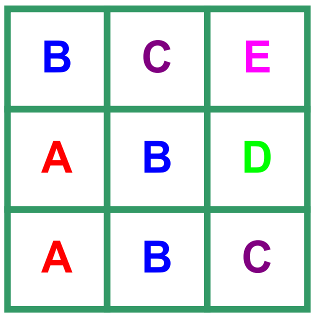

vs.



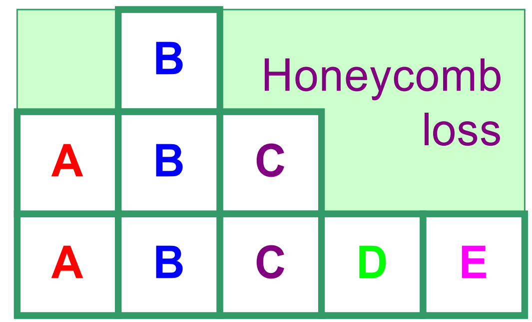

Square shape minimizes perimeter length for a given area, thus minimizing building costs

Aspect ratio of 2 (W = 2D) min. expected distance from I/O port to slots, thus minimizing handling costs

# Storage Trade-Off




vs.



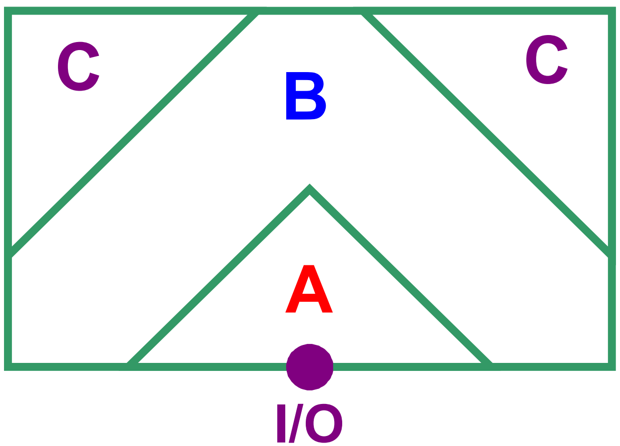

Maximizes cube utilization, but minimizes material accessibility

Making at least one unit of each item accessible decreases cube utilization

# Storage Policies

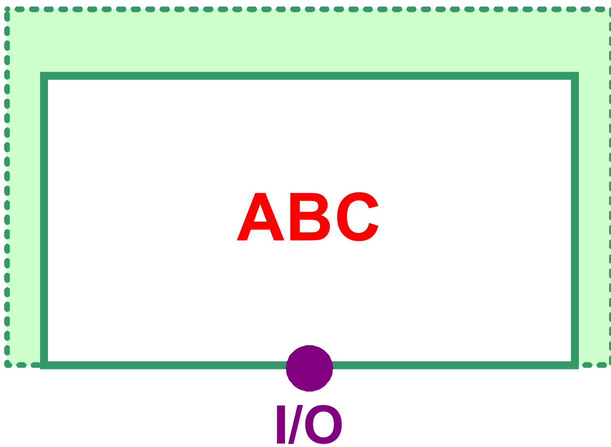
- A storage policy determines how the slots in a storage region are assigned to the different SKUs to the stored in the region.
- The differences between storage polices illustrate the trade-off between minimizing building cost and minimizing handling cost.
- Type of policies:
  - Dedicated
  - Randomized
  - Class-based

# Dedicated Storage



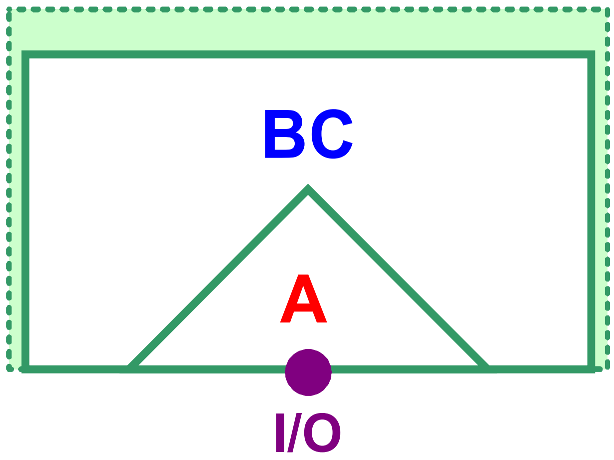

- Each SKU has a predetermined number of slots assigned to it.
- Total capacity of the slots assigned to each SKU must equal the storage space corresponding to the maximum inventory level of each *individual* SKU.
- Minimizes handling cost.
- Maximizes building cost.

