ISE 453: Design of PLS Systems

Michael G. Kay

Geometric Mean

- How many people can be crammed into a car?
  - Certainly more than one and less than 100: the average (50) seems to be too high, but the geometric mean (10) is reasonable

$$\text{Geometric Mean:} \quad X = \sqrt{LB \times UB} = \sqrt{1 \times 100} = 10$$

- Often it is difficult to directly estimate input parameter *X*, but is easy to estimate reasonable lower and upper bounds (LB and UB) for the parameter
  - Since the guessed LB and UB are usually orders of magnitude apart, use of the arithmetic mean would give too much weight to UB
  - Geometric mean gives a more reasonable estimate because it is a logarithmic average of LB and UB

Fermi Problems

- Involves "reasonable" (i.e., +/– 10%) *guesstimation* of input parameters needed and back-of-the-envelope type approximations
  - Goal is to have an answer that is within an order of magnitude of the correct answer (or what is termed a *zeroth-order approximation*)
  - Works because over- and under-estimations of each parameter tend to cancel each other out as long as there is no consistent bias
- How many McDonald's restaurants in U.S.? (actual 2013: 14,267)

Parameter	**LB**		**UB**	**Estimate**	
Annual per capita demand	1	1 order/person-day x 350 day/yr =	350	18.71	(order/person-yr)
U.S. population				300,000,000	(person)
Operating hours per day				16	(hr/day)
Orders per store per minute (in-store + drive-thru)				1	(order/store-min)
Analysis					
Annual U.S. demand		(person) x (order/person-yr) =		5,612,486,080	(order/yr)
Daily U.S. demand		(order/yr)/365 day/yr =		15,376,674	(order/day)
Daily demand per store		(hrs/day) x 60 min/hr x (order/store-min) =		960	(order/store-day)
Est. number of U.S. stores		(order/day) / (order/store-day) =		16,017	(store)

System Performance Estimation

- Often easy to estimate performance of a new system if can assume either perfect (LB) or no (UB) control
- Example: estimate waiting time for a bus
  - 8 min. avg. time (aka “headway”) between buses
  - Customers arrive at random
    - assuming no web-based bus tracking
  - Perfect control (LB): wait time = half of headway
  - No control (*practical* UB): wait time = headway
    - assuming buses arrive at random (Poisson process)

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{8}{2} \times 8} = 5.67 \text{ min}$$

  - Bad control can result in higher values than no control

http://www.nextbuzz.gatech.edu/

SELF-COORDINATING BUSES
REDUCE BUNCHING

HOME THE IDEA PROOF OF CONCEPT HOW IT WORKS CONTRIBUTORS

A BUS-HEADWAY CONTROLLER

A software system to coördinate buses on a route, based on an idea by John J. Bartholdi III and Donald D. Eisenstein. The current version of the software was designed and largely written by Loren K. Platzman. Implementation has been led by Russ Clark, Jin Lee, and David Williamson.

THE IDEA

Delaying buses briefly at certain checkpoints equalizes headways

Read more

PROOF OF CONCEPT

Coordinating trolleys on Georgia Tech's busiest route

Read more

HOW IT WORKS

Tablets, GPS, cellular networks, and web-based control

Read more

Ex 1: Geometric Mean

- If, during the morning rush, there are three buses operating on Wolfline Route 13 and it takes them 45 minutes, on average, to complete one circuit of the route, what is the estimated waiting time for a student who does not use TransLoc for real-time bus tracking?

Answer :

$$\text{Frequency (TH)} = \frac{WIP}{CT} = \frac{\text{3 bus/circuit}}{\text{45 min/circuit}} = \frac{1}{15}\ \text{bus/min}, \quad \text{Headway} = \frac{1}{\text{Freq.}} = 15\ \text{min/bus}$$

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{15}{2} \times 15} = 10.61\ \text{min}$$

Ex 2: Fermi Problem

- Estimate the average amount spent per trip to a grocery store. Total U.S. supermarket sales were recently determined to be $649,087,000,000, but it is not clear whether this number refers to annual sales, or monthly, or weekly sales.

$$\textbf{Answer:}\quad \frac{\$6.5e11}{3e8} \approx \$2,000\,/\,\text{person-yr}, \quad LB = 1\ \text{trips/wk}, UB = 7\ \text{trips/wk}$$

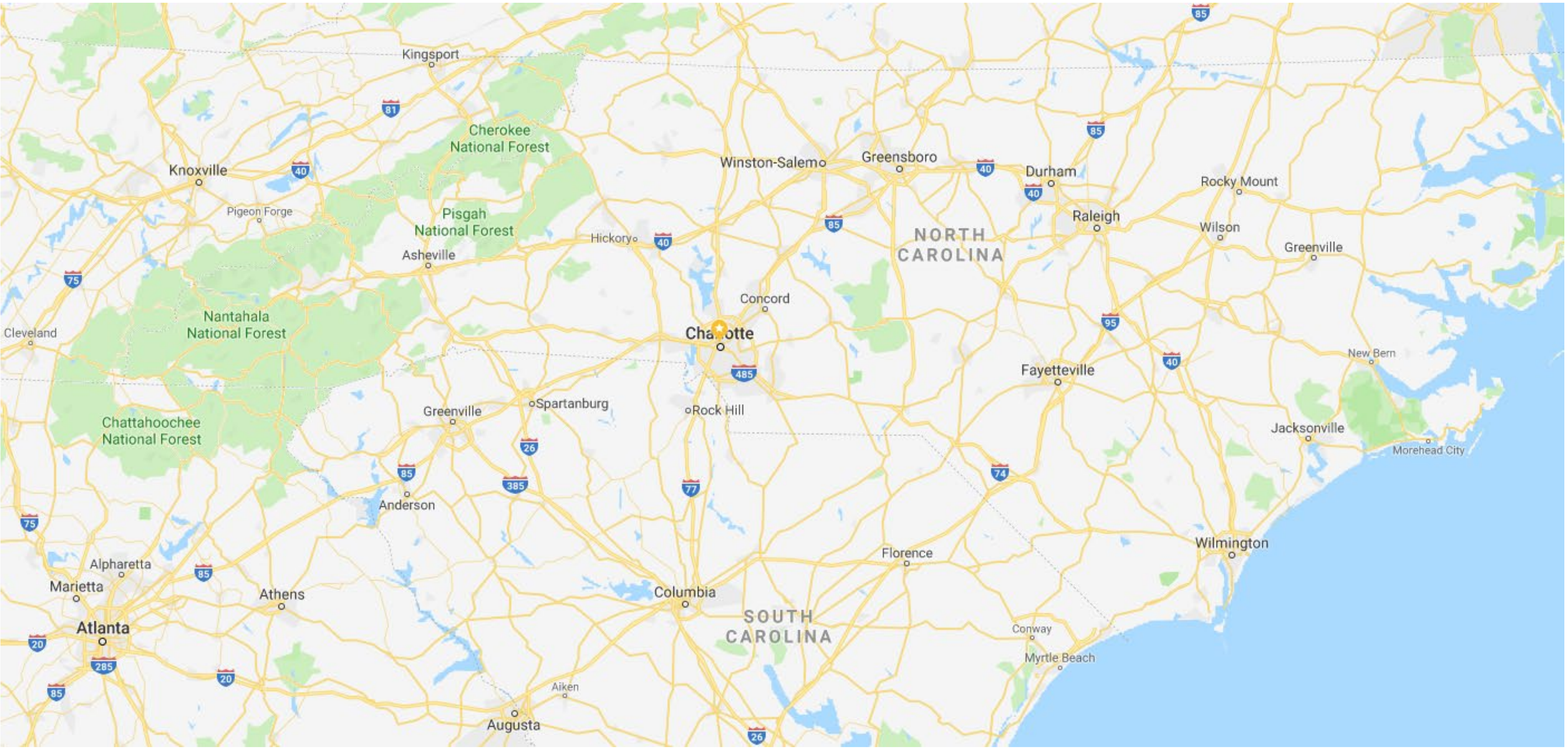
$$\Rightarrow \sqrt{1(7)} \times 52 \approx 2 \times 52 \approx 100\ \text{trips/yr} \Rightarrow \frac{\$2,000}{100} = \$20\ /\ \text{person-trip}$$

Supermarket / Grocery Store Statistics	Data
Total number of grocery store employees	3,400,000
Total supermarket sales in 2015	$649,087,000,000
Total supermarket sales in 2012	$602,609,000,000
Total number of grocery stores / supermarkets	37,053
Median weekly sales per supermarket store	$384,911
Average grocery store transaction amount	$27.30
Average number of grocery store trips per week a consumers makes	2.2
Average number of items carried in a supermarket	38,718

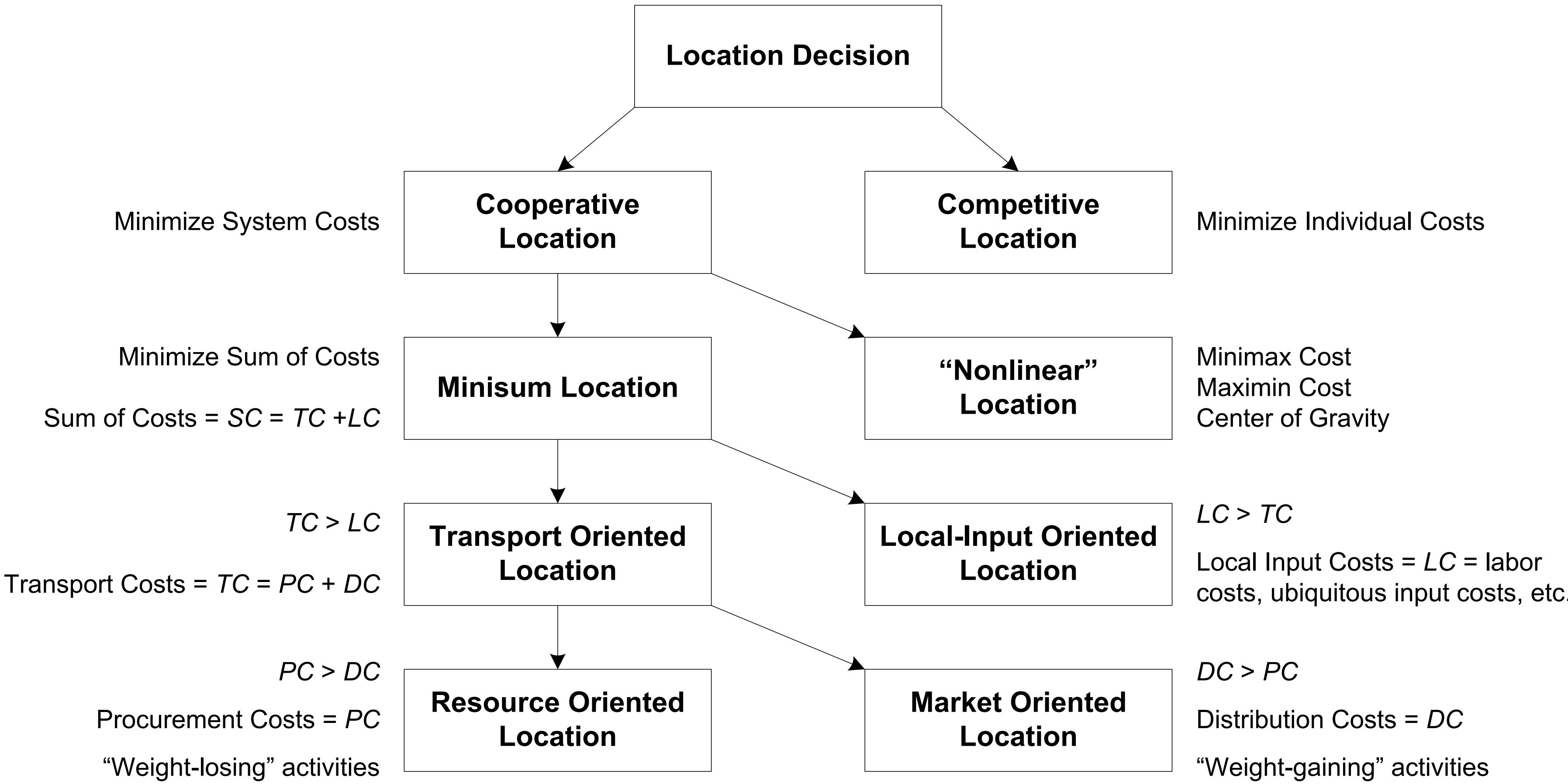
Levels of Modeling

0. Guesstimation (order of magnitude)
1. Mean value analysis (linear, ±20%)
2. Nonlinear models (incl. variance, ±5%)
3. Simulation models (complex interactions)
4. Prototypes/pilot studies
5. Build/do and then tweak it

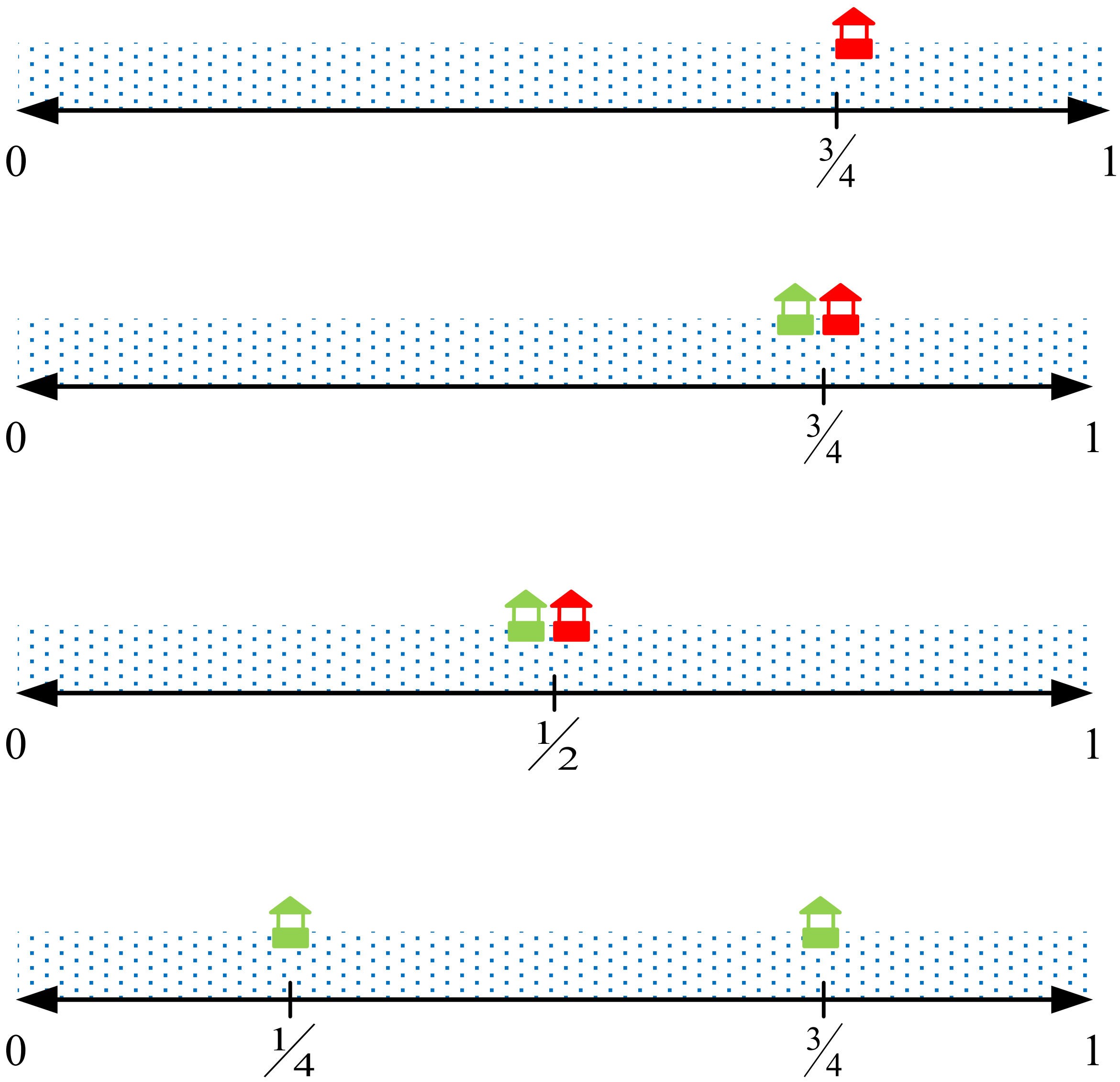
Why Are Cities Located Where They Are?



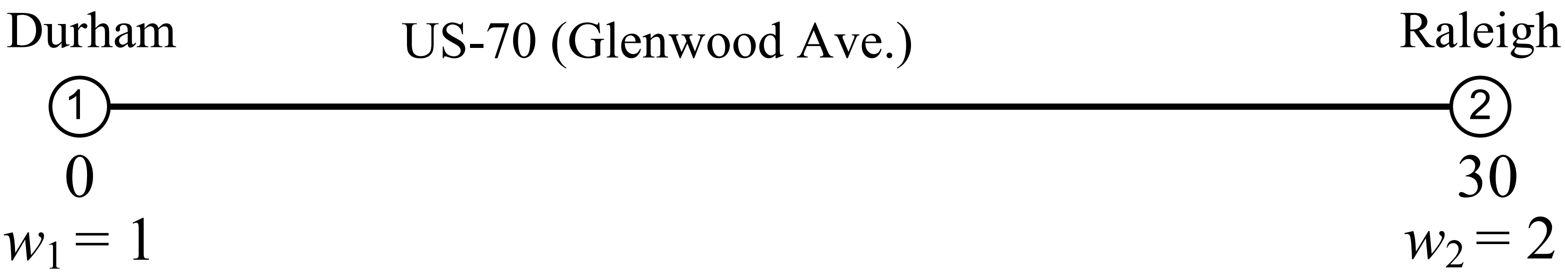
Taxonomy of Location Problems



Hotelling's Law



1-D Cooperative Location



$$\text{Min } TC = \sum w_i \, d_i$$

$$\text{Min } TC = \sum w_i \, d_i^2$$

$$\text{Min } TC = \sum w_i \, d_i^k$$

$$a_1 = 0, \quad a_2 = 30$$

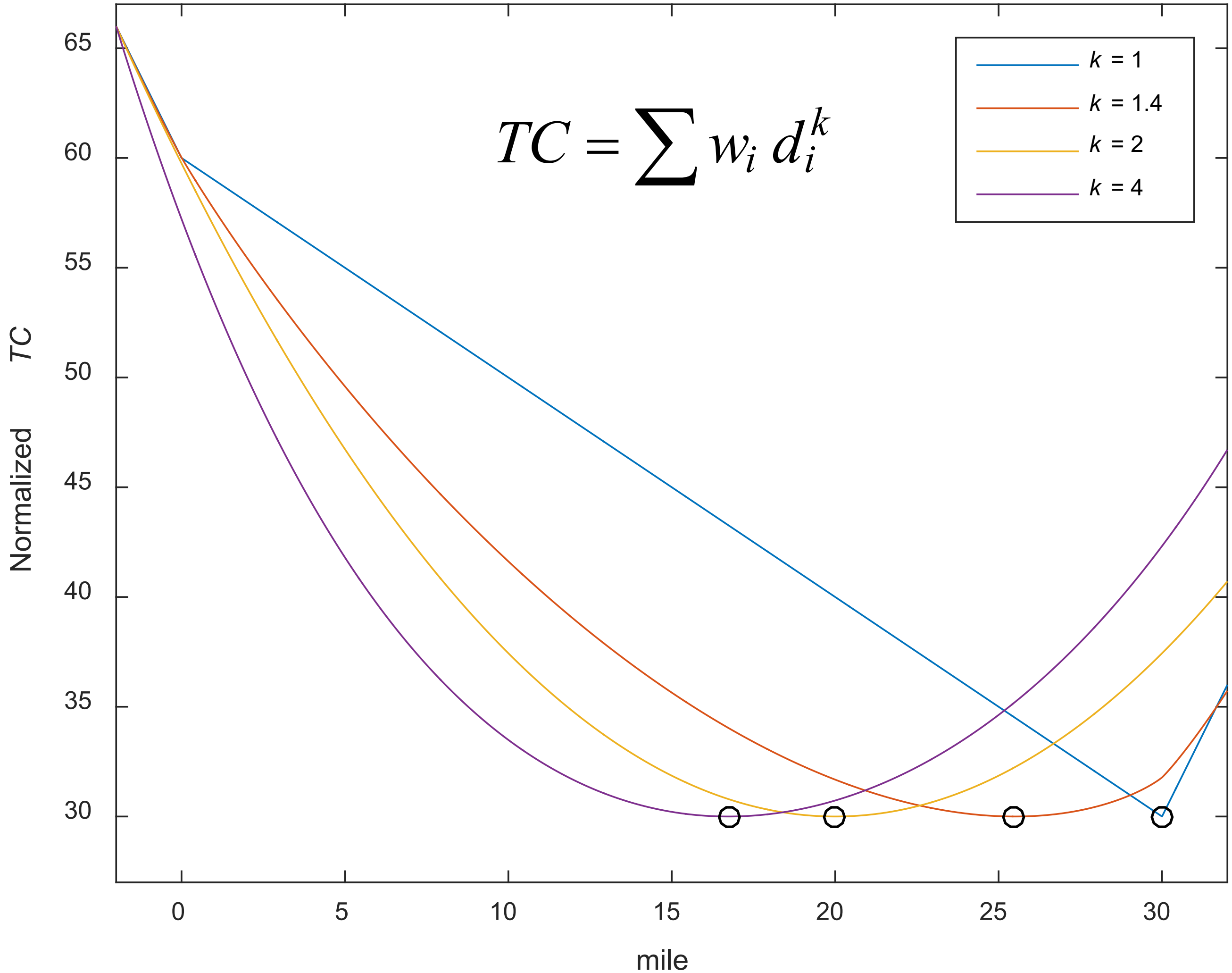
$$TC = \sum w_i \, d_i^2 = \sum w_i \left(x - a_i \right)^2$$

$$\frac{dTC}{dx} = 2 \sum w_i \left(x - a_i \right) = 0 \Rightarrow$$

$$x \sum w_i = \sum w_i \, a_i \Rightarrow$$

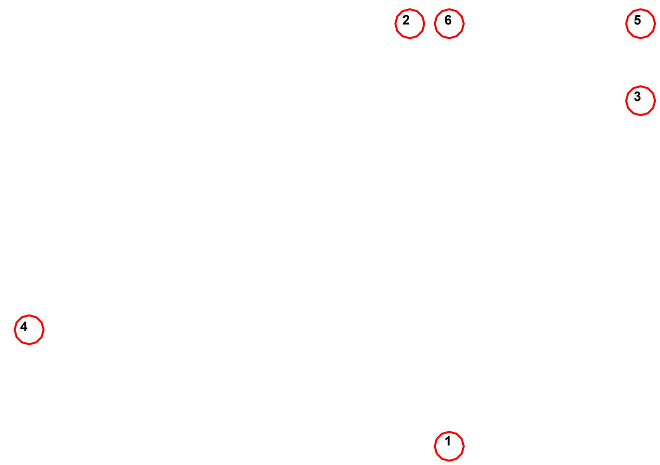
Squared−Euclidean Distance $\Rightarrow$ Center of Gravity: $$x^* = \frac{\sum w_i \, a_i}{\sum w_i} = \frac{1(0) + 2(30)}{1 + 2} = 20$$

“Nonlinear” Location



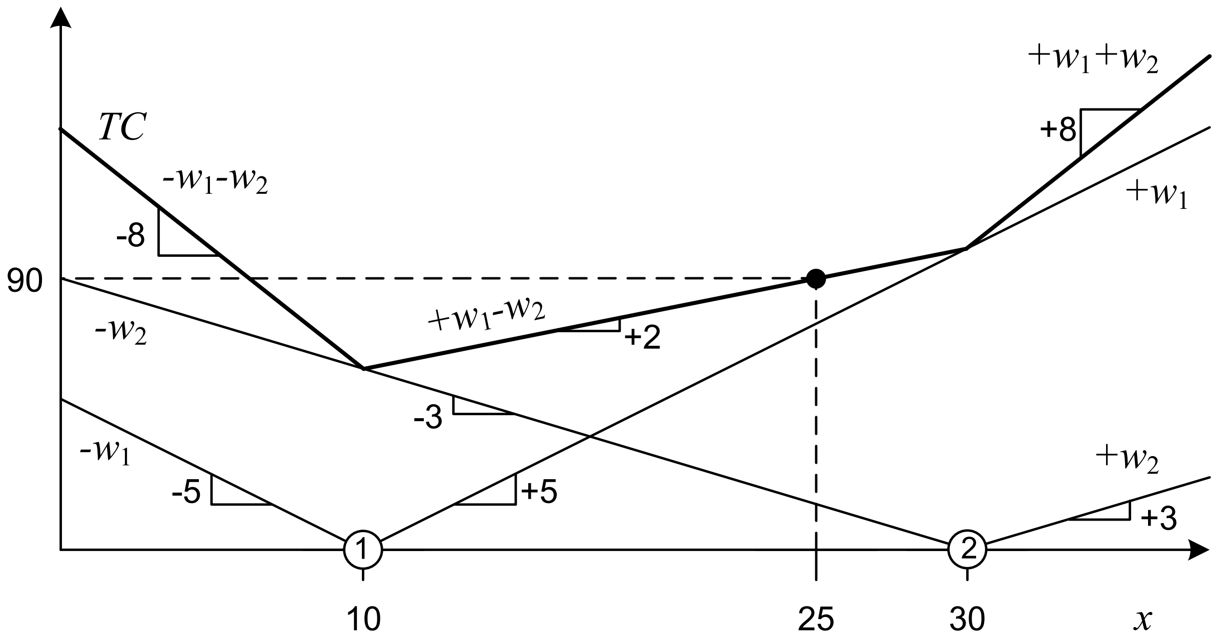
Minimax and Maximin Location

- Minimax
  - Min max distance
  - Set covering problem



- Maximin
  - Max min distance
  - AKA obnoxious facility location

2-EF Minisum Location



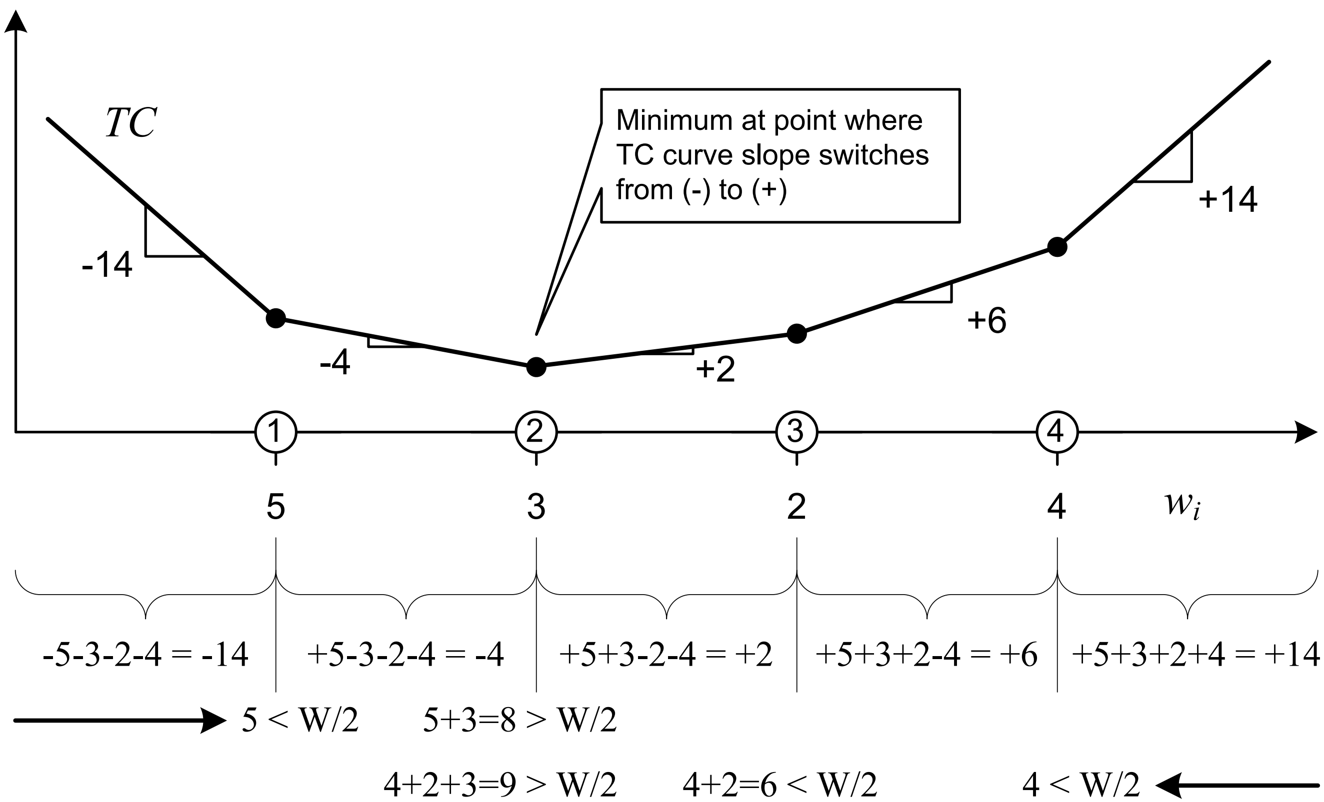
$$TC(x) = \sum w_i \, d_i = \beta_1(x - x_1) + \beta_2(x - x_2), \quad \text{where } \beta_i = \begin{cases} w_i, & \text{if } x \geq x_i \\ -w_i, & \text{if } x < x_i \end{cases}$$

$$\begin{aligned} TC(25) &= w_1(25-10) + (-w_2)(25-30) \\ &= 5(15) + (-3)(-5) = 90 \end{aligned}$$

Median Location: 1-D 4 EFs

Median location: For each dimension x of X:

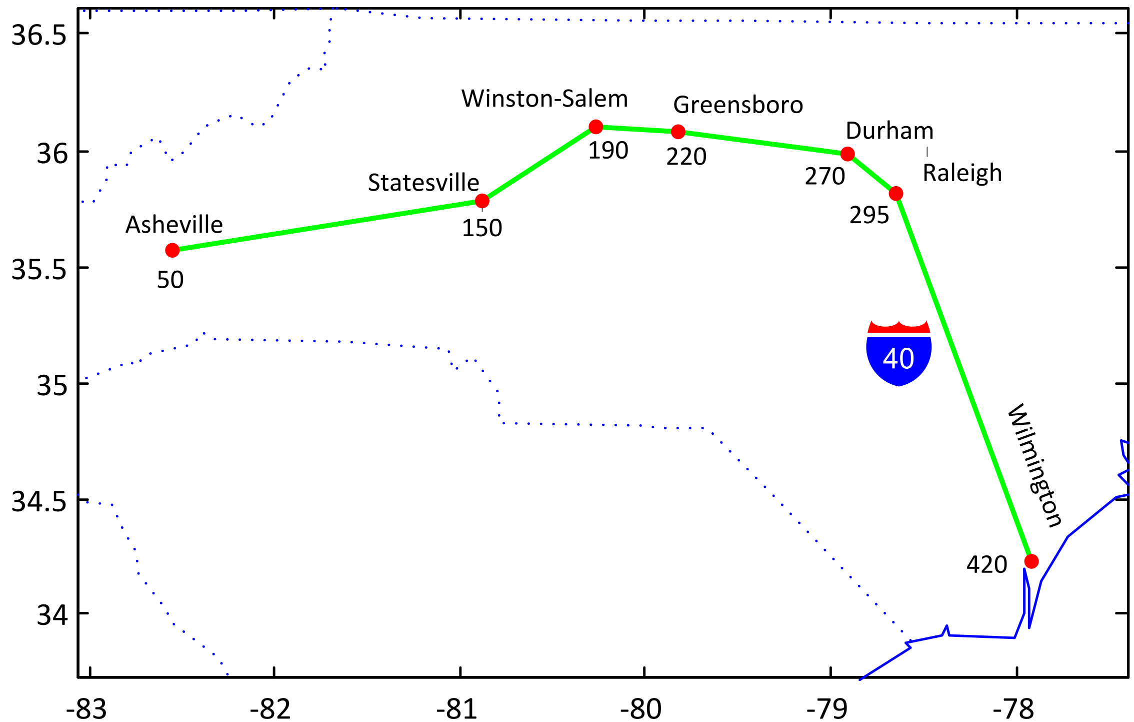
1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EF j where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

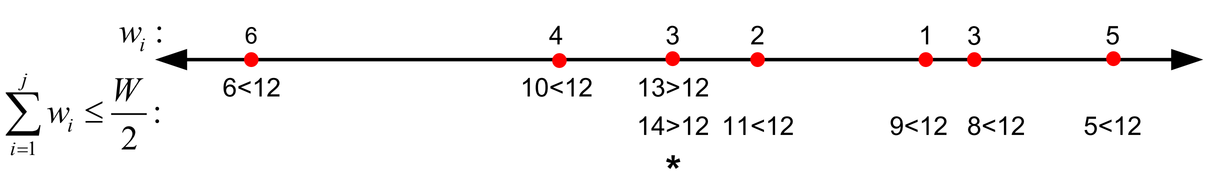


Median Location: 1-D 7 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EF j where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

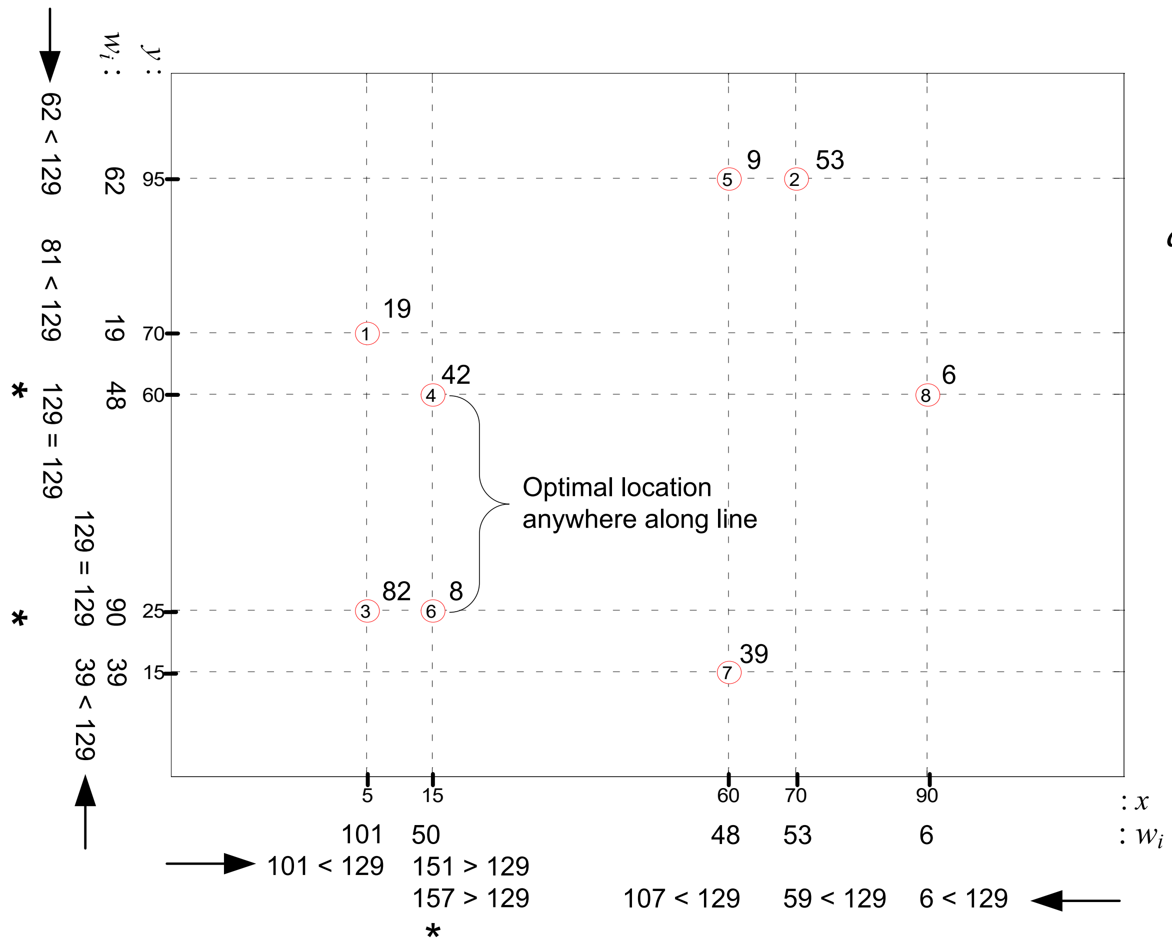




Median Location: 2-D Rectilinear Distance 8 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EF j where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

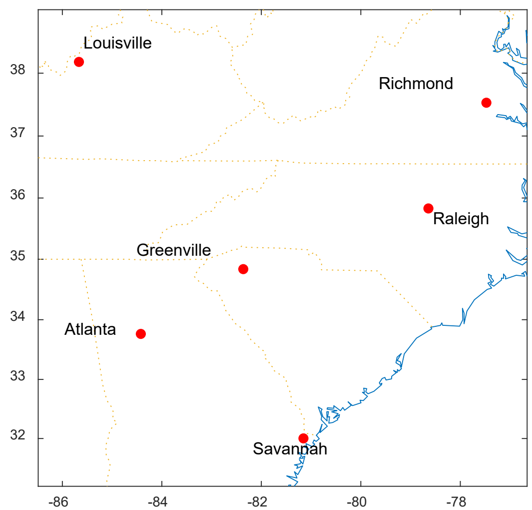


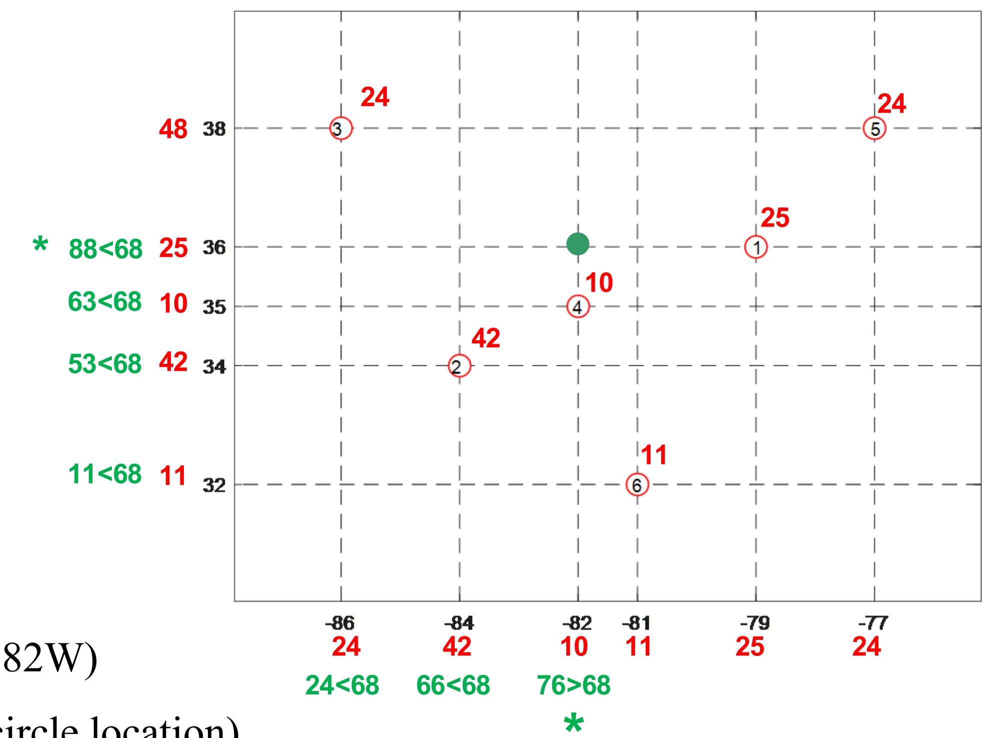
$$d_1(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$$

$$d_2(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Ex 3: 2D Loc with Rect Approx to GC Dist

- It is expected that 25, 42, 24, 10, 24, and 11 truckloads will be shipped each year from your DC to six customers located in Raleigh, NC (36N,79W), Atlanta, GA (34N,84W), Louisville, KY (38N,86W), Greenville, SC (35N, 82W), Richmond, VA (38N,77W), and Savannah, GA (32N,81W). Assuming that all distances are rectilinear, where should the DC be located in order to minimize outbound transportation costs?



