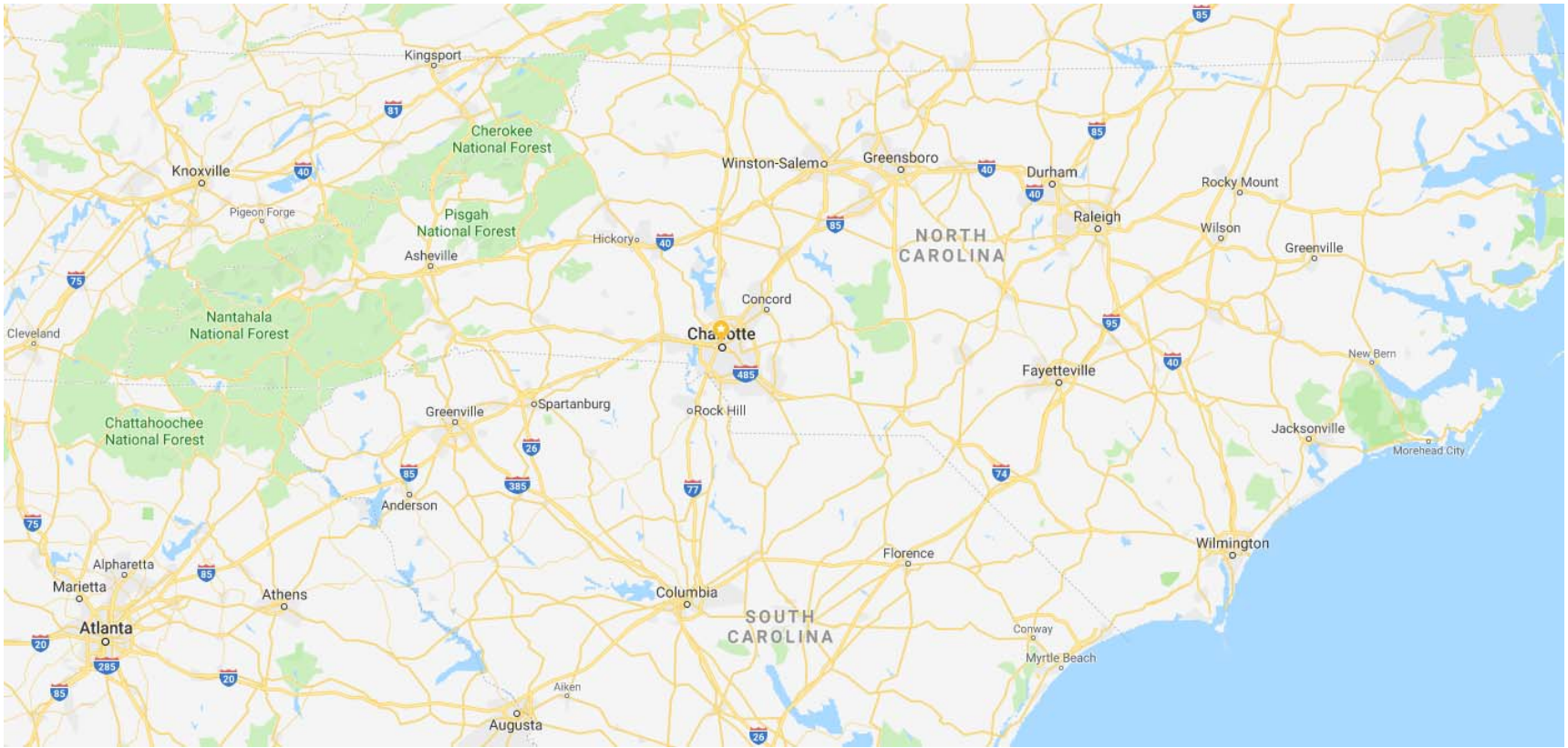