# Randomized Storage



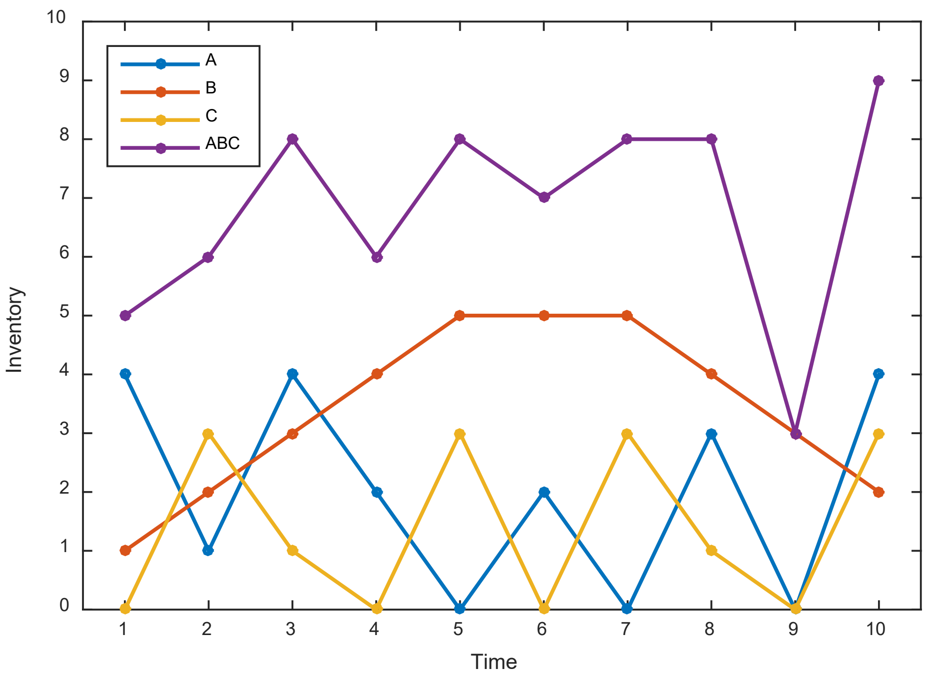

- Each SKU can be stored in any available slot.
- Total capacity of all the slots must equal the storage space corresponding to the maximum *aggregate* inventory level of all of the SKUs.
- Maximizes handling cost.
- Minimizes building cost.

# Class-based Storage




- Combination of dedicated and randomized storage, where each SKU is assigned to one of several different storage classes.
- Randomized storage is used for each SKU within a class, and dedicated storage is used between classes.
- Building and handling costs between dedicated and randomized.

