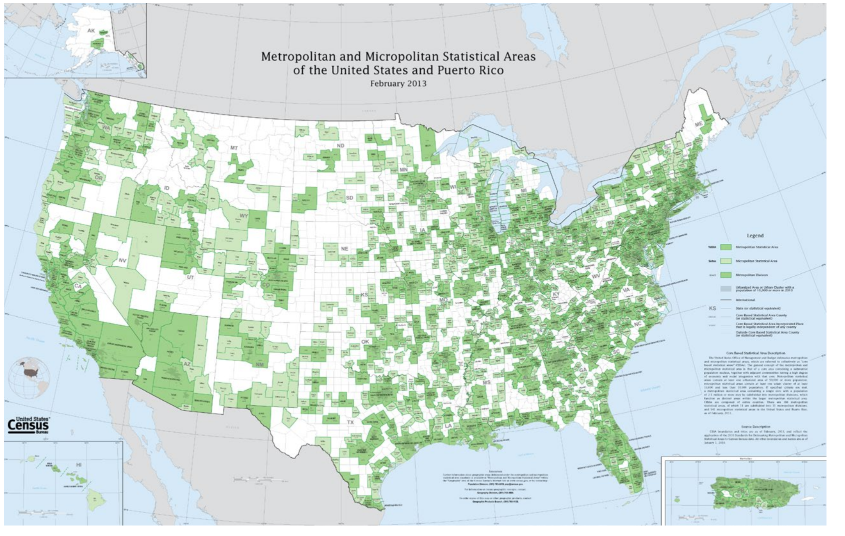
U.S. Geographic Statistical Areas

- Defined by Office of Management and Budget (OMB)
  - Each consists of one or more counties
- Top-to-bottom:
  1. Metropolitan divisions
  2. Combined statistical areas (CSAs)
  3. Core-based statistical areas (CBSAs)
  4. Metropolitian/ micropolitan statistical areas (MSAs)
  5. County (rural)



Aggregate Demand Point Data Sources

- Aggregate demand point: centroid of population + area + population
- Good rule of thumb: use at least 10x number of NFs (≈ 100 pts provides minimum coverage for locating ≈ 10 NFs)

1. City data: ONLY USE FOR LABELING!, not as aggregate demand points
2. 3-digit ZIP codes: ≈ 1000 pts covering U.S., = 20 pts NC
3. County data: ≈ 3000 pts covering U.S., = 100 pts NC
   - Grouped by state or CSA (Combined Statistical Area)
   - CSA = defined by set of counties (174 CSAs in U.S.)
   - FIPS code = 5-digit state-county FIPS code
     = 2-digit state code + 3-digit county code
     = 37183 = 37 NC FIPS + 183 Wake FIPS
   - CSA List: www2.census.gov/programs-surveys/metro-micro/geographies/reference-files/2017/delineation-files/list1.xls
4. 5-digit ZIP codes: > 35K pts U.S., ≈ 1000 pts NC
5. Census Block Group: > 220K pts U.S., ≈ 1000 pts Raleigh-Durham-Chapel Hill, NC CSA
   - Grouped by state, county, or CSA
   - Finest resolution aggregate demand data source