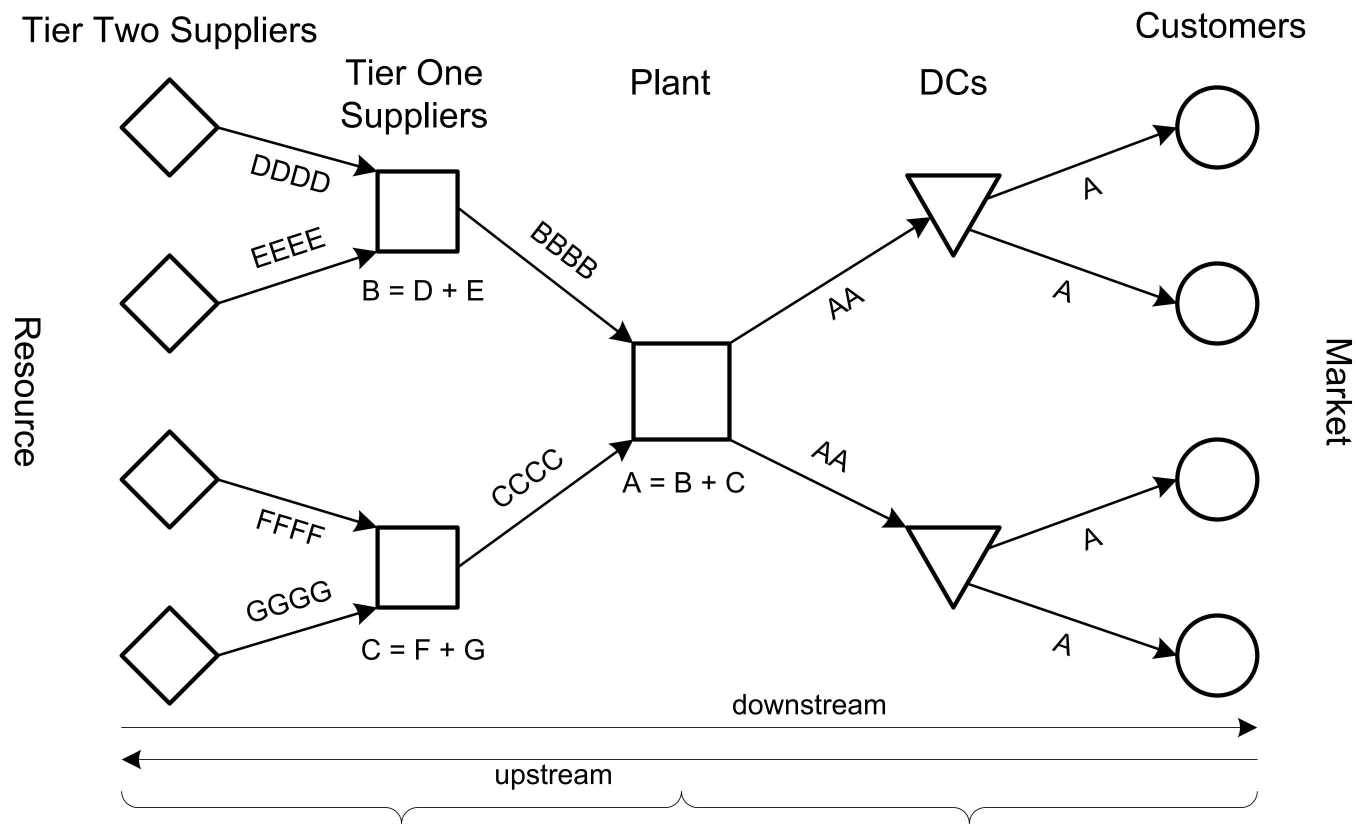


$$W = \sum w_i = 136, \quad \frac{W}{2} = 68$$

Answer : Optimal location (36N,82W)

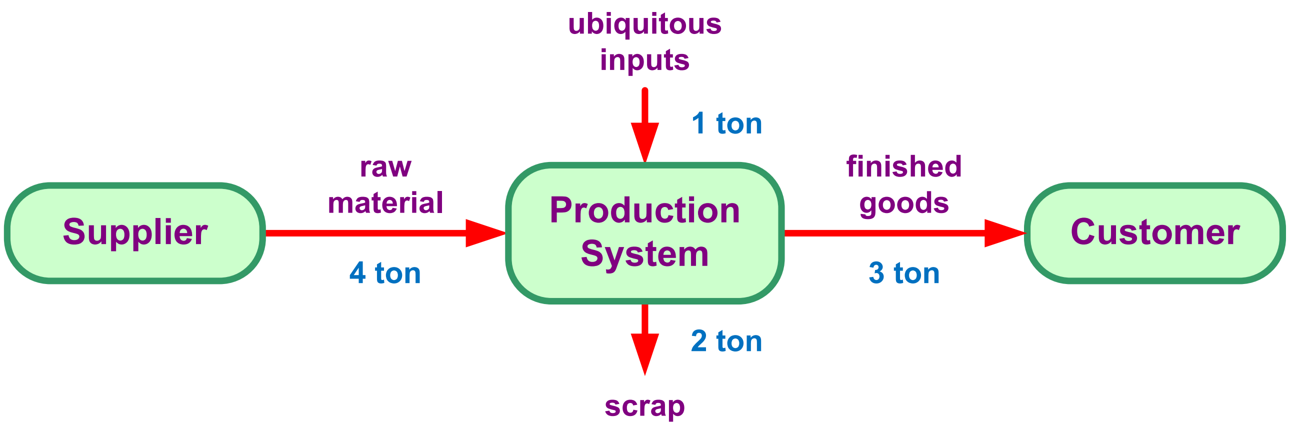
(65 mi from opt great-circle location)

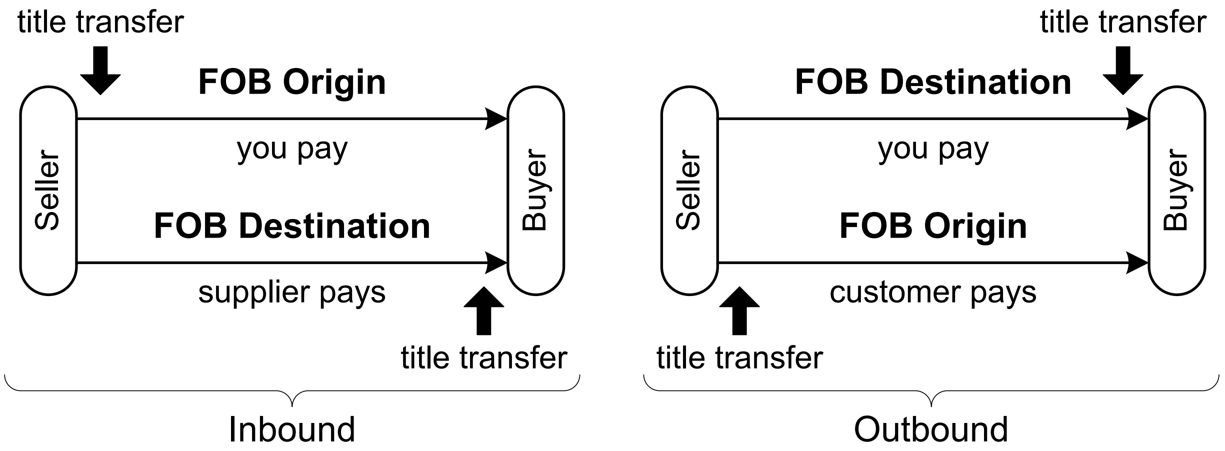
Logistics Network for a Plant



Assembly Network	vs.	Distribution Network
Procurement	vs.	Distribution
Inbound Logistics	vs.	Outbound Logistics
Raw Materials	vs.	Finished Goods

Basic Production System





FOB (free on board)

FOB and Location

- Choice of FOB terms (who directly pays for transport) usually does not impact location decisions:

Procurement cost	=	Landed cost at supplier	+	Inbound transport cost
Production cost	=	Procurement cost	+	Local resource cost (labor, etc.)
Total delivered cost	=	Production cost	+	Outbound transport cost
Transport cost (TC)	=	Inbound transport cost	+	Outbound transport cost

  – *Purchase price* from supplier and *sale price* to customer adjusted to reflect who is paying transport cost
  – Usually determined by who can provide the transport at the lowest cost
    - Savings in lower transport cost allocated (bargained) between parties

Monetary vs. Physical Weight

$$\min TC(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} d(X, P_i)$$

where

TC = total transport cost (\$/yr)

w_i = monetary weight (\$/mi-yr)

f_i = physical weight rate (ton/yr)

r_i = transport rate (\$/ton-mi)

$d(X, P_i)$ = distance between NF at X and EF_i at P_i (mi)

NF = new facility to be located

EF = existing facility

m = number of EFs

(Montetary) Weight Gaining: $\Sigma w_{\text{in}} < \Sigma w_{\text{out}}$

Physically Weight Losing: $\Sigma f_{\text{in}} > \Sigma f_{\text{out}}$

Minisum Location: TC vs. TD

- Assuming local input costs are
  - same at every location or
  - insignificant as compared to transport costs,

  the minisum transport-oriented single-facility location problem is to locate NF to minimize TC
- Can minimize total distance (TD) if transport rate is same:

$$\min TD(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} d(X, P_i)$$

where TD = total transport distance (mi/yr)

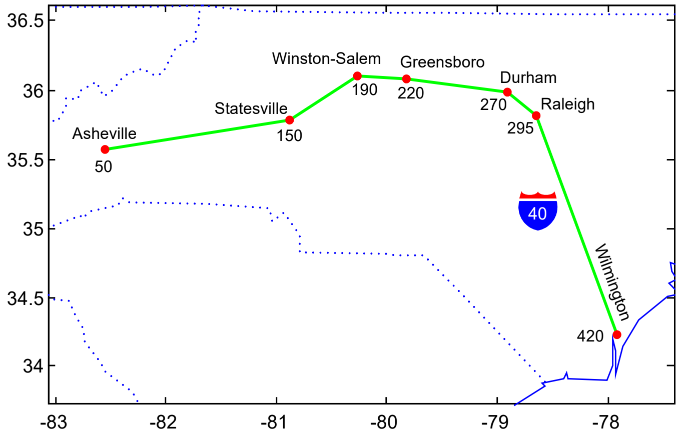
w_i = monetary weight (trip/yr)

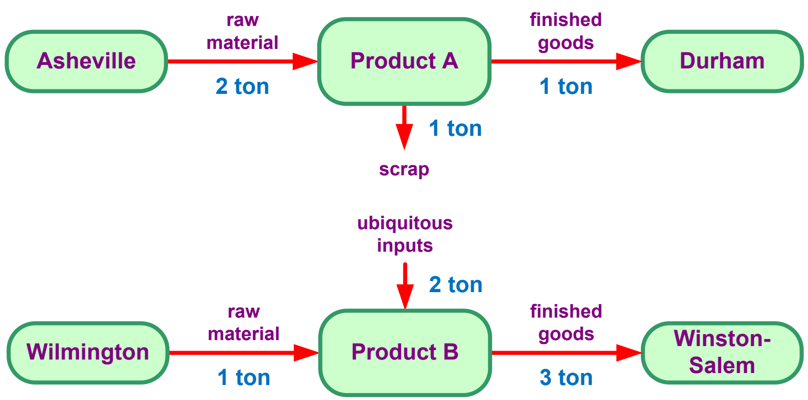
f_i = trips per year (trip/yr)

r_i = transport rate = 1

$d(X, P_i)$ = per-trip distance between NF and EF_i (mi/trip)

Ex 4: Single Supplier and Customer Location

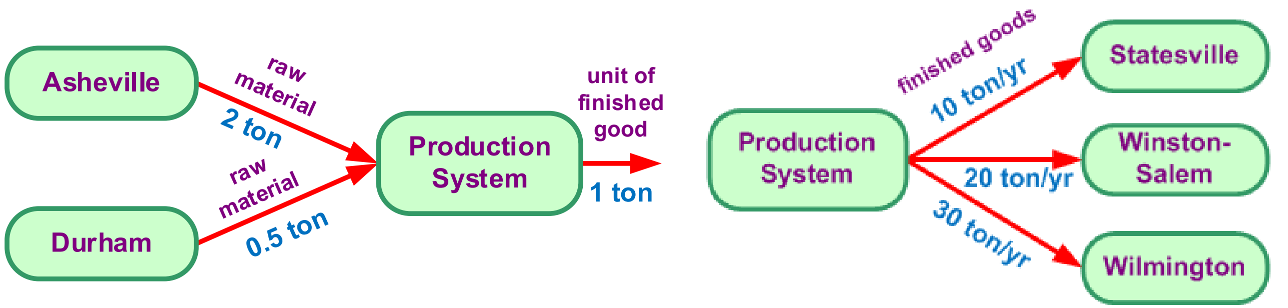




- The cost per ton-mile (i.e., the cost to ship one ton, one mile) for both raw materials and finished goods is the same (e.g., \$0.10).
  1. Where should the plant for each product be located?
  2. How would location decision change if customers paid for distribution costs (FOB Origin) instead of the producer (FOB Destination)?
     - In particular, what would be the impact if there were competitors located along I-40 producing the same product?
  3. Which product is weight gaining and which is weight losing?
  4. If both products were produced in a single shared plant, why is it now necessary to know each product's annual demand (f_i)?

$$TC(X) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} \, d(X, P_i)$$

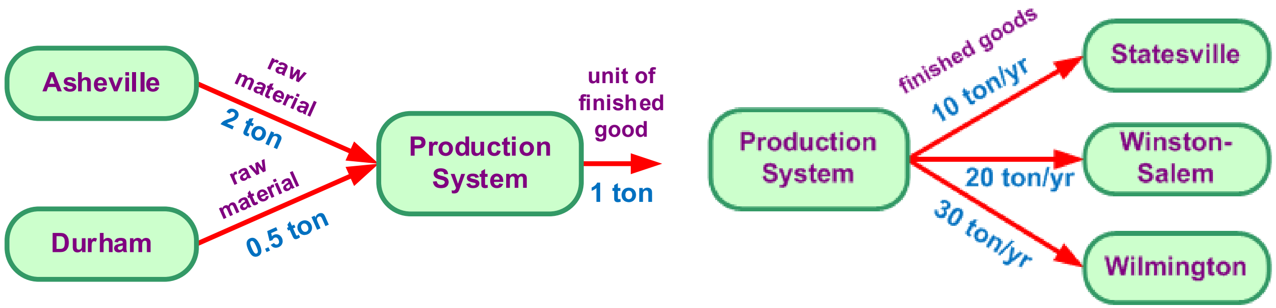
Ex 5: 1-D Location with Procurement and Distribution Costs



Assume: all scrap is disposed of locally

A product is to be produced in a plant that will be located along I-40. Two tons of raw materials from a supplier in Ashville and a half ton of a raw material from a supplier in Durham are used to produce each ton of finished product that is shipped to customers in Statesville, Winston-Salem, and Wilmington. The annual demand of these customers is 10, 20, and 30 tons, respectively, and it costs \$0.33 per ton-mile to ship raw materials to the plant and \$1.00 per ton-mile to ship finished goods from the plant. Determine where the plant should be located so that procurement and distribution costs (i.e., transportation costs to and from the plant) are minimized, and whether the plant is weight gaining or weight losing.

Ex 5: 1-D Location with Procurement and Distribution Costs



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

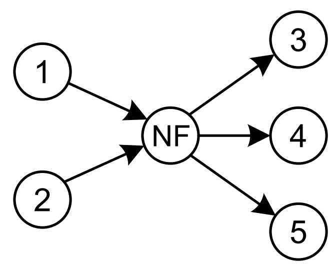
$$\overbrace{\underset{(\$/\text{mi-yr})}{w_i}}^{\text{monetary weight}} = \overbrace{\underset{(\text{ton/yr})}{f_i}}^{\text{physical weight}} \times \underset{(\$/\text{ton-mi})}{r_i}$$

Assume: all scrap is disposed of locally

$r_{\text{in}} = \$0.33/\text{ton-mi}$ $\qquad$ $r_{\text{out}} = \$1.00/\text{ton-mi}$

$f_1 = BOM_1 \sum f_{\text{out}} = 2(60) = 120, \quad w_1 = f_1 r_{\text{in}} = 40$

$f_2 = BOM_2 \sum f_{\text{out}} = 0.5(60) = 30, \quad w_2 = f_2 r_{\text{in}} = 10$



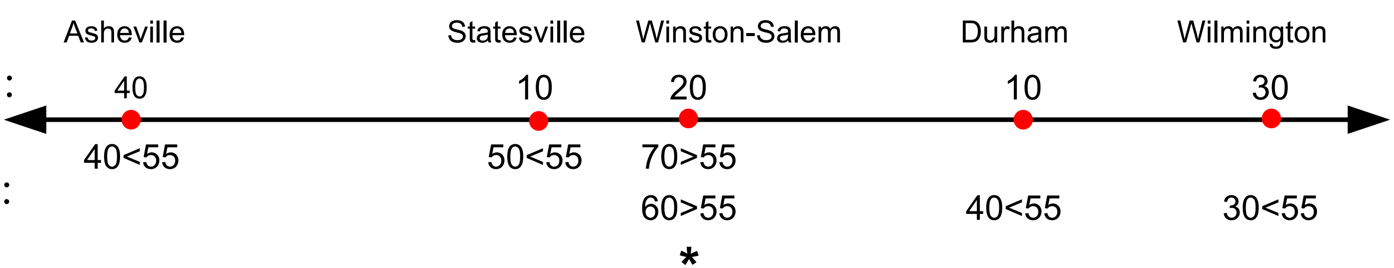
$f_3 = 10, \quad w_3 = f_3 r_{\text{out}} = 10$

$f_4 = 20, \quad w_4 = f_4 r_{\text{out}} = 20$

$f_5 = 30, \quad w_5 = f_5 r_{\text{out}} = 30$

w_i :

$\sum_{i=1}^{j} w_i \le \frac{W}{2}$:



(Montetary) Weight Gaining: $\Sigma w_{\text{in}} = 50 < \Sigma w_{\text{out}} = 60$

Physically Weight Losing: $\Sigma f_{\text{in}} = 150 > \Sigma f_{\text{out}} = 60$

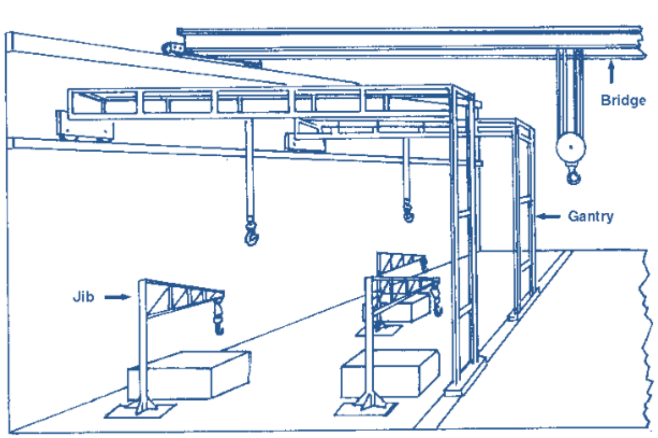
Metric Distances

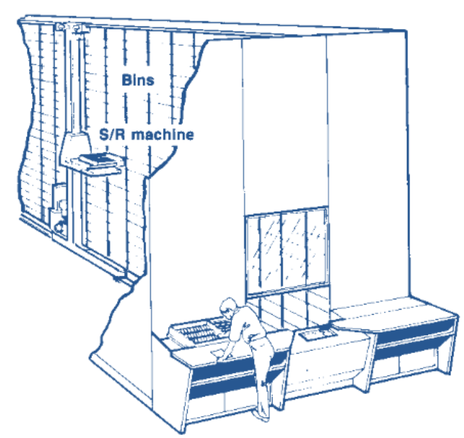
General l_p: $$d_p(P_1,P_2)=\left[|x_1-x_2|^p+|y_1-y_2|^p\right]^{\frac{1}{p}},\quad p\ge 1$$

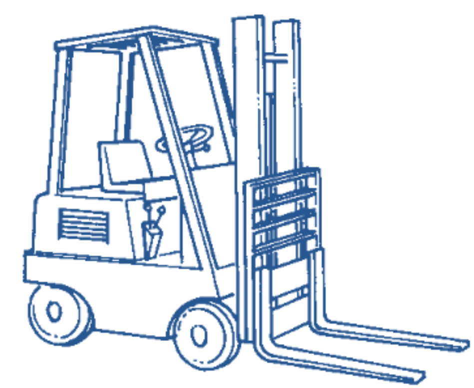
Rectilinear ($p=1$): $$d_1(P_1,P_2)=|x_1-x_2|+|y_1-y_2|$$

Euclidean ($p=2$): $$d_2(P_1,P_2)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

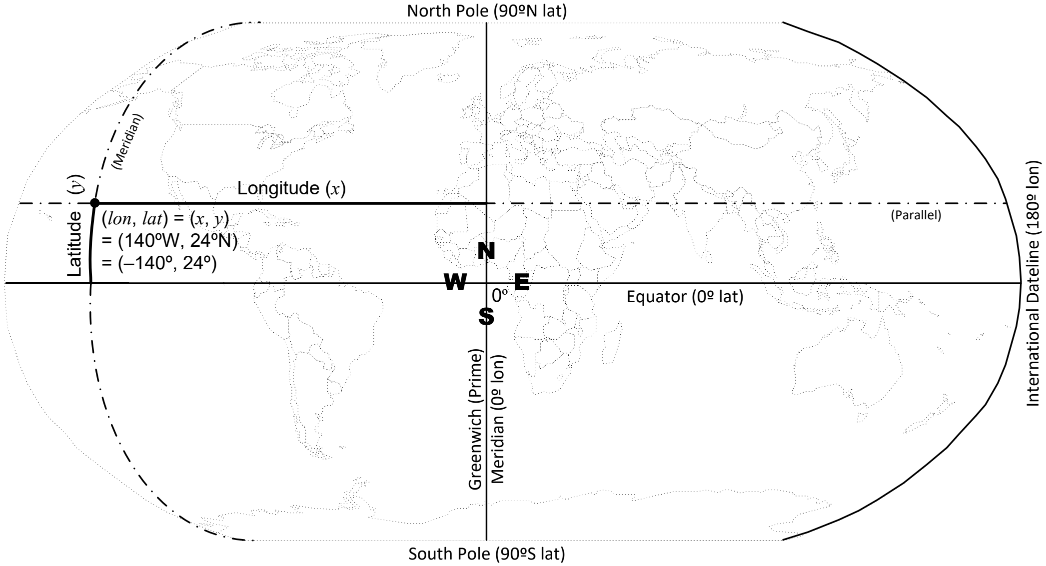
Chebychev ($p\to\infty$): $$d_\infty(P_1,P_2)=\max\left\{|x_1-x_2|,|y_1-y_2|\right\}$$

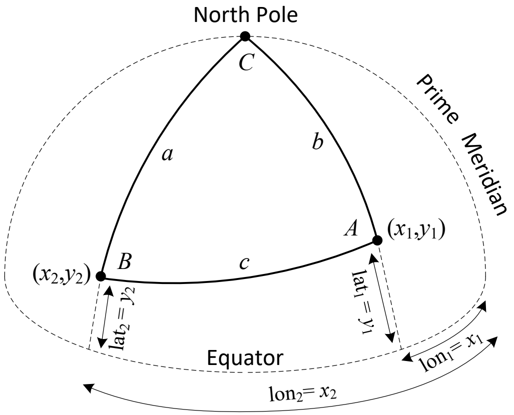


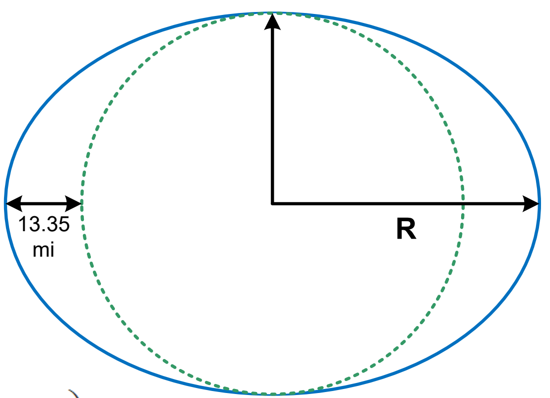




Great Circle Distances







$$(lon_1, lat_1) = (x_1, y_1), \quad (lon_2, lat_2) = (x_2, y_2)$$

$$d_{rad} = (\text{great circle distance in radians of a sphere})$$

$$= \cos^{-1}\left[\sin y_2 \sin y_1 + \cos y_2 \cos y_1 \cos(x_1 - x_2)\right]$$

$$R = (\text{radius of earth at equator}) - (\text{bulge from north pole to equator})$$

$$= 3{,}963.34 - 13.35 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ mi}, \quad = 6{,}378.388 - 21.476 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ km}$$

$$d_{\text{GC}} = \text{distance } (x_1, y_1) \text{ to } (x_2, y_2) = \boxed{d_{rad} \cdot R}$$

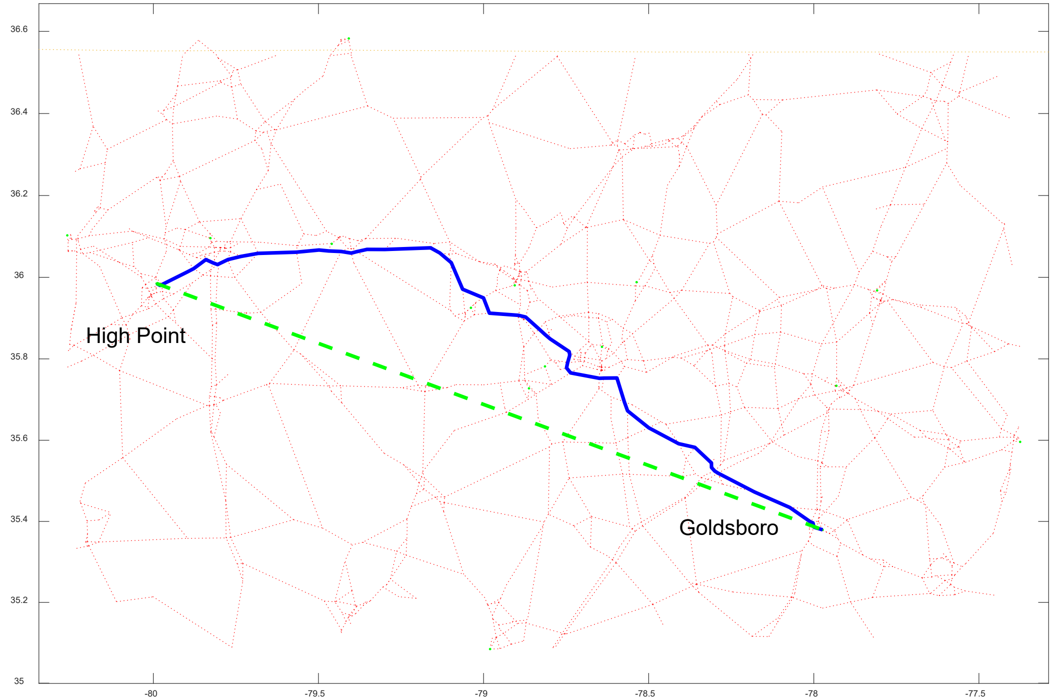
$$x_{\text{deg}} = \begin{cases} DD + \dfrac{MM}{60} + \dfrac{SS}{3{,}600}, & \text{if } E \text{ or } N \\ -DD - \dfrac{MM}{60} - \dfrac{SS}{3{,}600}, & \text{if } W \text{ or } S \end{cases}$$

Circuity Factor

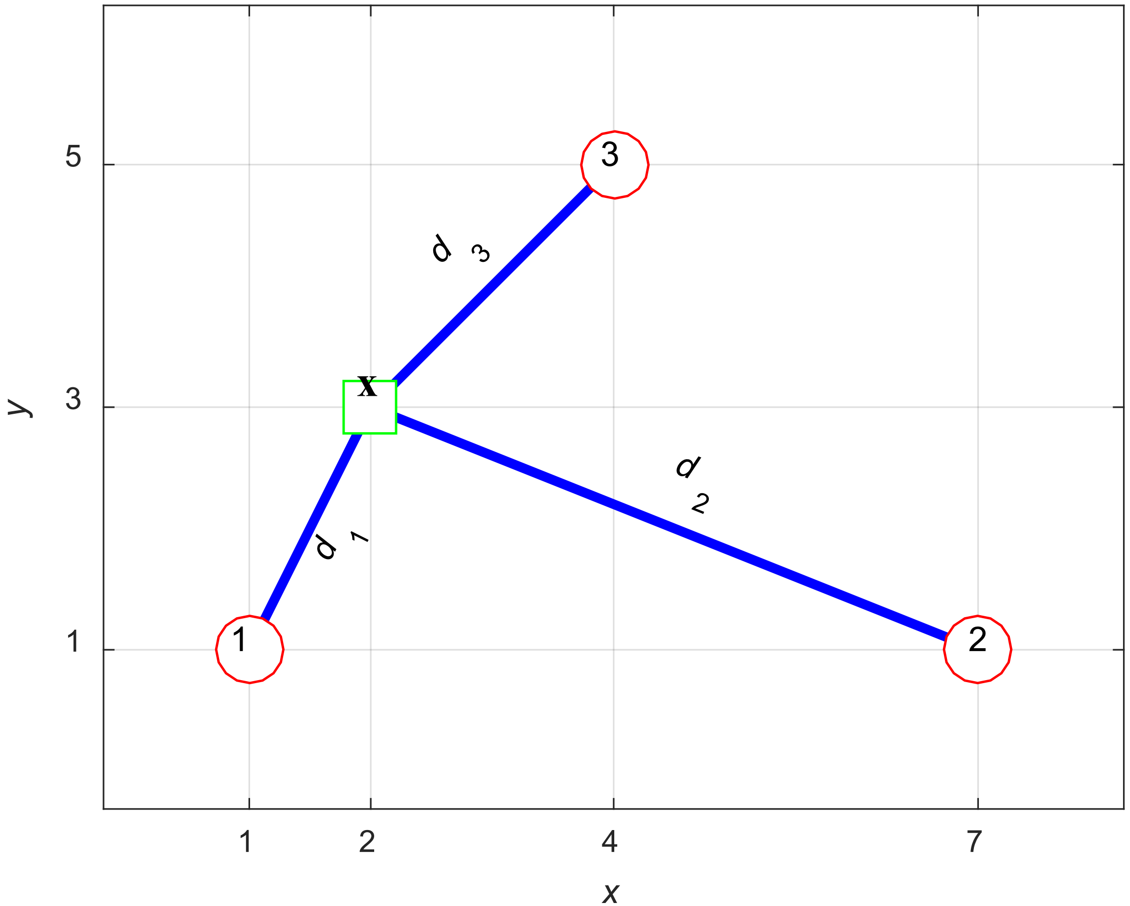
$$\textit{Circuity Factor:} \quad g = \sum \frac{d_{\text{road}_i}}{d_{GC_i}}, \quad \text{where usually } 1.15 \le g \le 1.5$$

$$d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2), \quad \text{estimated road distance from } P_1 \text{ to } P_2$$

From High Point to Goldsboro: Road = 143 mi, Great Circle = 121 mi, Circuity = 1.19



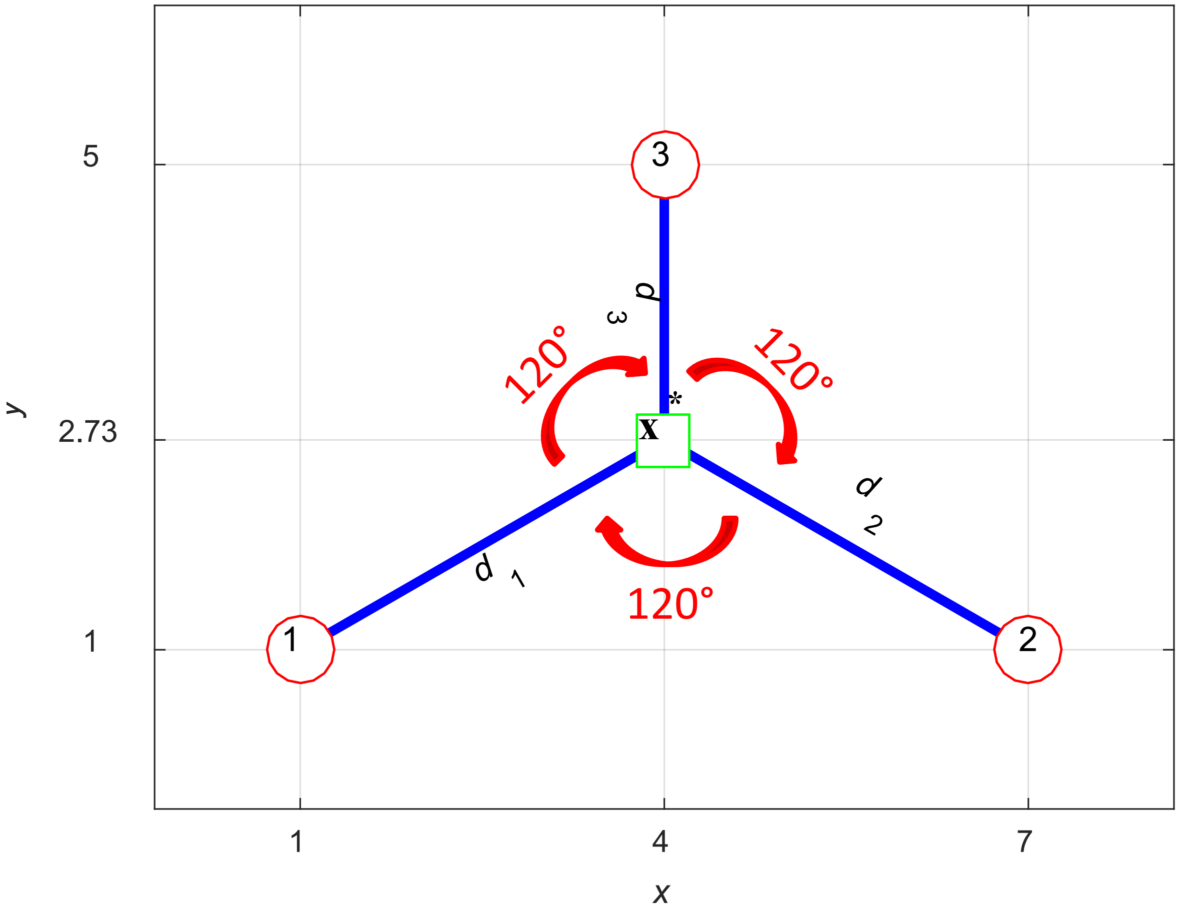
2-D Euclidean Distance



$$\mathbf{x} = \begin{bmatrix} 2 & 3 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_1 - p_{1,1}\right)^2 + \left(x_2 - p_{1,2}\right)^2} \\ \sqrt{\left(x_1 - p_{2,1}\right)^2 + \left(x_2 - p_{2,2}\right)^2} \\ \sqrt{\left(x_1 - p_{3,1}\right)^2 + \left(x_2 - p_{3,2}\right)^2} \end{bmatrix}$$

Minisum Distance Location



$$\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$d_i(\mathbf{x}) = \sqrt{\left(x_1 - p_{i,1}\right)^2 + \left(x_2 - p_{i,2}\right)^2}$$

$$TD(\mathbf{x}) = \sum_{i=1}^{3} d_i(\mathbf{x})$$

$$\mathrm{x}^* = \arg\min_{\mathbf{x}} TD(\mathbf{x})$$

$$TD^* = TD(\mathbf{x}^*)$$

Fermat's Problem (1629):

Given three points, find fourth (Steiner point) such that sum to others is minimized

(Solution: Optimal location corresponds to all angles = 120°)

Minisum Weighted-Distance Location

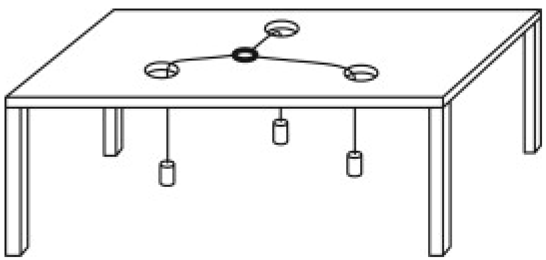
- Solution for 2-D+ and non-rectangular distances:
  - *Majority Theorem:* Locate NF at EF*j* if $w_j \geq \dfrac{W}{2}, \quad \text{where } W = \sum_{i=1}^{m} w_i$
  - Mechanical (Varigon frame)
  - 2-D rectangular approximation
  - Numerical: nonlinear unconstrained optimization
    - Analytical/estimated derivative (quasi-Newton, `fminunc`)
    - Direct, derivative-free (Nelder-Mead, `fminsearch`)

$$m = \text{number of EFs}$$

$$TC(\mathbf{x}) = \sum_{i=1}^{m} w_i d_i(\mathbf{x})$$

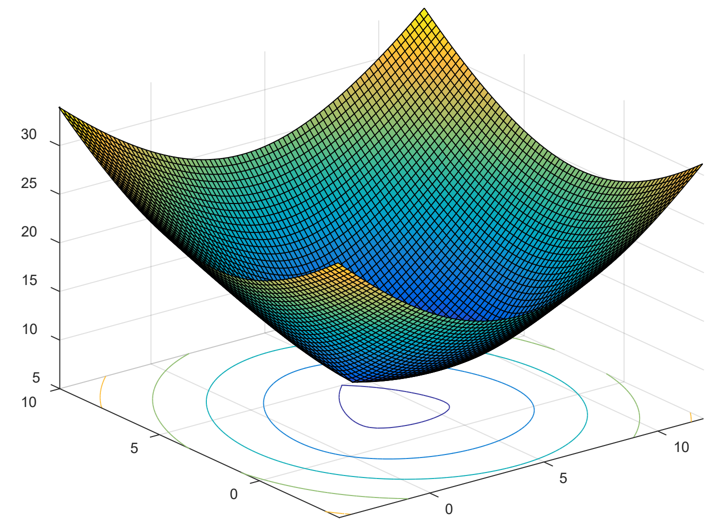
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

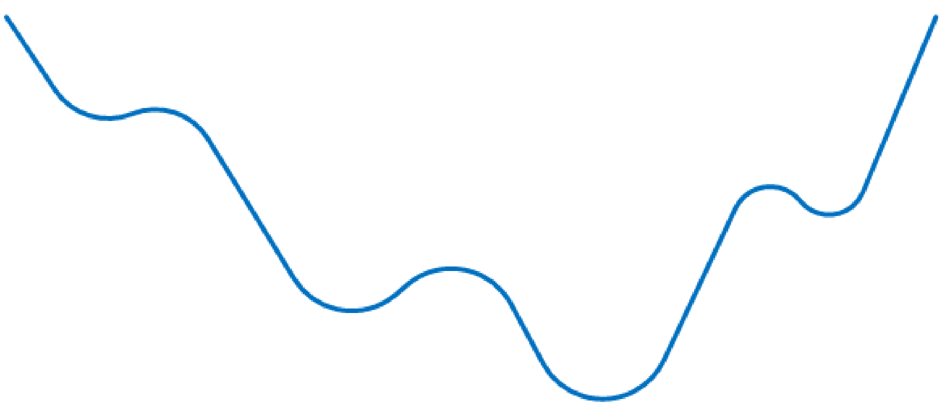
$$TC^* = TC(\mathbf{x}^*)$$



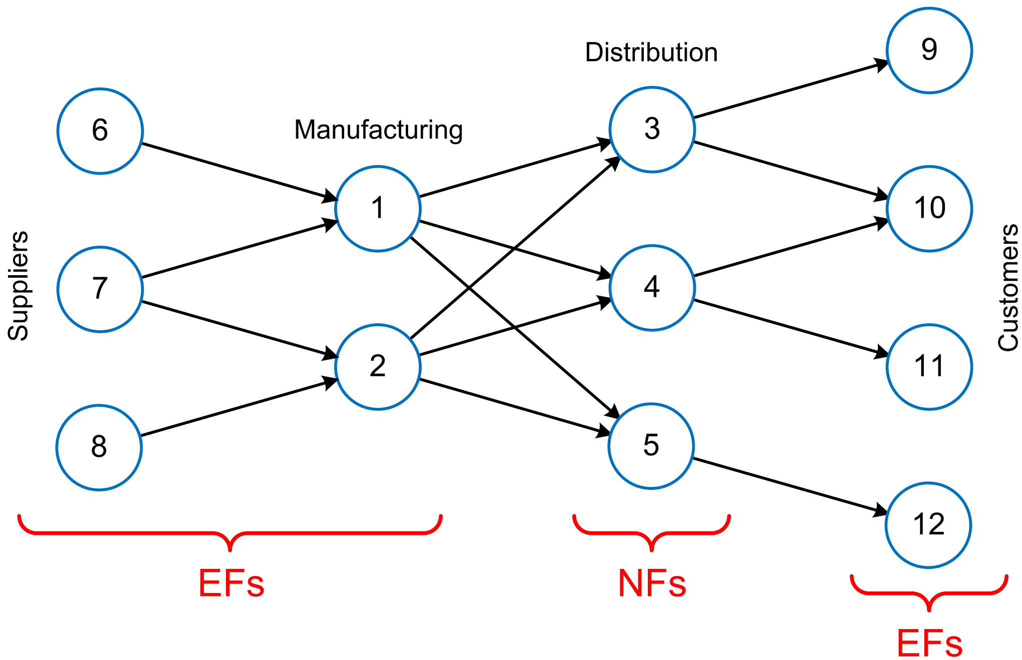
Varignon Frame

Convex vs Nonconvex Optimization





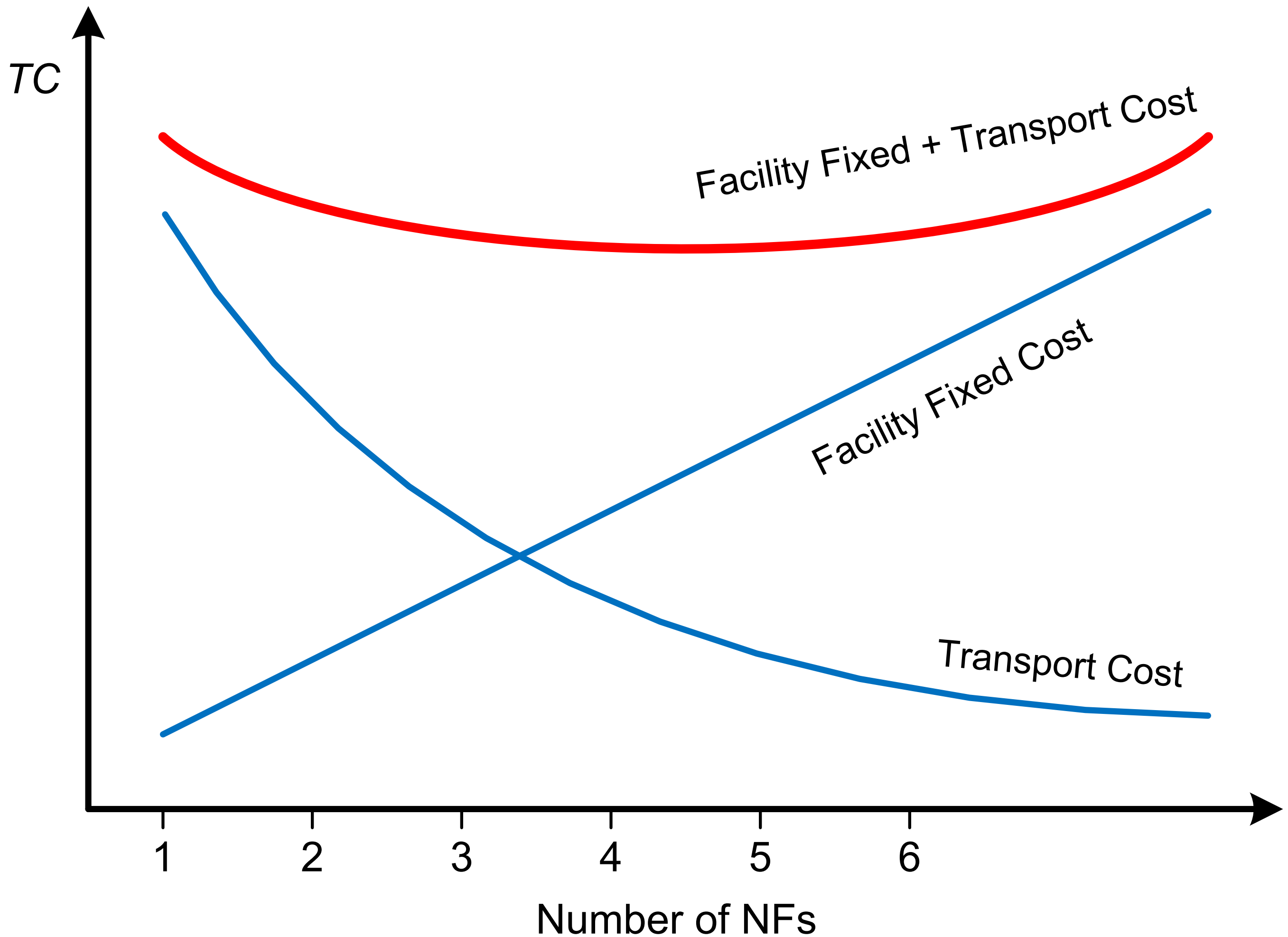
Multiple Single-Facility Location



Best Retail Warehouse Locations

Number of Locations	Average Transit Time (days)	Warehouse Location		
1	2.20	Bloomington, IN		
2	1.48	Ashland, KY	Palmdale, CA	
3	1.29	Allentown, PA	Palmdale, CA	McKenzie, TN
4	1.20	Edison, NJ Meridian, MS	Palmdale; CA	Chicago, IL
5	1.13	Madison, NJ Dallas, TX	Palmdale, CA Macon, GA	Chicago, IL
6	1.08	Madison, NJ Dallas, TX	Pasadena, CA Macon, GA	Chicago, IL Tacoma, WA
7	1.07	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA	Chicago, IL Tacoma, WA
8	1.05	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA
9	1.04	Madison, NJ Dallas, TX Lakeland. FL	Alhambra, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA Oakland, CA
10	1.04	Newark, NJ Palistine, TX Lakeland, FL Mansfield, OH	Alhambra, CA Gainesville, GA Denver, CO	Rockford, IL Tacoma, WA Oakland. CA

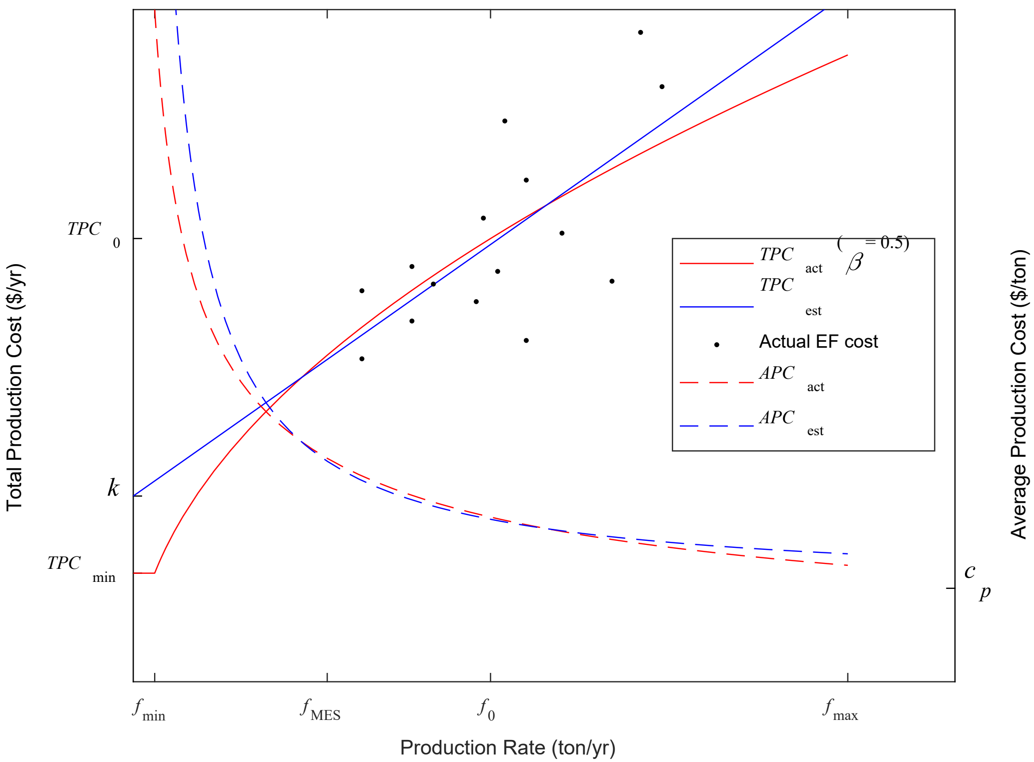
Optimal Number of NFs



Fixed Cost and Economies of Scale

- How to estimate facility fixed cost?
  - Cost data from existing facilities can be used to fit linear estimate
    - *y*-intercept is fixed cost, k
  - *Economies of scale* in production

    $\Rightarrow k > 0$ and $\beta < 1$



$$TPC_{\text{act}} = \max_{f < f_{\max}} \left\{ TPC_{\min},\, TPC_0 \left(\frac{f}{f_0} \right)^{\beta} \right\}$$

$$\beta = \begin{cases} 0.62, & \text{Hand tool mfg.} \\ 0.48, & \text{Construction} \\ 0.41, & \text{Chemical processing} \\ 0.23, & \text{Medical centers} \end{cases}$$

$$TPC_{\text{est}} = k + c_p f$$

$$APC_{\text{act}} = \frac{TPC_{\text{act}}}{f} = \frac{TPC_0}{f_0^{\beta}} f^{\beta - 1}$$

$$APC_{\text{est}} = \frac{k}{f} + c_p$$

k = fixed cost

c_p = constant unit production cost

$f_{\min}/f_{\max}$ = min/max feasible scale

f_{MES} = *Minimum Efficient Scale*

TPC_0/f_0 = base cost/rate

MILP

LP: max $\mathbf{c}'\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} \le \mathbf{b}$

$\mathbf{x} \ge 0$

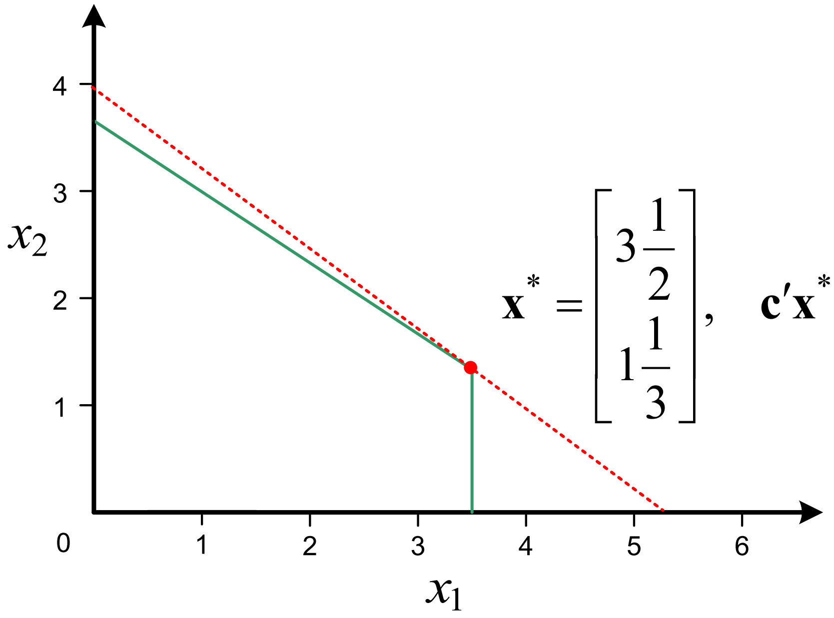
MILP: some x_i integer

ILP: $\mathbf{x}$ integer

BLP: $\mathbf{x} \in \{0,1\}$

$$\max\ 6x_1 + 8x_2$$
$$\text{s.t.}\ 2x_1 + 3x_2 \le 11$$
$$2x_1 \le 7$$
$$x_1, x_2 \ge 0$$

$$\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}$$
$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$



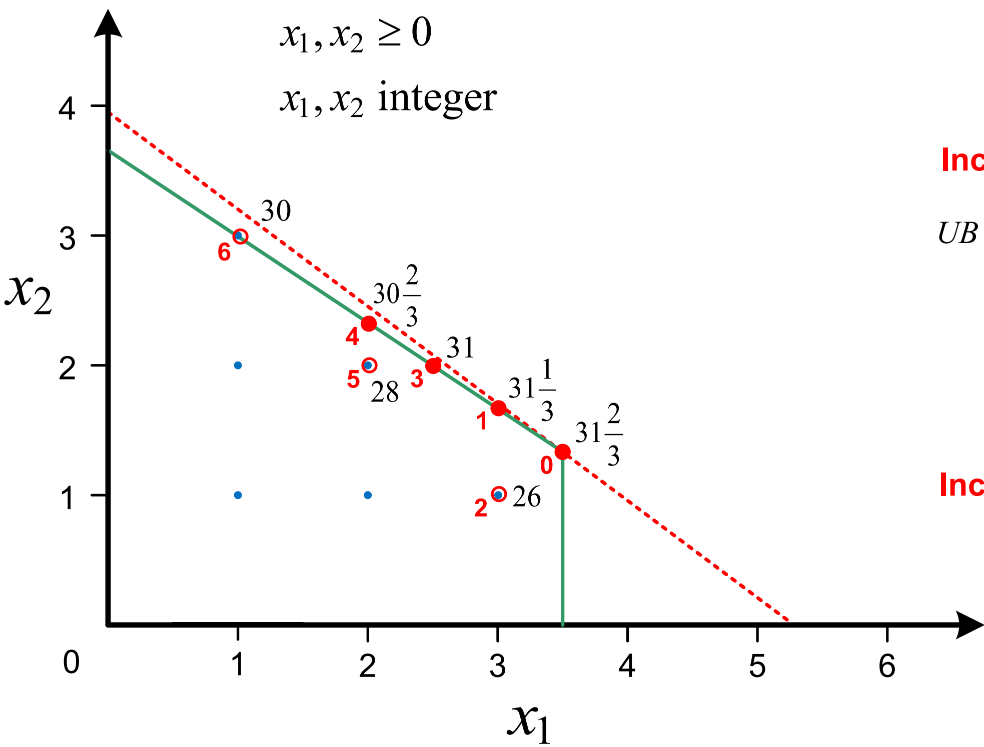
$$\mathbf{x}^* = \begin{bmatrix} 3\frac{1}{2} \\ 1\frac{1}{3} \end{bmatrix}, \quad \mathbf{c}'\mathbf{x}^* = 31\frac{2}{3}$$

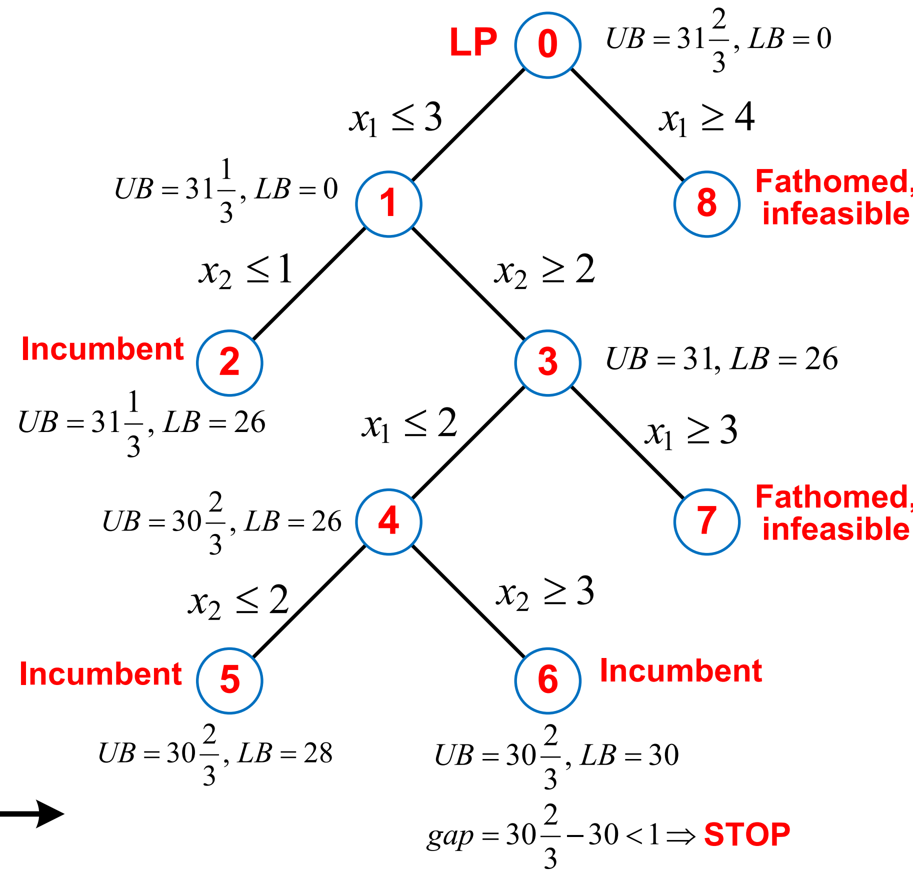
Branch and Bound

$$\begin{aligned} \max \quad & 6x_1 + 8x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\quad \le 7 \\ & x_1, x_2 \ge 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

$$\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$





MILP Formulation of UFL

$$
\begin{aligned}
\min \quad & \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} x_{ij} = 1, && j \in M \\
& y_i \geq x_{ij}, && i \in N, j \in M \\
& 0 \leq x_{ij} \leq 1, && i \in N, j \in M \\
& y_i \in \{0,1\}, && i \in N
\end{aligned}
$$

where

k_i = fixed cost of NF at site $i \in N = \{1, \ldots, n\}$

c_{ij} = variable cost from i to serve EF $j \in M = \{1, \ldots, m\}$

$$
y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}
$$

x_{ij} = fraction of EF j demand served from NF at site i.