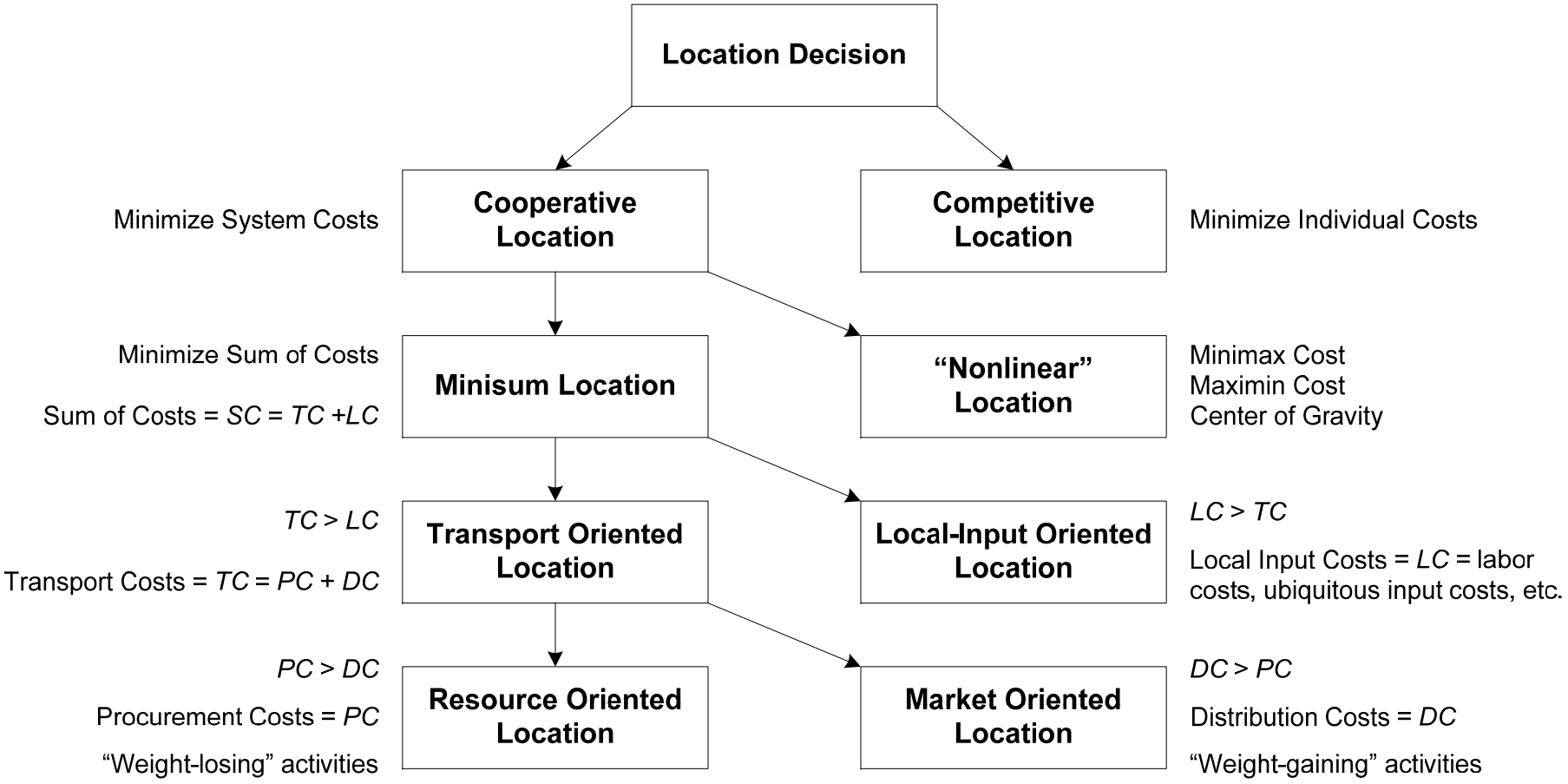
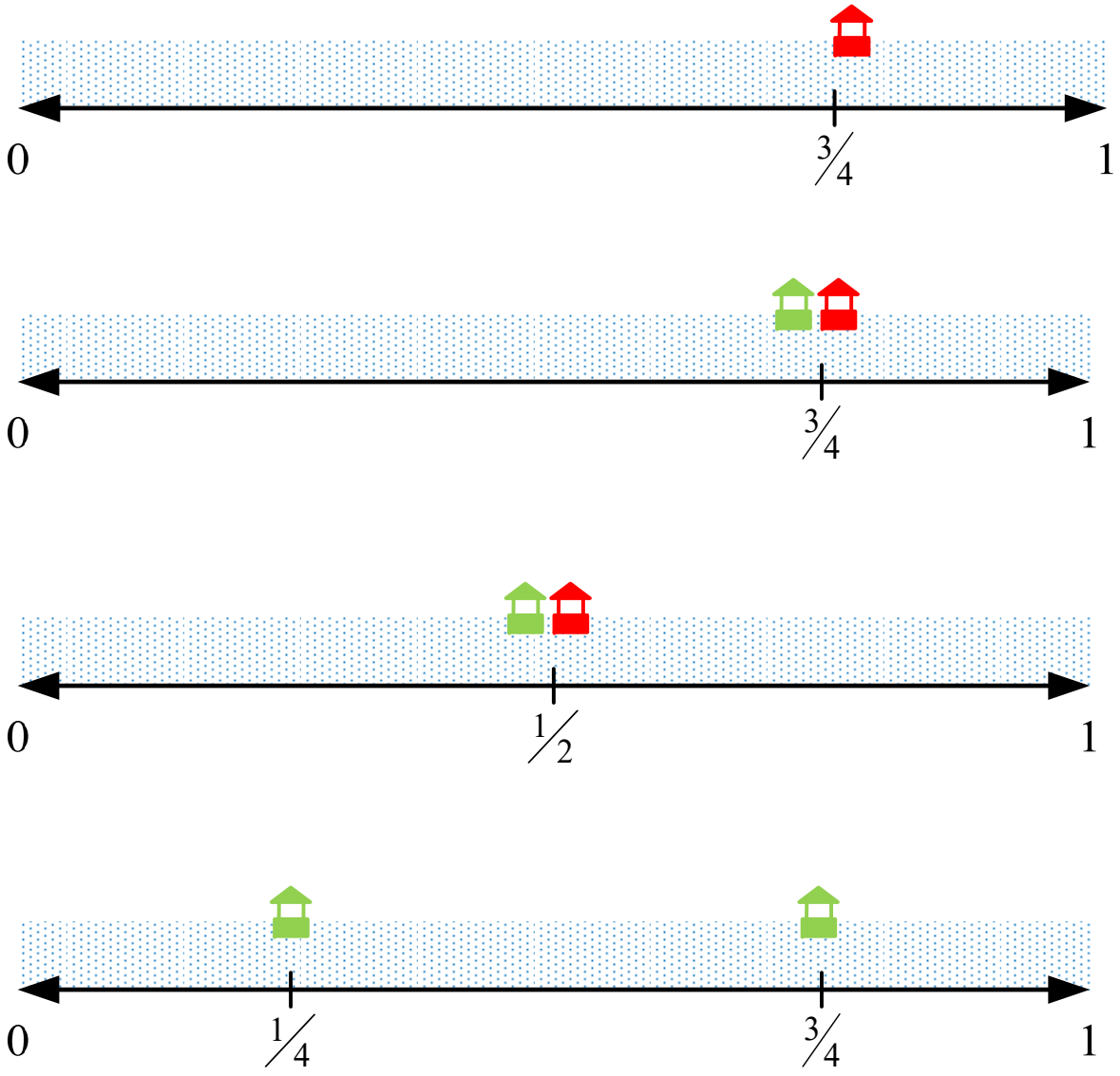
Why Are Cities Located Where They Are?