# Individual vs Aggregate SKUs



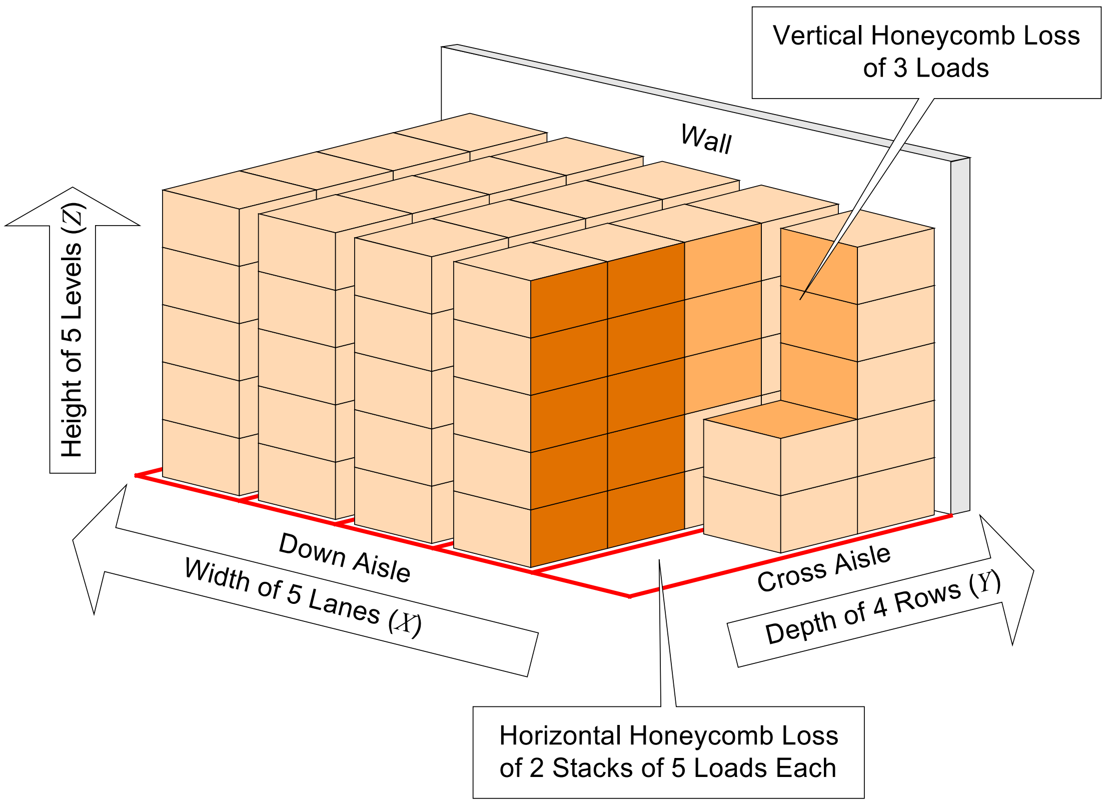

| | Dedicated | | | Random | Class-Based | | |
|---|---|---|---|---|---|---|---|
| **Time** | A | B | C | ABC | AB | AC | BC |
| 1 | 4 | 1 | 0 | 5 | 5 | 4 | 1 |
| 2 | 1 | 2 | 3 | 6 | 3 | 4 | 5 |
| 3 | 4 | 3 | 1 | 8 | 7 | 5 | 4 |
| 4 | 2 | 4 | 0 | 6 | 6 | 2 | 4 |
| 5 | 0 | 5 | 3 | 8 | 5 | 3 | 8 |
| 6 | 2 | 5 | 0 | 7 | 7 | 2 | 5 |
| 7 | 0 | 5 | 3 | 8 | 5 | 3 | 8 |
| 8 | 3 | 4 | 1 | 8 | 7 | 4 | 5 |
| 9 | 0 | 3 | 0 | 3 | 3 | 0 | 3 |
| 10 | 4 | 2 | 3 | 9 | 6 | 7 | 5 |
| $M_i$ | 4 | 5 | 3 | 9 | 7 | 7 | 8 |

# Cube Utilization

- *Cube utilization* is percentage of the total space (or “cube”) required for storage actually occupied by items being stored.
- There is usually a trade-off between cube utilization and material accessibility.
- Bulk storage using block stacking can result in the minimum cost of storage, but material accessibility is low since only the top of the front stack is accessible.
- Storage racks are used when support and/or material accessibility is required.

# Honeycomb Loss

- *Honeycomb loss*, the price paid for accessibility, is the unusable empty storage space in a lane or stack due to the storage of only a single SKU in each lane or stack



# Estimating Cube Utilization

- The (3-D) cube utilization for dedicated and randomized storage can estimated as follows:

$$\text{Cube utilization} = \frac{\text{item space}}{\text{total space}} = \frac{\text{item space}}{\text{item space} + \begin{pmatrix}\text{honeycomb}\\ \text{loss}\end{pmatrix} + \begin{pmatrix}\text{down aisle}\\ \text{space}\end{pmatrix}}$$

$$CU(\text{3-D}) = \begin{cases} \dfrac{x \cdot y \cdot z \cdot \sum_{i=1}^{N} M_i}{TS(D)}, & \text{dedicated} \\ \dfrac{x \cdot y \cdot z \cdot M}{TS(D)}, & \text{randomized} \end{cases}$$

$$CU(\text{2-D}) = \begin{cases} \dfrac{x \cdot y \cdot \sum_{i=1}^{N} \left\lceil \dfrac{M_i}{H} \right\rceil}{TA(D)}, & \text{dedicated} \\ \dfrac{x \cdot y \cdot \left\lceil \dfrac{M}{H} \right\rceil}{TA(D)}, & \text{randomized} \end{cases}$$