MILP Formulation of *p*-Median

$$
\begin{aligned}
\min \quad & \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} y_i = p \\
& \sum_{i \in N} x_{ij} = 1, \qquad j \in M \\
& y_i \ge x_{ij}, \qquad i \in N, j \in M \\
& 0 \le x_{ij} \le 1, \qquad i \in N, j \in M \\
& y_i \in \{0,1\}, \qquad i \in N
\end{aligned}
$$

where

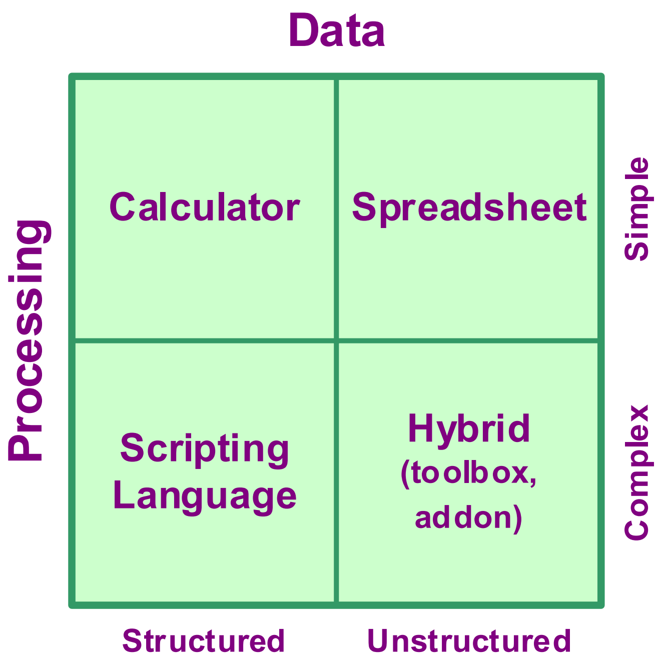
p = number of NF to establish

c_{ij} = variable cost from i to serve EF $j \in M = \{1, \ldots, m\}$

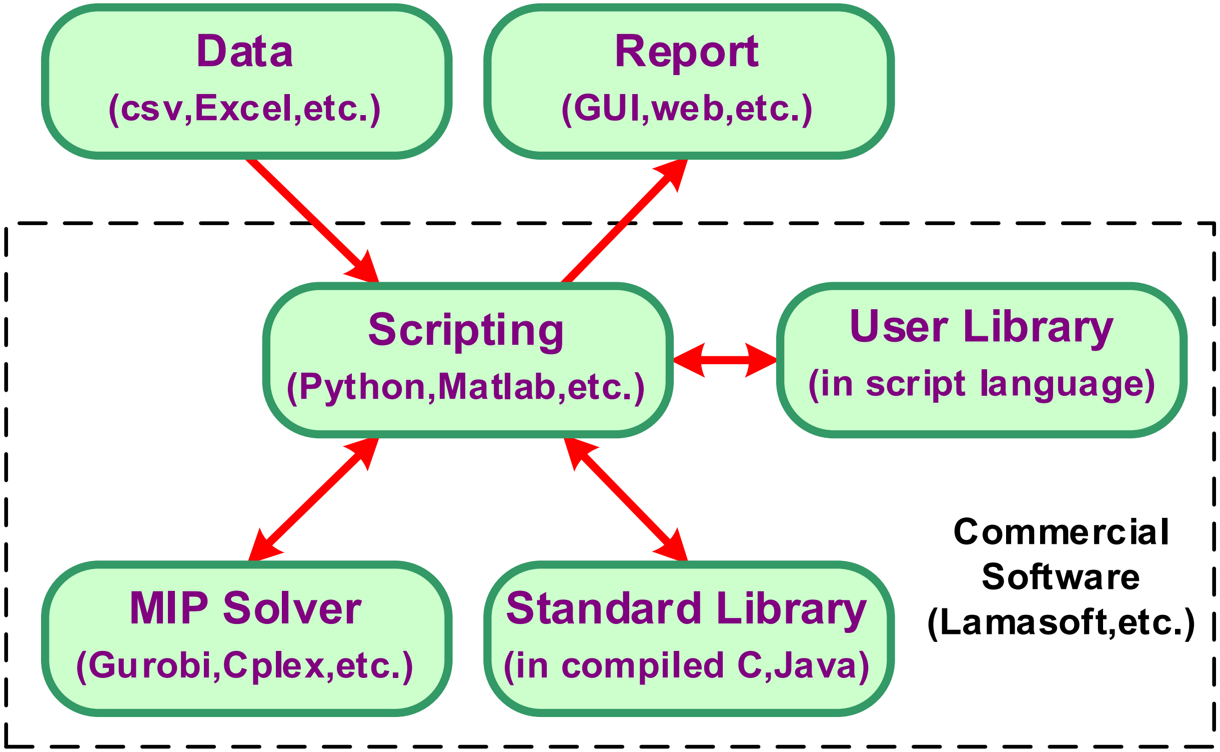
$$
y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}
$$

x_{ij} = fraction of EF j demand served from NF at site i.

Computational Tools



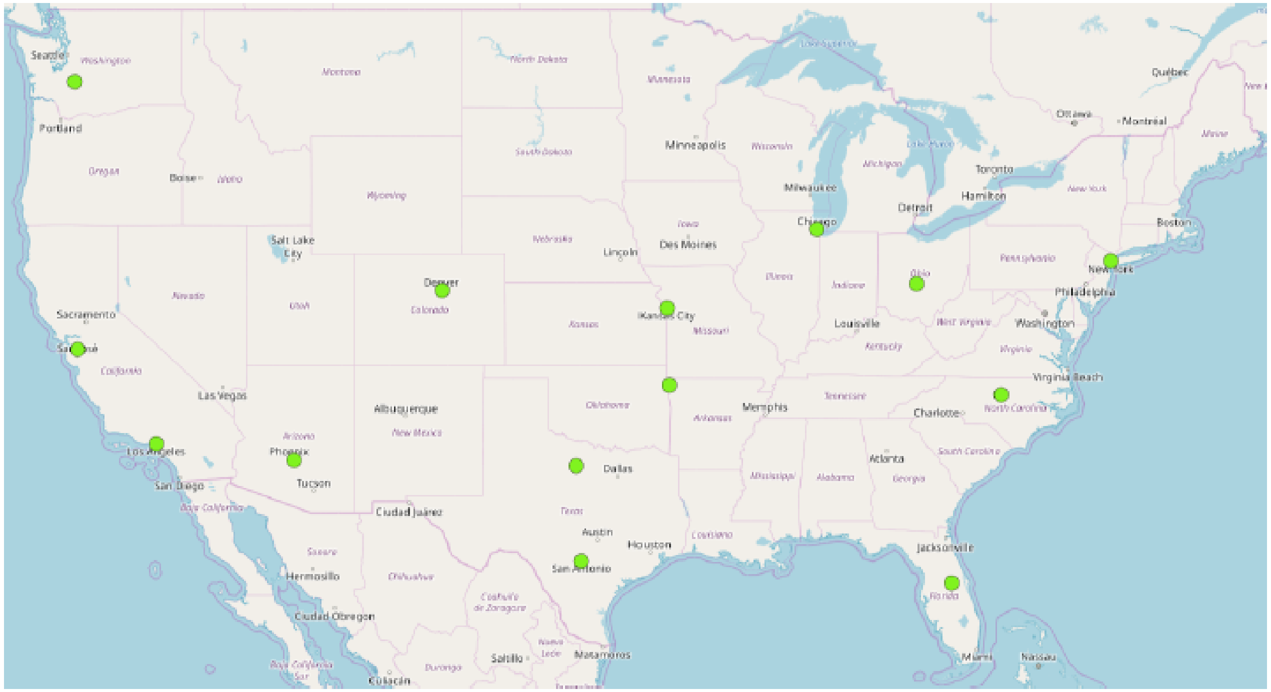
Logistics Software Stack



- New *Julia* (1.0) scripting language
  - (almost?) as fast as C and Java (but not FORTRAN)
  - does not require compiled standard library for speed
  - uses *multiple dispatch* to make type-specific versions of functions

PharmaCo Case Study

Exhibit A: Map of PharmaCo DC locations



2. Last year's P&L showed distribution operating costs of $109.3 million annually, subdivided into fixed DC operating, variable DC operating, and freight.

Exhibit B: Supply Chain Cost Profile

Supply Chain Costs (millions)				
Fixed Operating	Variable Operating	Transport-ation	Inventory Carrying	Total Supply Chain
$34.9	$42.0	$6.7	$25.7	$109.3

Logistics Engineering Design Constants

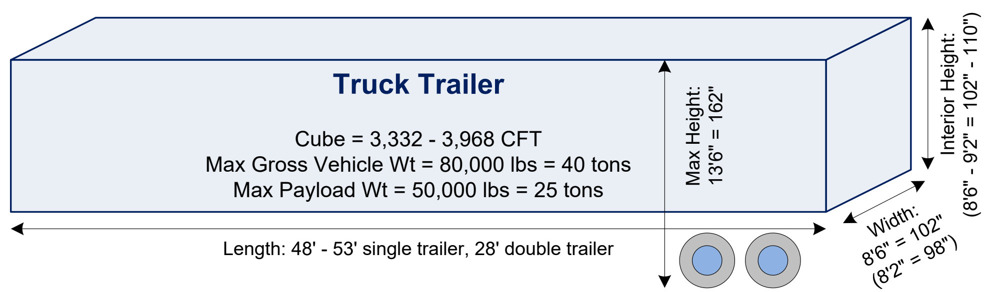
1. Circuity Factor: 1.2 (g)
   - 1.2 × GC distance ≈ actual road distance
2. Local vs. Intercity Transport:
   - Local: < 50 mi ⇒ use actual road distances
   - Intercity: > 50 mi ⇒ can estimate road distances
     - 50-250 mi ⇒ return possible (11 HOS)
     - > 250 mi ⇒ always one-way transport
     - > 500-750 mi ⇒ intermodal rail possible
3. Inventory Carrying Cost (h) = funds + storage + obsolescence
   - 16% average (no product information, per U.S. Total Logistics Costs)
     - (16% ≈ 5% funds + 6% storage + 5% obsolescence)
   - 5-10% low-value product (construction)
   - 25-30% general durable manufactured goods
   - 50% computer equipment
   - >> 100% perishable goods (produce)

Logistics Engineering Design Constants

4. $\dfrac{\text{Value}}{\text{Transport Cost}} \gg 1: \quad \color{red}{\$1\text{ ft}^3} \approx \dfrac{\$2{,}620 \text{ Shanghai-LA/LB shipping cost}}{2{,}400 \text{ ft}^3 \text{ 40' ISO container capacity}}$

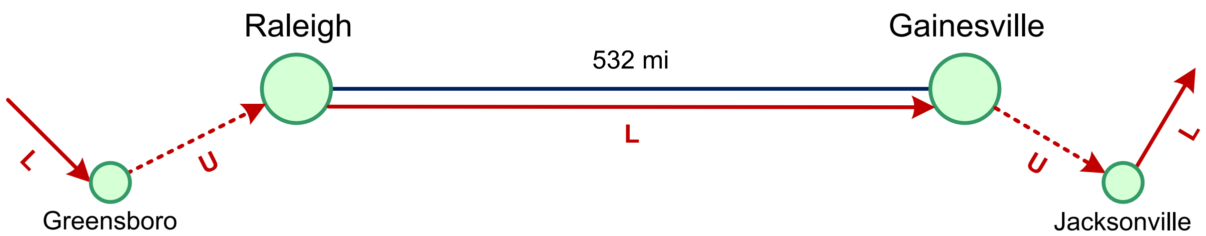
5. TL Weight Capacity: 25 tons (K_{wt})
   - (40 ton max per regulation) – (15 ton tare for tractor-trailer) = 25 ton max payload
   - Weight capacity = 100% of physical capacity

6. TL Cube Capacity: 2,750 ft^3 (K_{cu})
   - Trailer physical capacity = 3,332 ft^3
   - Effective capacity = $3{,}332 \times 0.80 \approx 2{,}750$ ft^3
   - Cube capacity = 80% of physical capacity



Logistics Engineering Design Constants

7. TL Revenue per Loaded Truck-Mile: $2/mi in 2004 (r)
   - TL revenue for the carrier is your TL cost as a shipper



$$15\%, \quad \text{average deadhead travel}$$

$$\$1.60, \quad \text{cost per mile in 2004}$$

$$\frac{\$1.60}{1-0.15} = \$1.88, \quad \text{cost per loaded-mile}$$

$$6.35\%, \quad \text{average operating margin for trucking}$$

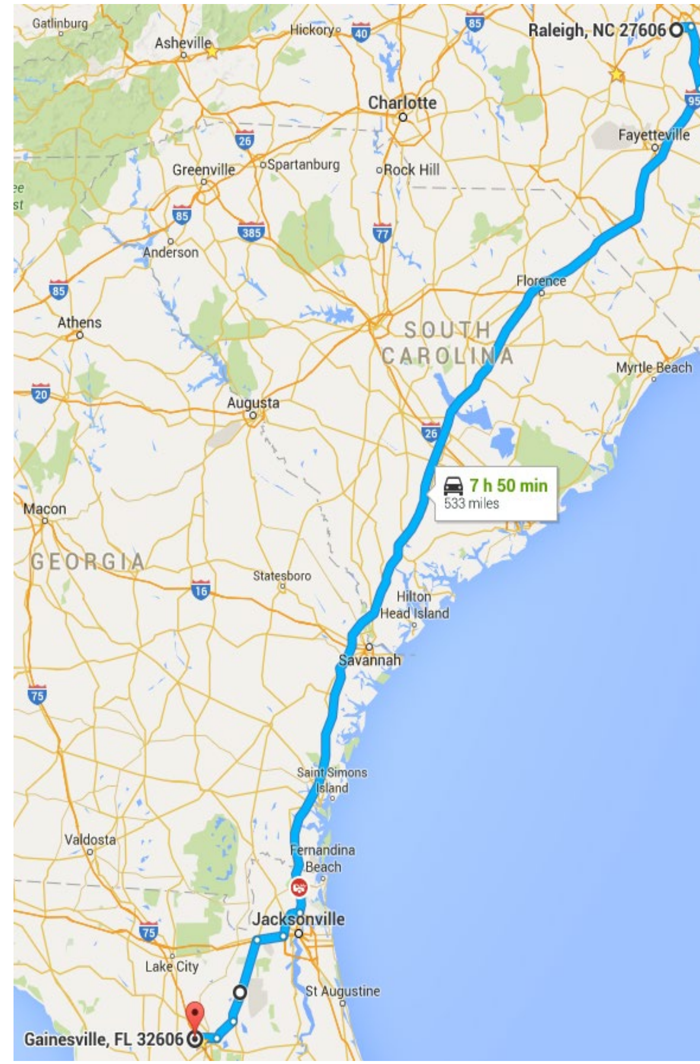
$$\frac{\$1.88}{1-0.0635} \approx \$2.00, \quad \text{revenue per loaded-mile}$$

One-Time vs Periodic Shipments

- **One-Time Shipments** (*operational* decision): know shipment size q
  - Know when and how much to ship, need to determine if TL and/or LTL to be used
  - Must contact carrier or have agreement to know charge
    - Can/should estimate charge before contacting carrier
- **Periodic Shipments** (*tactical* decision): know demand rate f, must determine size q
  - Need to determine how often and how much to ship
  - Analytical transport charge formula allow “optimal” size (and shipment frequency) to be estimated
    - U.S. Bureau of Labor Statistic's *Producer Price Index* (PPI) for TL and LTL used to estimate transport charges

Truck Shipment Example

- Product shipped in cartons from Raleigh, NC (27606) to Gainesville, FL (32606)
- Each identical unit weighs 40 lb and occupies 9 ft^3 (its *cube*)
  - Don't know linear dimensions of each unit for TL and LTL
- Units can be stacked on top of each other in a trailer
- Additional info/data is presented only when it is needed to determine answer



Truck Shipment Example: One-Time

1. Assuming that the product is to be shipped P2P TL, what is the maximum payload for each trailer used for the shipment?

$$q_{\max}^{wt} = K_{wt} = 25 \text{ ton}$$

$$K_{cu} = 2750 \text{ ft}^3$$

$$s = \frac{40 \text{ lb/unit}}{9 \text{ ft}^3\text{/unit}} = 4.4444 \text{ lb/ft}^3$$

$$K_{cu} = \frac{q_{\max}^{cu}}{\left(\frac{s}{2000}\right)} \Rightarrow q_{\max}^{cu} = \frac{sK_{cu}}{2000}$$

$$q_{\max} = \min\left\{q_{\max}^{wt}, q_{\max}^{cu}\right\} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\}$$

$$= \min\left\{25, \frac{4.4444(2750)}{2000}\right\} = 6.1111 \text{ ton}$$

Truck Shipment Example: One-Time

2. On Jan 10, 2018, 320 units of the product were shipped. How many truckloads were required for this shipment?

$$q = 320\frac{40}{2000} = 6.4 \text{ ton}, \quad \left\lceil \frac{q}{q_{\max}} \right\rceil = \left\lceil \frac{6.4}{6.1111} \right\rceil = 2 \text{ truckloads}$$

3. Before contacting the carrier (and using Jan 2018 PPI), what is the estimated TL transport charge for this shipment?

$$d = 532 \text{ mi}$$

$$r_{TL} = \frac{PPI_{TL}^{\text{Jan 2018}}}{PPI_{TL}^{2004}} \times r_{2004} = \frac{PPI_{TL}}{102.7} \times \$2.00 / \text{mi}$$

$$= \frac{131.0}{102.7} \times \$2.00 / \text{mi} = \$2.5511 / \text{mi}$$

$$c_{TL} = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d = \left\lceil \frac{6.4}{6.1111} \right\rceil (2.5511)(532) = \$2,714.39$$

Truck Shipment Example: One-Time

UNITED STATES DEPARTMENT OF LABOR

BUREAU OF LABOR STATISTICS

Home ▾ | Subjects ▾ | Data Tools ▾ | Publications ▾ | Economic Releases ▾ | Students ▾ | Beta ▾

Databases, Tables & Calculators by Subject

Change Output Options:

From: 2008 ▼ To: 2018 ▼

☐ include graphs ☐ include annual averages

Data extracted on: September 5, 2018 (4:22:19 PM)

PPI Industry Data

```
Series Id:     PCU484121484121
Series Title:  PPI industry data for General freight trucking, long-distance TL, not seasonally adjusted
Industry:      General freight trucking, long-distance TL
Product:       General freight trucking, long-distance TL
Base Date:     200312
```

Download: xlsx

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	116.0	115.9	116.5	117.8	120.5	123.0	124.0	124.0	121.8	121.3	117.8	115.1
2009	113.2	112.1	110.4	109.7	109.8	110.1	111.4	111.0	111.7	110.8	111.5	110.9
2010	110.8	111.0	111.9	112.2	113.2	113.5	113.4	113.7	113.8	114.4	115.8	116.1
2011	116.5	117.4	119.3	121.0	121.7	121.4	121.3	121.2	122.0	122.0	123.2	123.3
2012	124.0	124.6	126.2	126.7	127.0	125.8	125.6	126.8	127.4	127.2	126.9	127.0
2013	126.7	127.2	128.0	127.5	127.8	127.6	127.6	127.6	127.1	127.2	127.6	127.4
2014	127.9	128.2	128.7	129.5	130.6	130.8	130.3	130.4	130.4	129.7	129.8	128.9
2015	126.7	126.0	126.0	126.2	126.3	127.1	126.9	126.2	125.9	125.5	125.8	124.8
2016	124.6	123.4	123.2	123.6	122.8	122.7	123.0	123.0	123.3	124.1	124.1	124.2
2017	124.4	124.7	124.2	124.3	124.0	124.2	124.2	125.9	126.6	126.6	128.5	130.3
2018	131.0	132.0	132.0	132.6(P)	133.6(P)	135.9(P)	138.6(P)					

P : Preliminary. All indexes are subject to revision four months after original publication.

Truck Shipment Example: One-Time

4. Using the Jan 2018 PPI LTL rate estimate, what was the transport charge to ship the fractional portion of the shipment LTL (i.e., the last partially full truckload portion)?

$$q_{\text{frac}} = q - q_{\max} = 6.4 - 6.1111 = 0.2889 \text{ ton}$$

$$r_{LTL} = PPI_{LTL} \left[\frac{\frac{s^2}{8} + 14}{\left(q_{\text{frac}}^{\frac{1}{7}} d^{\frac{15}{29}} - \frac{7}{2} \right)\left(s^2 + 2s + 14 \right)} \right]$$

$$= 177.4 \left[\frac{\frac{4.44^2}{8} + 14}{\left(0.2889^{\frac{1}{7}} 532^{\frac{15}{29}} - \frac{7}{2} \right)\left(4.44^2 + 2(4.44) + 14 \right)} \right] = \$3.8014 / \text{ton-mi}$$

$$c_{LTL} = r_{LTL}\, q_{\text{frac}}\, d = 3.8014(0.2889)(532) = \$584.23$$

Truck Shipment Example: One-Time

5. What is the change in total charge associated with the combining TL and LTL as compared to just using TL?

$$\Delta c = c_{TL} - \left(c_{TL-1} + c_{LTL} \right)$$

$$= \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d - \left(\left\lfloor \frac{q}{q_{\max}} \right\rfloor r_{TL} d + r_{LTL}\, q_{\text{frac}}\, d \right)$$

$$= \$772.96$$

Truck Shipment Example: One-Time

6. What would the fractional portion have to be so that the TL and LTL charges are equal?

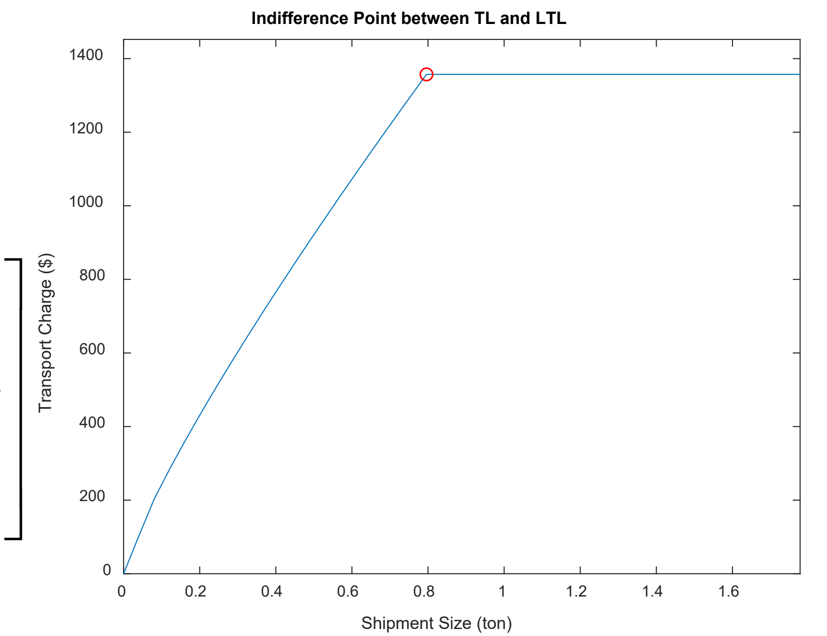
$$c_{TL}(q) = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d$$

$$r_{LTL}(q) = PPI_{LTL} \left[\frac{\frac{s^2}{8} + 14}{\left(q^{\frac{1}{7}} d^{\frac{15}{29}} - \frac{7}{2} \right) \left(s^2 + 2s + 14 \right)} \right]$$

$$c_{LTL}(q) = r_{LTL}(q)\, q\, d$$

$$q_I = \arg\min_q \left(\left| c_{TL}(q) - c_{LTL}(q) \right| \right)$$

$$= 0.7960 \text{ ton}$$



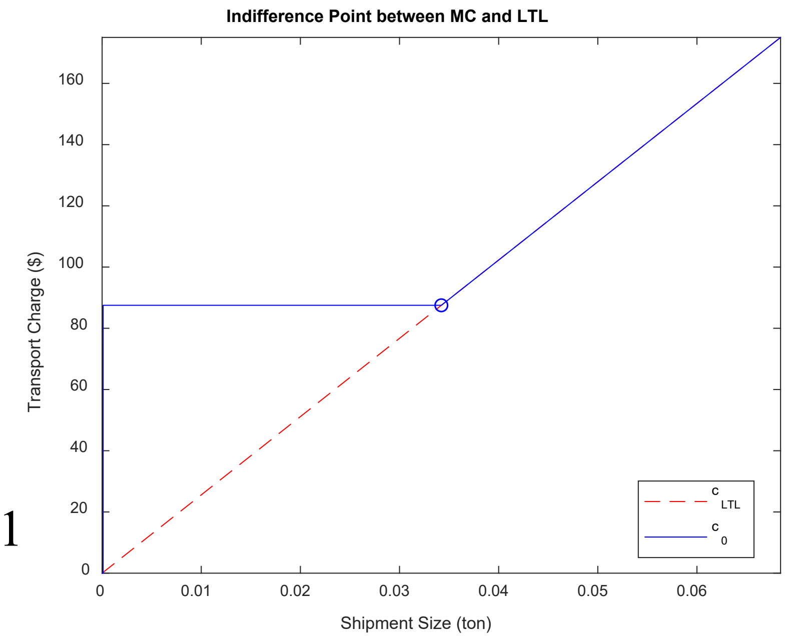
Truck Shipment Example: One-Time

7. What are the TL and LTL minimum charges?

$$MC_{TL} = \left(\frac{r_{TL}}{2}\right) 45 = \$57.40$$

$$MC_{LTL} = \left(\frac{PPI_{LTL}}{104.2}\right)\left(45 + \frac{d^{\frac{28}{19}}}{1625}\right)$$

$$= \left(\frac{177.4}{104.2}\right)\left(45 + \frac{532^{\frac{28}{19}}}{1625}\right) = \$87.51$$

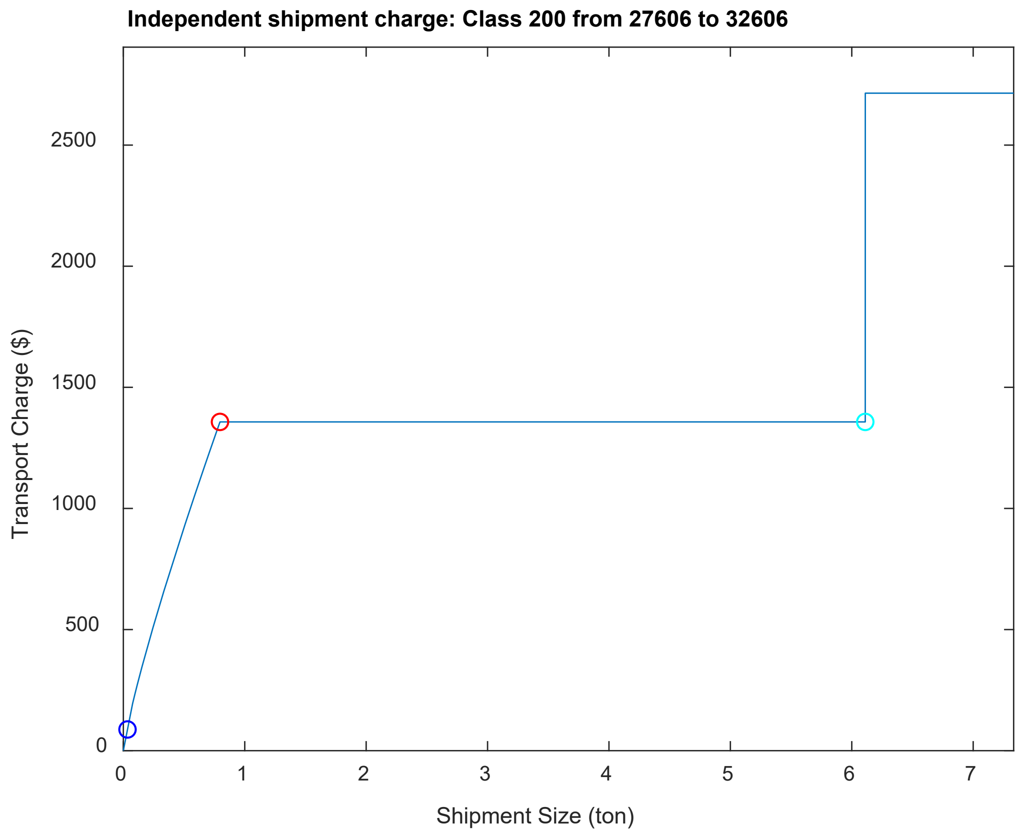


- Why do these charges not depend on the size of the shipment?
- Why does only the LTL minimum charge depend of the distance of the shipment?

Truck Shipment Example: One-Time

- Independent Transport Charge ($):

$$c_0(q) = \min\left\{\max\left\{c_{TL}(q), MC_{TL}\right\}, \max\left\{c_{LTL}(q), MC_{LTL}\right\}\right\}$$



Truck Shipment Example: One-Time

8. Using the same LTL shipment, find online one-time (spot) LTL rate quotes using the FedEx LTL website

$$q_{\text{frac}} = 0.2889 \text{ ton}$$

$$= 0.2889(2000) = 578 \text{ lb}$$

$$\text{no. units} = \left\lceil \frac{0.2889(2000)}{40} \right\rceil = 15 \text{ cartons}$$

- Most likely freight class:

$$s = \frac{40 \text{ lb/unit}}{9 \text{ ft}^3\text{/unit}} = 4.4444 \text{ lb/ft}^3$$

$$\Rightarrow \text{Class 200}$$

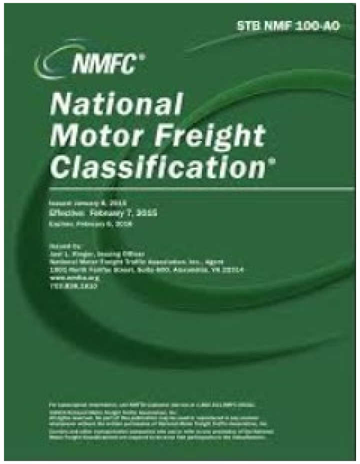
- What is the rate quote for the reverse trip from Gainesville (32606) to Raleigh (27606)?

Class-Density Relationship

Class	Load Density (lb/ft^3) Minimum	Average	Max Physical Weight (tons)	Max Effective Cube (ft^3)
500	–	0.52	0.72	2,750
400	1	1.49	2.06	2,750
300	2	2.49	3.43	2,750
250	3	3.49	4.80	2,750
200	4	4.49	6.17	2,750
175	5	5.49	7.55	2,750
150	6	6.49	8.92	2,750
125	7	7.49	10.30	2,750
110	8	8.49	11.67	2,750
100	9	9.72	13.37	2,750
92.5	10.5	11.22	15.43	2,750
85	12	12.72	17.49	2,750
77.5	13.5	14.22	19.55	2,750
70	15	18.01	24.76	2,750
65	22.5	25.50	25	1,961
60	30	32.16	25	1,555
55	35	39.68	25	1,260
50	50	56.18	25	890

Truck Shipment Example: One-Time

- The *National Motor Freight Classification* (NMFC) can be used to determine the product class
- Based on:
  1. Load density
  2. Special handling
  3. Stowability
  4. Liability



Item	Description	Class	NMFC	Sub
Abietic Acid	Abietic Acid, in drums	55	42605	-
Accordions	Accordions, in boxes	125	138820	-
Acetonitrile	Acetonitrile, in boxes or drums. See item 60000 for class dependent upon released value	85	42645	-
Acetylene	in steel cylinders	70	85520	-
Acid Fish Scrap	Fish Scrap, NOI, dry, not ground, pulverized nor screened, or Acid Fish Scrap, in bags	77.5	69980	-
Aircraft Parts	metal, struts, skins, panels	200	11790	01
Aluminum Channel	U channel	60	13340	-
Aluminum Table Set	aluminum table SU	200	82105	01
Ambulance Stretcher	stretcher	200	56920	06
Arches Support	Iron Steel	60	52460	-
Architectural Details	6 - 8 lbs per cubic foot	125	56290	05
Architectural Details	2 - 4 lbs per cubic ft	250	56290	03
Assembled Furniture	Bathroom cabinet set up	300	39220	01
Assembled Furniture	Highboys, dressers, wooden set up	125	80120	01
Assembled Furniture	Wood furniture 4-6 Lbs per cu ft	150	82270	04
Assembled Furniture	Chairs wooden setup w/out upholstery	300	80770	01
Assembled Furniture	Chairs wooden setup w/out upholstery KD	125	80770	03
Assembled Furniture	Couch w/ back & arms put together	175	80865	03
Assembled Furniture	Chairs put together w/ upholstery	200	79255	01
Assembled Furniture	Metal cabinets in boxes	110	39270	06
Assembled Furniture	18 gauge steel cabinet	70	39340	-
Assembled Furniture	Benches, cabinets, tables for workstations	125	23410	-
Assembled Furniture	Buffets, china cabinets put together	125	80080	-
Assembled Furniture	Cabinets of metal or plastic for storage	92.5	39235	-
Assembled Furniture	Tanning bed	150	109050	-
Assembled Furniture	Mattresses, in packages or boxes	200	79550	-
Athletic / Sporting Goods	Gym equipment, playground, sports items. Density Item			
Attachments: Backhoe	NOI: Attachments, backhoe (Backhoes), tractor or truck, on lift truck skids or pallets:	175	114217	01
Attachments: Backhoe	Attachments, backhoe (Backhoes), tractor or truck, on lift truck skids or pallets: Each shipped with all components secured to a single pallet, platform or skid, weighing 1100 pounds or more and having a density of 8 pounds or greater per cubic foot	100	114217	02

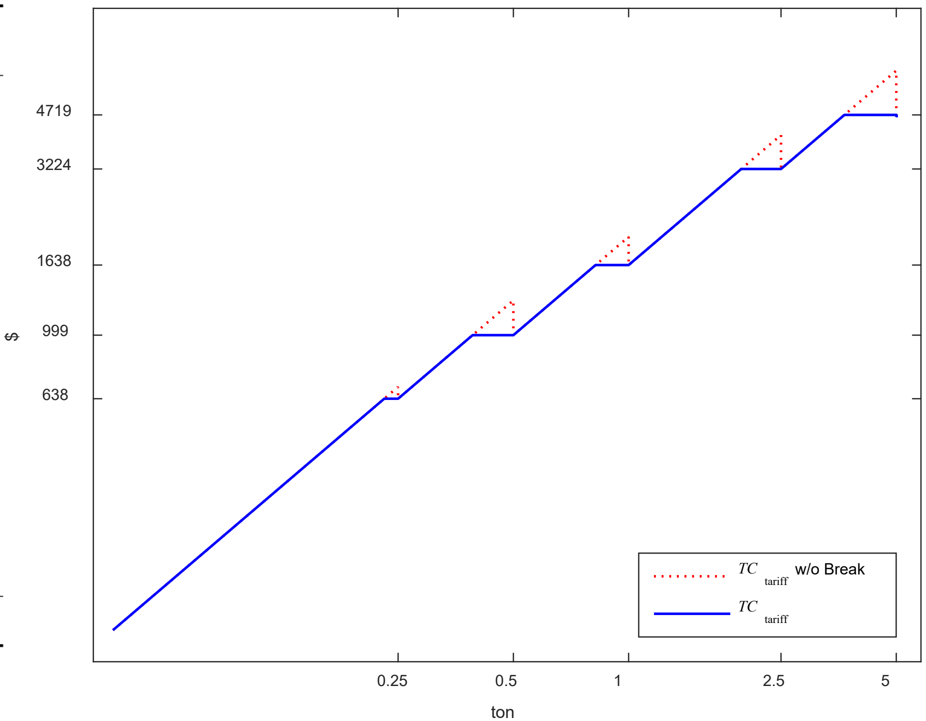
Truck Shipment Example: One-Time

- CzarLite tariff table for O-D pair 27606-32606

$$cwt = \text{hundredweight} = 100 \text{ lb} = \frac{100}{2000} = \frac{1}{20} \text{ ton}$$

Tariff (in \$/cwt) from Raleigh, NC (27606) to Gainesville, FL (32606)
(532 mi, CzarLite DEMOCZ02 04-01-2000, minimum charge = \$95.23)

Freight Class	Rate Breaks (i) 1	2	3	4	5	6	7	8	9&10
500	341.42	314.14	245.80	201.48	158.60	112.37	55.66	55.66	55.66
400	273.88	251.99	197.19	161.61	127.22	91.12	45.10	45.10	45.10
300	206.34	189.85	148.56	121.76	95.85	69.47	34.43	34.43	34.43
250	172.56	158.77	124.23	101.83	80.15	58.03	28.79	28.79	28.79
200	138.78	127.69	99.92	81.89	64.47	47.19	23.40	23.40	23.40
175	121.37	111.68	87.39	71.62	56.38	41.27	20.39	20.39	20.39
150	104.49	96.13	75.22	61.66	48.53	35.96	17.75	17.75	17.75
125	87.59	80.60	63.07	51.69	40.69	30.24	15.00	15.00	15.00
110	77.57	71.37	55.85	45.77	36.04	28.61	14.40	14.40	14.40
100	71.23	65.55	51.29	42.04	33.09	27.58	14.03	10.80	9.90
92	66.48	61.18	47.88	39.24	30.89	25.75	13.68	10.52	9.66
85	61.74	56.80	44.45	36.43	28.68	23.91	13.20	10.15	9.32
77	56.99	52.44	41.04	33.63	26.48	22.07	12.60	9.68	8.89
70	52.77	48.55	37.99	31.14	24.51	20.43	12.00	9.23	8.47
65	50.07	46.08	36.05	29.56	23.04	19.39	11.87	9.14	8.39
60	47.44	43.64	34.15	28.00	21.82	18.37	11.76	9.04	8.30
55	44.75	41.17	32.22	26.40	20.59	17.32	11.64	8.96	8.22
50	41.57	38.26	29.94	24.54	19.12	16.10	11.52	8.85	8.14
Tons (q_i^B)	0.25	0.5	1	2.5	5	10	15	20	∞



Truck Shipment Example: One-Time

9. Using the same LTL shipment, what is the transport cost found using the undiscounted CzarLite tariff?

$$q = 0.2889, \quad class = 200$$

$$disc = 0, \quad MC = 95.23$$

Freight Class	Rate Breaks (i) 1	2	3	4	5	6	7	8	9&10
500	341.42	314.14	245.80	201.48	158.60	112.37	55.66	55.66	55.66
400	273.88	251.99	197.19	161.61	127.22	91.12	45.10	45.10	45.10
300	206.34	189.85	148.56	121.76	95.85	69.47	34.43	34.43	34.43
250	172.56	158.77	124.23	101.83	80.15	58.03	28.79	28.79	28.79
200	138.78	127.69	99.92	81.89	64.47	47.19	23.40	23.40	23.40
175	121.37	111.68	87.39	71.62	56.38	41.27	20.39	20.39	20.39
50	41.57	38.26	29.94	24.54	19.12	16.10	11.52	8.85	8.14
Tons (q_i^B)	0.25	0.5	1	2.5	5	10	15	20	∞

$$i = \arg\left\{ q_i^B \,\middle|\, q_{i-1}^B \le q < q_i^B \right\}$$

$$= \arg\left\{ q_2^B \,\middle|\, q_1^B \le q < q_2^B \right\}$$

$$= \arg\left\{ q_2^B \,\middle|\, 0.25 \le 0.2889 < 0.5 \right\} = 2$$

$$c_{\text{tariff}} = (1 - disc)\max\left\{ MC, \min\left\{ OD(class, i)\, 20q,\; OD(class, i+1)\, 20 q_i^B \right\}\right\}$$

$$= (1 - 0)\max\left\{ 95.23, \min\left\{ OD(200, 2)\, 20(0.2889),\; OD(200, 3)\, 20(0.5) \right\}\right\}$$

$$= \max\left\{ 95.23, \min\left\{ (127.69)\, 20(0.2889),\; (99.92) 20(0.5) \right\}\right\}$$

$$= \max\left\{ 95.23, \min\left\{ 737.76,\; 999.20 \right\}\right\} = \$737.76$$

Truck Shipment Example: One-Time

10. What is the implied discount of the estimated charge from the CzarLite tariff cost?

$$disc = \frac{c_{\text{tariff}} - c_{LTL}}{c_{\text{tariff}}}$$

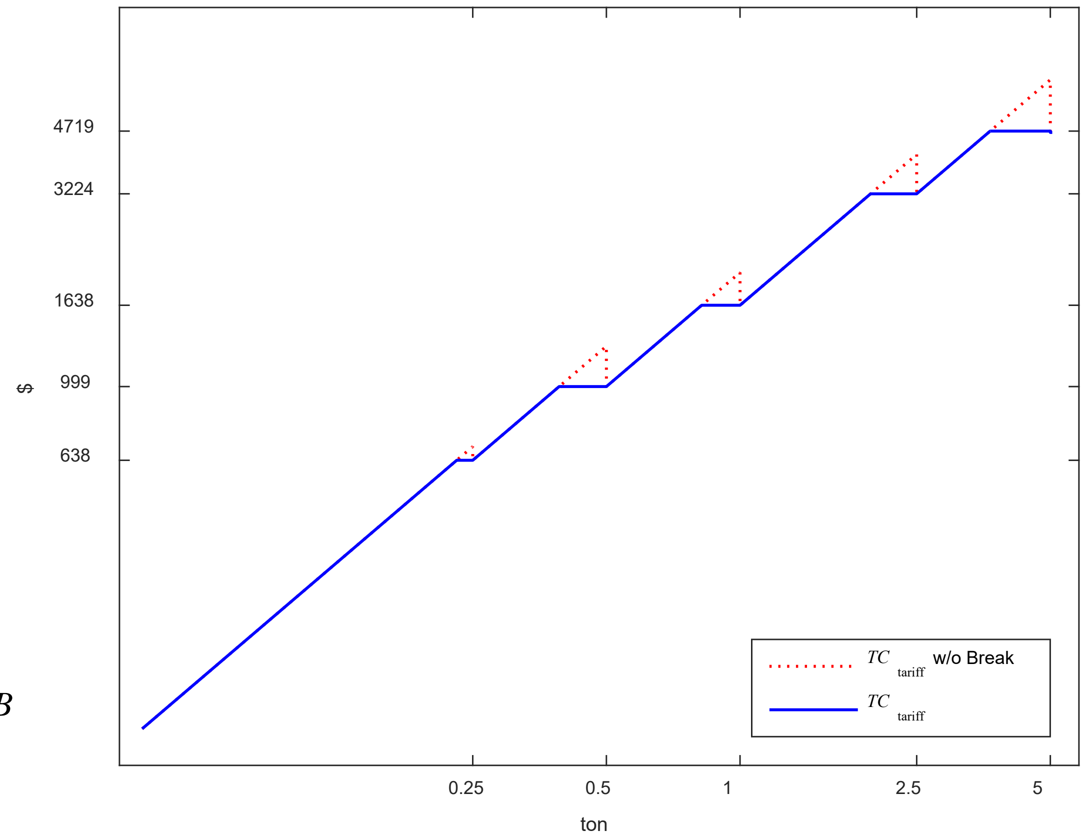
$$= \frac{737.76 - 584.23}{737,76}$$

$$= 20.81\%$$

- What is the weight break between the rate breaks?

$$q_i^W = \frac{OD(class, i+1)}{OD(class, i)} q_i^B$$

$$= \frac{99.92}{127.69}(0.5) = 0.3913 \text{ ton}$$



Truck Shipment Example: One-Time

- **PX: Package Express**
  - (Undiscounted) charge c_{PX} based rate tables, R, for each service (2-day ground, overnight, etc.)
  - Rate determined by on *chargeable weight*, wt_{chrg}, and *zone*
  - All PX carriers (FedEX, UPS, USPS, DHL) use *dimensional weight*, wt_{dim}
  - wt_{dim} > 150 lb is prorated per-lb rate
  - Actual weight 1–70 lb (UPS, FedEx home), 1–150 lb (FedEx commercial)
  - Carrier sets a *shipping factor*, which is min cubic volume per pound
  - Zone usually determined by O-D distance of shipment
  - Supplemental charges for home delivery, excess declared value, etc.

$$c_{PX} = R\left(wt_{\text{chrg}}, zone\right)$$

$$wt_{\text{chrg}} = \left\lceil \max\left\{wt_{\text{act}}, wt_{\text{dim}}\right\} \right\rceil \text{(lb)}$$

$$wt_{\text{act}} = \text{actual weight (1 to 150 lb)}$$

$$wt_{\text{dim}} = \frac{l \times w \times d \ (\text{in}^3)}{sf \ (\text{in}^3/\text{lb})} \text{(lb)}$$

$$l, w, d = \text{length, width, depth (in)}$$

$$l \geq w, \quad l \times w \times d \geq \text{actual cube}$$

$$sf = \text{shipping factor (in}^3/\text{lb)}$$

$$= 12^3 \big/ s, \text{ inverse of density}$$

$$= 139 \text{ FedEx (2019)}$$

$$\Rightarrow s = 12.43 \text{ lb/ft}^3 \text{ (Class 85)}$$

$$= 194 \text{ USPS} \ \Rightarrow s = 8.9 \text{ lb/ft}^3$$

Truck Shipment Example: One-Time

- (Undisc.) charge to ship a single carton via FedEx?

$$wt_{\text{act}} = 40 \text{ lb}, \, cu = 9 \text{ ft}^3$$

$$d = 532 \text{ mi} \Rightarrow zone = 4$$

$$\text{carton} \Rightarrow l \times w \times d = \text{actual cube} \Rightarrow$$

$$l \times w \times d = 9 \times 12^3 = 15,552 \text{ in}^3 = 32 \times 27 \times 18$$

$$wt_{\text{dim}} = \frac{l \times w \times d}{sf} = \frac{15,552}{139} = 111.9 \text{ lb}$$

$$wt_{\text{chrg}} = \left\lceil \max\left\{ wt_{\text{act}}, wt_{\text{dim}} \right\} \right\rceil$$

$$= \left\lceil \max\left\{ 40, 111.9 \right\} \right\rceil = 112 \text{ lb}$$

$$c_{PX} = R\left(wt_{\text{chrg}}, zone \right)$$

$$= R\left(112, 4\right) = \$64.27$$

FedEx Standard List Rates (eff. Jan. 7, 2019)

Service	FedEx Ground® and FedEx Home Delivery® (up to 70 lbs.)						
Delivery Commitment	1–5 days based on distance to destination						
Zones[1]	2	3	4	5	6	7	8
	0–150 miles	151–300 miles	301–600 miles	601–1,000 miles	1,001–1,400 miles	1,401–1,800 miles	1,801-plus miles
Maximum Weight in Lbs. 1 lb.	$ 7.85	$ 8.23	$ 8.96	$ 9.36	$ 9.68	$ 9.80	$ 9.96
2 lbs.	8.52	9.48	10.15	10.37	10.82	11.24	11.43
3	8.87	9.89	10.70	11.14	11.59	11.98	12.57
4	9.13	[illegible]	11.04	11.75	12.08	12.87	13.47
5	9.37	[illegible]	[illegible]	[illegible]	[illegible]	13.46	14.22
6	9.68	[illegible]	[illegible]	[illegible]	[illegible]	13.81	14.48
7	10.23	[illegible]	[illegible]	[illegible]	[illegible]	14.18	15.18
8	10.43	[illegible]	[illegible]	[illegible]	[illegible]	14.61	15.69
9	10.59	11.40	12.48	13.39	14.04	15.21	16.52
10	10.84	11.51	12.60	13.76	14.33	16.10	17.62
111	59.41	59.89	64.26	67.20	75.20	82.60	92.25
112	60.62	61.13	64.27	67.21	75.84	83.31	92.36
113	60.68	61.18	64.98	67.83	76.52	84.00	94.04
114	61.32	62.45	66.33	69.15	77.81	85.41	94.65
115	61.99	63.16	66.34	69.33	77.82	85.42	94.66
146	82.51	84.98	88.95	89.15	98.04	105.96	118.85
147	83.66	85.00	89.66	89.86	98.74	106.69	119.66
148	84.68	85.63	90.61	90.62	100.20	107.40	120.46
149	84.84	86.38	91.26	91.28	100.42	108.08	121.81
150[2]	84.85	87.16	92.76	94.33	100.95	108.83	122.60

Note: No Zone 1 (usually < 50 mi local)

Truck Shipment Example: Periodic

11. Continuing with the example: assuming a constant annual demand for the product of 20 tons, what is the number of full truckloads per year?

$$f = 20 \text{ ton/yr}$$

$$q = q_{\max} = 6.1111 \text{ ton/ TL} \quad (\text{full truckload} \Rightarrow q \equiv q_{\max})$$

$$n = \frac{f}{q} = \frac{20}{6.1111} = 3.2727 \text{ TL/yr}, \quad \text{average shipment frequency}$$

- Why should this number not be rounded to an integer value?