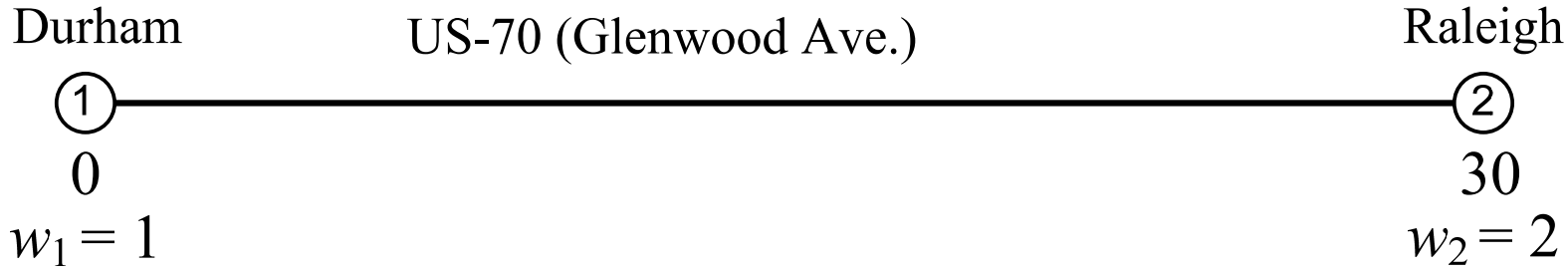
Taxonomy of Location Problems



Hotelling's Law



1-D Cooperative Location



$$\text{Min } TC = \sum w_i \, d_i$$

$$\text{Min } TC = \sum w_i \, d_i^2$$

$$\text{Min } TC = \sum w_i \, d_i^k$$

$$a_1 = 0, \quad a_2 = 30$$

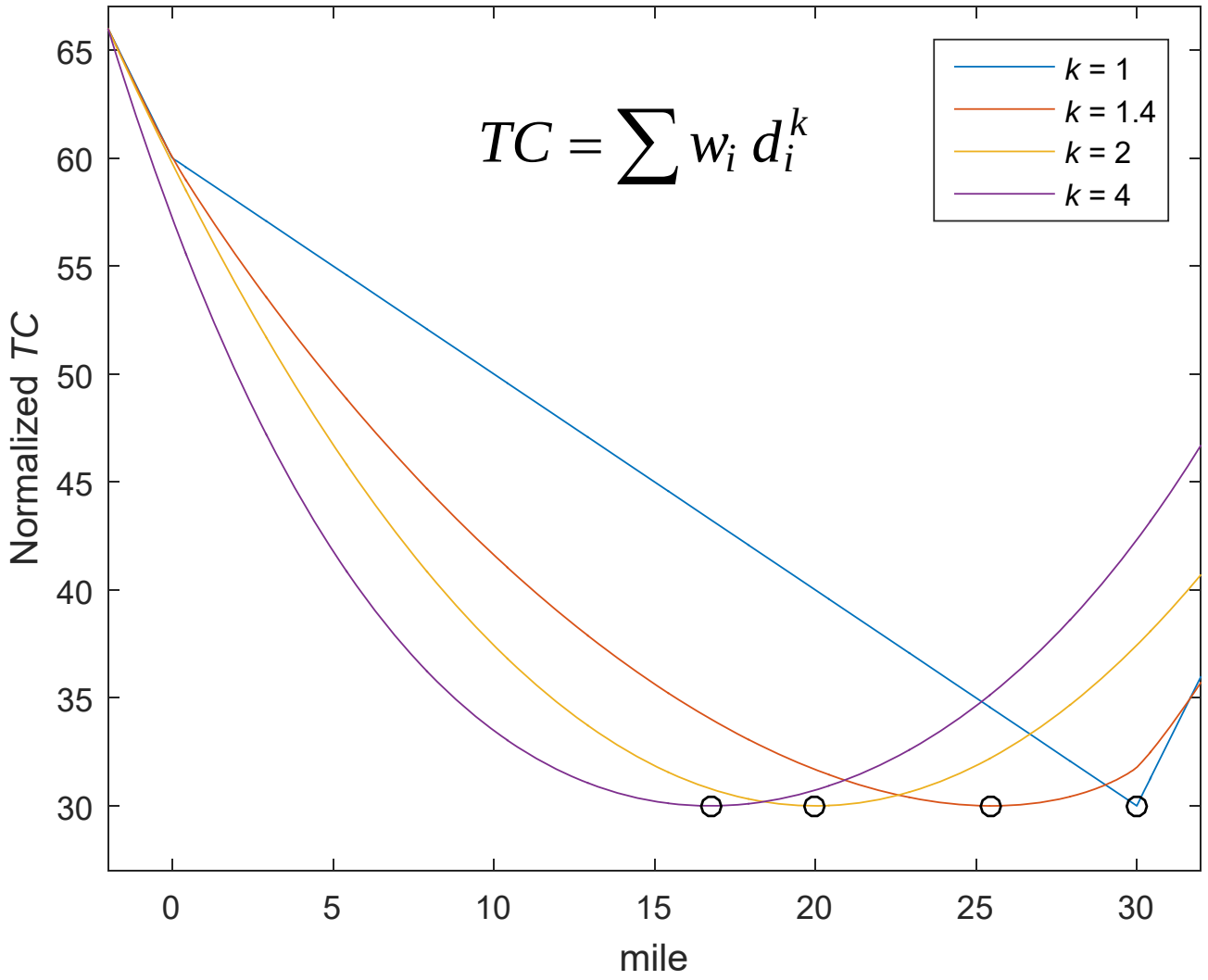
$$TC = \sum w_i \, d_i^2 = \sum w_i \left(x - a_i \right)^2$$