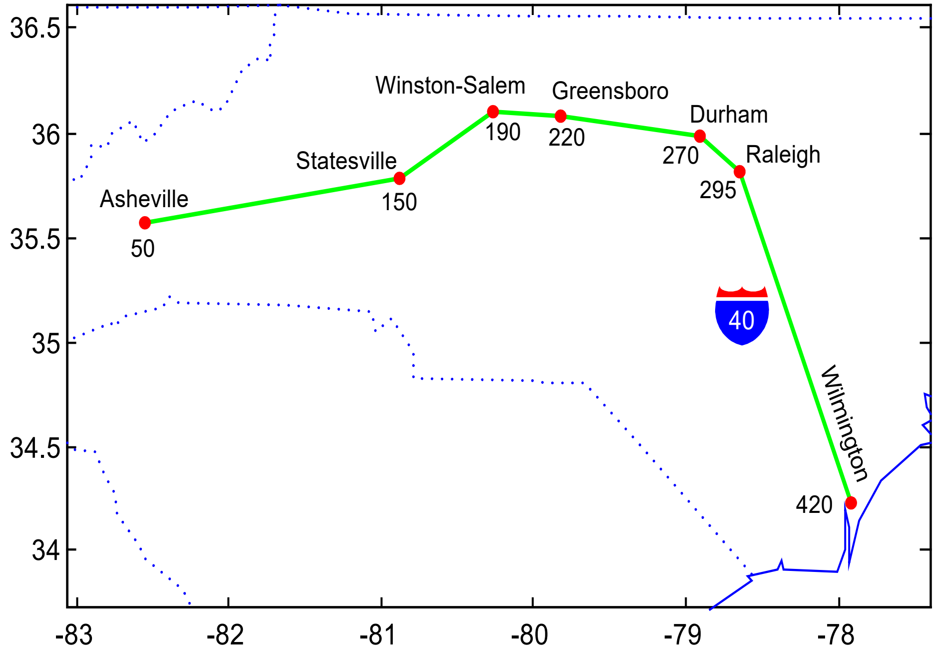
City vs CSA Population Data

Rank	City; State	2010 population	2012 population
1	New York City; New York	8,175,133	**8,336,697**
2	Los Angeles; California	3,792,621	**3,857,799**
3	Chicago; Illinois	2,695,598	**2,714,856**
4	Houston; Texas	2,099,451	**2,160,821**
5	Philadelphia; Pennsylvania	1,526,006	**1,547,607**
6	Phoenix; Arizona	1,445,632	**1,488,750**
7	San Antonio; Texas	1,327,407	**1,382,951**
8	San Diego; California	1,307,402	**1,338,348**
9	Dallas; Texas	1,197,816	**1,241,162**
10	San Jose; California	945,942	**982,765**
11	Austin; Texas	790,390	**842,592**
12	Jacksonville; Florida	821,784	**836,507**
13	Indianapolis; Indiana	820,445	**834,852**
14	San Francisco; California	805,235	**825,863**
15	Columbus; Ohio	787,033	**809,798**
16	Fort Worth; Texas	741,206	**777,992**
17	Charlotte; North Carolina	731,424	**775,202**
18	Detroit; Michigan	713,777	**701,475**
19	El Paso; Texas	649,121	**672,538**
20	Memphis; Tennessee	646,889	**655,155**

Metropolitan Area	2010 Population	City
New York-Northern NJ-Long Island, NY-NJ-PA	18,897,109	New York
Los Angeles-Long Beach-Santa Ana, CA	12,828,837	Los Angeles
Chicago-Joliet-Naperville, IL-IN-WI	9,461,105	Chicago
Dallas-Fort Worth-Arlington, TX	6,371,773	Dallas
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	5,965,343	Philadelphia
Houston-Sugar Land-Baytown, TX	5,946,800	Houston
Washington-Arlington-Alexandria, DC-VA-MD-WV	5,582,170	Washington
Miami-Fort Lauderdale-Pompano Beach, FL	5,564,635	Miami
Atlanta-Sandy Springs-Marietta, GA	5,268,860	Atlanta
Boston-Cambridge-Quincy, MA-NH	4,552,402	Boston
San Francisco-Oakland-Fremont, CA	4,335,391	San Francisco
Detroit-Warren-Livonia, MI	4,296,250	Detroit
Riverside-San Bernardino-Ontario, CA	4,224,851	Riverside
Phoenix-Mesa-Glendale, AZ	4,192,887	Phoenix
Seattle-Tacoma-Bellevue, WA	3,439,809	Seattle
Minneapolis-St. Paul-Bloomington, MN-WI	3,279,833	Minneapolis
San Diego-Carlsbad-San Marcos, CA	3,095,313	San Diego
St. Louis, MO-IL	2,812,896	St. Louis
Tampa-St. Petersburg-Clearwater, FL	2,783,243	Tampa
Baltimore-Towson, MD	2,710,489	Baltimore

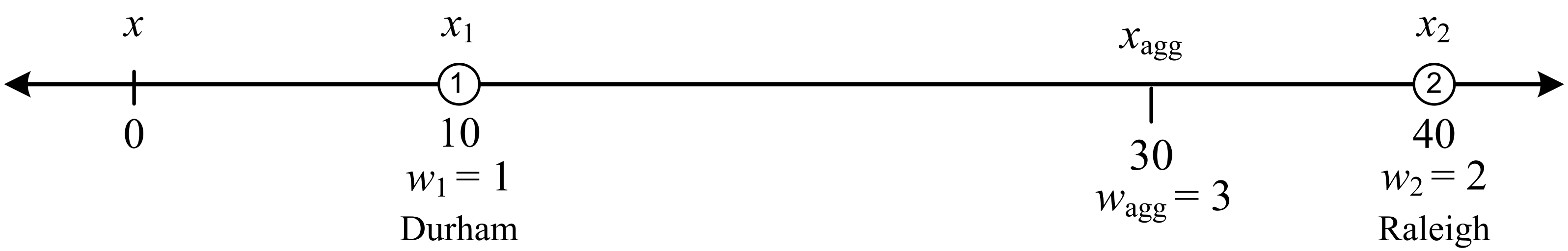
Demand Point Aggregation

- *Existing facility* (EF): actual physical location of demand source
- *Aggregate demand point*: single location representing multiple demand sources