Truck Shipment Example: Periodic

12. What is the shipment interval?

$$t = \frac{1}{n} = \frac{q}{f} = \frac{6.1111}{20} = 0.3056 \text{ yr/TL}, \quad \text{average shipment interval}$$

- How many days are there between shipments?

$$365.25 \text{ day/yr}$$

$$t \times 365.25 = \frac{365.25}{n} = 111.6042 \text{ day/TL}$$

Truck Shipment Example: Periodic

13. What is the annual full-truckload transport cost?

$$d = 532 \text{ mi}, \quad r_{TL} = \$2.5511 / \text{mi}$$

$$r_{FTL} = \frac{r_{TL}}{q_{\max}} = \frac{2.5511}{6.1111} = \$0.4175 / \text{ton-mi}$$

$$TC_{FTL} = f\, r_{FTL}\, d = n\, r_{TL} d \quad \left(= w\,d\,,\, w = \text{monetary weight in \$/mi}\right)$$

$$= 3.2727\,(2.5511)\,532 = \$4,441.73\text{/yr}$$

- What would be the cost if the shipments were to be made at least every three months?

$$t_{\max} = \frac{3}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 4 \text{ TL/yr} \quad \Rightarrow \quad q = \frac{f}{\max\left\{n, n_{\min}\right\}}$$

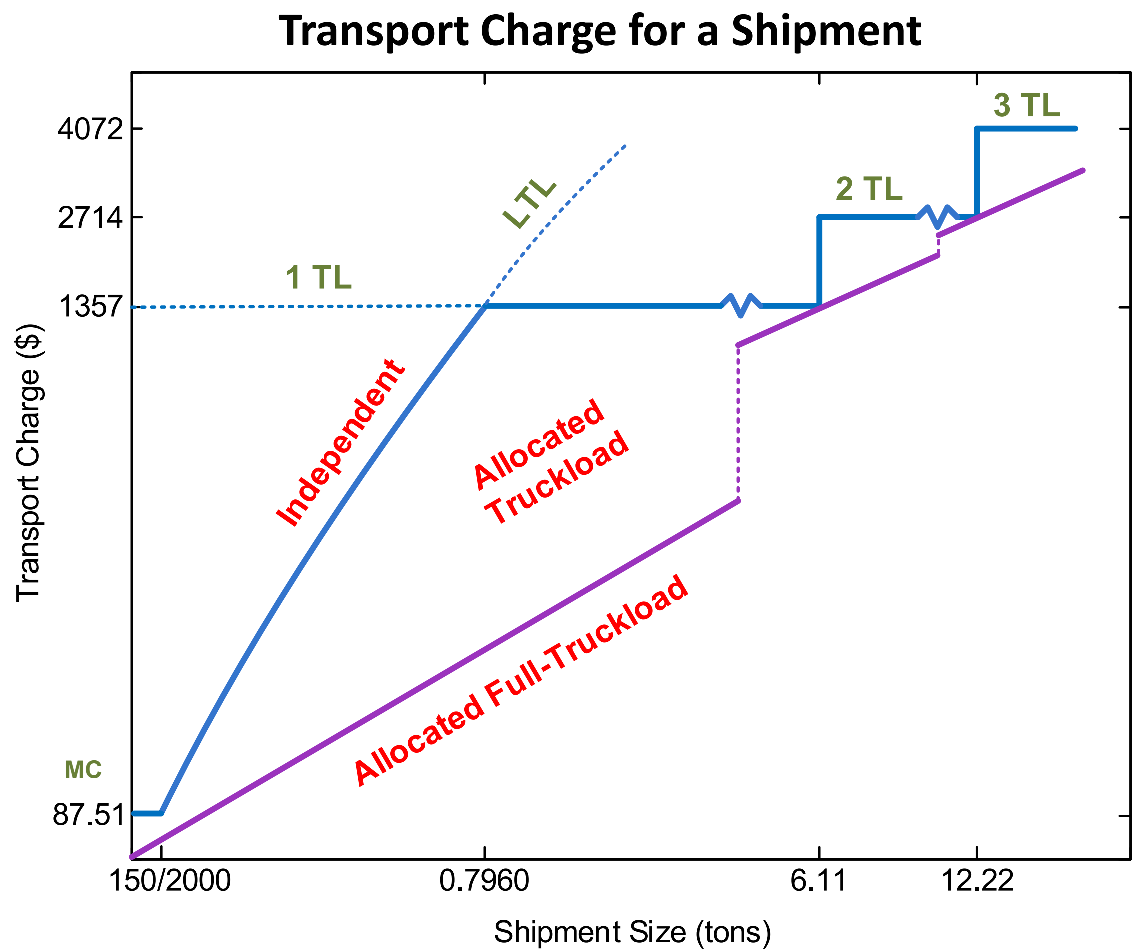
$$TC'_{FTL} = \max\left\{n, n_{\min}\right\} r_{TL} d$$

$$= \max\left\{3.2727, 4\right\} 2.5511(532) = \$5,428.78\text{/yr}$$

Truck Shipment Example: Periodic

- Independent and allocated full-truckload charges:

$$q \le q_{\max} \Rightarrow [UB, LB] = [c_0(q), q r_{FTL} d]$$



Truck Shipment Example: Periodic

- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$
\begin{aligned}
TLC &= TC + IC + PC \\
TC &= \text{transport cost} \\
IC &= \text{inventory cost} \\
&= IC_{\text{cycle}} + IC_{\text{pipeline}} + IC_{\text{safety}} \\
PC &= \text{purchase cost}
\end{aligned}
$$

- *Cycle inventory*: held to allow cheaper large shipments
- *Pipeline inventory*: goods in transit or awaiting transshipment
- *Safety stock*: held due to transport uncertainty
- *Purchase cost*: can be different for different suppliers

Truck Shipment Example: Periodic

- Same units of inventory can serve multiple roles at each position in a production process

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)
- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)
- *Safety stock:* held to buffer effects of uncertainty
  - (decoupling, MRO (maintenance, repair, and operations))

Truck Shipment Example: Periodic

14. Since demand is constant throughout the year, one half of a shipment is stored at the destination, on average. Assuming that the production rate is also constant, one half of a shipment will also be stored at the origin, on average. Assuming each ton of the product is valued at $25,000, what is a "reasonable estimate" for the total annual cost for this cycle inventory?

$$IC_{\text{cycle}} = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$

$$= (vh)(\alpha q)$$

$$v = \text{unit value of shipment (\$/ton)}$$

$$h = \text{inventory carrying rate, the cost per dollar of inventory per year (1/yr)}$$

$$\alpha = \text{average inter-shipment inventory fraction at Origin and Destination}$$

$$q = \text{shipment size (ton)}$$

Truck Shipment Example: Periodic

- **Inv. Carrying Rate** (h) = interest + warehousing + obsolescence
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate (*yr*) h = 0.3 $\Rightarrow h_{\text{obs}} \approx$ 0.2 (mfg product)
  - *Low* FGI cost (*yr*): $h = h_{\text{int}} + h_{\text{wh}} + h_{\text{obs}}$
  - *High* FGI cost (*hr*): $h \approx h_{\text{obs}}$, can ignore interest & warehousing
    - $(h_{\text{int}}+h_{\text{wh}})/H$ = (0.05+0.06)/2000 = 0.000055 (H = oper. hr/yr)
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h_{obs}

$$h_{\text{obs}} t_h v = x_h v \Rightarrow h_{\text{obs}} t_h = x_h \Rightarrow \boxed{h_{\text{obs}} = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h_{\text{obs}}}$$

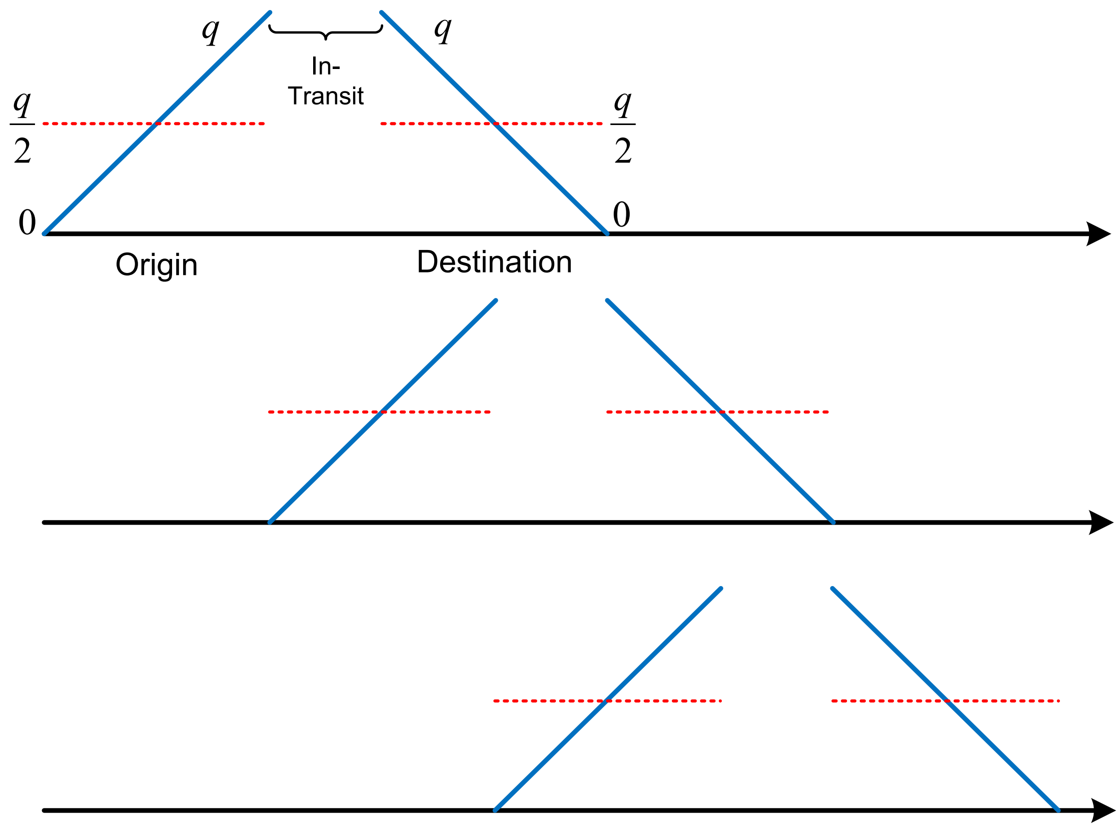
  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as production time, t_{CT}

Truck Shipment Example: Periodic

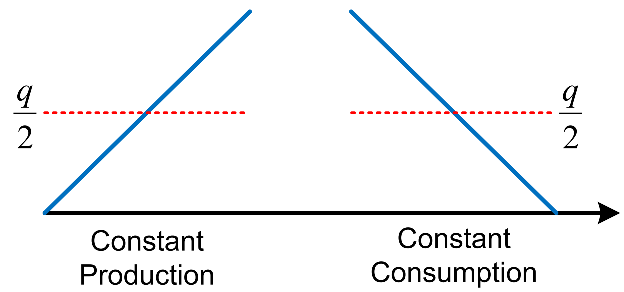
- Avg. annual cycle inventory level $= \frac{q}{2} + \frac{q}{2} = \left(\frac{1}{2} + \frac{1}{2}\right) q = (1) q \Rightarrow \alpha = 1$



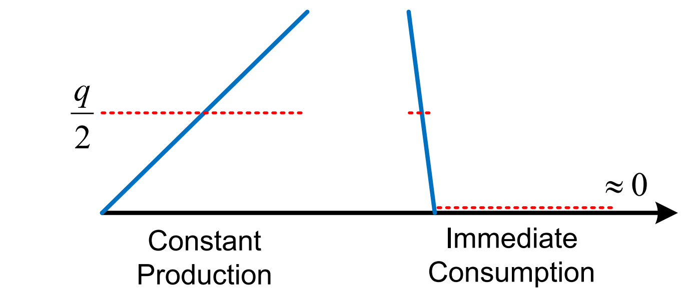
- Note: Cycle inventory is FGI at Origin and RMI at Destination

Truck Shipment Example: Periodic

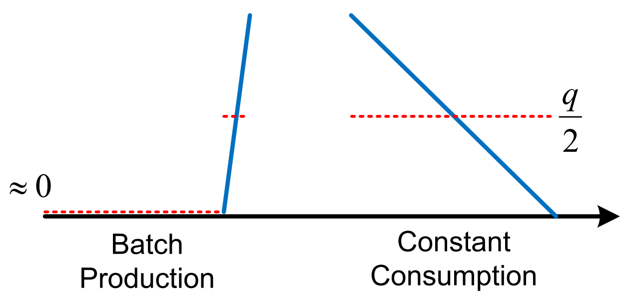
- Inter-shipment inventory fraction alternatives: $\alpha = \alpha_O + \alpha_D$



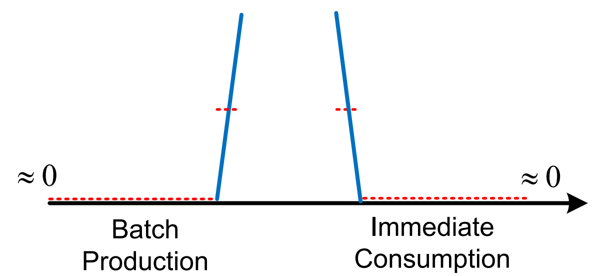
$$\alpha = \frac{1}{2} + \frac{1}{2} = 1$$



$$\alpha = \frac{1}{2} + 0 = \frac{1}{2}$$



$$\alpha = 0 + \frac{1}{2} = \frac{1}{2}$$



$$\alpha = 0 + 0 = 0$$

Truck Shipment Example: Periodic

- "Reasonable estimate" for the total annual cost for the cycle inventory:

$$
\begin{aligned}
IC_{\text{cycle}} &= \alpha v h q \\
&= (1)(25{,}000)(0.3)6.1111 \\
&= \$45{,}833.33\ /\ \text{yr}
\end{aligned}
$$

where

$$\alpha = \frac{1}{2} \text{at Origin} + \frac{1}{2} \text{at Destination} = 1$$

$$v = \$25{,}000 = \text{unit value of shipment (\$/ton)}$$

$$h = 0.3 = \text{estimated carrying rate for manufactured products (1/yr)}$$

$$q = q_{\text{max}} = 6.111 = \text{FTL shipment size (ton)}$$

Truck Shipment Example: Periodic

15. What is the annual total logistics cost (TLC) for these (necessarily P2P) full-truckload TL shipments?

$$
\begin{aligned}
TLC_{FTL} &= TC_{FTL} + IC_{\text{cycle}} \\
&= n\, r_{TL} d + \alpha v h q \\
&= 3.2727(2.5511)532 + (1)(25,000)(0.3)6.1111 \\
&= 4,441.73 + 45,833.33 \\
&= \$50,275.06 / yr
\end{aligned}
$$

- Problem: FTL may not minimize TLC
  - $\Rightarrow$ Can assume, for any periodic shipment, $q \le q_{\max}$
  - $\Rightarrow$ Assuming P2P TL, what to find q, q^*, that minimizes TLC

$$\Rightarrow \quad c_{TL}(q) = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d = r_{TL} d$$

Truck Shipment Example: Periodic

16. What is minimum possible annual total logistics cost for P2P TL shipments, where the shipment size can now be less than a full truckload?

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha vhq = \frac{f}{q} rd + \alpha vhq$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q_{TL}^* = \sqrt{\frac{f r_{TL} d}{\alpha vh}} = \sqrt{\frac{20(2.5511)532}{(1)25000(0.3)}} = 1.9024 \text{ ton}$$

$$TLC_{TL}(q_{TL}^*) = \frac{f}{q_{TL}^*} r_{TL} d + \alpha vh q_{TL}^*$$

$$= \frac{20}{1.8553}(2.5511)532 + (1)25000(0.3)1.8553$$

$$= 14,268.12 + 14,268.12$$

$$= \$28,536.25 / \text{yr}$$

Truck Shipment Example: Periodic

- Including the minimum charge and maximum payload restrictions:

$$q_{TL}^* = \min\left\{ \sqrt{\frac{f \max\left\{ r_{TL} d, MC_{TL} \right\}}{\alpha v h}}, q_{\max} \right\} \approx \sqrt{\frac{f r_{TL} d}{\alpha v h}}$$

- What is the TLC if this size shipment could be made as a (not-necessarily P2P) allocated full-truckload?

$$TLC_{AllocFTL}(q_{TL}^*) = \frac{f}{q_{TL}^*}\left(q_{TL}^* r_{FTL} d \right) + \alpha v h q_{TL}^* = f \frac{r_{TL}}{q_{\max}} d + \alpha v h q_{TL}^*$$

$$= 20 \frac{2.5511}{6.1111} 532 + (1) 25000 (0.3) 1.9024$$

$$= 4,441.73 + 14,268.12$$

$$= \$18,709.85 / \text{yr} \quad \left(\text{vs. } \$28,536.25 \text{ as independent P2P TL} \right)$$

Truck Shipment Example: Periodic

17. What is the optimal LTL shipment size?

$$TLC_{LTL}(q) = TC_{LTL}(q) + IC(q) = \frac{f}{q} c_{LTL}(q) + \alpha v h q$$

$$q^*_{LTL} = \arg\min_q TLC_{LTL}(q) = 0.7622 \text{ ton}$$

- Must be careful in picking starting point for optimization since LTL formula only valid for limited range of values:

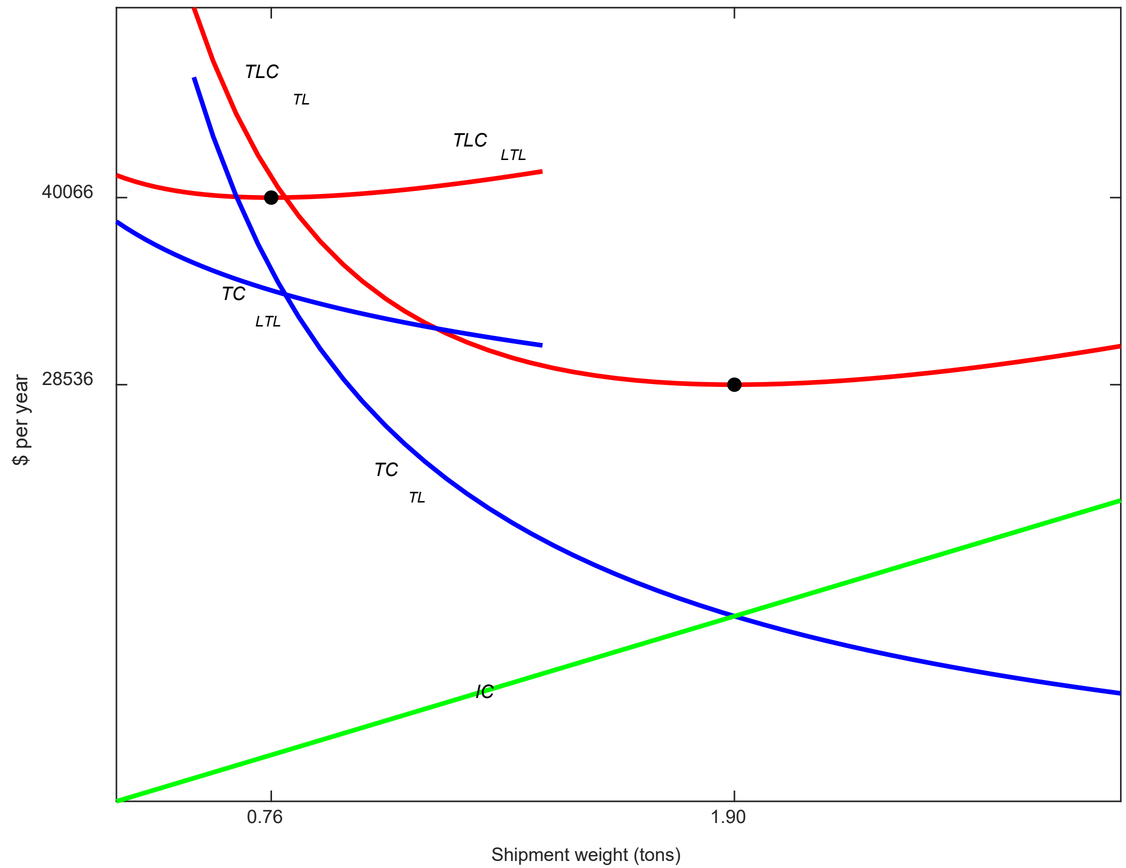
$$r_{LTL} = PPI_{LTL} \left[\frac{\frac{s^2}{8} + 14}{\left(q^{\frac{1}{7}} d^{\frac{15}{29}} - \frac{7}{2} \right)\left(s^2 + 2s + 14 \right)} \right], \quad \begin{cases} 37 \le d \le 3354 \text{ (dist)} \\ \frac{150}{2,000} \le q \le \frac{10,000}{2,000} \text{ (wt)} \\ 2000 \frac{q}{s} \le 650 \text{ ft}^3 \text{ (cube)} \end{cases}$$

$$\frac{150}{2000} \le q \le \min\left\{ \frac{10,000}{2,000}, \frac{650s}{2000} \right\} \Rightarrow 0.075 \le q \le 1.44$$

Truck Shipment Example: Periodic

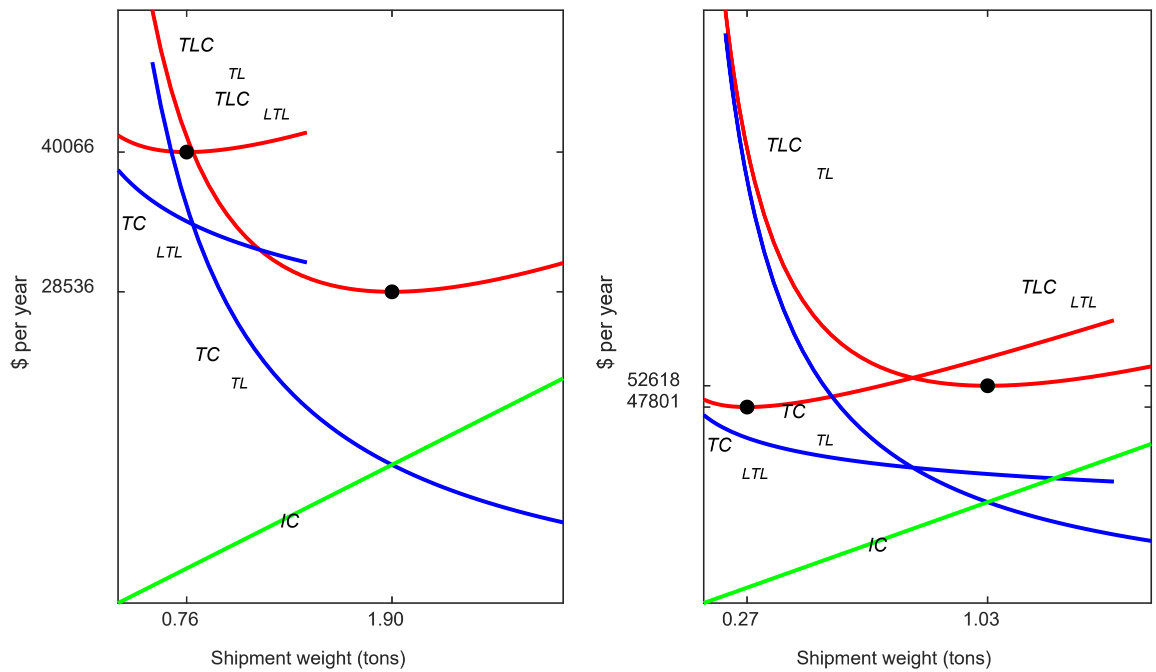
18. Should the product be shipped TL or LTL?

$$TLC_{LTL}(q^*_{LTL}) = TC_{LTL}(q^*_{LTL}) + IC(q^*_{LTL}) = 34,349.19 + 5,716.40 = \$40,065.59 / \text{yr}$$



Truck Shipment Example: Periodic

19. If the value of the product increased to $85,000 per ton, should the product be shipped TL or LTL?



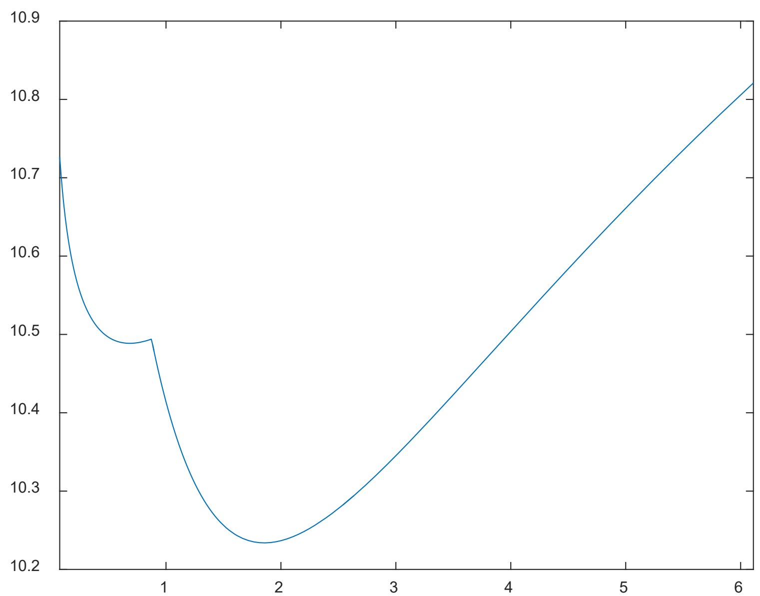
(a) $25000 value per ton

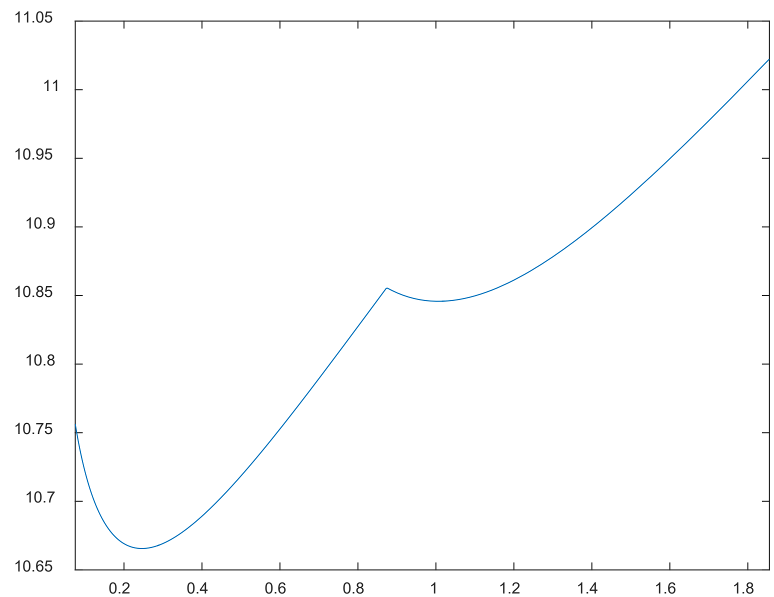
(b) $85000 value per ton

Truck Shipment Example: Periodic

- Better to pick from separate optimal TL and LTL because independent charge has two local minima:

$$q_0^* = \arg\min_q \{TLC_{TL}(q), TLC_{LTL}(q)\} \qquad q_0^* \overset{!}{=} \arg\min_q \left\{ \frac{f}{q} c_0(q) + \alpha v h q \right\}$$





Truck Shipment Example: Periodic

20. What is optimal independent shipment size to ship 80 tons per year of a Class 60 product valued at $5000 per ton, with the same inventory fraction and carrying rate, between Raleigh and Gainesville?

$$s = 32.16 \text{ lb/ft}^3$$

$$q_0^* = \arg\min_q \left\{ TLC_{TL}(q), TLC_{LTL}(q) \right\} = 8.5079 \text{ ton}$$

$$TLC_{TL}(q_0^*) = \$25,523.60 \text{ / yr} < TLC_{LTL}(q_0^*)$$

Truck Shipment Example: Periodic

21. What is the optimal shipment size if both shipments will always be shipped together on the same truck (with same shipment interval)?

$$d_1 = d_2, \quad h_{\text{agg}} = h_1 = h_2, \quad \alpha_{\text{agg}} = \alpha_1 = \alpha_2$$

$$f_{\text{agg}} = f_1 + f_2 = 20 + 80 = 100 \text{ ton}$$

$$s_{\text{agg}} = \frac{(\text{aggregate weight, in lb})}{(\text{aggregate cube, in ft}^3)} = \frac{f_{\text{agg}}}{\frac{f_1}{s_1} + \frac{f_2}{s_2}} = \frac{100}{\frac{20}{4.44} + \frac{80}{32.16}} = 14.31 \text{ lb/ft}^3$$

$$v_{\text{agg}} = \frac{f_1}{f_{\text{agg}}} v_1 + \frac{f_2}{f_{\text{agg}}} v_2 = \frac{20}{100} 85,000 + \frac{80}{100} 5000 = \$21,000 / \text{ton}$$

$$q_{TL}^* = \sqrt{\frac{f_{\text{agg}} r_{TL} d}{\alpha_{\text{agg}} v_{\text{agg}} h_{\text{agg}}}} = \sqrt{\frac{100(2.5511)532}{(1)21000(0.3)}} = 4.6414 \text{ ton}$$

Truck Shipment Example: Periodic

- Summary of results:

	f	s	v	qmax	TLC	q	t
1:	20	4.44	85,000	6.11	47,801.01	0.27	5.00
2:	80	32.16	5,000	25.00	25,523.60	8.51	38.84
1+2:					73,324.60		
Aggregate:	100	14.31	21,000	19.68	58,481.90	4.64	16.95

Ex 6: FTL vs Interval Constraint

- On average, 200 tons of components are shipped 750 miles from your fabrication plant to your assembly plant each year. The components are produced and consumed at a constant rate throughout the year. Currently, full truckloads of the material are shipped. What would be the impact on total annual logistics costs if TL shipments were made every two weeks? The revenue per loaded truck-mile is \$2.00; a truck's cubic and weight capacities are 3,000 ft^3 and 24 tons, respectively; each ton of the material is valued at \$5,000 and has a density of 10 lb per ft^3; the material loses 30% of its value after 18 months; and in-transit inventory costs can be ignored.

$$f = 200, \quad d = 750, \quad \alpha = \frac{1}{2} + \frac{1}{2} = 1, \quad r_{TL} = 2, \quad K_{cu} = 3000, \quad K_{wt} = 24, \quad v = 5000, \quad s = 10$$

$$h_{\text{obs}} = \frac{x_h}{t_h} = \frac{0.3}{1.5} = 0.2 \Rightarrow h = 0.05 + 0.06 + 0.2 = 0.31, \quad q_{FTL} = q_{\max} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\} = 15$$

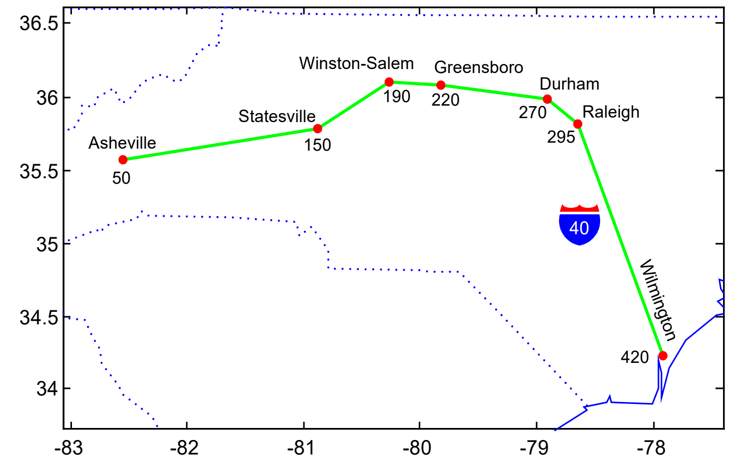
$$n_{FTL} = \frac{f}{q_{FTL}} = 13.33, TLC_{FTL} = n_{FTL} r_{TL} d + \alpha v h q_{FTL} = 43,250, \quad \text{2-wk TL} \Rightarrow \text{LTL not considered}$$

$$t_{\max} = \frac{2 \cdot 7}{365.25} \Rightarrow n_{\min} = 26.09, \quad q_{2\text{wk}} = \frac{f}{n_{\min}} = 7.67, \quad TLC_{2\text{wk}} = n_{\min} r_{TL} d + \alpha v h q_{2\text{wk}} = 51,016$$

$$\Delta TLC = TLC_{2\text{wk}} - TLC_{FTL} = \$7,766 \text{ per year increase with two-week interval constraint}$$

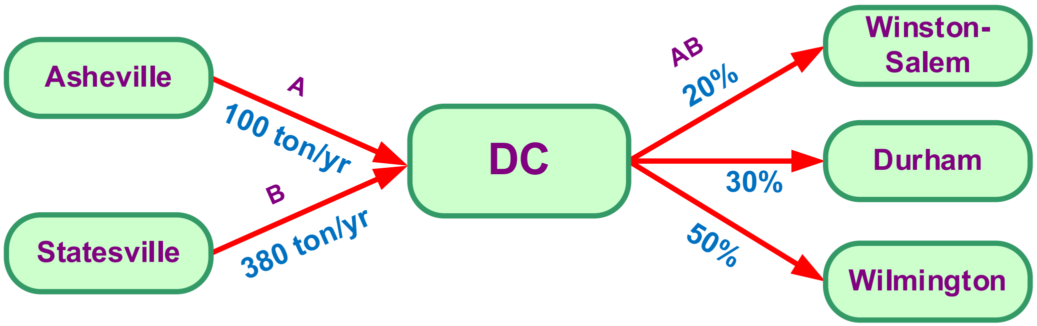
Ex 7: FTL Location

- Where should a DC be located in order to minimize transportation costs, given:
  1. FTLs containing mix of products A and B shipped P2P from DC to customers in Winston-Salem, Durham, and Wilmington
  2. Each customer receives 20, 30, and 50% of total demand



  3. 100 tons/yr of A shipped FTL P2P to DC from supplier in Asheville
  4. 380 tons/yr of B shipped FTL P2P to DC from Statesville
  5. Each carton of A weighs 30 lb, and occupies 10 ft^3
  6. Each carton of B weighs 120 lb, and occupies 4 ft^3
  7. Revenue per loaded truck-mile is $2
  8. Each truck's cubic and weight capacity is 2,750 ft^3 and 25 tons, respectively

Ex 7: FTL Location



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

$$\underset{(\$/\text{mi-yr})}{w_i} = \underset{(\text{ton/yr})}{f_i} \times \underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \underset{(\text{TL/yr})}{n_i} \times \underset{(\$/\text{TL-mi})}{r_i}$$

$$\underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \frac{r}{q_{\max}}, \quad n = \frac{f}{q_{\max}}$$

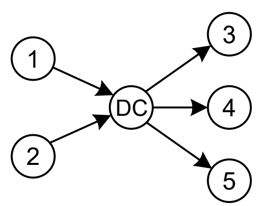
$$r = \$2 / \text{TL-mi}, \quad f_{\text{agg}} = f_A + f_B = 100 + 380 = 480 \text{ ton/yr}, \quad s_{\text{agg}} = \frac{f_{\text{agg}}}{\frac{f_A}{s_A} + \frac{f_B}{s_B}} = \frac{480}{\frac{100}{3} + \frac{380}{30}} = 10.4348 \text{ lb/ft}^3, \quad q_{\max} = \left\{25, \frac{10.4348(2750}{2000}\right\} = 14.3478$$

$$s_1 = \frac{30}{10} = 3 \text{ lb/ft}^3, \; q_{\max} = \min\left\{25, \frac{3(2750)}{2000}\right\} = 4.125 \text{ ton}$$

$$f_1 = 100, \; n_1 = \frac{100}{4.125} = 24.24, \; w_1 = 24.24(2) = 48.48$$

$$s_2 = \frac{120}{4} = 30 \text{ lb/ft}^3, \; q_{\max} = \min\left\{25, \frac{30(2750)}{2000}\right\} = 25 \text{ ton}$$

$$f_2 = 380, \; n_2 = \frac{380}{25} = 15.2, \; w_2 = 15.2(2) = 30.4$$

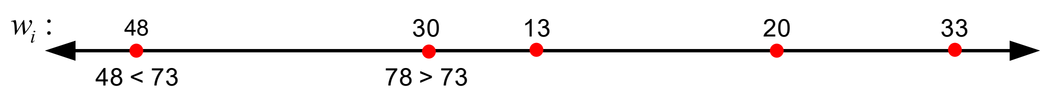


$$f_3 = 0.20 f_{\text{agg}} = 96, \; n_3 = \frac{96}{14.3478} = 6.69, \; w_3 = 6.69(2) = 13.38$$

$$f_4 = 0.30 f_{\text{agg}} = 144, \; n_4 = \frac{144}{14.3478} = 10.04, \; w_4 = 10.04(2) = 20.07$$

$$f_5 = 0.50 f_{\text{agg}} = 240, \; n_5 = \frac{240}{14.3478} = 16.73, \; w_5 = 16.73(2) = 33.45$$

$$W = 146, \quad \frac{W}{2} = 73$$



$$\text{(Montetary) Weight Losing:} \quad \Sigma w_{\text{in}} = 79 > \Sigma w_{\text{out}} = 67 \quad \left(\Sigma n_{\text{in}} = 39 > \Sigma n_{\text{out}} = 33\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$$

Ex 7: FTL Location

- Include monthly outbound frequency constraint:
  - Outbound shipments must occur at least once each month
  - Implicit means of including inventory costs in location decision

$$t_{\max} = \frac{1}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 12 \text{ TL/yr}$$

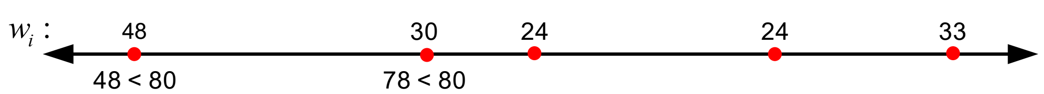
$$TC'_{FTL} = \max\left\{n, n_{\min}\right\} r d$$

$$n_3 = \max\left\{6.69, 12\right\} = 12, w_3 = 12(2) = 24$$

$$n_4 = \max\left\{10.04, 12\right\} = 12, w_4 = 12(2) = 24$$

$$n_5 = \max\left\{16.73, 12\right\} = 16.73, w_5 = 16.73(2) = 33.45$$

$$W = 160, \quad \frac{W}{2} = 80$$



(Montetary) Weight Gaining: $\Sigma w_{\text{in}} = 79 < \Sigma w_{\text{out}} = 81 \quad \left(\Sigma n_{\text{in}} = 39 < \Sigma n_{\text{out}} = 41\right)$

Physically Weight Unchanging (DC): $\Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$

Location and Transport Costs

- Monetary weights *w* used for location are, in general, a function of the location of a NF
  - Distance *d* appears in optimal TL size formula
  - TC & IC functions of location $\Rightarrow$ Need to minimize TLC instead of TC
  - FTL (since size is fixed at max payload) results in only constant weights for location $\Rightarrow$ Need to only minimize TC since IC is constant in TLC

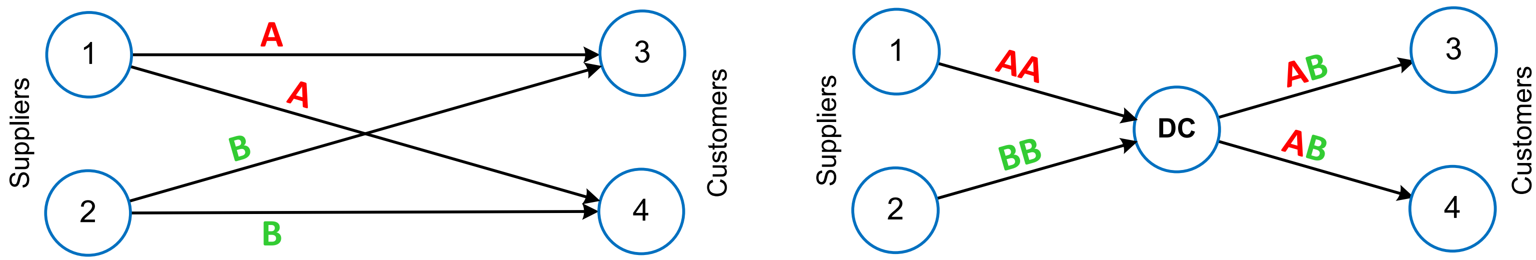
$$TLC_{TL}(\mathbf{x}) = \sum_{i=1}^{m} w_i(\mathbf{x}) d_i(\mathbf{x}) + \alpha v h q_i(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_i(\mathbf{x})} r d_i(\mathbf{x}) + \alpha v h\, q_i(\mathbf{x})$$

$$= \sum_{i=1}^{m} \frac{f_i}{\sqrt{\dfrac{f_i r d_i(\mathbf{x})}{\alpha v h}}} r d_i(\mathbf{x}) + \alpha v h \sqrt{\frac{f_i r d_i(\mathbf{x})}{\alpha v h}} = \sum_{i=1}^{m} \sqrt{f_i r d_i(\mathbf{x})} \left(\frac{1}{\sqrt{\alpha v h}} + \sqrt{\alpha v h} \right)$$

$$TLC_{FTL}(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_{\max}} r d_i(\mathbf{x}) + \alpha v h\, q_{\max} = \sum_{i=1}^{m} w_i\, d_i(\mathbf{x}) + \alpha v h\, q_{\max} = TC_{FTL}(\mathbf{x}) + \text{constant}$$

Transshipment

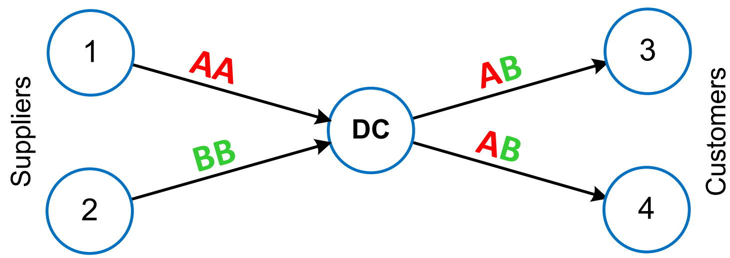
- *Direct*: P2P shipments from Suppliers to Customers



- *Transshipment*: use DC to consolidate outbound shipments
  - *Uncoordinated*: determine separately each optimal inbound and outbound shipment $\Rightarrow$ hold inventory at DC
  - *(Perfect) Cross-dock*: use single shipment interval for all inbound and outbound shipments $\Rightarrow$ no inventory at DC (usually only cross-dock a selected subset of shipments)