where

$x$ = lane/unit-load width

$y$ = unit-load depth

$z$ = unit-load height

$M_i$ = maximum number of units of SKU $i$

$M$ = maximum number of units of all SKUs

$N$ = number of different SKUs

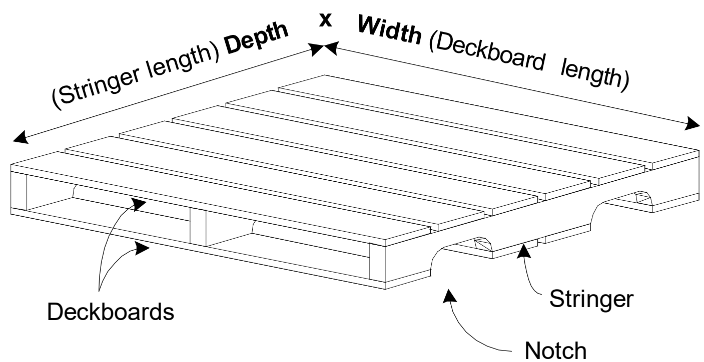
$D$ = number of rows

$TS(D)$ = total 3-D *space* (given $D$ rows of storage).

$TA(D)$ = total 2-D *area* (given $D$ rows of storage).

# Unit Load

- *Unit load*: single unit of an item, or multiple units restricted to maintain their integrity
- Linear dimensions of a unit load:




*Depth* (stringer length) × *Width* (deckboard length)

$$y \times x$$

- Pallet height (5 in.) + load height gives $z$: $y \times x \times z$

# Cube Utilization for Dedicated Storage

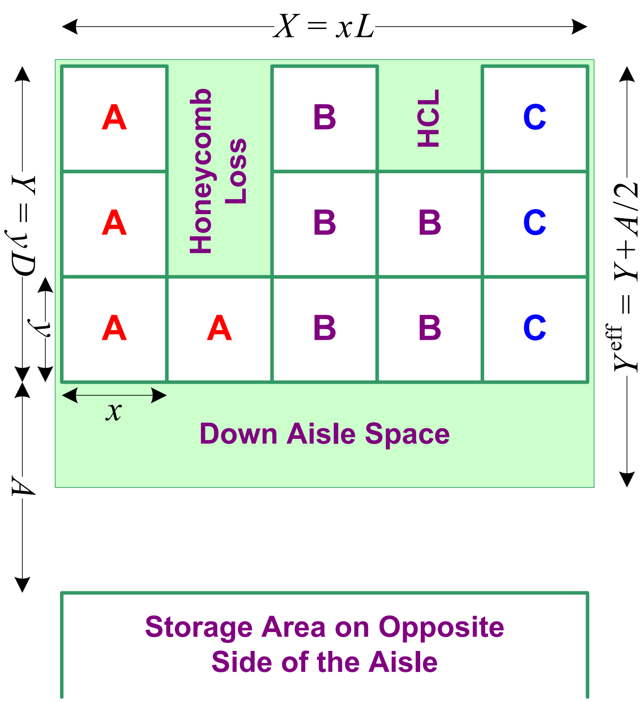
| | Storage Area at Different Lane Depths | Item Space | Lanes | Total Space | Cube Util. |
|---|---|---|---|---|---|
| $D = 1$<br>$A/2 = 1$ | A A A A B B B B B C C C | 12 | 12 | 24 | 50% |
| $D = 2$<br>$A/2 = 1$ | A A B B   C<br>A A B B B C C | 12 | 7 | 21 | 57% |
| $D = 3$<br>$A/2 = 1$ | A   B   C<br>A   B B C<br>A A B B C | 12 | 5 | 20 | 60% |

# Total Space/Area

- The total space required, as a function of lane depth *D*:

$$\text{Total space (3-D):}\quad TS(D) = X \cdot \underbrace{\left(Y + \frac{A}{2}\right)}_{\text{Eff. lane depth}} \cdot Z = xL(D) \cdot \left(yD + \frac{A}{2}\right) \cdot zH$$

$$\text{Total area (2-D):}\quad TA(D) = \frac{TS(D)}{Z} = X \cdot Y^{\text{eff}} = xL(D) \cdot \left(yD + \frac{A}{2}\right)$$




where

$X =$ width of storage region (row length)

$Y =$ depth of storage region (lane depth)

$Z =$ height of storage region (stack height)

$A =$ down aisle width

$L(D) =$ number of lanes (given $D$ rows of storage)

$H =$ number of levels.