$$\frac{dTC}{dx} = 2 \sum w_i \left(x - a_i \right) = 0 \Rightarrow$$

$$x \sum w_i = \sum w_i \, a_i \Rightarrow$$

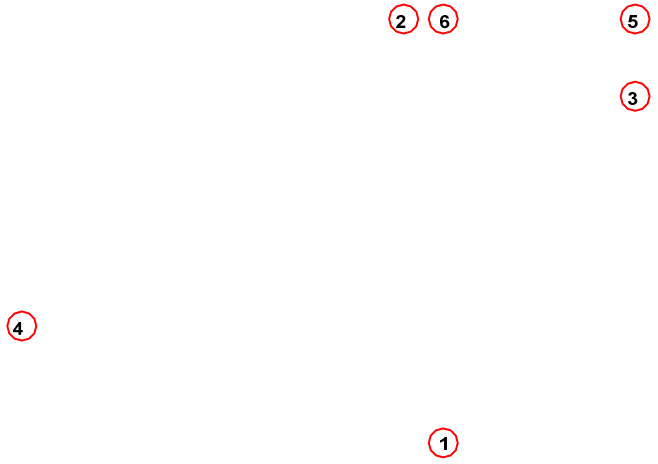
Squared−Euclidean Distance $\Rightarrow$ Center of Gravity: $$x^* = \frac{\sum w_i \, a_i}{\sum w_i} = \frac{1(0) + 2(30)}{1 + 2} = 20$$