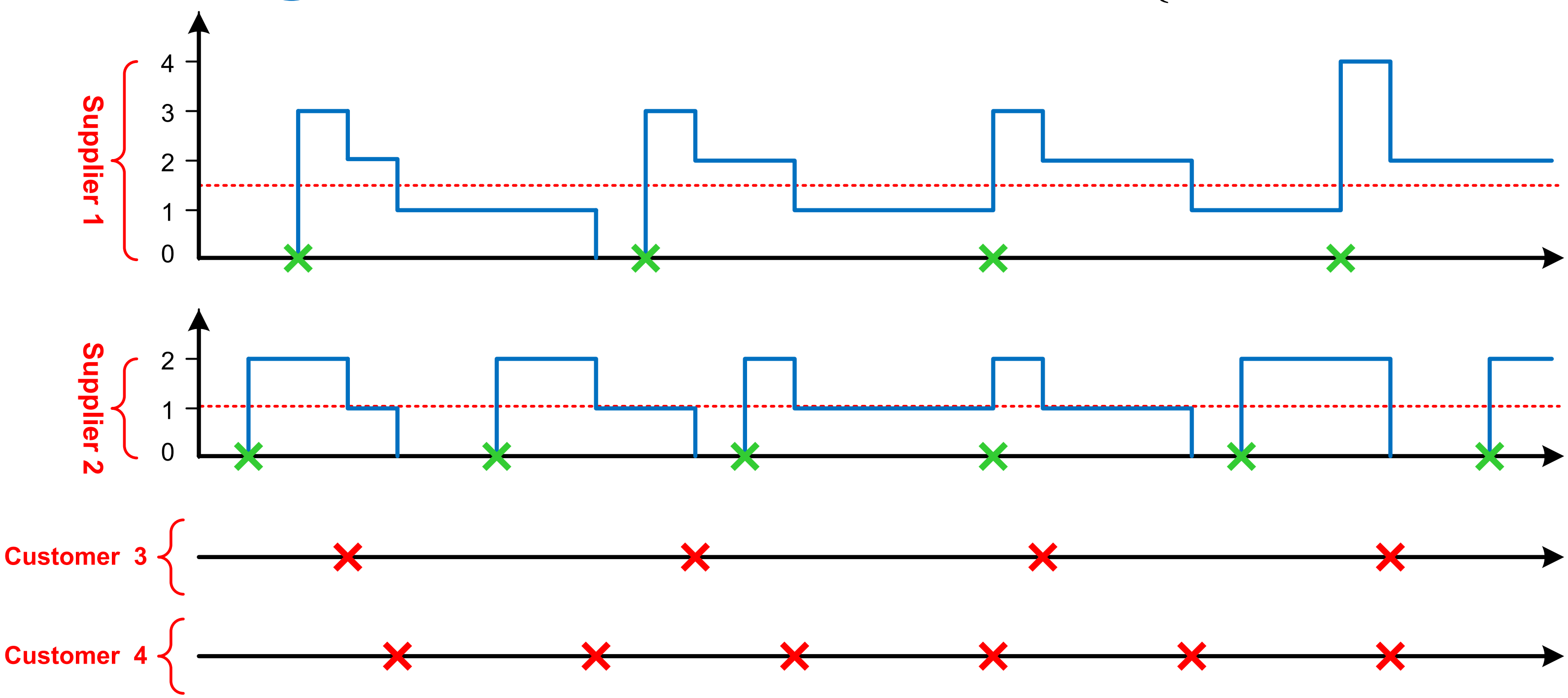
Uncoordinated Inventory

- Average pipeline inventory level at DC:



$$\alpha = \alpha_O + \alpha_D$$

$$= \begin{cases} \alpha_O + \frac{1}{2}, & \text{inbound} \\ 0 + \alpha_D, & \text{outbound} \end{cases}$$



$1.55 \approx \frac{q}{2}$

$1.11 \approx \frac{q}{2}$

TLC with Transshipment

- Uncoordinated: $TLC_i = TLC$ of supplier/customer i

$$q_i^* = \arg\min_q TLC_i(q)$$

$$TLC^* = \sum TLC_i\left(q_i^*\right)$$

- Cross-docking:

$$t = \frac{q}{f}, \quad \text{shipment interval}$$

$$TLC_i(t) = \frac{c_0(t)}{t} + \alpha v h f t \quad \left(\text{cf. } TLC_i(q) = \frac{f}{q} c_0(q) + \alpha v h q\right)$$

$$c_0(t) = \text{independent transport charge as function of } t$$

$$\alpha = \begin{cases} \alpha_O + 0, & \text{inbound} \\ 0 + \alpha_D, & \text{outbound} \end{cases}$$

$$t^* = \arg\min_t \sum TLC_i(t)$$

$$TLC^* = \sum TLC_i\left(t^*\right)$$

Economic Analysis

- Two aspects of economic analysis are important in production system design:
  1. *Costing*: determine the unit cost of a production activity (e.g., $2 per mile for TL shipments (actually $1.60/mi))
     - Termed "should-cost" analysis when used to guide procurement negotiations with suppliers
  2. *Project justification*: formal means of evaluating alternate projects that involve significant capital expenditures

Costing

- Capital recovery cost used to make *one-time* investment costs and salvage values commensurate with *per-period* operating costs via discounting

$$\textit{Effective cost}: \quad IV^{\text{eff}} = IV - SV(1+i)^{-N}$$

$$\textit{Capital recovery cost}: \quad K = IV^{\text{eff}}\left[\frac{i}{1-(1+i)^{-N}}\right] = (IV - SV)\left[\frac{i}{1-(1+i)^{-N}}\right] + SV \cdot i$$

$$\textit{Average Cost}: \quad AC = \frac{K + OC}{q} \neq \frac{IV^{\text{eff}} + PV \text{ of } OC}{Nq}$$

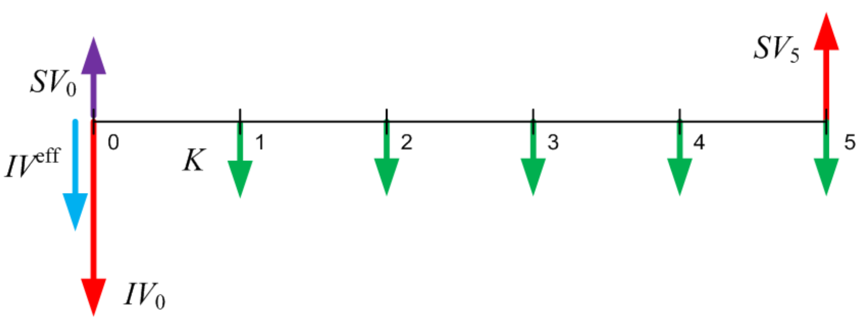
where

IV = initial one-time investment cost

SV = one-time salvage value at time N

OC = operating cost per period

q = units per period



Project Justification

- If cash flows are uniform, can use simple formulas; otherwise, need to use spreadsheet to discount each period's cash flows
- In practice, the payback period is used to evaluate most small projects:

$$\boxed{\textit{Payback period} = \frac{IV_0}{OP}, \quad \text{for } OP > 0}$$

where

$$IV_0 = IV_{\text{new}} - SV_{\text{current}}, \quad \text{net intital investment expenditure at time 0 for project}$$

$$IV_{\text{new}} = \text{initial investment cost at time 0 for (new) project}$$

$$SV_{\text{current}} = \text{salvage value of current project (if any) at time 0}$$

$$OP = \begin{cases} OR - OC, & \text{uniform operating profit per period from project} \\ OC_{\text{current}} - OC_{\text{new}}, & \text{net uniform operating cost } \textit{savings} \text{ per period} \end{cases}$$

$$OR = \text{ uniform operating revenue per period from project}$$

$$OC = \text{ uniform operating cost per period of project}$$

Discounting

- NPV and NAV equivalent methods for evaluating projects
- Project accepted if NPV ≥ 0 or NAV ≥ 0

$$\textit{Weighted Average Cost of Capital: } i = (\%\ \text{debt})\, i_{\text{debt}} + (\%\ \text{equity})\, i_{\text{equity}}$$

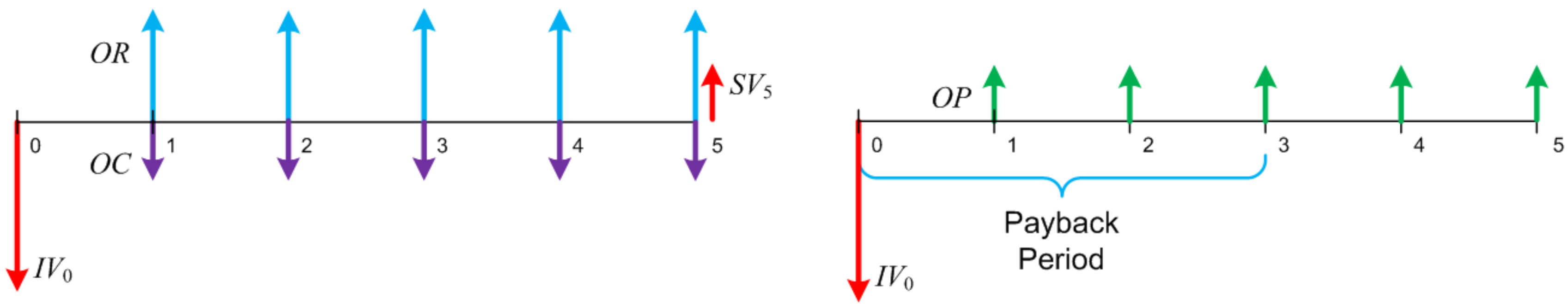
$$= (0.5)\, 0.06 + (0.5)\, 0.30 = 0.18$$

$$NPV = PV \text{ of } OP - IV^{\text{eff}}$$

$$\textit{Net Present Value: } \quad = OP\left[\frac{1-(1+i)^{-N}}{i}\right] - IV^{\text{eff}}, \quad i \neq 0$$

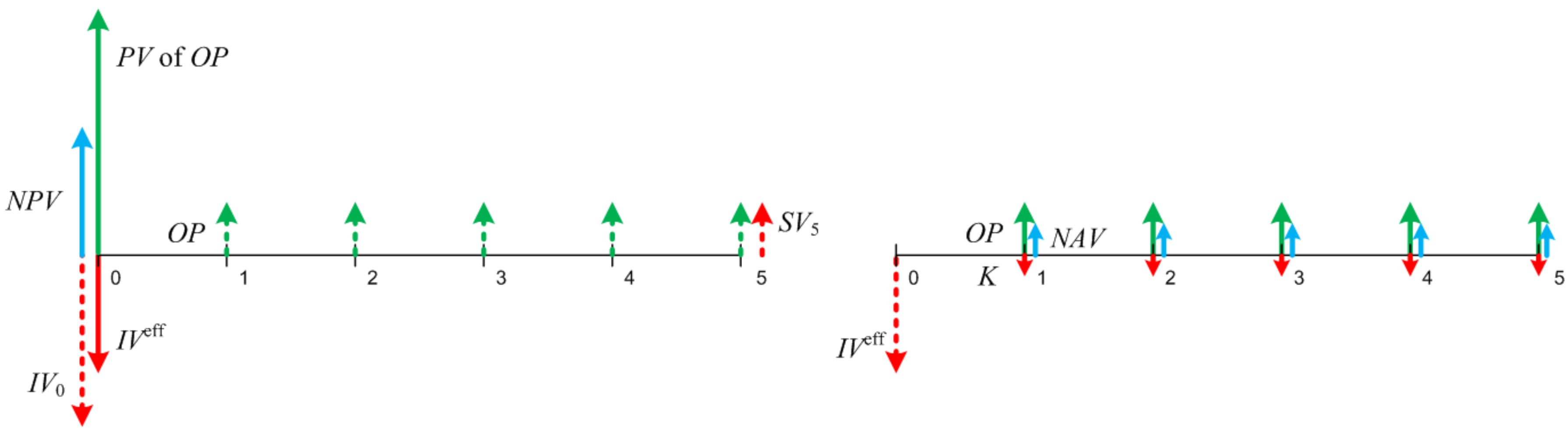
$$\textit{Net Annual (Periodic) Value: } \quad NAV = OP - K$$

Project with Uniform Cash Flows



(a) Actual cash flows.

(b) Payback method.



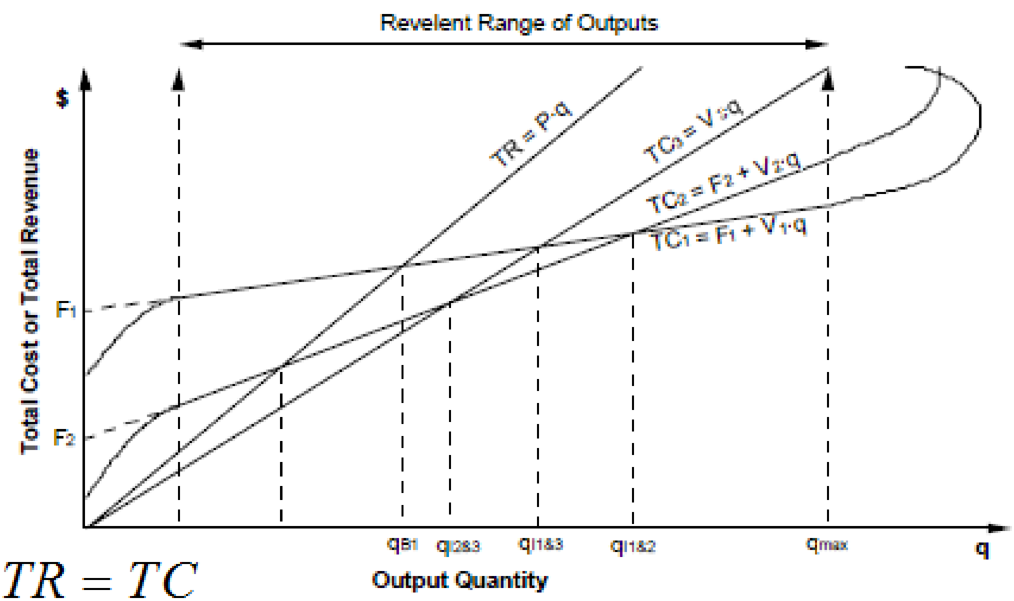
(c) Net present value (NPV).

(d) Net annual value (NAV).

Cost Reduction Example

Common				
Cost of Capital	(i)	8%	8%	
Economic Life	(N, yr)	15	15	
Annual Demand	(q/yr)	500,000	500,000	
Sale Price	($/q)			
Project		**Current**	**New**	**Net**
Investment Cost	(IV, $)	2,000,000	5,000,000	3,000,000
Salvage Percentage		25%	25%	
Salvage Value	(SV, $)	500,000	1,250,000	750,000
Eff. Investment Cost	(IV^{eff}, $)	1,842,379	4,605,948	2,763,569
Cost Cap Recovery	(K, $/yr)	215,244	538,111	322,866
Oper Cost per Unit	($/q)	1.25	0.50	(0.75)
Operating Cost	(OC, $/yr)	625,000	250,000	(375,000)
Operating Revenue	(OR, $/yr)	0	0	0
Operating Profit (OR - OC)	(OP, $/yr)	(625,000)	(250,000)	375,000
Analysis				
Payback Period (IV/OP)	(yr)			8.00
PV of OP	($)	(5,349,674)	(2,139,870)	3,209,805
NPV (PV of OP - IV^{eff})	($)	(7,192,053)	(6,745,818)	446,236
NAV (OP - K)	($/yr)	(840,244)	(788,111)	52,134
Average Cost ((K + OC)/q)	($/$q$)	1.68	1.58	

(Linear) Break-Even and Cost Indifference Pts.



$$TR = TC$$

$$P \cdot q = F + V \cdot q$$

$$\left(P - V\right)q = F$$

$$\textit{Break-Even Point:} \quad q_B = \frac{F}{P - V}$$

$$TC_1 = TC_2$$

$$F_1 + V_1 \cdot q = F_2 + V_2 \cdot q$$

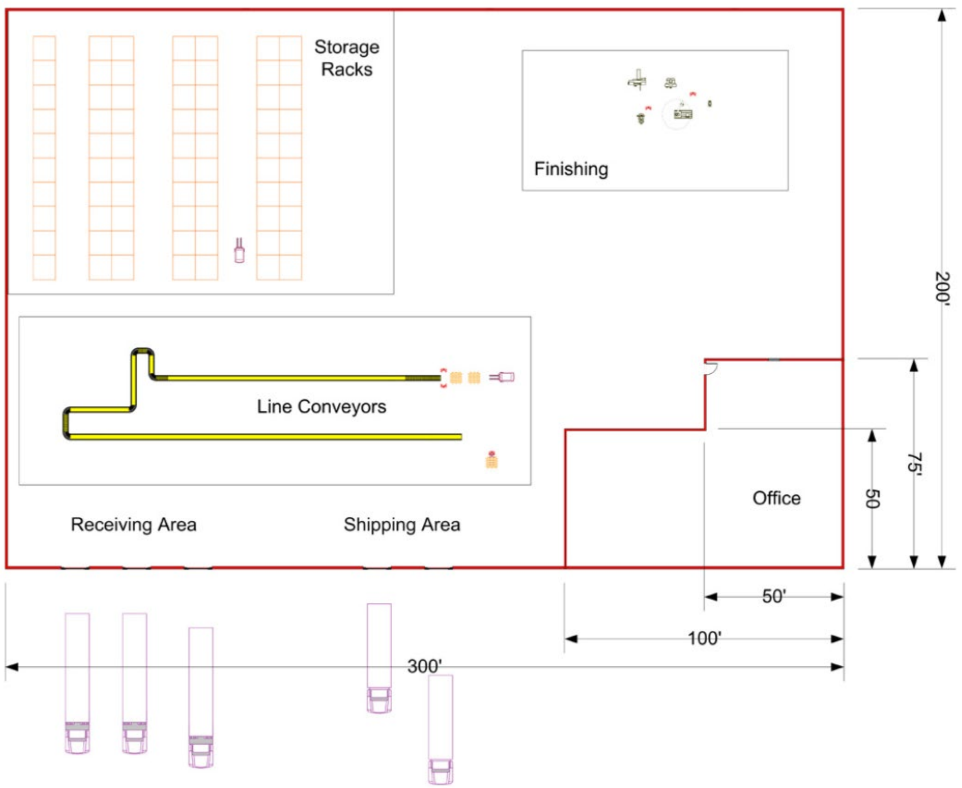
$$F_1 - F_2 = \left(V_2 - V_1\right)q$$

$$\textit{Cost Indifference Point:} \quad q_{I1\&2} = \frac{F_1 - F_2}{V_2 - V_1}$$

If output q is in units produced, then $F = K$ and $V = \frac{OC}{q}$.

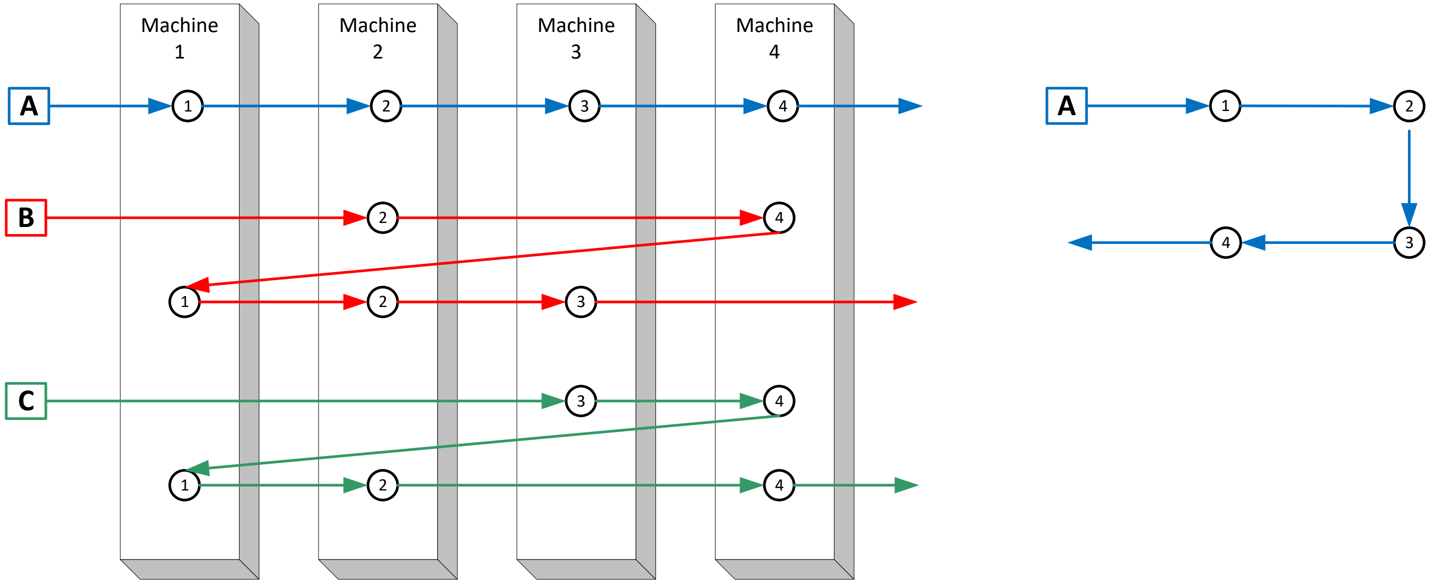
Facility Layout

- Two levels of layout problems:
  - *Machine*: determine assignment of machines to (fixed) sites
  - *Departmental*: determine space requirements of each department (or room) and its shape and relation of other departments



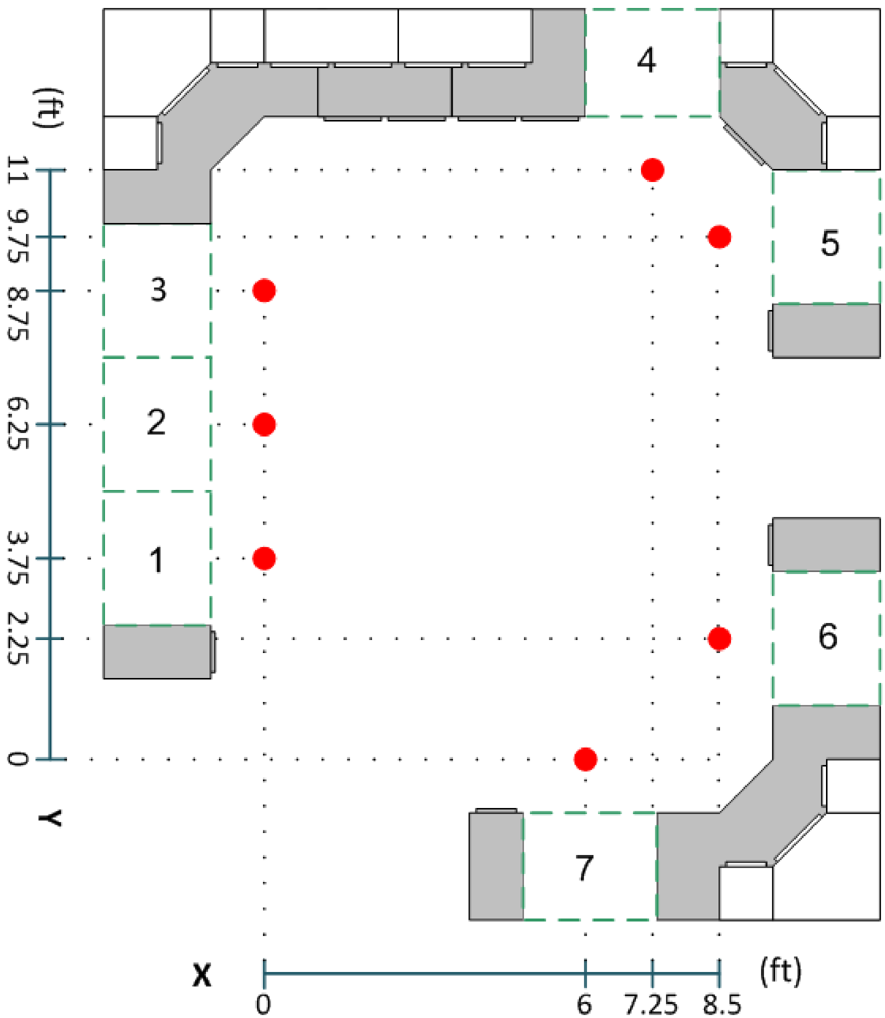
Machine Layout

- A *routing* is the sequence of W/S (or M/C) that work visits during its production
  - Dedicated M/C ⇒ single routing ⇒ single flow of material ⇒ layout only involves choice of straight-line or U-shaped layout
  - Shared M/C ⇒ multiple routings ⇒ multiple flows of material ⇒ layout involves complex problem of finding assignment of M/C to Sites corresponding to the dominate flow

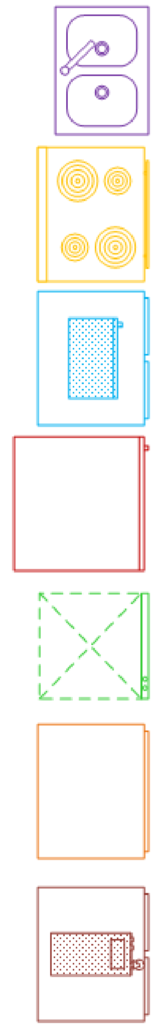


Example: Kitchen Layout

Site Locations



Appliances



1	T/K	Trash/Sink
2	S	Stove
3	M	Microwave
4	R	Refrigerator
5	W/D	Worksurface/ Dishwasher
6	P	Pantry
7	C	Cupboard/ Coffeemaker

Example: Kitchen Layout

Table 1. Site-to-Site Distances

Site	1	2	3	4	5	6	7
1	0.0	2.5	5.0	10.3	10.4	8.6	7.1
2	2.5	0.0	2.5	8.7	9.2	9.4	8.7
3	5.0	2.5	0.0	7.6	8.6	10.7	10.6
4	10.3	8.7	7.6	0.0	1.8	8.8	11.1
5	10.4	9.2	8.6	1.8	0.0	7.5	10.1
6	8.6	9.4	10.7	8.8	7.5	0.0	3.4
7	7.1	8.7	10.6	11.1	10.1	3.4	0.0

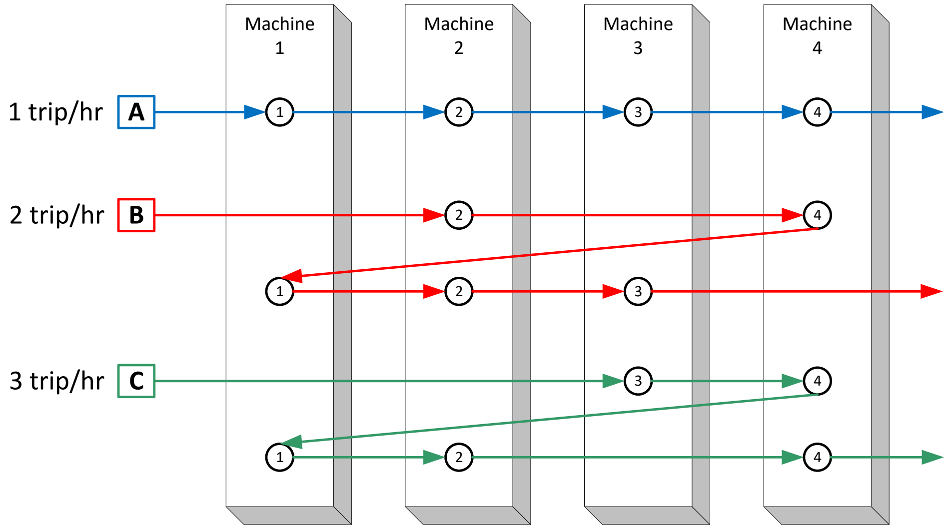
Table 2. Distance from Location (0,0) to Sites

Site	1	2	3	4	5	6	7
(0,0)	3.8	6.3	8.8	13.2	12.9	8.8	6.0

Table 3. Meals Prepared During Each Week

Meal	Freq.	Sequence
Snack	25	R–M (4–3)
Drink	10	C–R–W–T (7–4–5–1)
Breakfast	7	C–T–C–R–C–K (7–1–7–4–7–1)
Lunch	2	R–W–M–W–R–S–T (4–5–3–5–4–2–1)
Dinner	6	P–W–R–K-W-S-M-W-T (6–5–4–1–5–2–3–5–1)
Cleanup	8	K–D–K–R–K–D (1–5–1–4–1–5)

From/To Chart



From \ To	1	2	3	4
1	—	1+2+3		
2		—	1+2	2+3
3			—	1+3
4	2+3			—

From \ To	1	2	3	4
1	—	6		
2		—	3	5
3			—	4
4	5			—

Total Cost of Material Flow

Equivalent Flow Volume: $w_{ij} = \sum_{k=1}^{P} f_{ijk} h_{ijk}$ (machine-to-machine)

where f_{ijk} = moves between machines i and j for item k

h_{ijk} = equivalance factor for moves between machines i and j for item k

Total Cost of Material Flow: $TC_{MF} = \sum_{i=1}^{M} \sum_{j=1}^{M} w_{a_i a_j} d_{ij}$

where a_i = machine assigned to site i

d_{ij} = distance between sites i and j (site-to-site)

M = number of sites and machines

Equivalent Factors

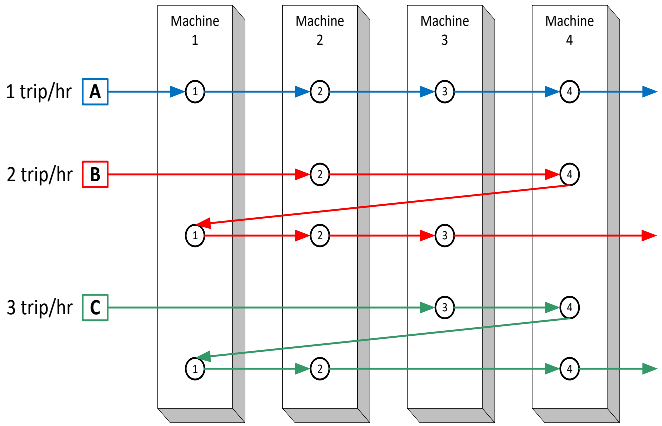
- Problem: Cost of move of item k from site i to j (h_{ijk}) usually depends on layout
  - equivalent factor used to represent likely “cost” differences due to, e.g., item volume

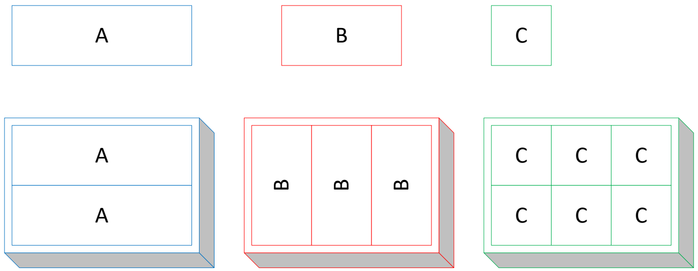
$$\text{All } h_{ijk} = 1 \Rightarrow \left[w_{ij} \right] = \begin{bmatrix} 0 & 6 & 0 & 0 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 4 \\ 5 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[f_{ijA} \right] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \left[f_{ijB} \right] = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \quad \left[f_{ijC} \right] = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[h_{ijA} \right] = 3 \quad \left[h_{ijB} \right] = 2 \quad \left[h_{ijC} \right] = 1$$

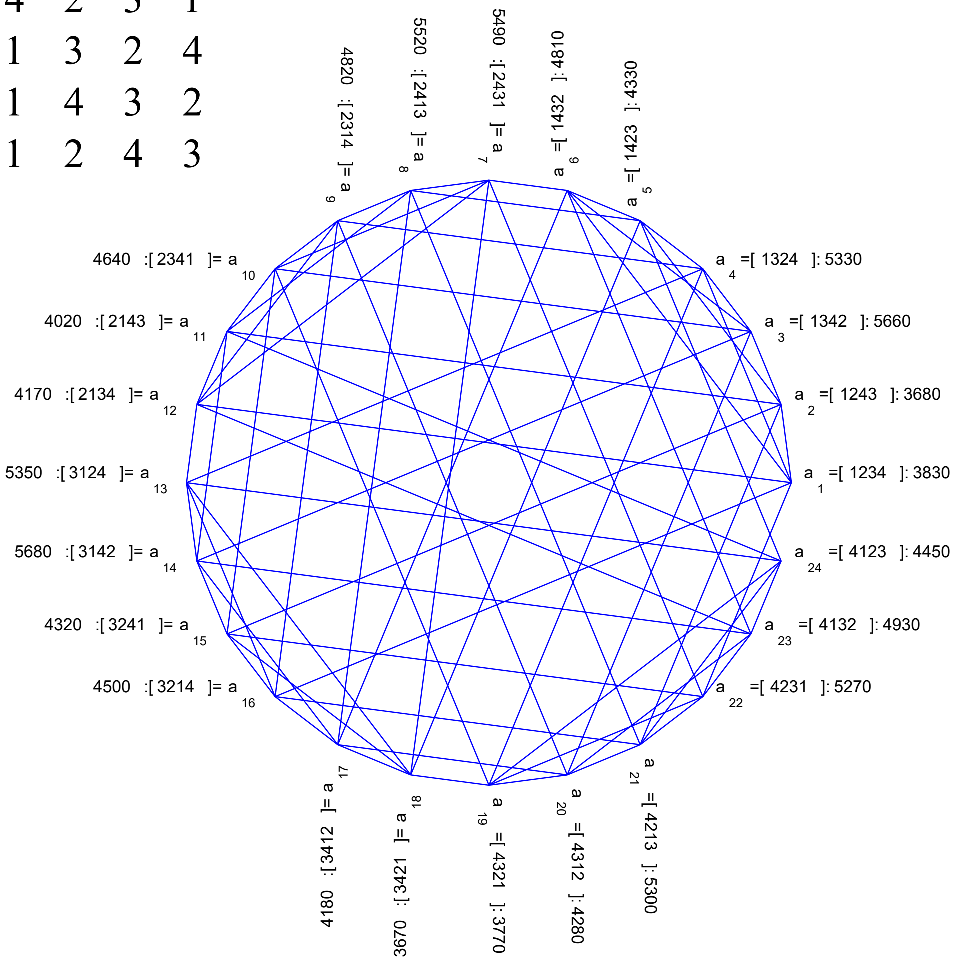
$$\left[w_{ij} \right] = \begin{bmatrix} 0 & 10 & 0 & 0 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 0 & 6 \\ 7 & 0 & 0 & 0 \end{bmatrix}$$





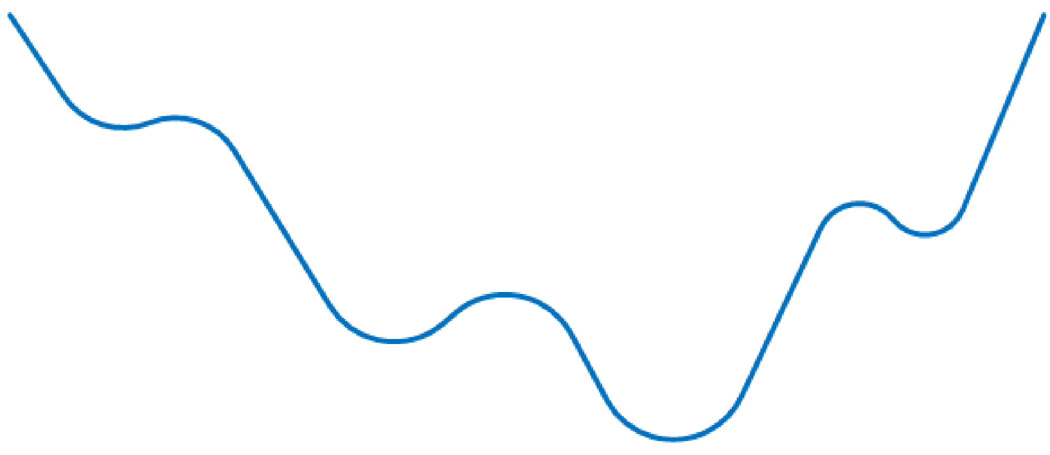
SDPI Heuristic

Interchange	1	2	3	4
1,2	2	1	3	4
1,3	3	2	1	4
1,4	4	2	3	1
2,3	1	3	2	4
2,4	1	4	3	2
3,4	1	2	4	3

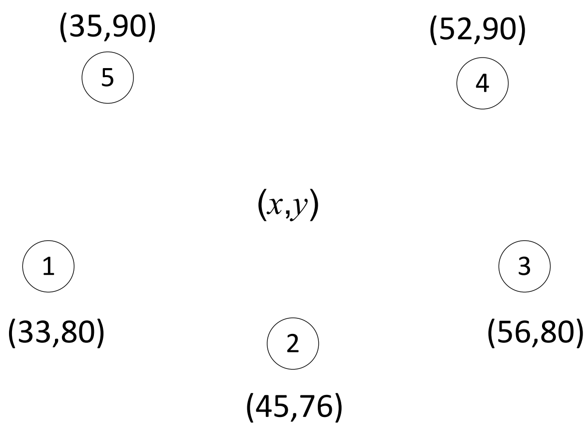


		1	2	3	4	*TC*
$\mathbf{a}_{14}$		3	1	4	2	5680
	$\mathbf{a}_{3}$	1	3	4	2	5660
	$\mathbf{a}_{23}$	4	1	3	2	4930
	$\mathbf{a}_{11}$	2	1	4	3	4020
	$\mathbf{a}_{17}$	3	4	1	2	4180
	$\mathbf{a}_{15}$	3	2	4	1	4320
	$\mathbf{a}_{13}$	3	1	2	4	5350
$\mathbf{a}_{11}$		2	1	4	3	4020
	$\mathbf{a}_{2}$	1	2	4	3	3680
	$\mathbf{a}_{24}$	4	1	2	3	4450
	$\mathbf{a}_{14}$	3	1	4	2	5680
	$\mathbf{a}_{8}$	2	4	1	3	5520
	$\mathbf{a}_{10}$	2	3	4	1	4640
	$\mathbf{a}_{12}$	2	1	3	4	4170
$\mathbf{a}_{2}$		1	2	4	3	3680
	$\mathbf{a}_{11}$	2	1	4	3	4020
	$\mathbf{a}_{21}$	4	2	1	3	5300
	$\mathbf{a}_{15}$	3	2	4	1	4320
	$\mathbf{a}_{5}$	1	4	2	3	4330
	$\mathbf{a}_{3}$	1	3	2	2	5660
	$\mathbf{a}_{1}$	1	2	3	4	3830

SDPI Heuristic

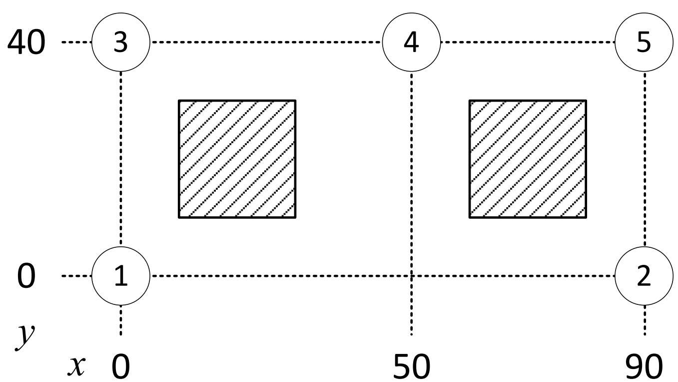


Layout Distances: Metric



(a) Open space.

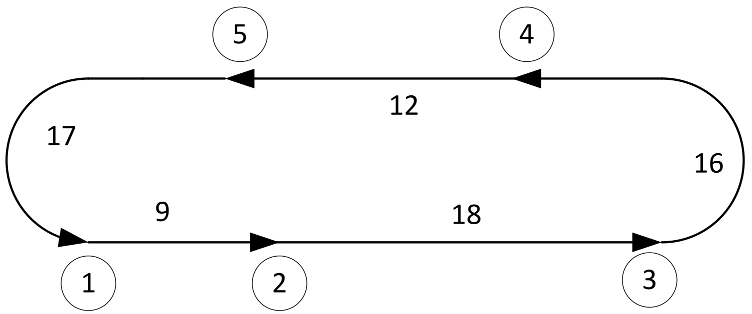
```
XY =
    33    80
    45    76
    56    80
    52    90
    35    90

D =
          0   12.6491   23.0000   21.4709   10.1980
    12.6491         0   11.7047   15.6525   17.2047
    23.0000   11.7047         0   10.7703   23.2594
    21.4709   15.6525   10.7703         0   17.0000
    10.1980   17.2047   23.2594   17.0000         0
```



(b) Rectangular grid.

```
XY =
     0     0
    90     0
     0    40
    50    40
    90    40
D =
     0    90    40    90   130
    90     0   130    80    40
    40   130     0    50    90
    90    80    50     0    40
   130    40    90    40     0
```

Layout Distances: Network



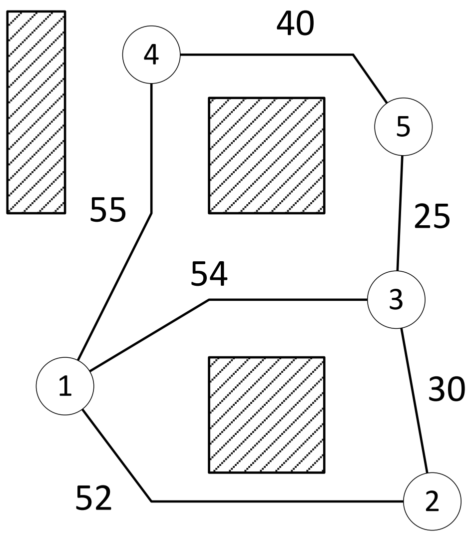
(c) Circulating conveyor.

IJD =

1	2	9
2	3	18
3	4	16
4	5	12
5	1	17

D =

0	9	27	43	55
63	0	18	34	46
45	54	0	16	28
29	38	56	0	12
17	26	44	60	0



(d) General network.