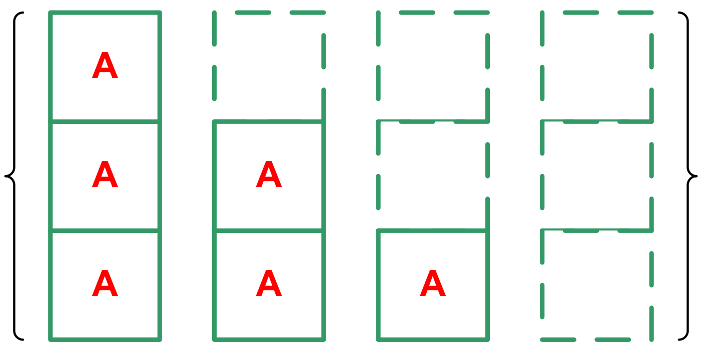
# Number of Lanes

- Given *D*, estimated total number of lanes in region:

$$\text{Number of lanes:} \quad L(D) = \begin{cases} \displaystyle\sum_{i=1}^{N} \left\lceil \frac{M_i}{DH} \right\rceil, & \text{dedicated} \\ \left\lceil \dfrac{M + NH\left(\dfrac{D-1}{2}\right) + N\left(\dfrac{H-1}{2}\right)}{DH} \right\rceil, & \text{randomized } (N > 1) \end{cases}$$

- Estimated HCL:

$D = 3$




doesn't occur because slots are used by another SKU

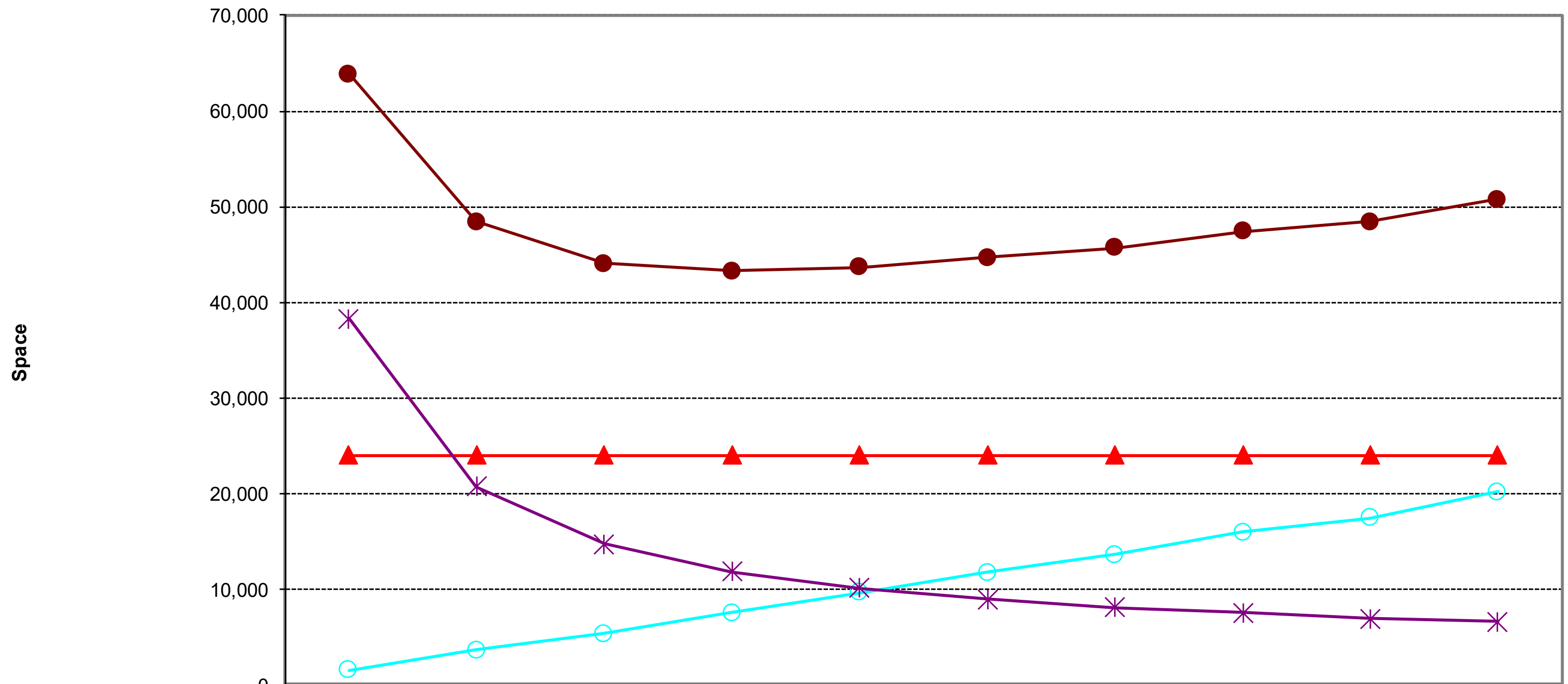
$$\begin{array}{lccccc} \text{Probability:} & \dfrac{1}{D} & & \dfrac{1}{D} & & \dfrac{1}{D} \\ & \times & + & \times & + & \times \\ \text{Unit Honeycomb Loss:} & 0 & & (D-2) & & (D-1) \end{array}$$