“Nonlinear” Location



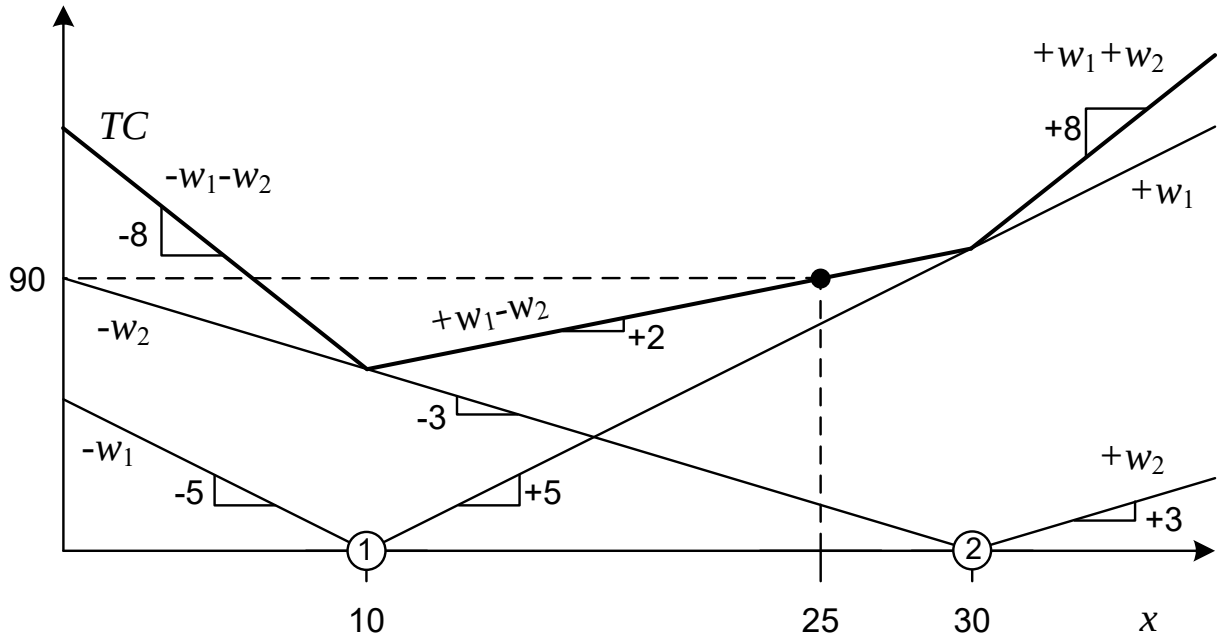
Minimax and Maximin Location

- Minimax
  - Min max distance
  - Set covering problem



- Maximin
  - Max min distance
  - AKA obnoxious facility location

2-EF Minisum Location



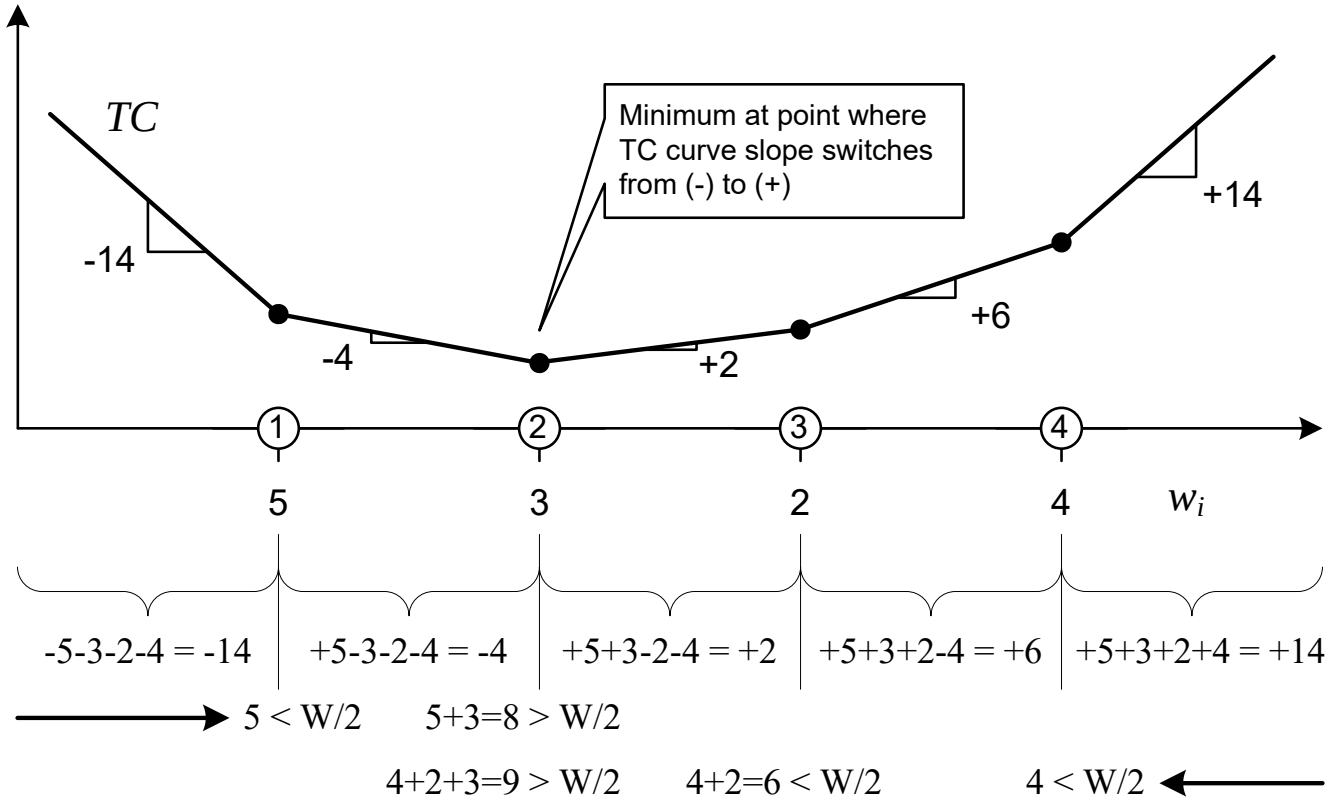
$$TC(x) = \sum w_i\, d_i = \beta_1(x - x_1) + \beta_2(x - x_2), \quad \text{where } \beta_i = \begin{cases} w_i, & \text{if } x \geq x_i \\ -w_i, & \text{if } x < x_i \end{cases}$$

$$TC(25) = w_1(25-10) + (-w_2)(25-30)$$
$$= 5(15) + (-3)(-5) = 90$$

Median Location: 1-D 4 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$



Median Location: 1-D 7 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$