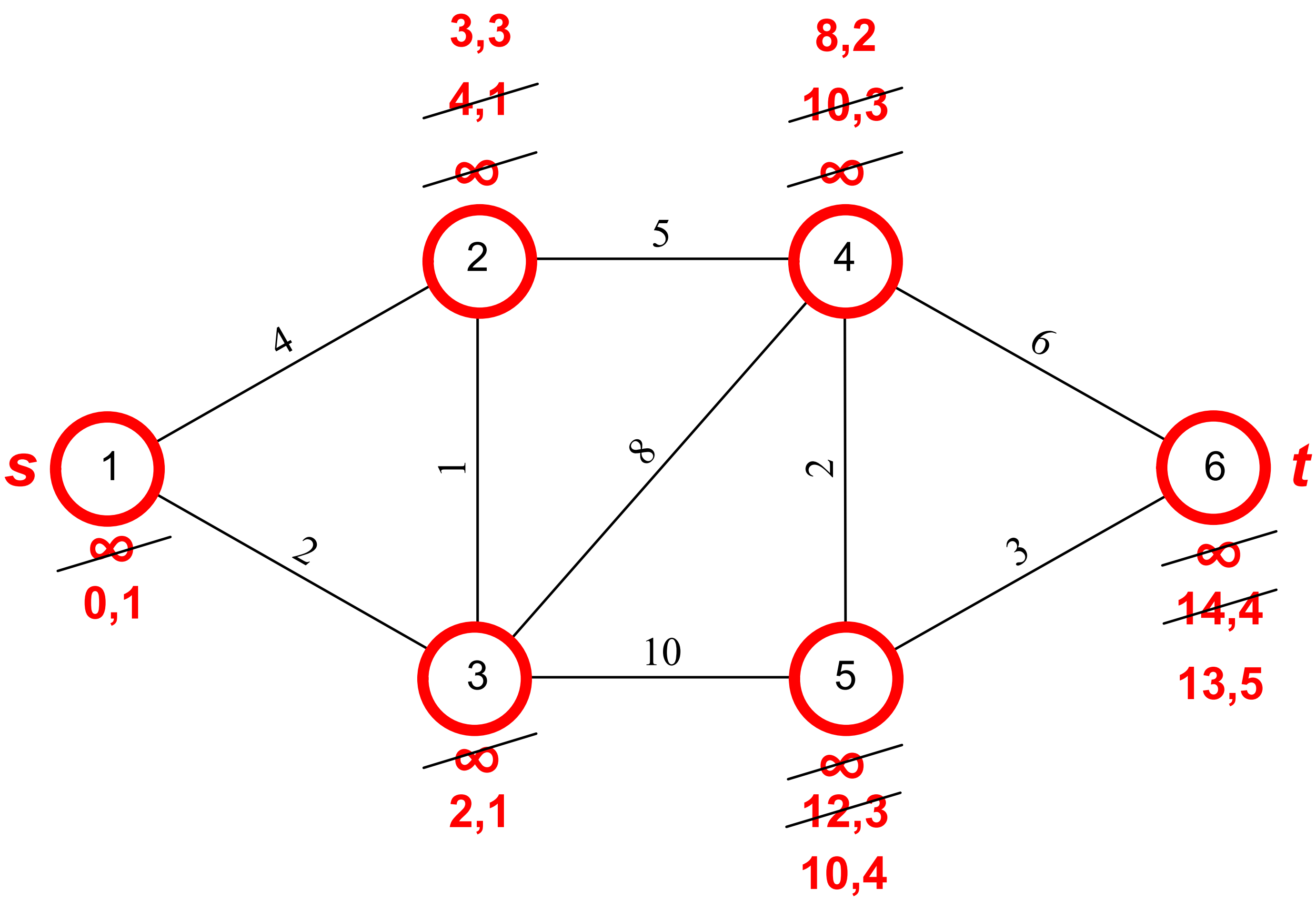
IJD =

1	-2	52
1	-3	54
1	-4	55
2	-3	30
3	-5	25
4	-5	40

D =

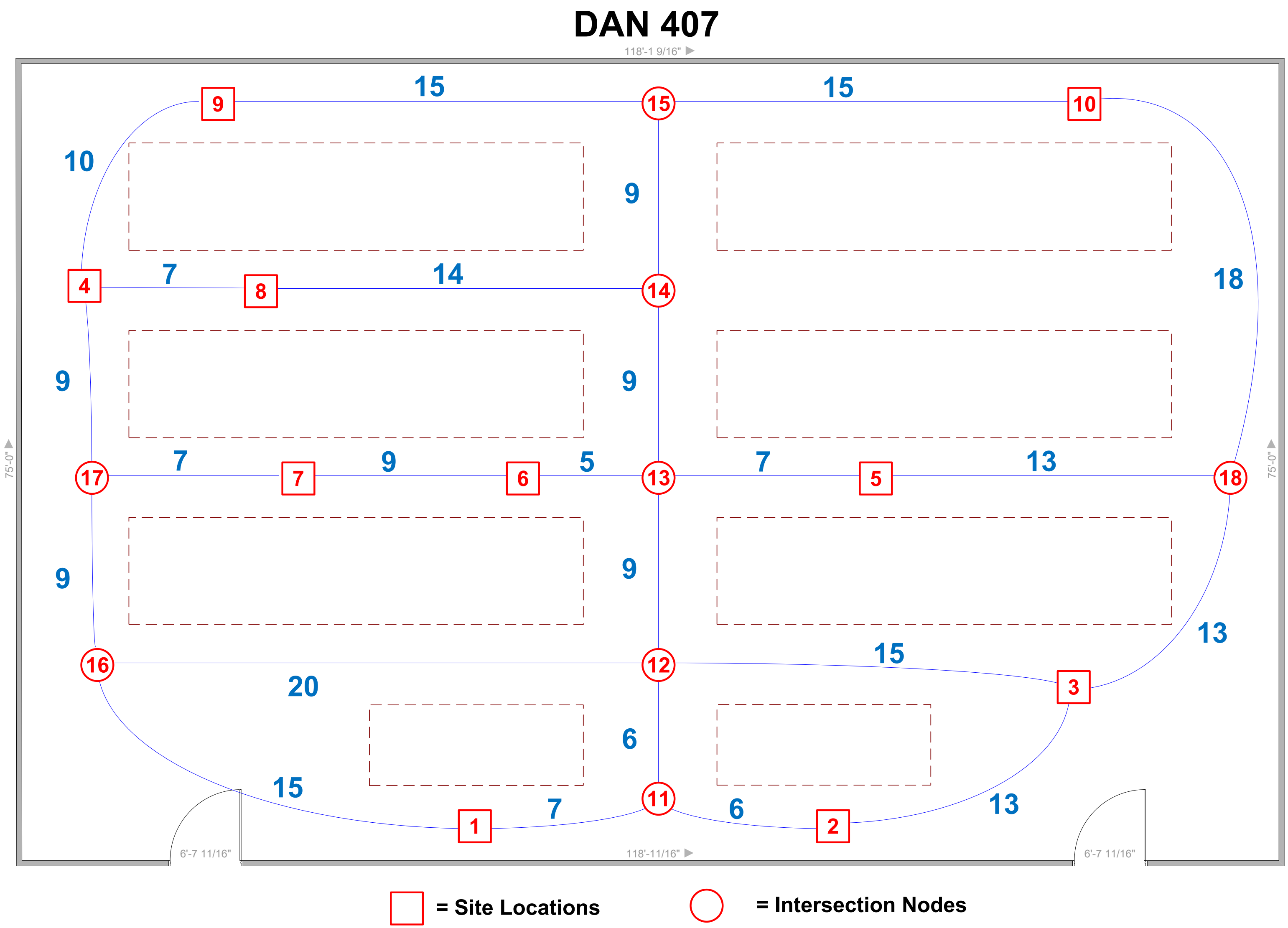
0	52	54	55	79
52	0	30	95	55
54	30	0	65	25
55	95	65	0	40
79	55	25	40	0

Dijkstra Shortest Path Procedure



Path: $1 \leftarrow 3 \leftarrow 2 \leftarrow 4 \leftarrow 5 \leftarrow 6:\quad 13$

General Network Distances



General Network Distances

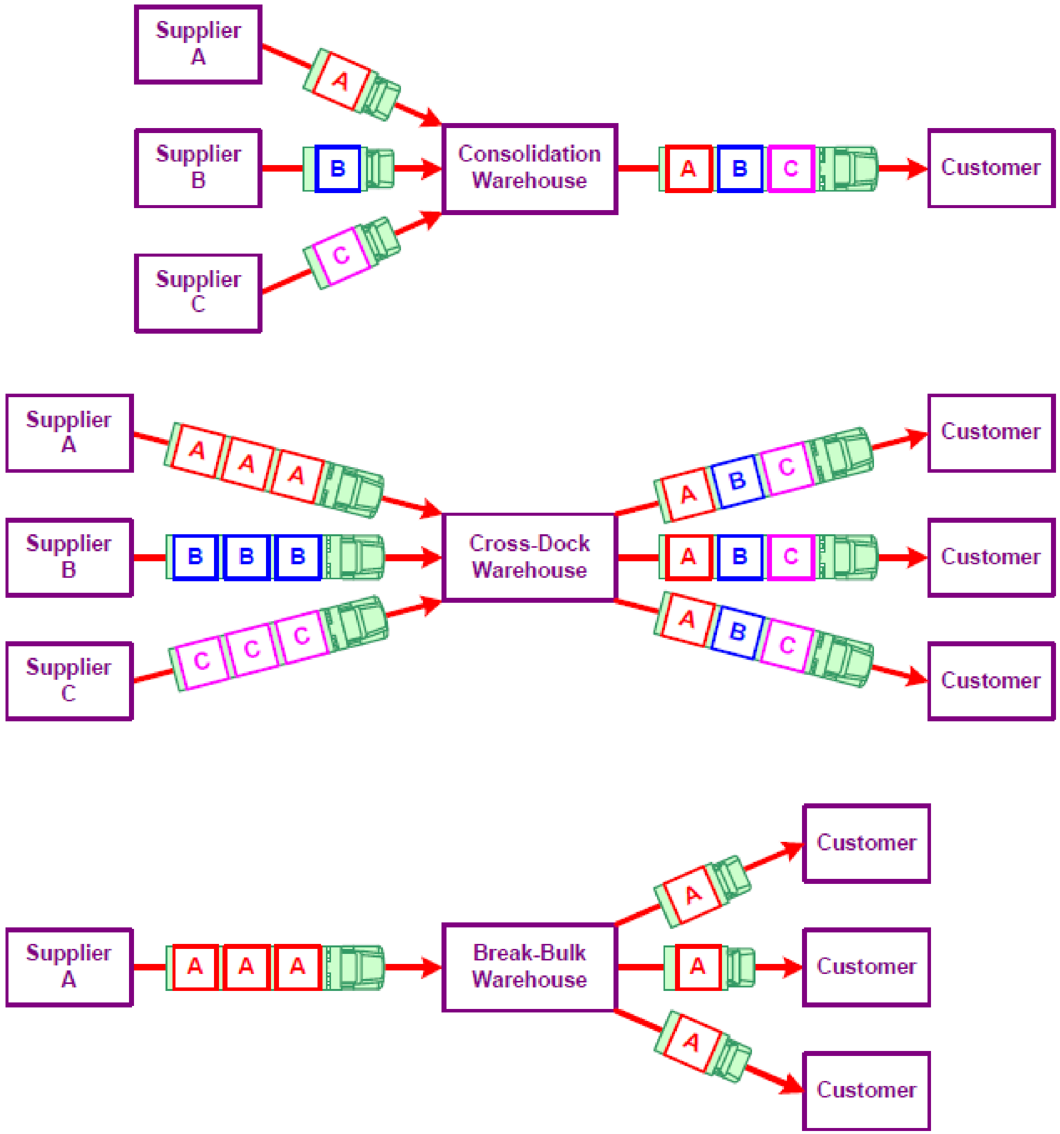
- Only need 10 × 10 distances between site locations, can throw away distances between intersection nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	13	26	33	29	27	31	40	43	55	7	13	22	31	40	15	24	39
2	13	0	13	46	28	26	35	44	54	44	6	12	21	30	39	28	37	26
3	26	13	0	53	26	29	38	47	57	31	19	15	24	33	42	35	44	13
4	33	46	53	0	37	25	16	7	10	40	40	38	30	21	25	18	9	50
5	29	28	26	37	0	12	21	30	40	31	22	16	7	16	25	36	28	13
6	27	26	29	25	12	0	9	28	35	38	20	14	5	14	23	25	16	25
7	31	35	38	16	21	9	0	23	26	47	29	23	14	23	32	16	7	34
8	40	44	47	7	30	28	23	0	17	38	38	32	23	14	23	25	16	43
9	43	54	57	10	40	35	26	17	0	30	48	42	33	24	15	28	19	48
10	55	44	31	40	31	38	47	38	30	0	48	42	33	24	15	58	49	18
11	7	6	19	40	22	20	29	38	48	48	0	6	15	24	33	22	31	32
12	13	12	15	38	16	14	23	32	42	42	6	0	9	18	27	20	29	28
13	22	21	24	30	7	5	14	23	33	33	15	9	0	9	18	29	21	20
14	31	30	33	21	16	14	23	14	24	24	24	18	9	0	9	38	30	29
15	40	39	42	25	25	23	32	23	15	15	33	27	18	9	0	43	34	33
16	15	28	35	18	36	25	16	25	28	58	22	20	29	38	43	0	9	48
17	24	37	44	9	28	16	7	16	19	49	31	29	21	30	34	9	0	41
18	39	26	13	50	13	25	34	43	48	18	32	28	20	29	33	48	41	0

Warehousing

- *Warehousing* are the activities involved in the design and operation of warehouses
- A *warehouse* is the point in the supply chain where raw materials, work-in-process (WIP), or finished goods are stored for varying lengths of time.
- Warehouses can be used to add value to a supply chain in two basic ways:
  1. Storage. Allows product to be available where and when its needed.
  2. Transport Economies. Allows product to be collected, sorted, and distributed efficiently.
- A *public warehouse* is a business that rents storage space to other firms on a month-to-month basis. They are often used by firms to supplement their own *private warehouses*.

Types of Warehouses



Warehouse Design Process

- The objectives for warehouse design can include:
  - maximizing cube utilization
  - minimizing total storage costs (including building, equipment, and labor costs)
  - achieving the required storage throughput
  - enabling efficient order picking
- In planning a storage layout: either a storage layout is required to fit into an existing facility, or the facility will be designed to accommodate the storage layout.

Warehouse Design Elements

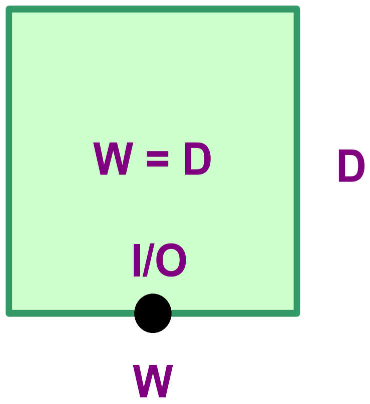
- The design of a new warehouse includes the following elements:
  1. Determining the layout of the storage locations (i.e., the warehouse layout).
  2. Determining the number and location of the input/output (I/O) ports (e.g., the shipping/receiving docks).
  3. Assigning items (stock-keeping units or *SKUs*) to storage locations (*slots*).
- A typical objective in warehouse design is to minimize the overall storage cost while providing the required levels of service.

Design Trade-Off

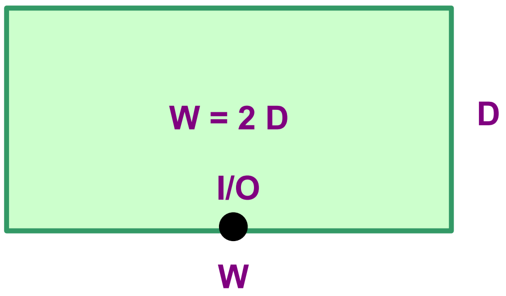
- Warehouse design involves the trade-off between building and handling costs:

min Building Costs	vs.	min Handling Costs
⇕		⇕
max Cube Utilization	vs.	max Material Accessibility

Shape Trade-Off



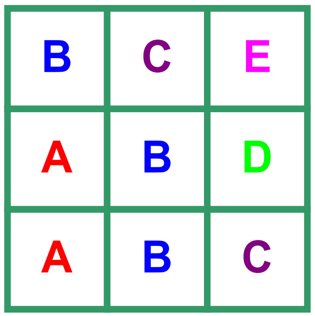
vs.



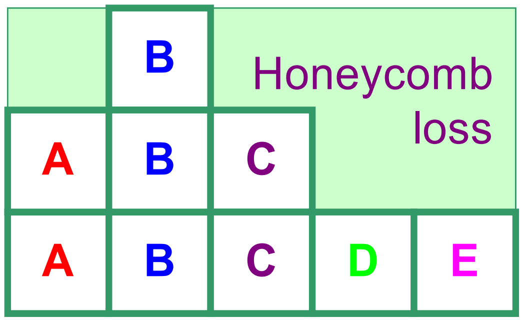
Square shape minimizes perimeter length for a given area, thus minimizing building costs

Aspect ratio of 2 (W = 2D) min. expected distance from I/O port to slots, thus minimizing handling costs

Storage Trade-Off



vs.



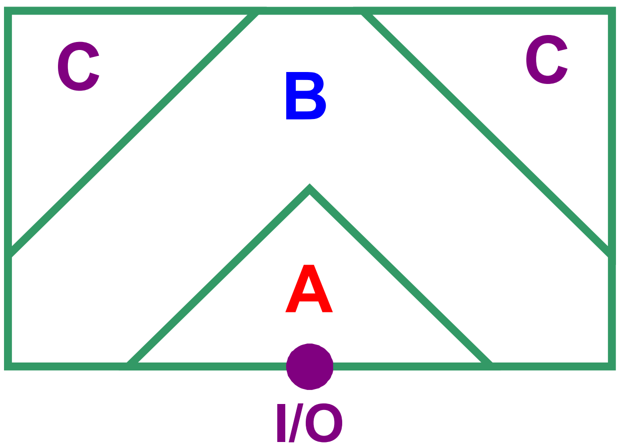
Maximizes cube utilization, but minimizes material accessibility

Making at least one unit of each item accessible decreases cube utilization

Storage Policies

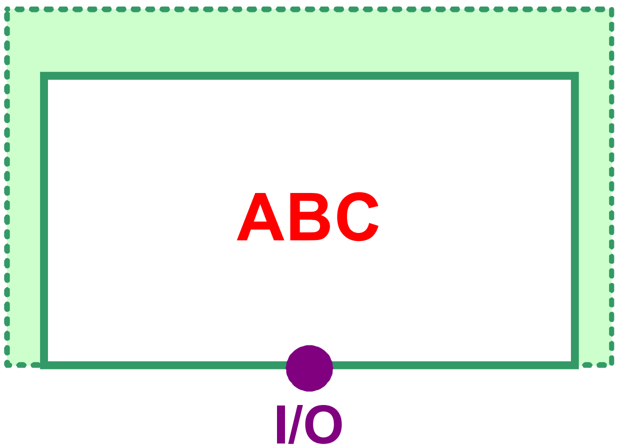
- A storage policy determines how the slots in a storage region are assigned to the different SKUs to the stored in the region.
- The differences between storage polices illustrate the trade-off between minimizing building cost and minimizing handling cost.
- Type of policies:
  - Dedicated
  - Randomized
  - Class-based

Dedicated Storage



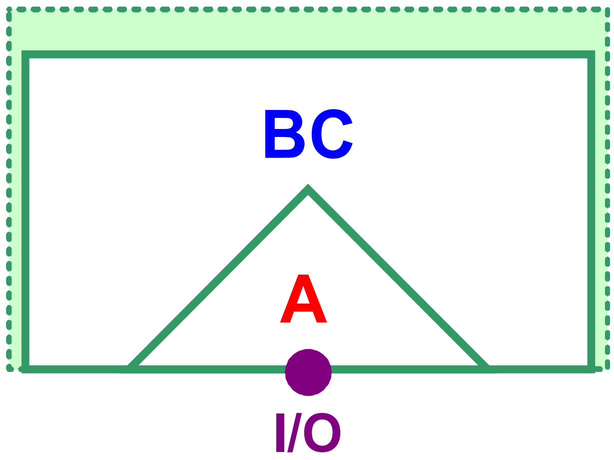
- Each SKU has a predetermined number of slots assigned to it.
- Total capacity of the slots assigned to each SKU must equal the storage space corresponding to the maximum inventory level of each *individual* SKU.
- Minimizes handling cost.
- Maximizes building cost.

Randomized Storage



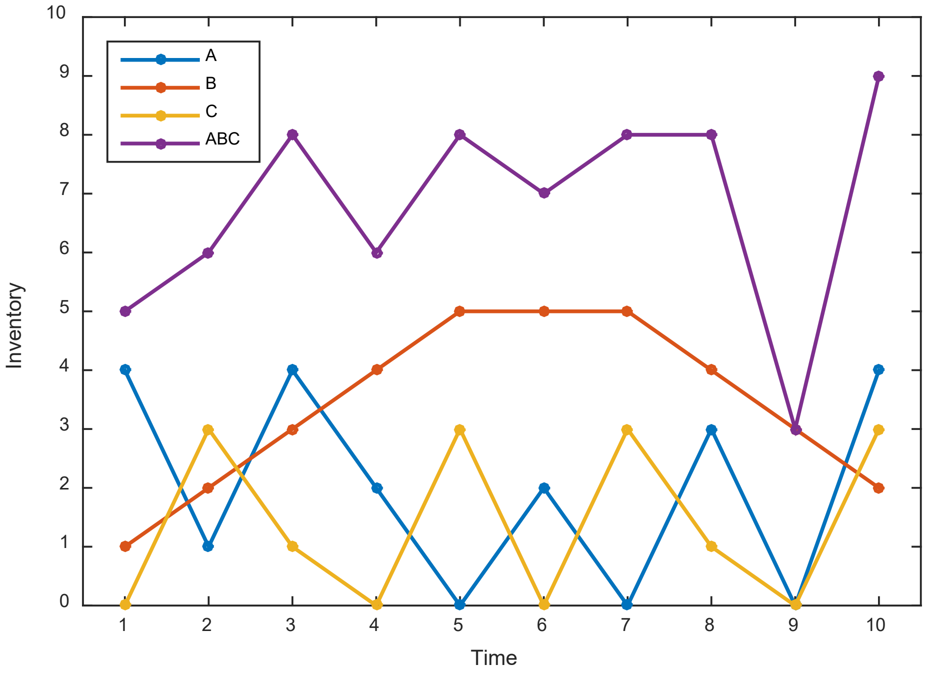
- Each SKU can be stored in any available slot.
- Total capacity of all the slots must equal the storage space corresponding to the maximum *aggregate* inventory level of all of the SKUs.
- Maximizes handling cost.
- Minimizes building cost.

Class-based Storage



- Combination of dedicated and randomized storage, where each SKU is assigned to one of several different storage classes.
- Randomized storage is used for each SKU within a class, and dedicated storage is used between classes.
- Building and handling costs between dedicated and randomized.

Individual vs Aggregate SKUs



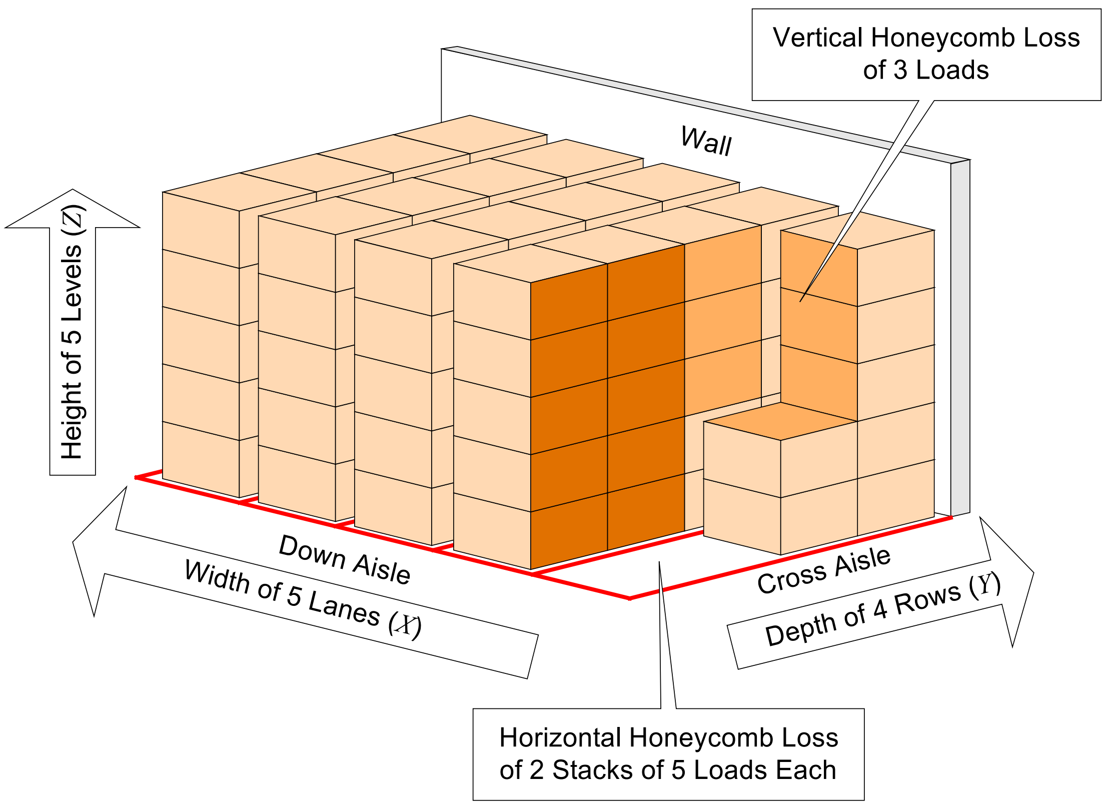
	Dedicated			Random	Class-Based		
Time	A	B	C	ABC	AB	AC	BC
1	4	1	0	5	5	4	1
2	1	2	3	6	3	4	5
3	4	3	1	8	7	5	4
4	2	4	0	6	6	2	4
5	0	5	3	8	5	3	8
6	2	5	0	7	7	2	5
7	0	5	3	8	5	3	8
8	3	4	1	8	7	4	5
9	0	3	0	3	3	0	3
10	4	2	3	9	6	7	5
M_i	4	5	3	9	7	7	8

Cube Utilization

- *Cube utilization* is percentage of the total space (or "cube") required for storage actually occupied by items being stored.
- There is usually a trade-off between cube utilization and material accessibility.
- Bulk storage using block stacking can result in the minimum cost of storage, but material accessibility is low since only the top of the front stack is accessible.
- Storage racks are used when support and/or material accessibility is required.

Honeycomb Loss

- *Honeycomb loss*, the price paid for accessibility, is the unusable empty storage space in a lane or stack due to the storage of only a single SKU in each lane or stack



Estimating Cube Utilization

- The (3-D) cube utilization for dedicated and randomized storage can estimated as follows:

$$\text{Cube utilization} = \frac{\text{item space}}{\text{total space}} = \frac{\text{item space}}{\text{item space} + \begin{pmatrix}\text{honeycomb} \\ \text{loss}\end{pmatrix} + \begin{pmatrix}\text{down aisle} \\ \text{space}\end{pmatrix}}$$

$$CU(\text{3-D}) = \begin{cases} \dfrac{x \cdot y \cdot z \cdot \sum_{i=1}^{N} M_i}{TS(D)}, & \text{dedicated} \\ \dfrac{x \cdot y \cdot z \cdot M}{TS(D)}, & \text{randomized} \end{cases}$$

$$CU(\text{2-D}) = \begin{cases} \dfrac{x \cdot y \cdot \sum_{i=1}^{N} \left\lceil \dfrac{M_i}{H} \right\rceil}{TA(D)}, & \text{dedicated} \\ \dfrac{x \cdot y \cdot \left\lceil \dfrac{M}{H} \right\rceil}{TA(D)}, & \text{randomized} \end{cases}$$

where

x = lane/unit-load width

y = unit-load depth

z = unit-load height

M_i = maximum number of units of SKU i

M = maximum number of units of all SKUs

N = number of different SKUs

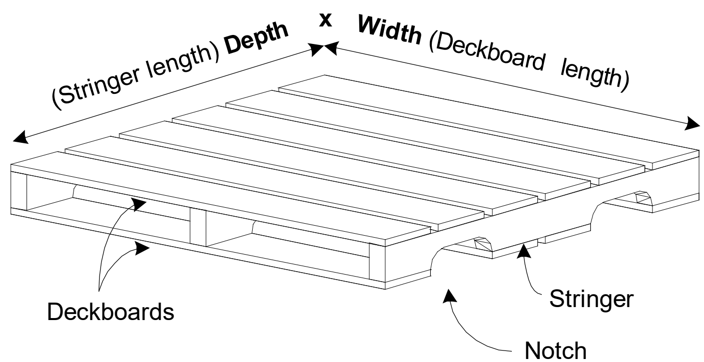
D = number of rows

$TS(D)$ = total 3-D *space* (given D rows of storage).

$TA(D)$ = total 2-D *area* (given D rows of storage).

Unit Load

- *Unit load*: single unit of an item, or multiple units restricted to maintain their integrity
- Linear dimensions of a unit load:

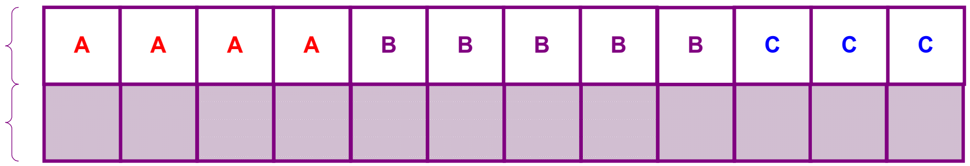
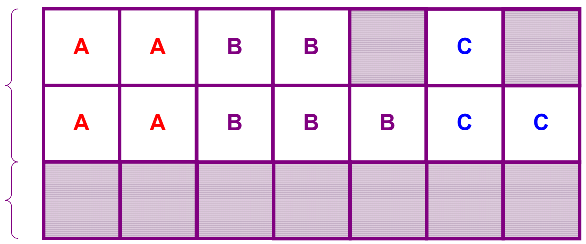
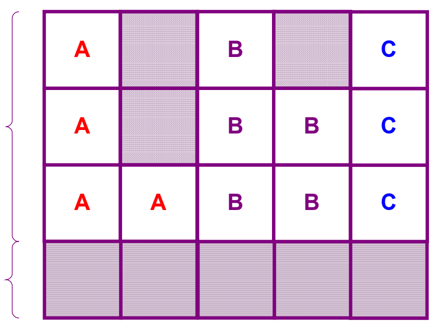


Depth (stringer length) × *Width* (deckboard length)

$$y \times x$$

- Pallet height (5 in.) + load height gives z: $y \times x \times z$

Cube Utilization for Dedicated Storage

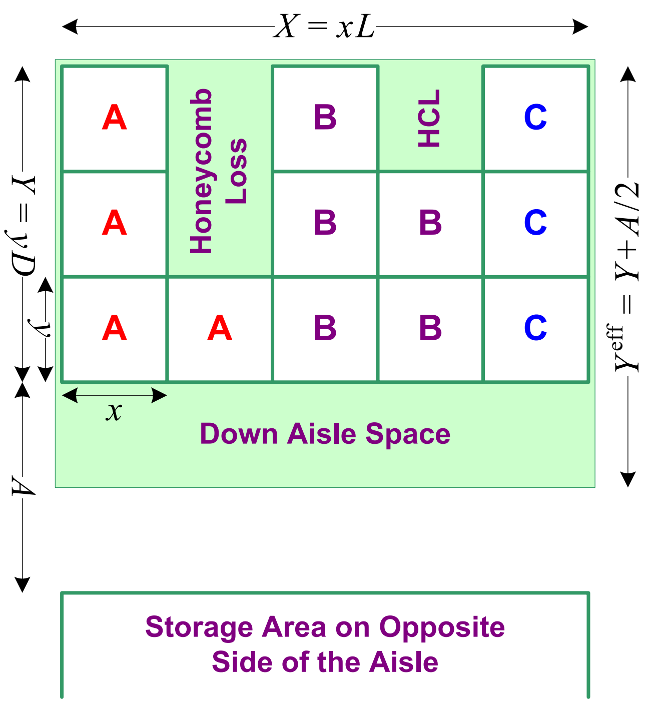
Storage Area at Different Lane Depths	Item Space	Lanes	Total Space	Cube Util.
$D = 1$ $A/2 = 1$ 	12	12	24	50%
$D = 2$ $A/2 = 1$ 	12	7	21	57%
$D = 3$ $A/2 = 1$ 	12	5	20	60%

Total Space/Area

- The total space required, as a function of lane depth *D*:

$$\text{Total space (3-D):} \quad TS(D) = X \cdot \underbrace{\left(Y + \frac{A}{2}\right)}_{\text{Eff. lane depth}} \cdot Z = xL(D) \cdot \left(yD + \frac{A}{2}\right) \cdot zH$$

$$\text{Total area (2-D):} \quad TA(D) = \frac{TS(D)}{Z} = X \cdot Y^{\text{eff}} = xL(D) \cdot \left(yD + \frac{A}{2}\right)$$



where

$X =$ width of storage region (row length)

$Y =$ depth of storage region (lane depth)

$Z =$ height of storage region (stack height)

$A =$ down aisle width

$L(D) =$ number of lanes (given D rows of storage)

$H =$ number of levels.

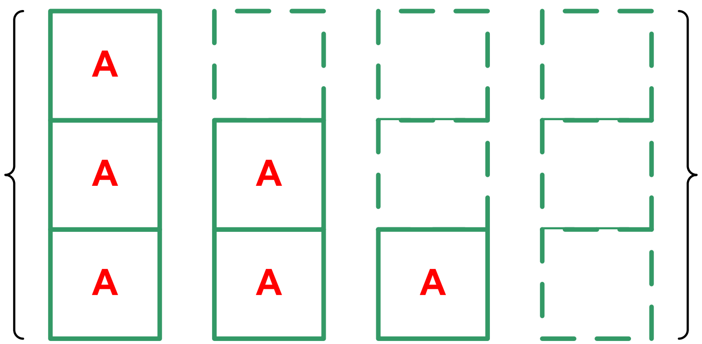
Number of Lanes

- Given *D*, estimated total number of lanes in region:

$$\text{Number of lanes:}\quad L(D)=\begin{cases}\displaystyle\sum_{i=1}^{N}\left\lceil \frac{M_i}{DH}\right\rceil, & \text{dedicated}\\[2ex] \left\lceil \dfrac{M+NH\left(\frac{D-1}{2}\right)+N\left(\frac{H-1}{2}\right)}{DH}\right\rceil, & \text{randomized } (N > 1)\end{cases}$$

- Estimated HCL:

$D=3$



doesn't occur because slots are used by another SKU

$$\text{Probability:}\quad \frac{1}{D}\quad\times\quad + \quad\frac{1}{D}\quad\times\quad + \quad\frac{1}{D}\quad\times$$

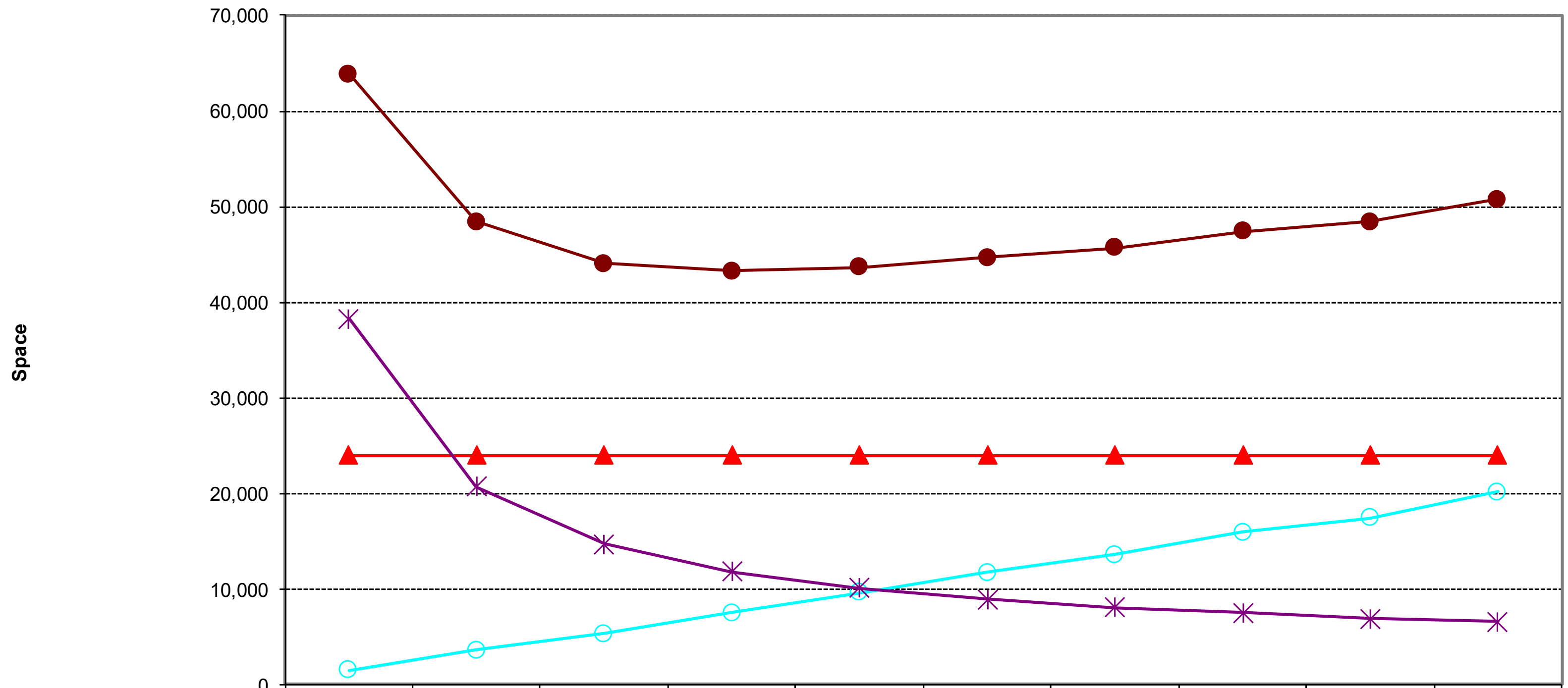
$$\text{Unit Honeycomb Loss:}\quad 0 \qquad (D-2) \qquad (D-1)$$

$$\text{Expected Loss:}\ \frac{1}{D}\big[(D-2)+(D-1)\big]=\frac{1}{D}(1+2)=\frac{1}{D}\sum_{i=1}^{D-1} i=\frac{1}{D}\left(\frac{(D-1)D}{2}\right)=\frac{D-1}{2}=1$$

Optimal Lane Depth

- Solving for D in $dTS(D)/dD = 0$ results in:

Optimal lane depth for randomized storage (in rows): $D^* = \left\lfloor \sqrt{\dfrac{A(2M-N)}{2NyH}} + \dfrac{1}{2} \right\rfloor$



	1	2	3	4	5	6	7	8	9	10
Item Space	24,000	24,000	24,000	24,000	24,000	24,000	24,000	24,000	24,000	24,000
Honeycomb Loss	1,536	3,648	5,376	7,488	9,600	11,712	13,632	15,936	17,472	20,160
Aisle Space	38,304	20,736	14,688	11,808	10,080	8,928	8,064	7,488	6,912	6,624
Total Space	63,840	48,384	44,064	43,296	43,680	44,640	45,696	47,424	48,384	50,784

Lane Depth (in Rows)

Max Aggregate Inventory Level

- Usually can determine max inventory level for each SKU:
  - M_i = maximum number of units of SKU *i*
- Since usually don't know M directly, but can estimate it **if**
  - SKUs' inventory levels are uncorrelated
  - Units of each item are either stored or retrieved at a constant rate

$$M = \left\lfloor \sum_{i=1}^{N} \frac{M_i}{2} + \frac{1}{2} \right\rfloor$$

- Can add include safety stock for each item, SS_i
  - For example, if the order size of three SKUs is 50 units and 5 units of each item are held as safety stock

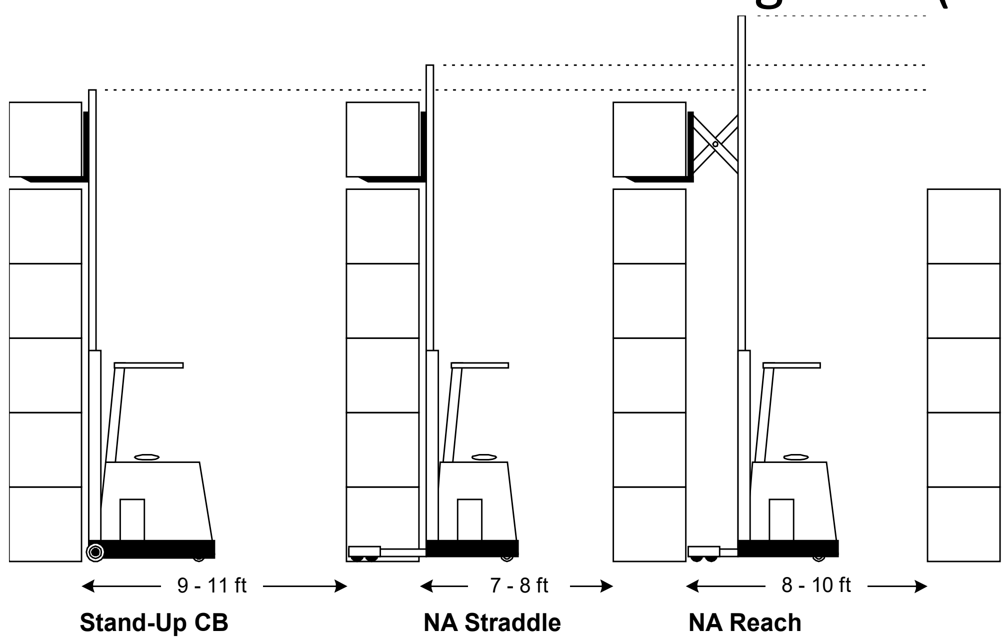
$$M = \left\lfloor \sum_{i=1}^{N} \left(\frac{M_i - SS_i}{2} + SS_i \right) + \frac{1}{2} \right\rfloor = \left\lfloor 3\left(\frac{50}{2} + 5 \right) + \frac{1}{2} \right\rfloor = 90$$

Steps to Determine Area Requirements

1. For randomized storage, assumed to know N, H, x, y, z, A, and all M_i
   - Number of levels, H, depends on building clear height (for block stacking) or shelf spacing
   - Aisle width, A, depends on type of lift trucks used
2. Estimate maximum aggregate inventory level, M
3. If D not fixed, estimate optimal land depth, D^*
4. Estimate number of lanes required, $L(D^*)$
5. Determine total 2-D area, $TA(D^*)$

Aisle Width Design Parameter

- Typically, A (and sometimes H) is a parameter used to evaluate different overall design alternatives
- Width depends on type of lift trucks used, a narrower aisle truck
  - reduces area requirements (building costs)
  - costs more and slows travel and loading time (handling costs)



Example 1: Area Requirements

Units of items A, B, and C are all received and stored as 42 × 36 × 36 in. ($y \times x \times z$) pallet loads in a storage region that is along one side of a 10-foot-wide down aisle in the warehouse of a factory. The shipment size received for each item is 31, 62, and 42 pallets, respectively. Pallets can be stored up to three deep and four high in the region.

$$x = \frac{36}{12} = 3' \qquad M_A = 31 \qquad A = 10'$$

$$y = 3.5' \qquad M_B = 62 \qquad D = 3$$

$$z = 3' \qquad M_C = 42 \qquad H = 4$$

$$N = 3$$

Example 1: Area Requirements

1. If a dedicated policy is used to store the items, what is the 2-D cube utilization of this storage region?

$$L(D) = L(3) = \sum_{i=1}^{N} \left\lceil \frac{M_i}{DH} \right\rceil = \left\lceil \frac{31}{3(4)} \right\rceil + \left\lceil \frac{62}{3(4)} \right\rceil + \left\lceil \frac{42}{3(4)} \right\rceil = 3 + 6 + 4 = 13 \text{ lanes}$$

$$TA(3) = xL(D) \cdot \left(yD + \frac{A}{2} \right) = 3(13) \cdot \left(3.5(3) + \frac{10}{2} \right) = 605 \text{ ft}^2$$

$$CU(3) = \frac{\text{item space}}{TA(3)} = \frac{x \cdot y \cdot \sum_{i=1}^{N} \left\lceil \frac{M_i}{H} \right\rceil}{TA(3)} = \frac{3 \cdot 3.5 \cdot \left(\left\lceil \frac{31}{4} \right\rceil + \left\lceil \frac{62}{4} \right\rceil + \left\lceil \frac{42}{4} \right\rceil \right)}{605} = 61\%$$

Example 1: Area Requirements

2. If the shipments of each item are uncorrelated with each other, no safety stock is carried for each item, and retrievals to the factory floor will occur at a constant rate, what is an estimate the maximum number of units of all items that would ever occur?

$$M = \left\lfloor \sum_{i=1}^{N} \frac{M_i}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{31+62+42}{2} + \frac{1}{2} \right\rfloor = 68$$

Example 1: Area Requirements

3. If a randomized policy is used to store the items, what is total 2-D area needed for the storage region?

$$D = 3$$

$$L(3) = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{68 + 3(4)\left(\frac{3-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 8 \text{ lanes}$$

$$TA(3) = xL(D) \cdot \left(yD + \frac{A}{2} \right) = 3(8) \cdot \left(3.5(3) + \frac{10}{2} \right) = 372 \text{ ft}^2$$

Example 1: Area Requirements

4. What is the optimal lane depth for randomized storage?

$$D^* = \left\lfloor \sqrt{\frac{A(2M-N)}{2NyH}} + \frac{1}{2} \right\rfloor = \left\lfloor \sqrt{\frac{10(2(68)-3)}{2(3)3.5(4)}} + \frac{1}{2} \right\rfloor = 4$$

5. What is the change in total area associated with using the optimal lane depth as opposed to storing the items three deep?

$$D = 4 \Rightarrow L(4) = \left\lceil \frac{68 + 3(4)\left(\frac{4-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 6 \text{ lanes}$$

$$\Rightarrow TA(4) = 3(6)\cdot\left(3.5(4) + \frac{10}{2}\right) = 342 \text{ ft}^2$$

$$D = 3 \Rightarrow TA(3) = 372 \text{ ft}^2$$

Example 2: Trailer Loading

How many identical 48 × 42 × 30 in. four-way containers can be shipped in a full truckload? Each container load:

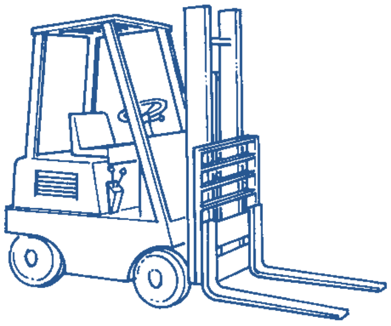
1. Weighs 600 lb
2. Can be stacked up to six high without causing damage from crushing
3. Can be rotated on the trucks with respect to their width and depth.