$$\text{Expected Loss:} \frac{1}{D}\left[(D-2)+(D-1)\right] = \frac{1}{D}(1+2) = \frac{1}{D}\sum_{i=1}^{D-1} i = \frac{1}{D}\left(\frac{(D-1)D}{2}\right) = \frac{D-1}{2} = 1$$

# Optimal Lane Depth

- Solving for $D$ in $dTS(D)/dD = 0$ results in:

Optimal lane depth for randomized storage (in rows): $D^* = \left\lfloor \sqrt{\dfrac{A(2M-N)}{2NyH}} + \dfrac{1}{2} \right\rfloor$




| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Item Space | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 | 24,000 |
| Honeycomb Loss | 1,536 | 3,648 | 5,376 | 7,488 | 9,600 | 11,712 | 13,632 | 15,936 | 17,472 | 20,160 |
| Aisle Space | 38,304 | 20,736 | 14,688 | 11,808 | 10,080 | 8,928 | 8,064 | 7,488 | 6,912 | 6,624 |
| Total Space | 63,840 | 48,384 | 44,064 | 43,296 | 43,680 | 44,640 | 45,696 | 47,424 | 48,384 | 50,784 |

Lane Depth (in Rows)

# Max Aggregate Inventory Level

- Usually can determine max inventory level for each SKU:
  - $M_i$ = maximum number of units of SKU *i*
- Since usually don't know $M$ directly, but can estimate it **if**
  - SKUs' inventory levels are uncorrelated
  - Units of each item are either stored or retrieved at a constant rate

$$M = \left\lfloor \sum_{i=1}^{N} \frac{M_i}{2} + \frac{1}{2} \right\rfloor$$

- Can add include safety stock for each item, $SS_i$
  - For example, if the order size of three SKUs is 50 units and 5 units of each item are held as safety stock