X	98/12 =	8.166667	8.166667	ft
Y		53	53	ft
Z	110/12 =	9.166667	9.166667	ft
x	[48,42]/12 =	4	3.5	ft
y	[42,48]/12 =	3.5	4	ft
z	30/12 =	2.5	2.5	ft
L	floor(X/x) =	2	2	
D	floor(Y/y) =	15	13	
H	min(6,floor(Z/z)) =	3	3	
LDH	L*D*H =	90	78	units
wt		600	600	lb
unit/TL	min(LDH, floor(50000/wt)) =	83	78	

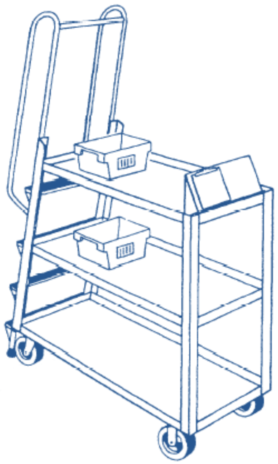
Max of 83 units per TL

Storage and Retrieval Cycle

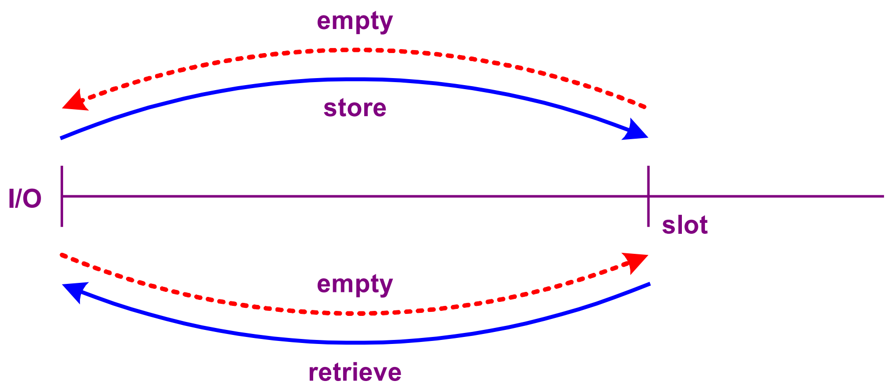
- A storage and retrieval (S/R) cycle is one complete roundtrip from an I/O port to slot(s) and back to the I/O
- Type of cycle depends on load carrying ability:
  - Carrying one load at-a-time (load carried on a pallet):
    - Single command
    - Dual command



  - Carrying multiple loads (order picking of small items):
    - Multiple command



Single-Command S/R Cycle



Expected time for each SC S/R cycle:

$$t_{SC} = \frac{d_{SC}}{v} + t_L + t_U = \frac{d_{SC}}{v} + 2t_{L/U}$$

where

d_{SC} = expected distance per SC cycle

v = average travel speed (e.g.: 2 mph = 176 fpm walking; 7 mph = 616 fpm riding)

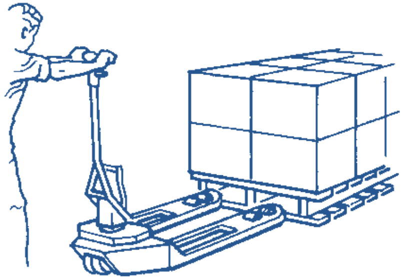
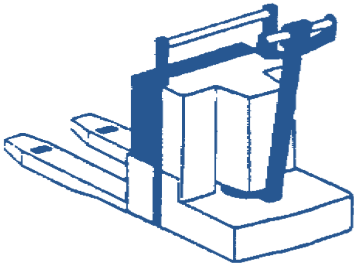
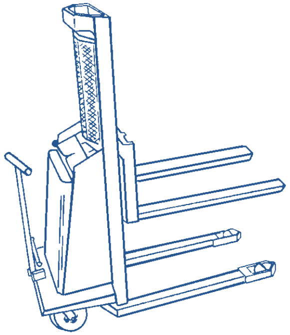
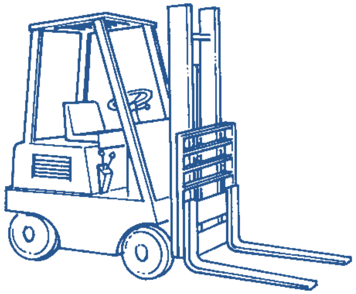
t_L = loading time

t_U = unloading time

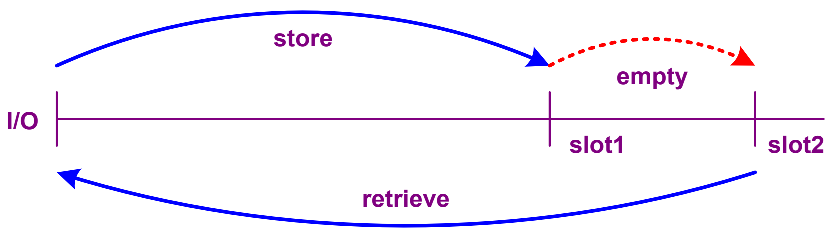
$t_{L/U}$ = loading/unloading time, if same value

- Single-command (SC) cycles:
  - Storage: carry one load to slot for storage and return empty back to I/O port, or
  - Retrieval: travel empty to slot to retrieve load and return with it back to I/O port

Industrial Trucks: Walk vs. Ride

Walk (2 mph = 176 fpm)	Ride (7 mph = 616 fpm)
	
Pallet Jack	Pallet Truck
	
Walkie Stacker	Sit-down Counterbalanced Lift Truck

Dual-Command S/R Cycle

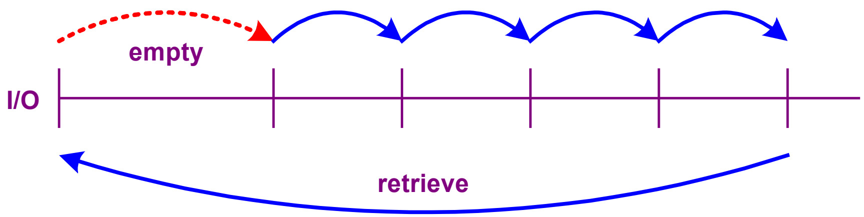


Expected time for each SC S/R cycle:

$$t_{DC} = \frac{d_{DC}}{v} + 2t_L + 2t_U = \frac{d_{DC}}{v} + 4t_{L/U}$$

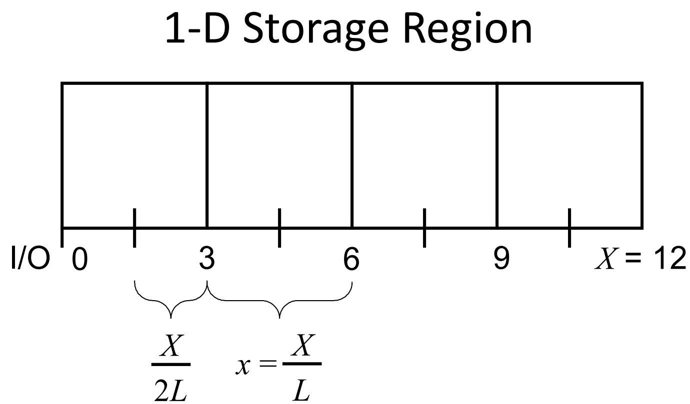
- Dual-command (DC):
- Combine storage with a retrieval:
  - store load in slot 1, travel empty to slot 2 to retrieve load
- Can reduce travel distance by a third, on average
- Also termed task "interleaving"

Multi-Command S/R Cycle



- Multi-command: multiple loads can be carried at the same time
- Used in case and piece order picking
- Picker routed to slots
  - Simple VRP procedures can be used

1-D Expected Distance



$$TD_{1-way} = \sum_{i=1}^{L}\left(i\frac{X}{L} - \frac{X}{2L}\right) = \frac{X}{L}\sum_{i=1}^{L} i - \frac{X}{2L}(1)$$

$$= \frac{X}{L}\left(\frac{L(L+1)}{2}\right) - \frac{X}{2L}(L)$$

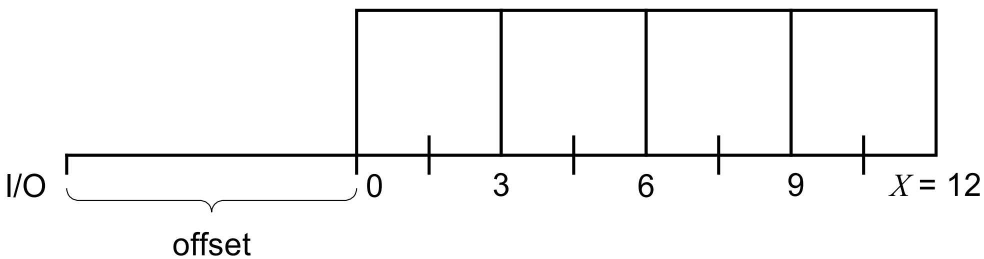
$$= \frac{XL + X - X}{2} = \frac{XL}{2}$$

$$ED_{1-way} = \frac{TD_{1-way}}{L} = \frac{X}{2}$$

$$d_{SC} = 2(ED_{1-way}) = X$$

- Assumptions:
  - All single-command cycles
  - Rectilinear distances
  - Each slot is region used with equal frequency (i.e., randomized storage)
- Expected distance is the average distance from I/O port to midpoint of each slot
  - e.g., [2(1.5) + 2(4.5) + 2(6.5) + 2(10.5)]/4 = 12

Off-set I/O Port



$$d_{SC} = 2(d_{\text{offset}}) + X$$

- If the I/O port is off-set from the storage region, then 2 times the distance of the offset is added the expected distance within the slots

2-D Expected Distances

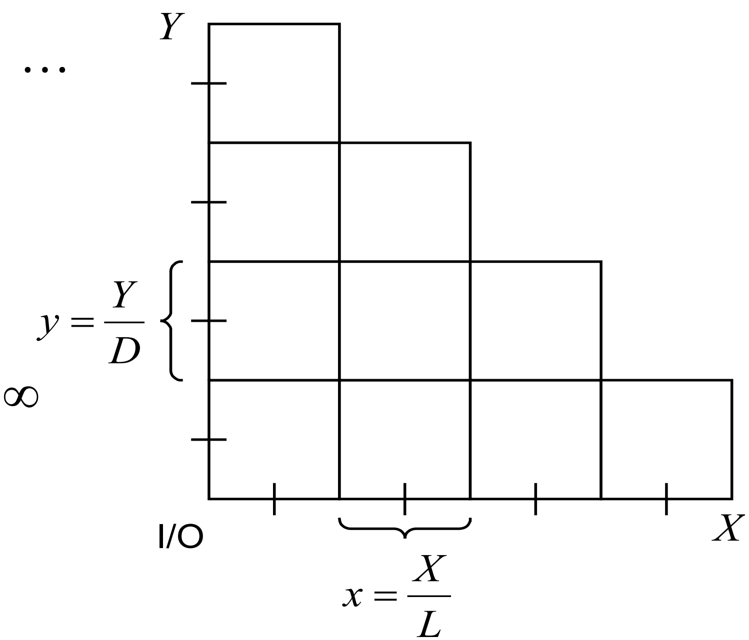
- Since dimensions *X* and *Y* are independent of each other for rectilinear distances, the expected distance for a 2-D rectangular region with the I/O port in a corner is just the sum of the distance in *X* and in *Y*: $d_{SC}^{rect} = X + Y$
- For a triangular region with the I/O port in the corner:

$$TD_{\text{1-way}} = \sum_{i=1}^{L} \sum_{j=1}^{L-i+1} \left[\left(i\frac{X}{L} - \frac{X}{2L} \right) + \left(j\frac{X}{L} - \frac{X}{2L} \right) \right] = \cdots$$

$$= \frac{X}{6}\left(2L^2 + 3L + 1\right)$$

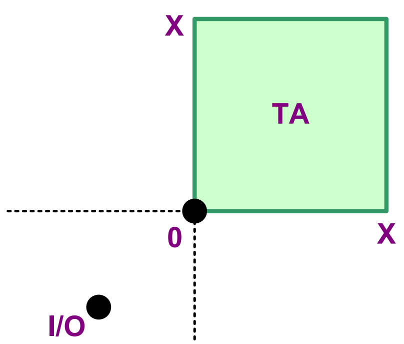
$$ED_{\text{1-way}} = \frac{TD_{\text{1-way}}}{\frac{L(L+1)}{2}} = \frac{2}{3}X + \frac{X}{3L} = \frac{2}{3}X, \quad \text{as } L \to \infty$$

$$d_{SC}^{tri} = 2\left(\frac{2}{3}X\right) = 2\left(\frac{1}{3}X + \frac{1}{3}Y\right) = \frac{2}{3}(X + Y)$$



I/O-to-Side Configurations

Rectangular

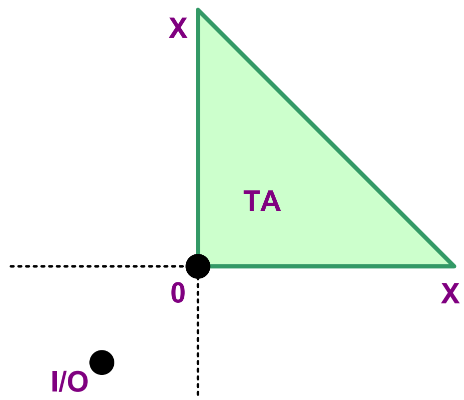


$$TA = X^2$$

$$\Rightarrow X = \sqrt{TA}$$

$$\Rightarrow d_{SC} = 2\sqrt{TA}$$

Triangular



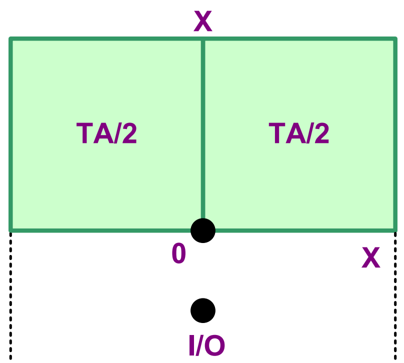
$$TA = \frac{1}{2} X^2$$

$$\Rightarrow X = \sqrt{2TA} = \sqrt{2}\sqrt{TA}$$

$$\Rightarrow d_{SC} = \frac{4}{3}\sqrt{2}\sqrt{TA} = 1.886\sqrt{TA}$$

I/O-at-Middle Configurations

Rectangular

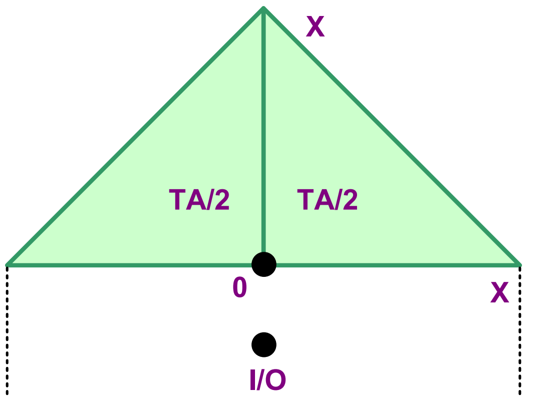


$$\frac{TA}{2} = X^2$$

$$\Rightarrow X = \sqrt{\frac{TA}{2}} = \frac{\sqrt{TA}}{\sqrt{2}}$$

$$\Rightarrow d_{SC} = \sqrt{2}\sqrt{TA} = 1.414\sqrt{TA}$$

Triangular



$$\frac{TA}{2} = \frac{1}{2}X^2$$

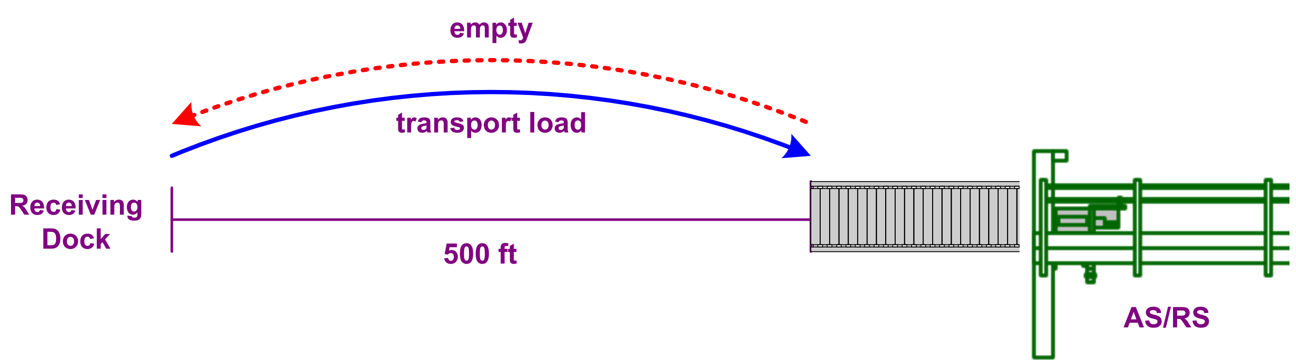
$$\Rightarrow X = \sqrt{TA}$$

$$\Rightarrow d_{SC} = \frac{4}{3}\sqrt{TA} = 1.333\sqrt{TA}$$

Example 3: Handling Requirements

Pallet loads will be unloaded at the receiving dock of a warehouse and placed on the floor. From there, they will be transported 500 feet using a dedicated pallet truck to the in-floor induction conveyor of an AS/RS. Given

a. It takes 30 sec to load each pallet at the dock
b. 30 sec to unload it at the induction conveyor
c. There will be 80,000 loads per year on average
d. Operator rides on the truck (because a pallet truck)
e. Facility will operate 50 weeks per year, 40 hours per week



Example 3: Handling Requirements

1. Assuming that it will take 30 seconds to load each pallet at the dock and 30 seconds to unload it at the induction conveyor, what is the expected time required for each single-command S/R cycle?

$$d_{SC} = 2(500) = 1000 \text{ ft/mov}$$

$$t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{1000 \text{ ft/mov}}{616 \text{ ft/min}} + 2\left(\frac{30}{60}\right) \text{min/mov}$$

$$= 2.62 \text{ min/mov} \quad = \frac{2.62}{60} \text{ hr/mov}$$

(616 fpm because operator rides on a pallet truck)

Example 3: Handling Requirements

2. Assuming that there will be 80,000 loads per year on average and that the facility will operate for 50 weeks per year, 40 hours per week, what is the minimum number of trucks needed?

$$r_{avg} = \frac{80{,}000 \text{ mov/yr}}{50(40) \text{ hr/yr}} = 40 \text{ mov/hr}$$

$$m = \lfloor r_{avg} t_{SC} + 1 \rfloor$$

$$= \left\lfloor 40\left(\frac{2.62}{60}\right) + 1 \right\rfloor = \lfloor 1.75 + 1 \rfloor$$

$$= 2 \text{ trucks}$$

Example 3: Handling Requirements

3. How many trucks are needed to handle a peak expected demand of 80 moves per hour?

$$r_{peak} = 80 \text{ mov/hr}$$

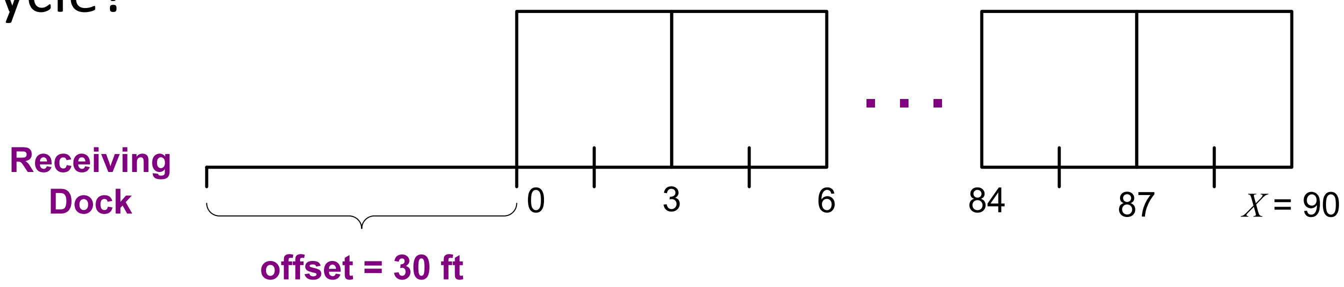
$$m = \lfloor r_{peak} t_{SC} + 1 \rfloor$$

$$= \left\lfloor 80\left(\frac{2.62}{60}\right) + 1 \right\rfloor = \lfloor 3.50 + 1 \rfloor$$

$$= 4 \text{ trucks}$$

Example 3: Handling Requirements

4. If, instead of unloading at the conveyor, the 3-foot-wide loads are placed side-by-side in a staging area along one side of 90-foot aisle that begins 30 feet from the dock, what is the expected time required for each single-command S/R cycle?



$$d_{SC} = 2(d_{\text{offset}}) + X = 2(30) + 90 = 150 \text{ ft}$$

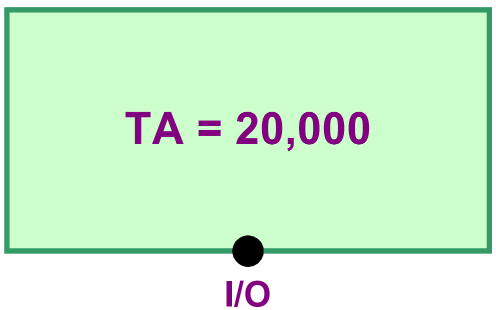
$$t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{150 \text{ ft/mov}}{616 \text{ ft/min}} + 2\left(\frac{30}{60}\right) \text{min/mov}$$

$$= 1.24 \text{ min/mov} \quad = \frac{1.24}{60} \text{ hr/mov}$$

Estimating Handling Costs

- Warehouse design involves the trade-off between building and handling cost.
- Maximizing the cube utilization of a storage region will help minimize building costs.
- Handling costs can be estimated by determining:
  1. Expected time required for each move based on an average of the time required to reach each slot in the region.
  2. Number of vehicles needed to handle a target *peak demand* for moves, e.g., moves per hour.
  3. *Operating costs per hour of vehicle operation*, e.g., labor, fuel (assuming the operators can perform other productive tasks when not operating a truck)
  4. Annual operating costs based on *annual demand* for moves.
  5. Total handling costs as the sum of the annual capital recovery costs for the vehicles and the annual operating costs.

Example 4: Estimating Handing Cost



$$\Uparrow$$

Add 20% Cross aisle:

$$TA = TA' \times 1.2$$

$$= 20,000\ \text{ft}^2$$

$$\Uparrow$$

Total Storage Area:

$$D^* \Rightarrow L(D^*) \Rightarrow TA'$$

Expected Distance: $d_{SC} = \sqrt{2}\sqrt{TA} = \sqrt{2}\sqrt{20,000} = 200\ \text{ft}$

Expected Time: $t_{SC} = \dfrac{d_{SC}}{v} + 2t_{L/U}$

$$= \frac{200\ \text{ft}}{200\ \text{fpm}} + 2(0.5\ \text{min}) = 2\ \text{min per move}$$

Peak Demand: $r_{\text{peak}} = 75\ \text{moves per hour}$

Annual Demand: $r_{\text{year}} = 100,000\ \text{moves per year}$

Number of Trucks: $m = \left\lfloor r_{\text{peak}} \dfrac{t_{SC}}{60} + 1 \right\rfloor = \lfloor 3.5 \rfloor = 3\ \text{trucks}$

Handling Cost: $TC_{\text{hand}} = mK_{\text{truck}} + r_{\text{year}} \dfrac{t_{SC}}{60} C_{\text{labor}}$

$$= 3(\$2,500\ /\ \text{tr-yr}) + 100,000 \frac{2}{60} (\$10\ /\ \text{hr})$$

$$= \$7,500 + \$33,333 = \$40,833\ \text{per year}$$

Dedicated Storage Assignment (DSAP)

- The assignment of items to slots is termed *slotting*
  - With randomized storage, all items are assigned to all slots
- DSAP (dedicated storage assignment problem):
  - Assign N items to slots to minimize total cost of material flow
- DSAP solution procedure:
  1. *Order Slots:* Compute the expected cost for each slot and then put into nondecreasing order
  2. *Order Items:* Put the flow density (flow per unit of volume, the reciprocal of which is the “cube per order index” or COI) for each item i into nonincreasing order

$$\frac{f_{[1]}}{M_{[1]}s_{[1]}} \geq \frac{f_{[2]}}{M_{[2]}s_{[2]}} \geq \cdots \geq \frac{f_{[N]}}{M_{[N]}s_{[N]}}$$

  3. *Assign Items to Slots:* For $i = 1, \ldots, N$, assign item $[i]$ to the first slots with a total volume of at least $M_{[i]}s_{[i]}$

1-D Slotting Example

		A	B	C
Max units	M	4	5	3
Space/unit	s	1	1	1
Flow	f	24	7	21
Flow Density	f/(M x s)	6.00	1.40	7.00

Flow Density	1-D Slot Assignments	Expected Distance		Flow		Total Distance
$\frac{21}{3} = 7.00$	I/O: C C C (0–3)	2(0) + 3 = 3	×	21	=	63
$\frac{24}{4} = 6.00$	I/O: A A A A (-3, 0–4)	2(3) + 4 = 10	×	24	=	240
$\frac{7}{5} = 1.40$	I/O: B B B B B (-7, 0–5)	2(7) + 5 = 19	×	7	=	133
	I/O: C C C A A A A B B B B B (0, 3, 7, 12)					436

1-D Slotting Example (cont)

		Dedicated			Random	Class-Based		
		A	B	C	ABC	AB	AC	BC
Max units	M	4	5	3	9	7	7	8
Space/unit	s	1	1	1	1	1	1	1
Flow	f	24	7	21	52	31	45	28
Flow Density	f/(M x s)	6.00	1.40	7.00	5.78	4.43	6.43	3.50

		1-D Slot Assignments												Total Distance	Total Space
Dedicated (flow density)	I/O	C	C	C	A	A	A	A	B	B	B	B	B	436	12
Dedicated (flow only)	I/O	A	A	A	A	C	C	C	B	B	B	B	B	460	12
Class-based	I/O	C	C	C	AB	AB	AB	AB	AB	AB	AB			466	10
Randomized	I/O	ABC	ABC	ABC	ABC	ABC	ABC	ABC	ABC	ABC				468	9

2-D Slotting Example

		A	B	C
Max units	M	4	5	3
Space/unit	s	1	1	1
Flow	f	24	7	21
Flow Density	f/(M x s)	6.00	1.40	7.00

8	7	6	5	4	5	6	7	8
7	6	5	4	3	4	5	6	7
6	5	4	3	2	3	4	5	6
5	4	3	2	1	2	3	4	5
4	3	2	1	0	1	2	3	4

Distance from I/O to Slot

	C	C	B	
C	A	A	B	B
A	A	I/O	B	B

Original Assignment (TD = 215)

	B	B	B	
B	A	C	A	B
A	C	I/O	C	A

Optimal Assignment (TD = 177)

DSAP Assumptions

1. All SC S/R moves
2. For item *i*, probability of move to/from each slot assigned to item is the same
3. The *factoring assumption*:
   a. Handling cost and distances (or times) for each slot are identical for all items
   b. Percent of S/R moves of item stored at slot *j* to/from I/O port *k* is identical for all items

- Depending of which assumptions not valid, can determine assignment using other procedures

$$\left[\left(\frac{f_i}{M_i}\cdot d_j\right)x_{ij}\right] DSAP \subset \underset{\left(c_{ij}x_{ij}\right)}{LAP} \subset LP \subset \underset{\underset{TSP}{\cup}}{QAP}\left(c_{ijkl}x_{ij}x_{kl}\right)$$

Example 5: 1-D DSAP

- What is the change in the minimum expected total distance traveled if dedicated, as compared to randomized, block stacking is used, where
  a. Slots located on one side of 10-foot-wide down aisle
  b. All single-command S/R operations
  c. Each lane is three-deep, four-high
  d. 40×36 in. two-way pallet used for all loads
  e. Max inventory levels of SKUs A, B, C are 94, 64, and 50
  f. Inventory levels are uncorrelated and retrievals occur at a constant rate
  g. Throughput requirements of A, B, C are 160, 140, 130
  h. Single I/O port is located at the end of the aisle

Example 5: 1-D DSAP

- Randomized:

$$M = \left\lfloor \frac{M_A + M_B + M_C}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{94 + 64 + 50}{2} + \frac{1}{2} \right\rfloor = 104$$

$$L_{rand} = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{104 + 3(4)\left(\frac{3-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 11 \text{ lanes}$$

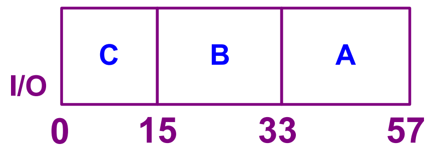
$$X = xL_{rand} = 3(11) = 33 \text{ ft}$$

$$d_{SC} = X = 33 \text{ ft}$$

$$TD_{rand} = \left(f_A + f_B + f_C\right) X = \left(160 + 140 + 130\right) 33 = 14{,}190 \text{ ft}$$

Example 5: 1-D DSAP

- Dedicated:



$$\frac{f_A}{M_A} = \frac{160}{94} = 1.7, \frac{f_B}{M_B} = \frac{140}{64} = 2.19, \frac{f_C}{M_C} = \frac{130}{50} = 2.6 \quad \Rightarrow \quad C > B > A$$

$$L_A = \left\lceil \frac{M_A}{DH} \right\rceil = \left\lceil \frac{94}{3(4)} \right\rceil = 8, L_B = \left\lceil \frac{M_B}{DH} \right\rceil = \left\lceil \frac{64}{3(4)} \right\rceil = 6, L_C = \left\lceil \frac{M_C}{DH} \right\rceil = \left\lceil \frac{50}{3(4)} \right\rceil = 5$$

$$X_C = xL_C = 3(5) = 15, X_B = xL_B = 3(6) = 18, X_A = xL_A = 3(8) = 24$$

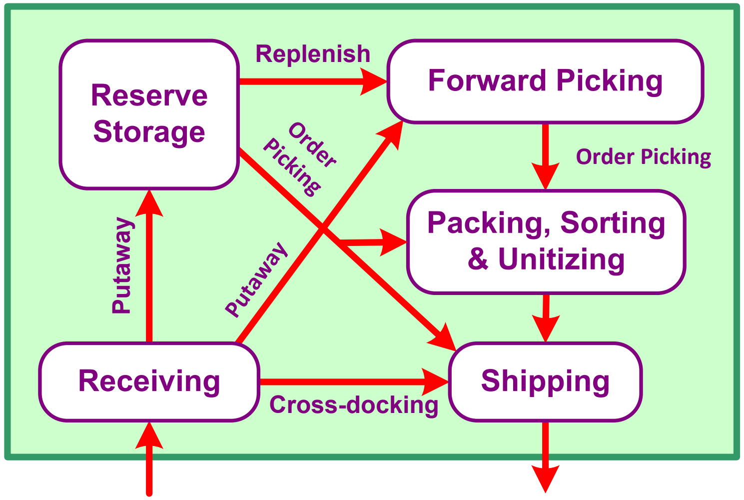
$$d_{SC}^C = X_C = 3(5) = 15 \text{ ft}$$

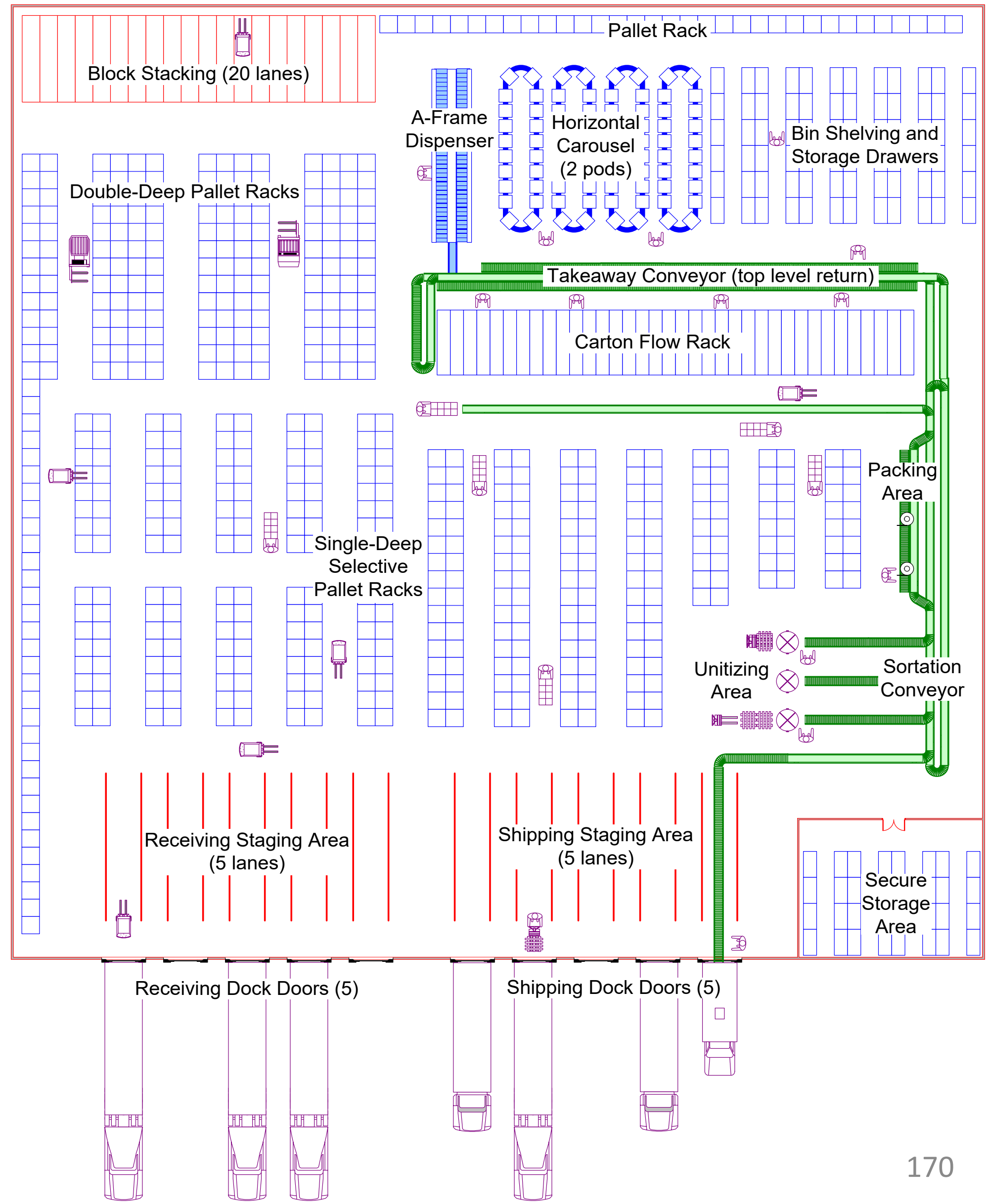
$$d_{SC}^B = 2(X_C) + X_B = 2(15) + 18 = 48 \text{ ft}$$

$$d_{SC}^A = 2(X_C + X_B) + X_A = 2(15 + 18) + 24 = 90 \text{ ft}$$

$$TD_{ded} = f_A d_{SC}^A + f_B d_{SC}^B + f_C d_{SC}^C = 160(90) + 140(48) + 130(15) = 23{,}070 \text{ ft}$$

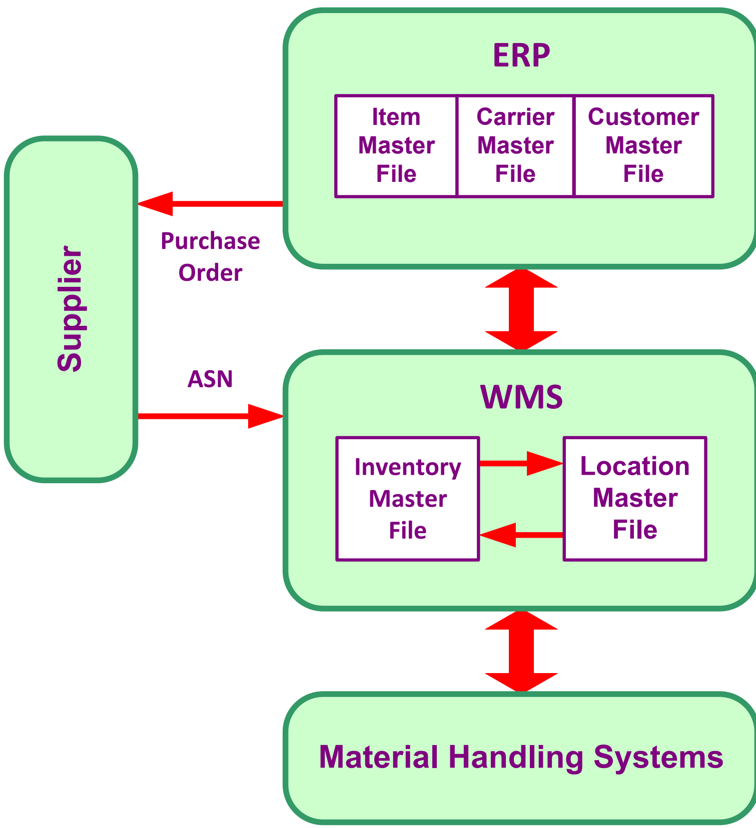
Warehouse Operations





Warehouse Management System

- WMS interfaces with a corporation's enterprise resource planning (ERP) and the control software of each MHS

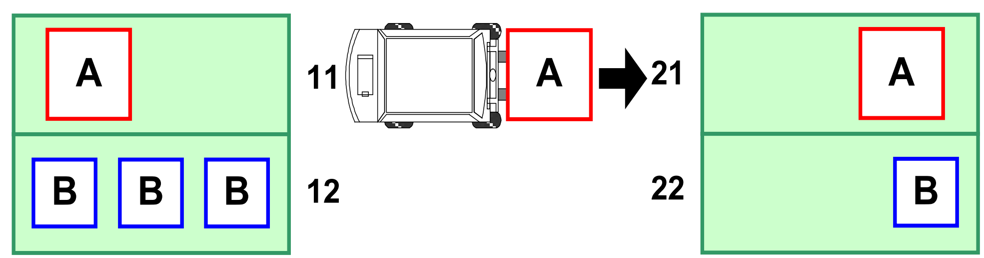


Inventory Master File

Item	On-Hand Balance	In-Transit Qty.	Locations
A	2	1	11,21
B	4	0	12,22

Location Master File

Location	Item	On-Hand Balance	In-Transit Qty.
11	A	1	0
12	B	3	0
21	A	1	1
22	B	1	0



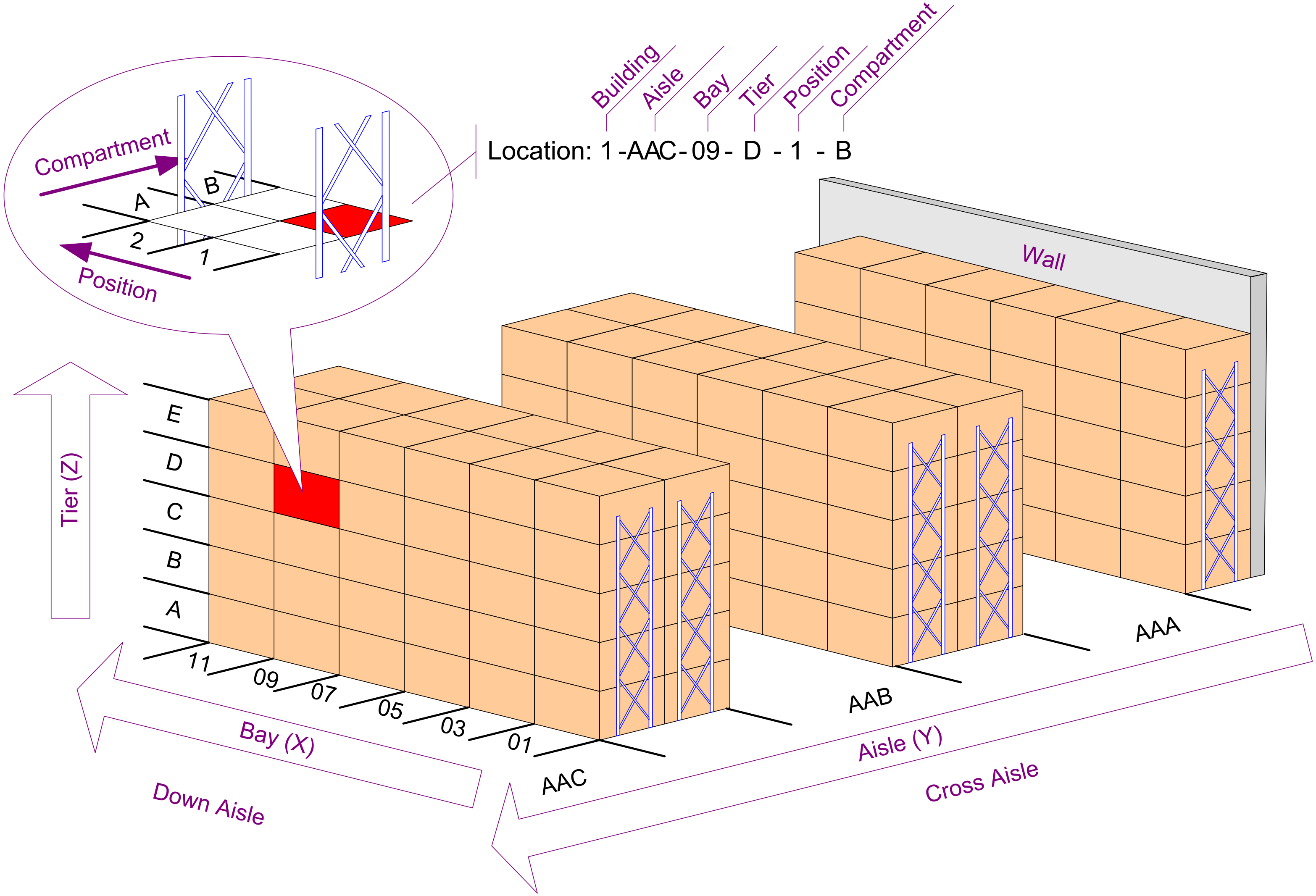
- Advance shipping notice (ASN) is a standard format used for communications

Logistics-related Codes

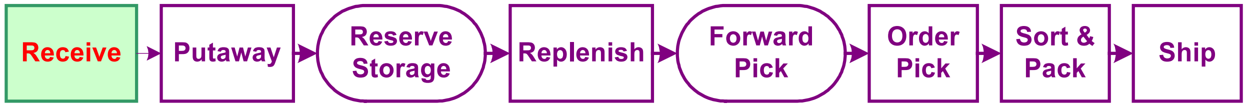
	Commodity Code	Item Code	Unit Code
Level	Category	Class	Instance
Description	Grouping of similar objects	Grouping of identical objects	Unique physical object
Function	Product classification	Inventory control	Object tracking
Names	—	Item number, Part number, SKU, SKU + Lot number	Serial number, License plate
Codes	UNSPSC, GPC	GTIN, UPC, ISBN, NDC	EPC, SSCC

UNSPSC: United Nations Standard Products and Services Code
GPC: Global Product Catalogue
GTIN: Global Trade Item Number (includes UPC, ISBN, and NDC)
UPC: Universal Product Code
ISBN: International Standard Book Numbering
NDC: National Drug Code
EPC: Electronic Product Code (globally unique serial number for physical objects identified using RFID tags)
SSCC: Serial Shipping Container Code (globally unique serial number for identifying *movable units* (carton, pallet, trailer, etc.))

Identifying Storage Locations



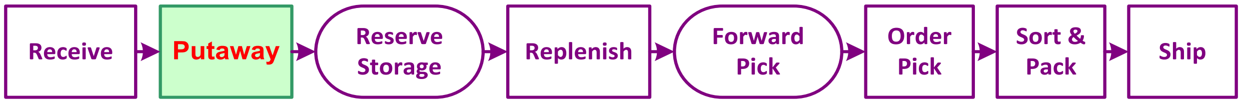
Receiving



- Basic steps:

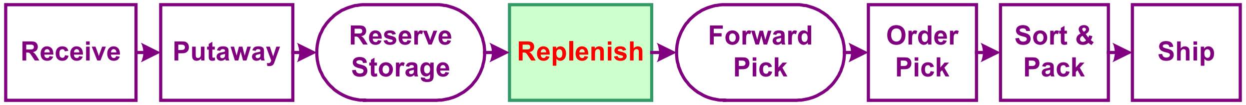
1. Unload material from trailer.
2. Identify supplier with ASN, and associate material with each moveable unit listed in ASN.
3. Assign inventory attributes to movable unit from item master file, possibly including repackaging and assigning new serial number.
4. Inspect material, possibly including holding some or all of the material for testing, and report any variances.
5. Stage units in preparation for putaway.
6. Update item balance in inventory master and assign units to a receiving area in location master.
7. Create receipt confirmation record.
8. Add units to putaway queue

Putaway

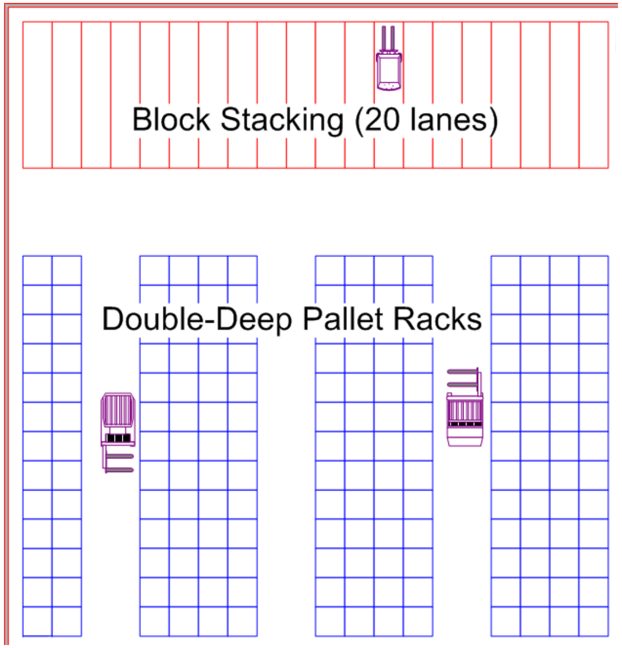


- A putaway algorithm is used in WMS to search for and validate locations where each movable unit in the putaway queue can be stored
- Inventory and location attributes used in the algorithm:
  - *Environment* (refrigerated, caged area, etc.)
  - *Container type* (pallet, case, or piece)
  - *Product processing type* (e.g., floor, conveyable, nonconveyable)
  - *Velocity* (assign to A, B, C based on throughput of item)
  - Preferred putaway zone (item should be stored in same zone as related items in order to improve picking efficiency)

Replenishment

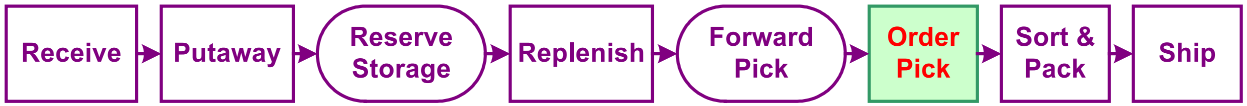


- Replenishment is the process of moving material from reserve storage to a forward picking area so that it is available to fill customer orders efficiently
- Other types of in-plant moves include:
  - Consolidation: combining several partially filled storage locations of an item into a single location
  - Rewarehousing: moving items to different storage locations to improve handling efficiency

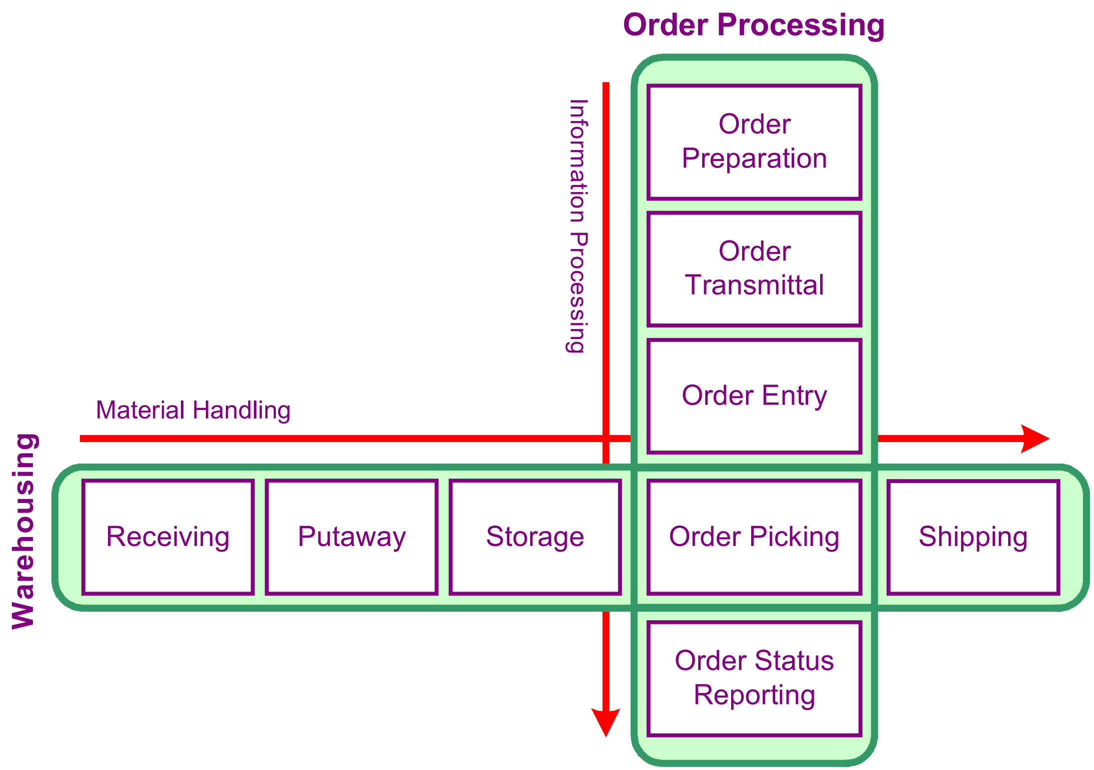


Reserve Storage Area

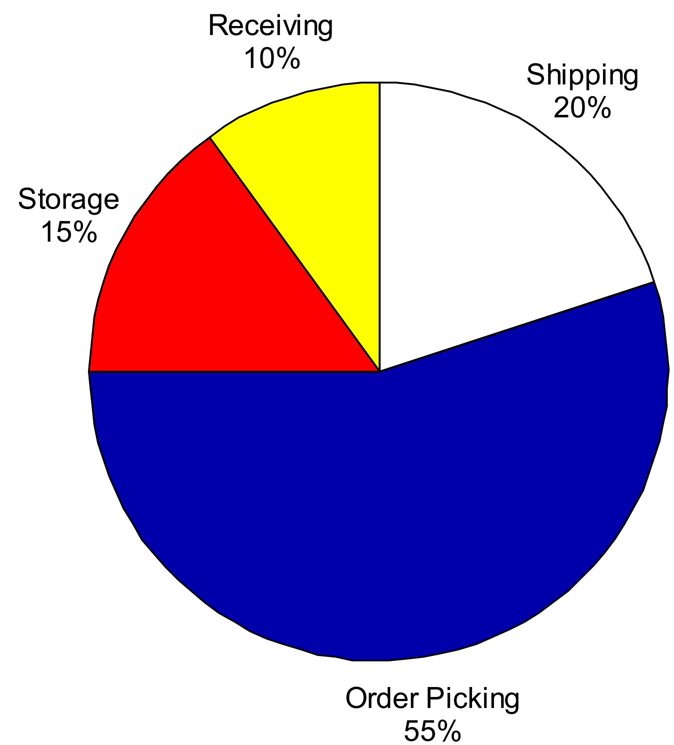
Order Picking



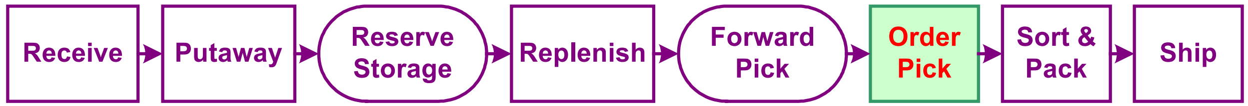
- Order picking is at the intersection of warehousing and order processing



WH Operating Costs

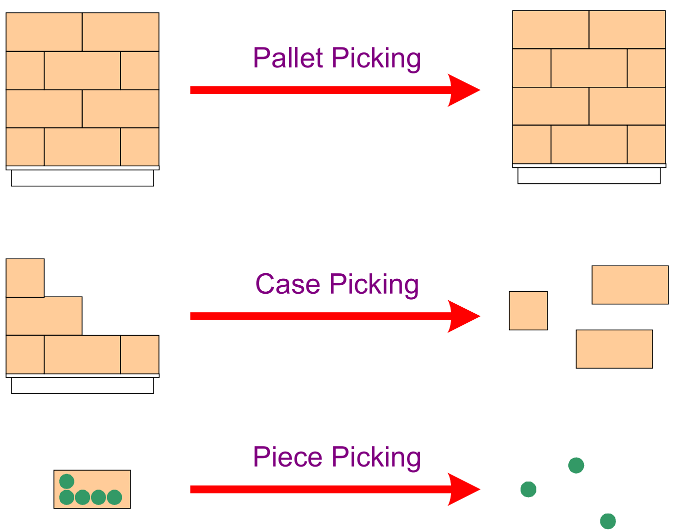


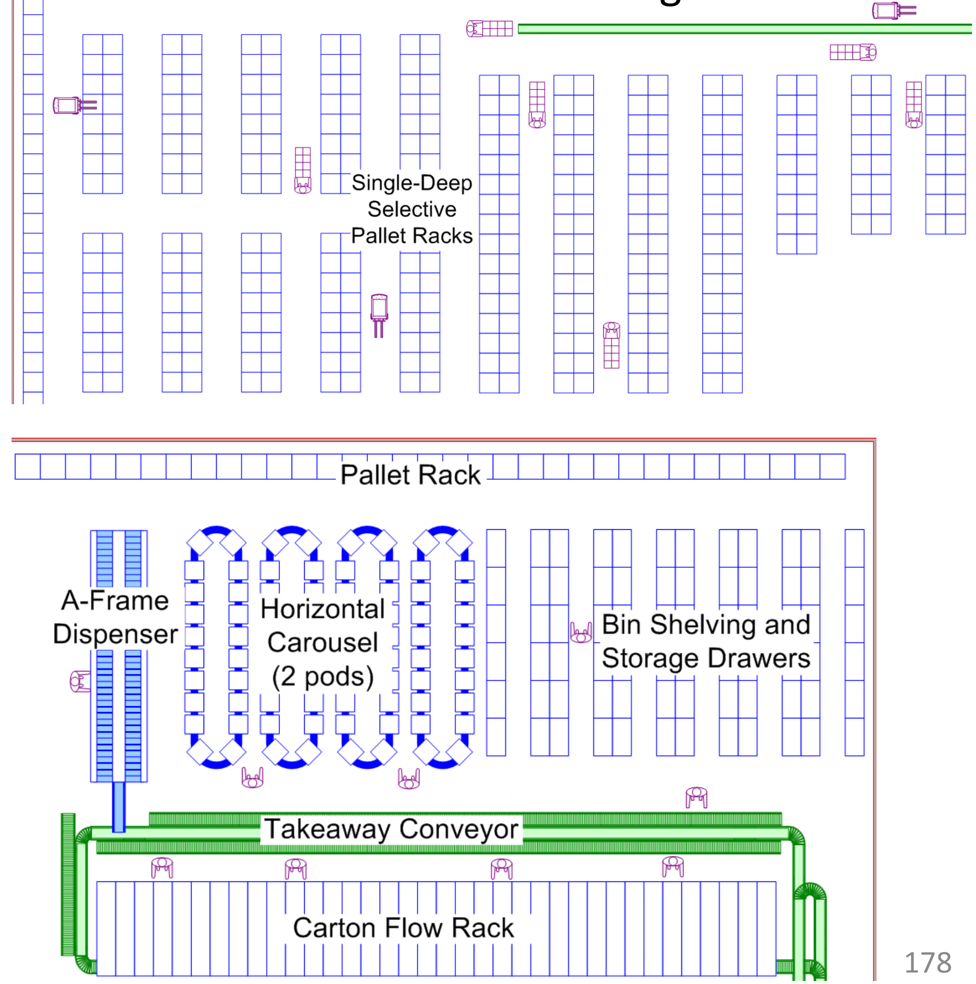
Order Picking



Pallet and Case Picking Area

Levels of Order Picking



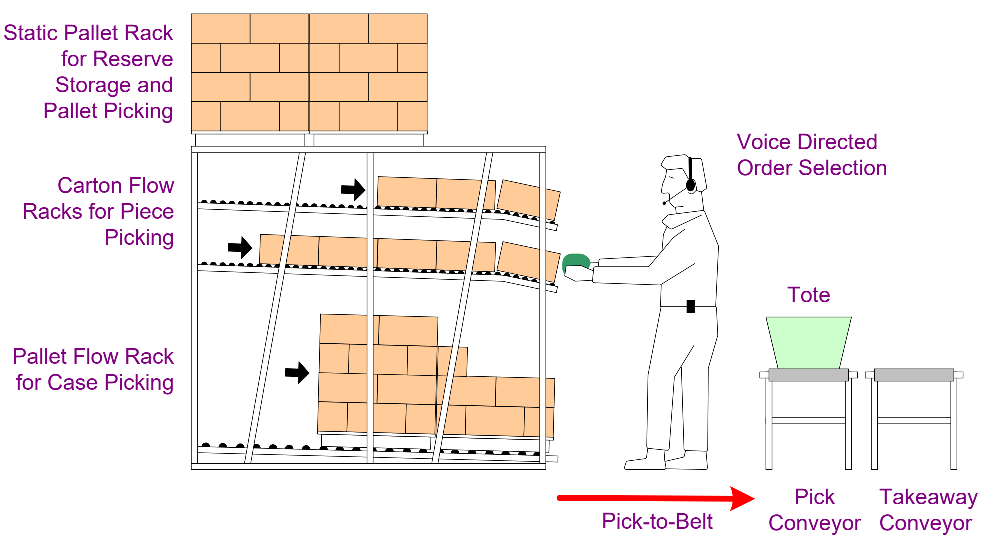


Forward Piece Picking Area

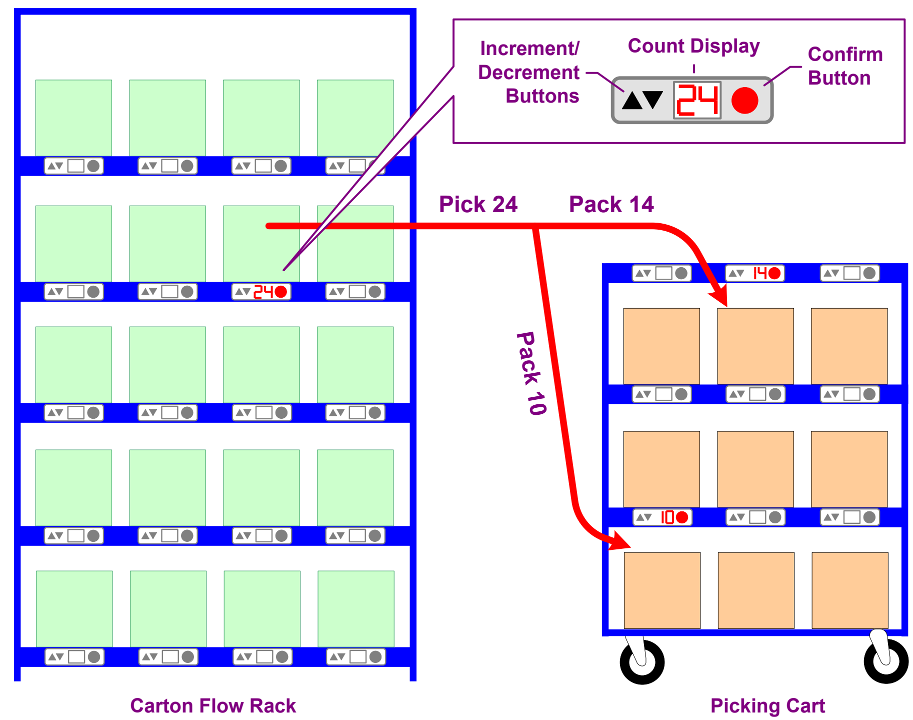
Order Picking

Receive → Putaway → Reserve Storage → Replenish → Forward Pick → **Order Pick** → Sort & Pack → Ship

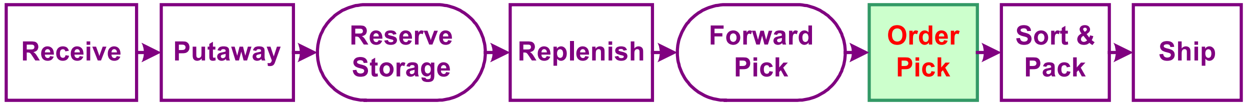
Voice-Directed Piece and Case Picking



Pick-to-Light Piece Picking



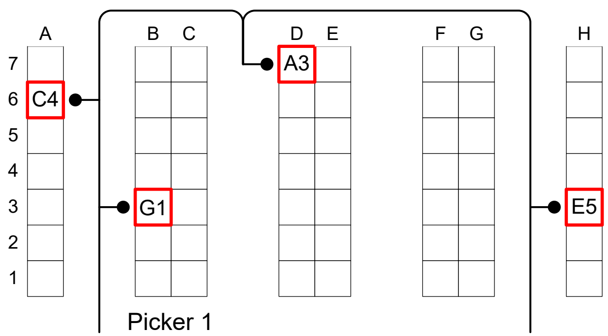
Order Picking



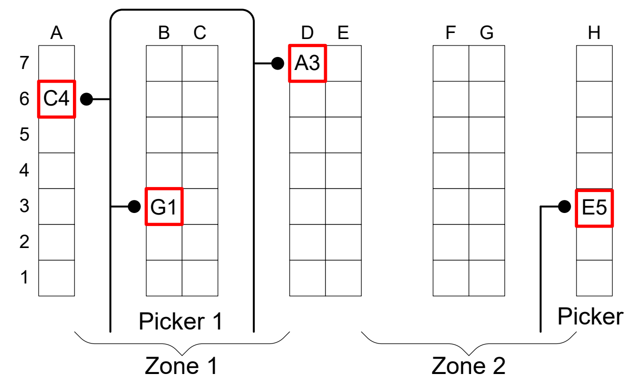
Methods of Order Picking

Method	Pickers per Order	Orders per Picker
Discrete	Single	Single
Zone	Multiple	Single
Batch	Single	Multiple
Zone-Batch	Multiple	Multiple

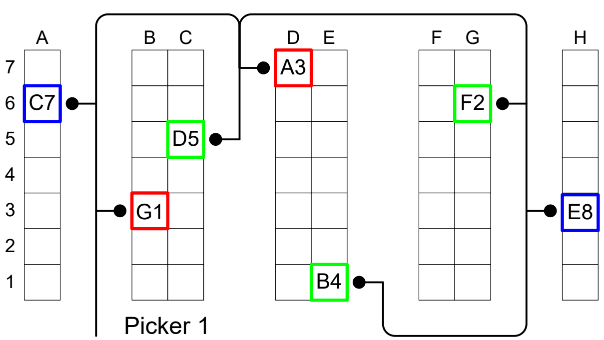
Discrete



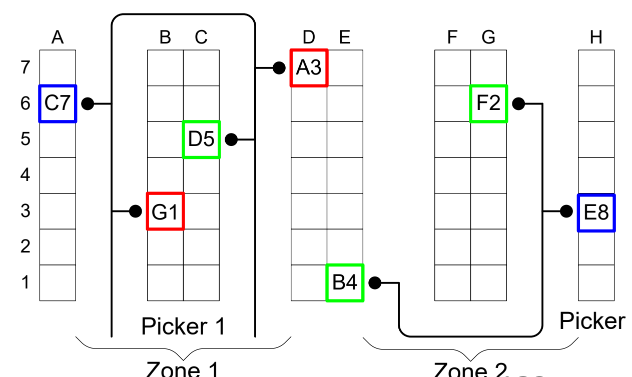
Zone



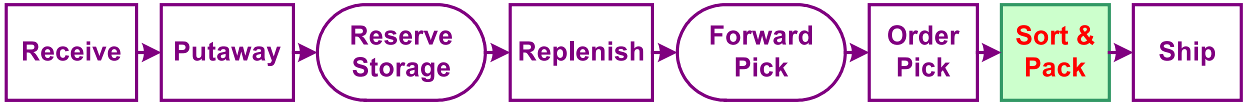
Batch

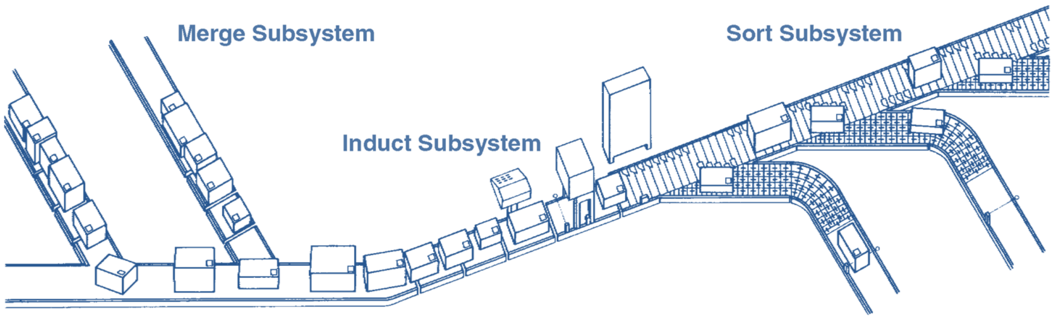


Zone-Batch

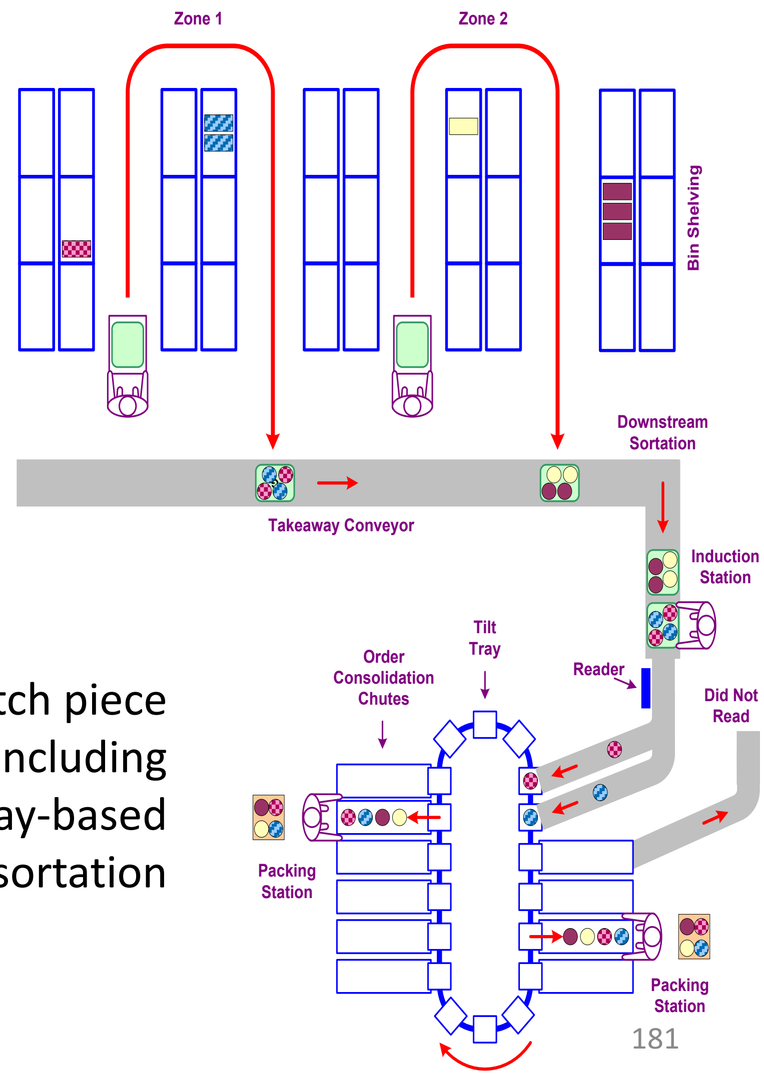


Sortation and Packing



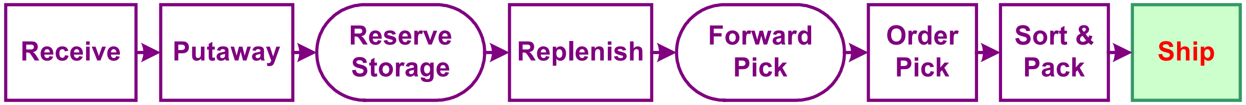


Case Sortation System

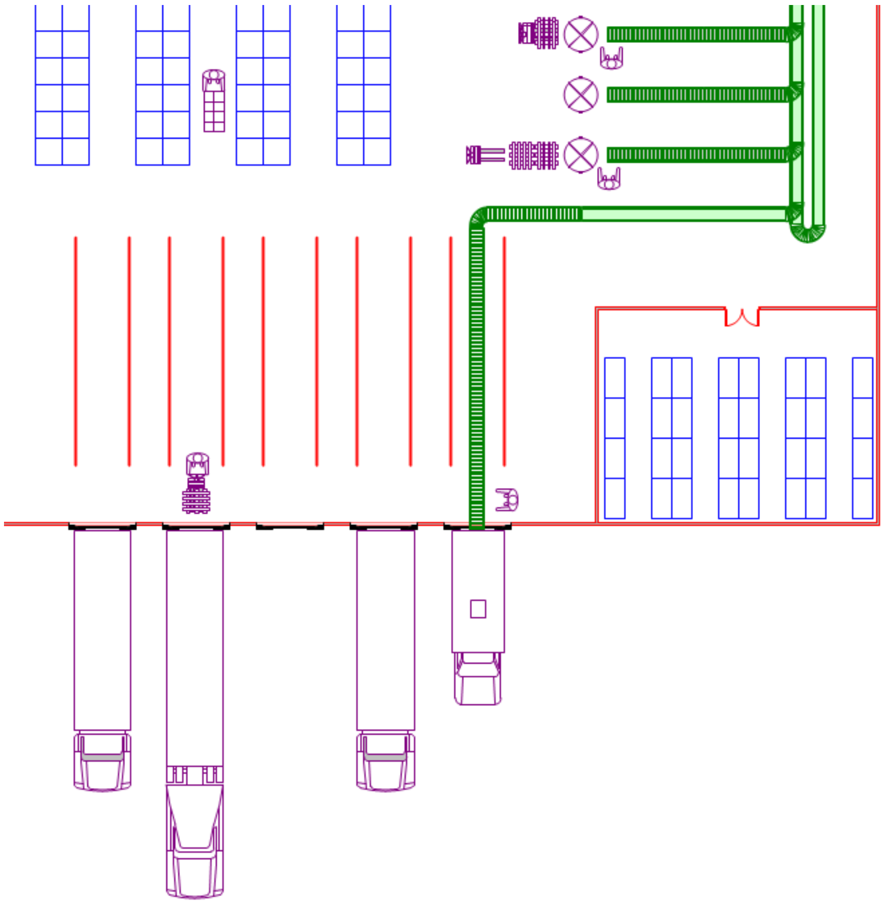


Wave zone-batch piece picking, including downstream tilt-tray-based sortation

Shipping



- Staging, verifying, and loading orders to be transported
  - ASN for each order sent to the customer
  - Customer-specific shipping instructions retrieved from customer master file
  - Carrier selection is made using the rate schedules contained in the carrier master file



Shipping Area

Activity Profiling

- *Total Lines*: total number of lines for all items in all orders
- *Lines per Order*: average number of different items (lines/SKUs) in order
- *Cube per Order*: average total cubic volume of all units (pieces) in order
- *Flow per Item*: total number of S/R operations performed for item
- *Lines per Item* (*popularity*): total number of lines for item in all orders
- *Cube Movement*: total unit demand of item time x cubic volume
- *Demand Correlation*: percent of orders in which both items appear

Customer Orders

Order: 1	
SKU	Qty
A	5
B	3
C	2
D	6

Order: 2	
SKU	Qty
A	4
C	1

Order: 3	
SKU	Qty
A	2

Order: 4	
SKU	Qty
B	2

Order: 5	
SKU	Qty
C	1
D	12
E	6

Item Master

SKU	Length	Width	Depth	Cube	Weight
A	5	3	2	30	1.25
B	3	2	4	24	4.75
C	8	6	5	180	9.65
D	4	4	3	32	6.35
E	6	4	5	120	8.20

Total Lines = 11

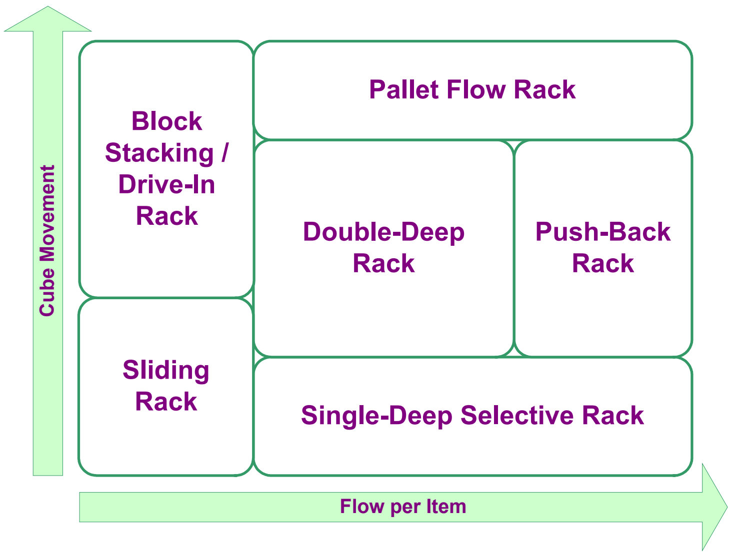
Lines per Order = 11/5 = 2.2

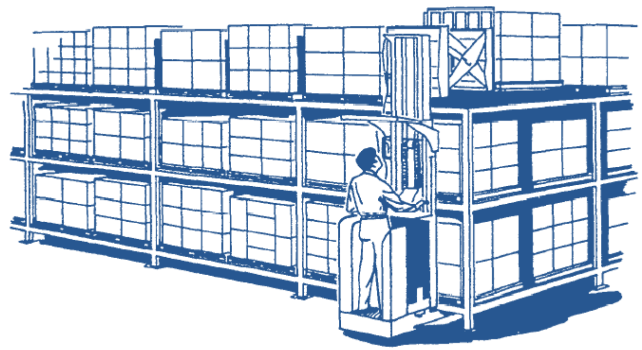
Cube per Order = 493.2

SKU	Flow per Item	Lines per Item	Cube Movement
A	11	3	330
B	5	2	120
C	4	3	720
D	18	2	576
E	6	1	720

Demand Correlation Distibution					
SKU	A	B	C	D	E
A		0.2	0.4	0.2	0.0
B			0.2	0.2	0.0
C				0.4	0.2
D					0.2
E					

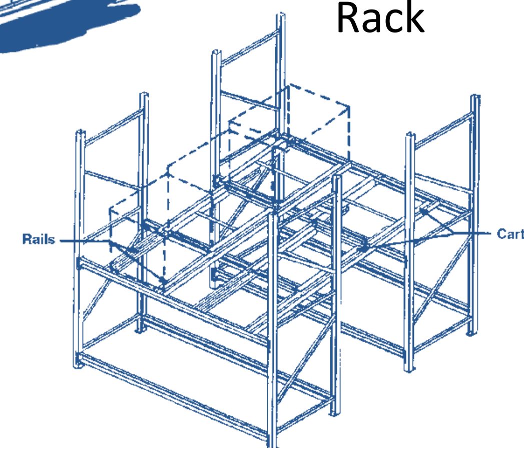
Pallet Picking Equipment

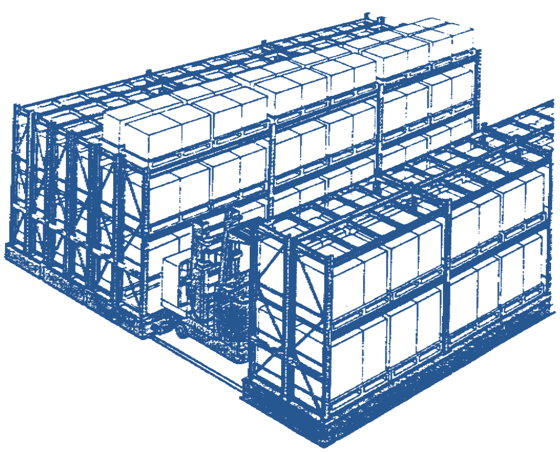




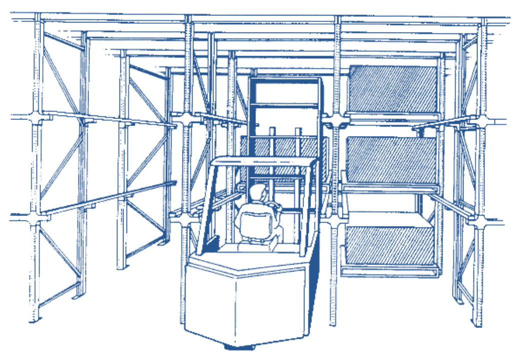
Double-Deep Rack

Push-Back Rack

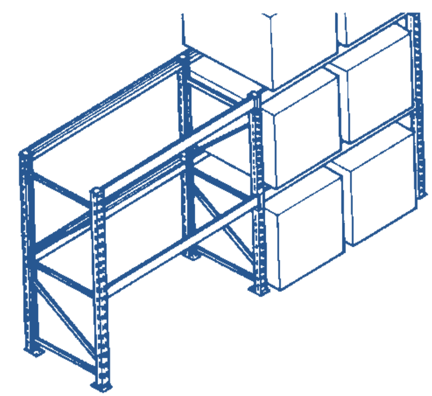




Sliding Rack

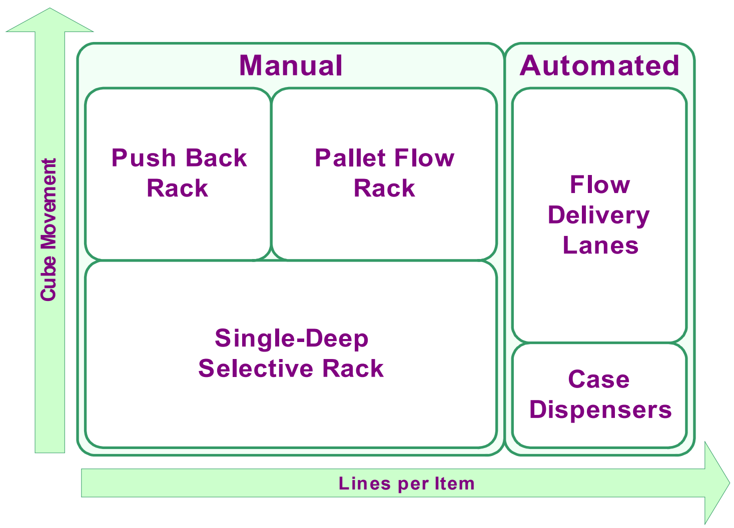


Drive-In Rack

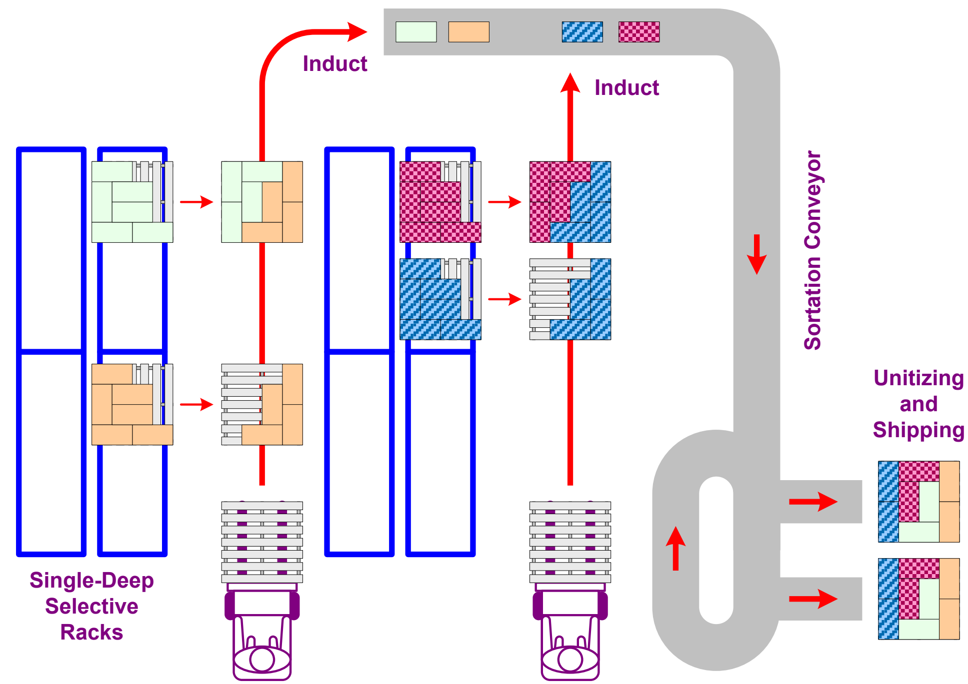


Single-Deep Selective Rack

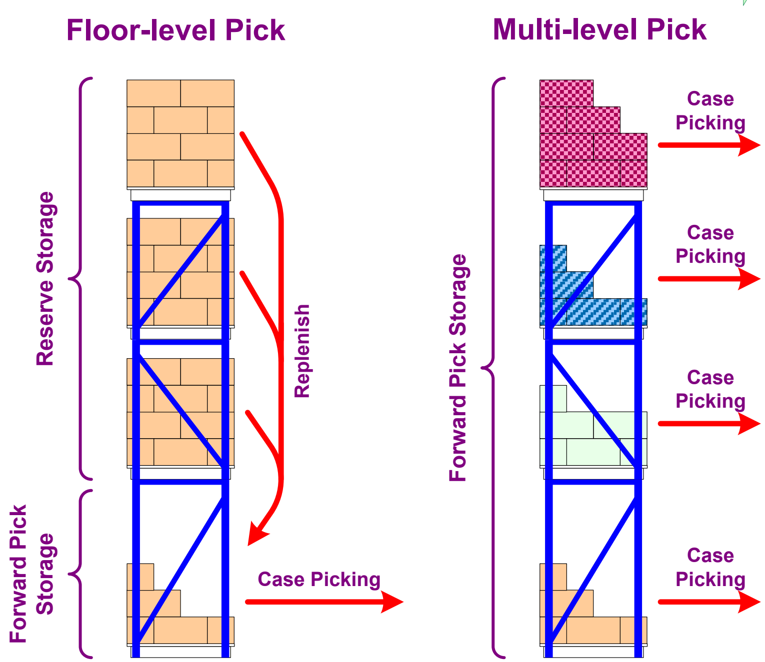
Case Picking



Case Picking Equipment

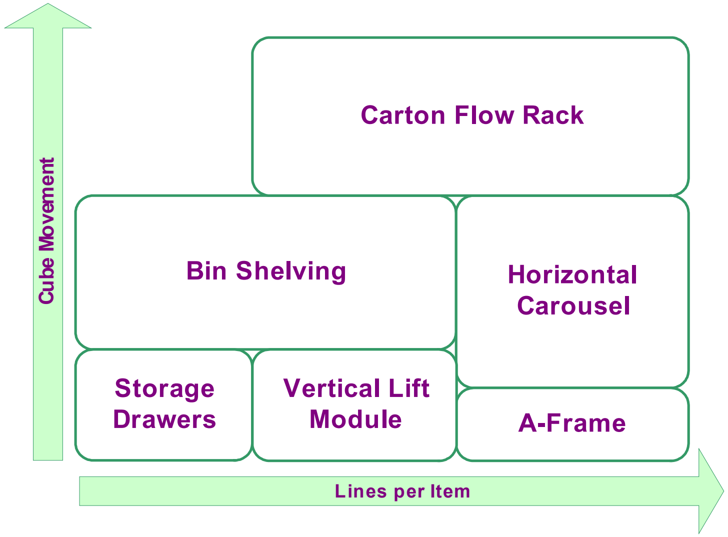


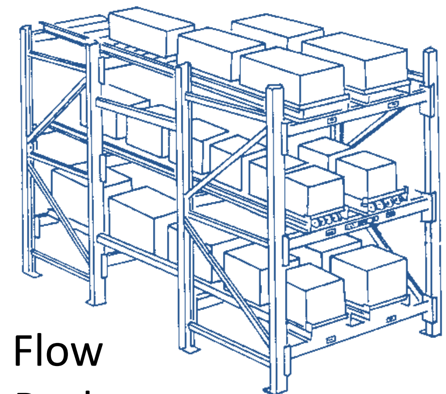
Zone-Batch Pick to Pallet



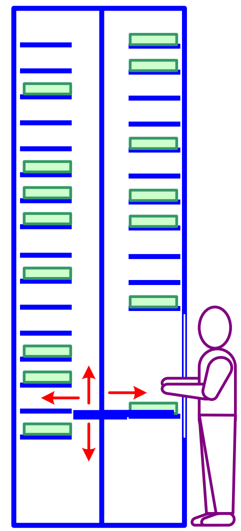
Floor- vs. Multi-level Pick to Pallet

Piece Picking Equipment

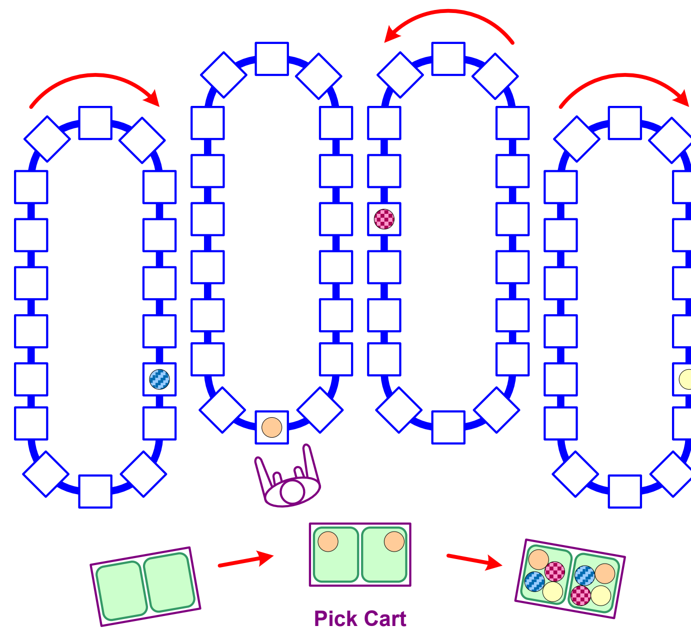




Carton Flow Rack

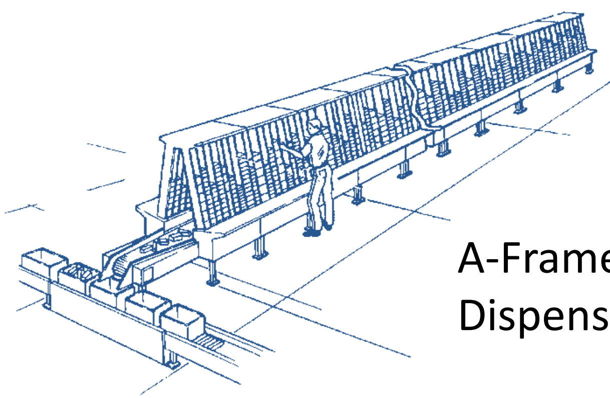


Vertical Lift Module



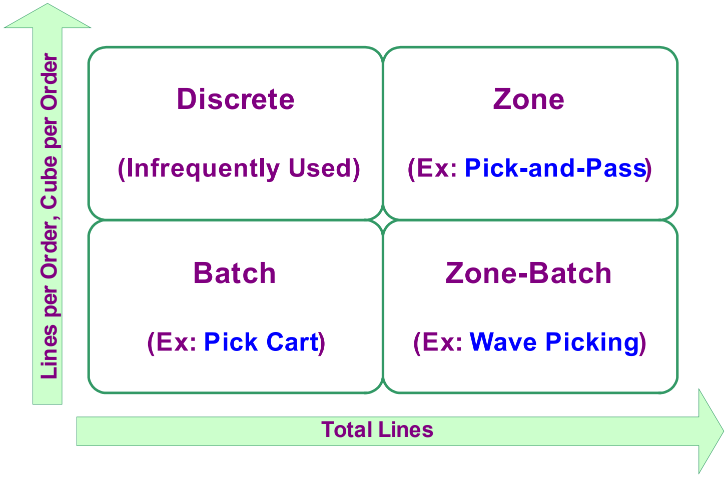
Carousel

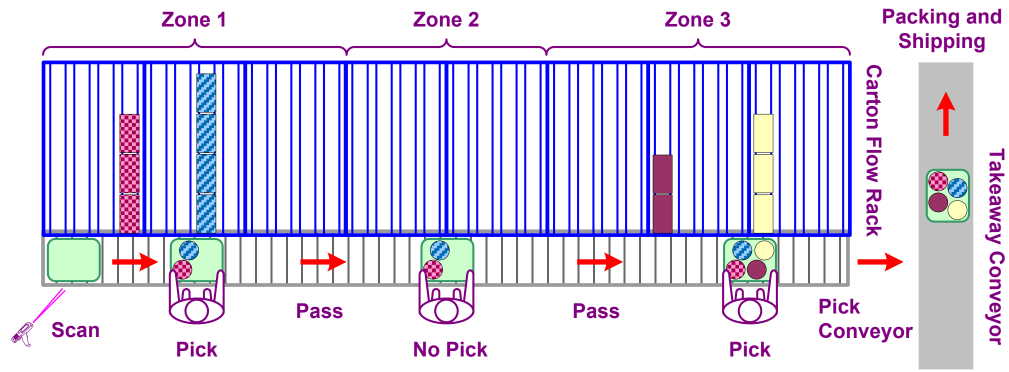
Drawers/Bins



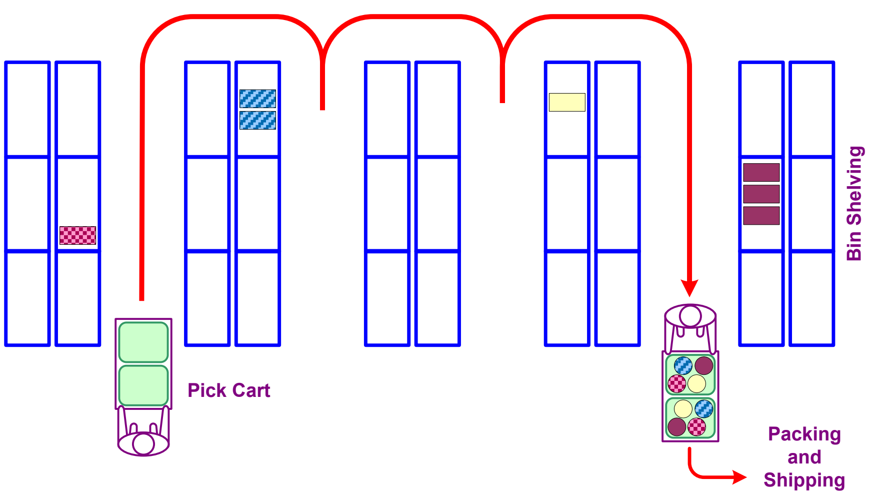
A-Frame Dispenser

Methods of Piece Picking

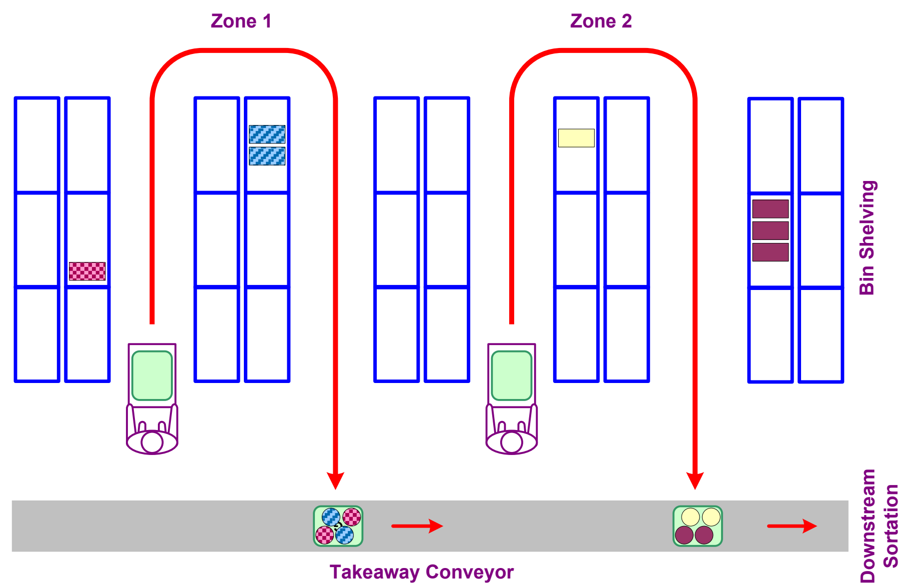




Pick-and-Pass Zone Piece Picking



Pick-cart Batch Piece Picking



Wave Zone-Batch Piece Picking

Warehouse Automation

- Historically, warehouse automation has been a craft industry, resulting highly customized, one-off, high-cost solutions
- To survive, need to
  - adapt mass-market, consumer-oriented technologies in order to realize to economies of scale
  - replace mechanical complexity with software complexity
- How much can be spent for automated equipment to replace one material handler:

$$\$45,432\left(\frac{1-1.017^{-5}}{0.017}\right)=\$45,432(4.75)=\$216,019$$

  - \$45,432: median moving machine operator annual wage + benefits
  - 1.7% average real interest rate 2005-2009 (real = nominal – inflation)
  - 5-year service life with no salvage (service life for Custom Software)

KIVA Mobile-Robotic Fulfillment System

- Goods-to-man order picking and fulfillment system
- Multi-agent-based control
  - Developed by Peter Wurman, former NCSU CSC professor
- Kiva now called Amazon Robotics
  - purchased by Amazon in 2012 for $775 million

Public WH Design (Problem 24)

- A *public warehouse* is a business that rents storage space to other firms on a month-to-month basis. They are often used by firms to supplement their own *private warehouses*.
- Min cost = Avg move cost (\$/move) + storage time cost (\$/slot-yr)

$$(a) \qquad AC_{\text{\$/mov}} = \frac{TC_{\text{\$/yr}}}{f_{\text{mov/yr}}} = \frac{TC_{\text{\$/yr}}}{2{,}000{,}000} \Rightarrow$$

$$TC_{\text{\$/yr}} = m_{\text{tr}} K_{\text{\$/tr-yr}} + \left(m_{\text{tr}} + 12\right) c_{\text{\$/lab-yr}}^{\text{lab}} + c_{\text{\$/hr}}^{\text{fuel}} f_{\text{mov/yr}}\, t_{\text{hr/mov}}$$

$$= m_{\text{tr}} K_{\text{\$/tr-yr}} + \left(m_{\text{tr}} + 12\right) c_{\text{\$/lab-yr}}^{\text{lab}} + 2.75(2{,}000{,}000)\frac{t_{\text{min/mov}}}{60} \Rightarrow$$

$$t_{\text{min/mov}} = t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{d_{SC}}{616} + 2\left(\frac{35}{60}\right) \Rightarrow d_{SC} = \sqrt{2}\sqrt{TA'}$$

Still need to determine: $m_{\text{tr}}, K_{\text{\$/tr-yr}}, c_{\text{\$/lab-yr}}^{\text{lab}}, TA'$

Public WH Design (Problem 24)

$$(b)\quad AC_{\$/\text{slot-yr}} = \frac{K_{\$/\text{yr}}}{M_{\text{slot}}}$$

Demand assumed uncorrelated since it belongs to different customers $\Rightarrow$

$$M = \left\lfloor \sum_{i=1}^{N} \left(\frac{M_i - SS_i}{2} + SS_i \right) + \frac{1}{2} \right\rfloor$$

$$= \left\lfloor 4,800 \left(\frac{250 - 0.06(250)}{2} + 15 \right) + \frac{1}{2} \right\rfloor = 636,000 \text{ slots}$$

$$IV_{0,\text{bldg}} = SV_{N,\text{bldg}} \Rightarrow K_{\$/\text{yr}} = i\, IV_{0,\text{bldg}} = 0.05\, IV_{0,\text{bldg}}$$

$$IV_{0,\text{bldg}} = \$15.50\, TA' \Rightarrow TA' = 1.15\, TA \Rightarrow$$

$$TA(D) = xL(D) \cdot \left(yD + \frac{A}{2} \right) = \frac{42}{12} L(D) \cdot \left(\frac{40}{12} D + \frac{7}{2} \right) \Rightarrow$$

Public WH Design (Problem 24)

$(b, cont)$

$$L(D) = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{636,000 + 4800H\left(\frac{D-1}{2}\right) + 4800\left(\frac{H-1}{2}\right)}{DH} \right\rceil \Rightarrow$$

$$H = \left\lfloor \frac{18}{z} \right\rfloor = \left\lfloor \frac{18}{42/12} \right\rfloor = 5 \quad \text{(building clear-height constraint)}$$

$$D = D^* = \left\lfloor \sqrt{\frac{A(2M-N)}{2NyH}} + \frac{1}{2} \right\rfloor = \left\lfloor \sqrt{\frac{7(2(636,000) - 4800)}{2(4800)\frac{40}{12}(5)}} + \frac{1}{2} \right\rfloor = 7$$

Public WH Design (Problem 24)

$(b, cont)$

$$\Rightarrow L = 20{,}503 \Rightarrow TA = 1{,}925{,}573 \Rightarrow TA' = 2{,}214{,}409 \Rightarrow$$

$$\Rightarrow IV_{0,\text{bldg}} = \$15.50\, TA' = \$15.50\,(2{,}214{,}409) = \$34{,}323{,}346$$

$$\Rightarrow K_{\$/\text{yr}} = 0.05\, IV_{0,\text{bldg}} = \$1{,}716{,}167 \Rightarrow$$

$$AC_{\$/\text{slot-yr}} = \frac{K_{\$/\text{yr}}}{M_{\text{slot}}} = \$2.70 \text{ per slot-yr}$$

Public WH Design (Problem 24)

$(a, cont)$

$$TA' = 2,214,409 \text{ ft}^2 \Rightarrow$$

$$d_{SC} = \sqrt{2}\sqrt{TA'} = \sqrt{2}\sqrt{2,214,409} = 2,104 \Rightarrow$$

$$t_{\text{min/mov}} = \frac{d_{SC}}{616} + 2\left(\frac{35}{60}\right) = 4.58$$

$$H' = 2(8)5(50) = 4000 \text{ hr/yr} \quad \text{(already using } H\text{)}$$

$$r_{peak} = 1.25\frac{f_{\text{mov/yr}}}{H'} = 1.25\frac{2,000,000}{4000} = 625 \text{ mov/hr}$$

$$m_{\text{tr}} = \lfloor r_a t_e + 1 \rfloor = \lfloor r_{peak} t_{\text{hr/mov}} + 1 \rfloor = \left\lfloor 625\frac{4.58}{60} + 1 \right\rfloor = 48 \text{ tr}$$

$$IV^{eff} = IV_0 - SV\left(1+i\right)^{-N} = 35,000 - 0.25(35,000)\left(1+0.05\right)^{-10}$$

$$= \$29,628$$

Public WH Design (Problem 24)

$(a, cont)$

$$K_{\text{tr/yr}} = IV^{eff}\left[\frac{i}{1-(1+i)^{-N}}\right] = 29,628\left[\frac{0.05}{1-(1+0.05)^{-10}}\right] = \$3,837$$

$$c_{\text{\$/lab-yr}}^{\text{lab}} = 15.00H' = \$60,000$$

$$TC_{\text{\$/yr}} = m_{\text{tr}}K_{\text{\$/tr-yr}} + \left(m_{\text{tr}} + 12\right)c_{\text{\$/lab-yr}}^{\text{lab}} + 2.75(2{,}000{,}000)\frac{t_{\text{min/mov}}}{60}$$

$$= 48(3,837) + \left(48+12\right)60,000 + 2.75(2{,}000{,}000)\frac{4.58}{60}$$

$$= \$4,204,286.27 \Rightarrow$$

$$AC_{\text{\$/mov}} = \frac{TC_{\text{\$/yr}}}{f_{\text{mov/yr}}} = \frac{4,204,286.27}{2{,}000{,}000} = \$2.10 \text{ per move}$$

Public WH Design (Problem 24)

- (c) What are other costs that should be added to each charge to better reflect the true costs of each activity?
  - most significant missing costs are the facility non-move-related operating costs, which should be added to the slot-year charge
- What about average unit cost of $46.75?
  - only possible impact of unit cost would be for any insurance coverage provided by the warehouse for items stored in the warehouse
- *Note*: Number of slots of max inventory, M, used to determine $AC_{\text{\$/slot-yr}}$ instead of the total slots in warehouse since unused HCL slots would underestimate cost:

$$\text{Total Slots} = L \times D \times H = 717,605$$

$$M = 636,000$$

$$HCL = 81,605$$