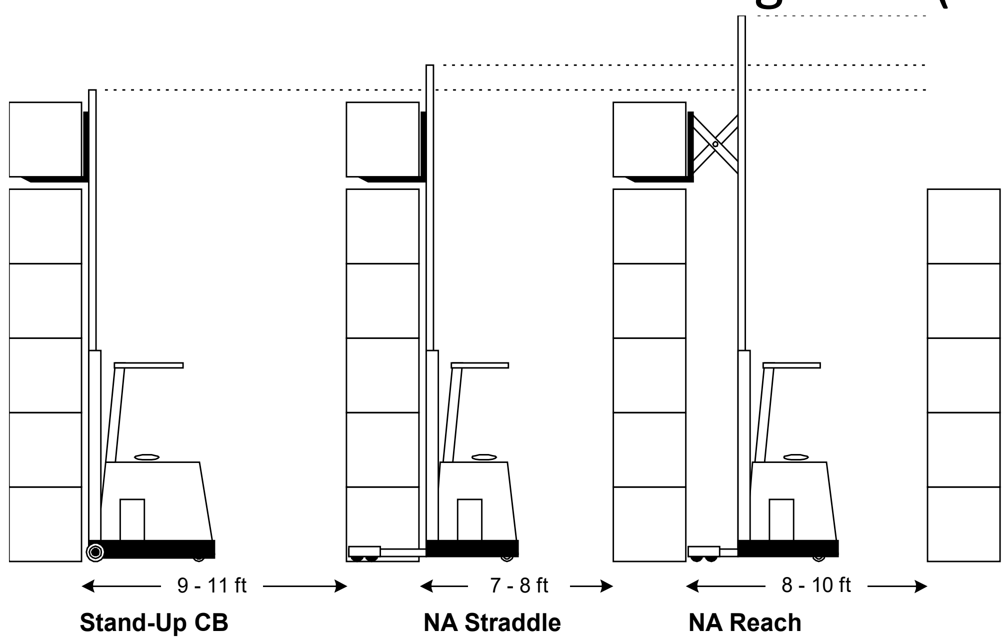
$$M = \left\lfloor \sum_{i=1}^{N} \left( \frac{M_i - SS_i}{2} + SS_i \right) + \frac{1}{2} \right\rfloor = \left\lfloor 3\left( \frac{50}{2} + 5 \right) + \frac{1}{2} \right\rfloor = 90$$

# Steps to Determine Area Requirements

1. For randomized storage, assumed to know $N$, $H$, $x$, $y$, $z$, $A$, and all $M_i$
   - Number of levels, $H$, depends on building clear height (for block stacking) or shelf spacing
   - Aisle width, $A$, depends on type of lift trucks used
2. Estimate maximum aggregate inventory level, $M$
3. If $D$ not fixed, estimate optimal land depth, $D^*$
4. Estimate number of lanes required, $L(D^*)$
5. Determine total 2-D area, $TA(D^*)$

# Aisle Width Design Parameter

- Typically, $A$ (and sometimes $H$) is a parameter used to evaluate different overall design alternatives
- Width depends on type of lift trucks used, a narrower aisle truck
  - reduces area requirements (building costs)
  - costs more and slows travel and loading time (handling costs)



# Example 1: Area Requirements

Units of items A, B, and C are all received and stored as 42 × 36 × 36 in. ($y \times x \times z$) pallet loads in a storage region that is along one side of a 10-foot-wide down aisle in the warehouse of a factory. The shipment size received for each item is 31, 62, and 42 pallets, respectively. Pallets can be stored up to three deep and four high in the region.

$$x = \frac{36}{12} = 3' \qquad M_A = 31 \qquad A = 10'$$

$$y = 3.5' \qquad M_B = 62 \qquad D = 3$$

$$z = 3' \qquad M_C = 42 \qquad H = 4$$

$$N = 3$$

# Example 1: Area Requirements

1. If a dedicated policy is used to store the items, what is the 2-D cube utilization of this storage region?

$$L(D) = L(3) = \sum_{i=1}^{N} \left\lceil \frac{M_i}{DH} \right\rceil = \left\lceil \frac{31}{3(4)} \right\rceil + \left\lceil \frac{62}{3(4)} \right\rceil + \left\lceil \frac{42}{3(4)} \right\rceil = 3 + 6 + 4 = 13 \text{ lanes}$$

$$TA(3) = xL(D) \cdot \left( yD + \frac{A}{2} \right) = 3(13) \cdot \left( 3.5(3) + \frac{10}{2} \right) = 605 \text{ ft}^2$$

$$CU(3) = \frac{\text{item space}}{TA(3)} = \frac{x \cdot y \cdot \sum_{i=1}^{N} \left\lceil \frac{M_i}{H} \right\rceil}{TA(3)} = \frac{3 \cdot 3.5 \cdot \left( \left\lceil \frac{31}{4} \right\rceil + \left\lceil \frac{62}{4} \right\rceil + \left\lceil \frac{42}{4} \right\rceil \right)}{605} = 61\%$$

# Example 1: Area Requirements

2. If the shipments of each item are uncorrelated with each other, no safety stock is carried for each item, and retrievals to the factory floor will occur at a constant rate, what is an estimate the maximum number of units of all items that would ever occur?

$$M = \left\lfloor \sum_{i=1}^{N} \frac{M_i}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{31+62+42}{2} + \frac{1}{2} \right\rfloor = 68$$

# Example 1: Area Requirements

3. If a randomized policy is used to store the items, what is total 2-D area needed for the storage region?

$$D = 3$$

$$L(3) = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{68 + 3(4)\left(\frac{3-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 8 \text{ lanes}$$

$$TA(3) = xL(D) \cdot \left( yD + \frac{A}{2} \right) = 3(8) \cdot \left( 3.5(3) + \frac{10}{2} \right) = 372 \text{ ft}^2$$

# Example 1: Area Requirements

4. What is the optimal lane depth for randomized storage?

$$D^* = \left\lfloor \sqrt{\frac{A\left(2M - N\right)}{2NyH}} + \frac{1}{2} \right\rfloor = \left\lfloor \sqrt{\frac{10\left(2(68) - 3\right)}{2(3)3.5(4)}} + \frac{1}{2} \right\rfloor = 4$$

5. What is the change in total area associated with using the optimal lane depth as opposed to storing the items three deep?

$$D = 4 \Rightarrow L(4) = \left\lceil \frac{68 + 3(4)\left(\frac{4-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 6 \text{ lanes}$$

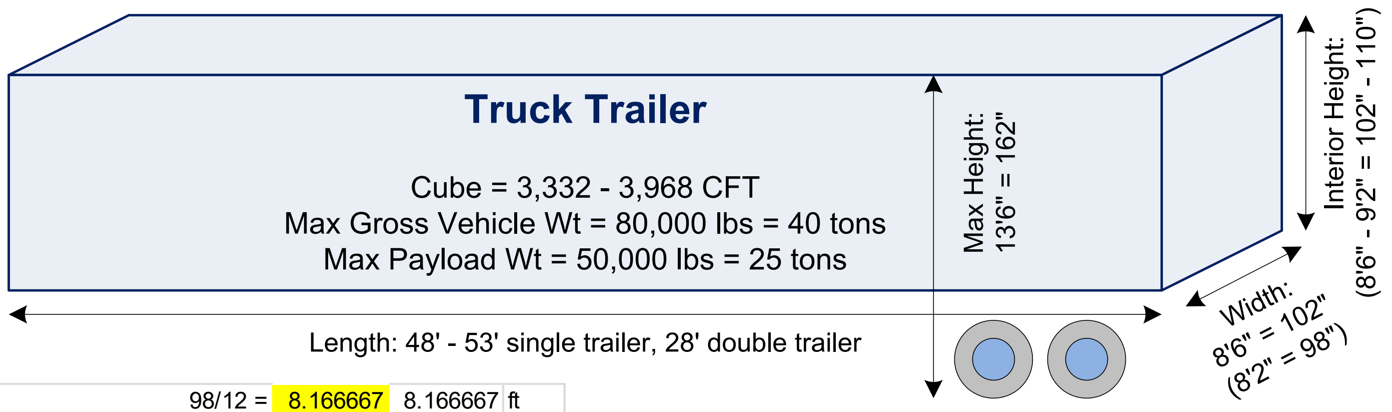
$$\Rightarrow TA(4) = 3(6) \cdot \left(3.5(4) + \frac{10}{2}\right) = 342 \text{ ft}^2$$

$$D = 3 \Rightarrow TA(3) = 372 \text{ ft}^2$$

# Example 2: Trailer Loading

How many identical 48 × 42 × 30 in. four-way containers can be shipped in a full truckload? Each container load:

1. Weighs 600 lb
2. Can be stacked up to six high without causing damage from crushing
3. Can be rotated on the trucks with respect to their width and depth.




| | | | | |
|---|---|---|---|---|
| X | 98/12 = | 8.166667 | 8.166667 | ft |
| Y | | 53 | 53 | ft |
| Z | 110/12 = | 9.166667 | 9.166667 | ft |
| x | [48,42]/12 = | 4 | 3.5 | ft |
| y | [42,48]/12 = | 3.5 | 4 | ft |
| z | 30/12 = | 2.5 | 2.5 | ft |
| L | floor(X/x) = | 2 | 2 | |
| D | floor(Y/y) = | 15 | 13 | |
| H | min(6,floor(Z/z)) = | 3 | 3 | |
| LDH | L*D*H = | 90 | 78 | units |
| wt | | 600 | 600 | lb |
| unit/TL | min(LDH, floor(50000/wt)) = | 83 | 78 | |

Max of 83 units per TL