Demand Point Aggregation

- Calculation of aggregate point depends on objective



- For minisum location, would like for any location x:

$$\left(w_1 + w_2\right) d(x, x_{\text{agg}}) = w_1 d(x, x_1) + w_2 d(x, x_2), \quad \text{let } x = 0,\, x_1, x_2 > 0$$

$$\left(w_1 + w_2\right) x_{\text{agg}} = w_1 x_1 + w_2 x_2$$

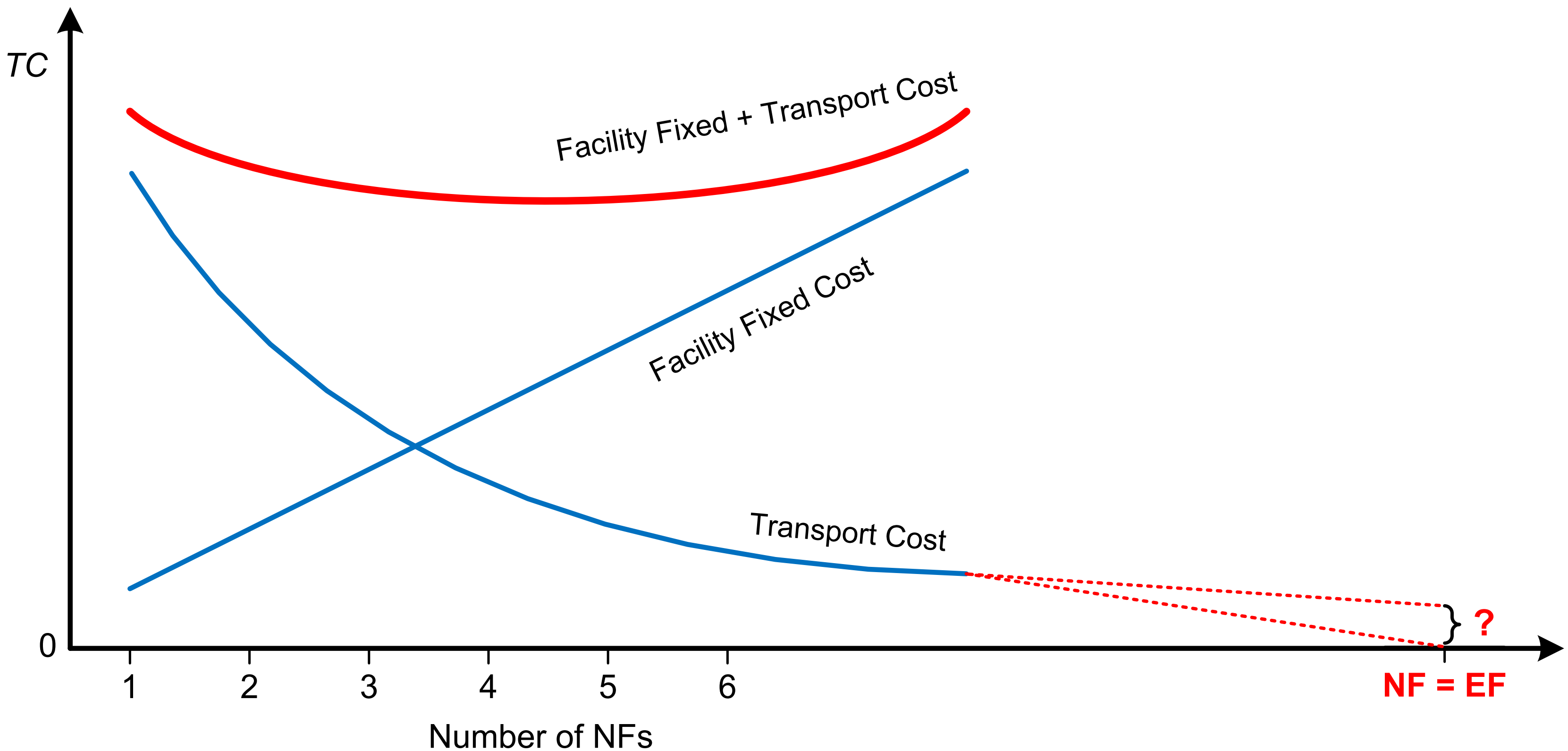
$$x_{\text{agg}} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2} \Rightarrow \quad \text{centroid}$$

Note: if $x_1 < x < x_2$, then x_{agg} not centroid

- For squared distance: $\left(w_1 + w_2\right) x_{\text{agg}}^2 = w_1 x_1^2 + w_2 x_2^2$

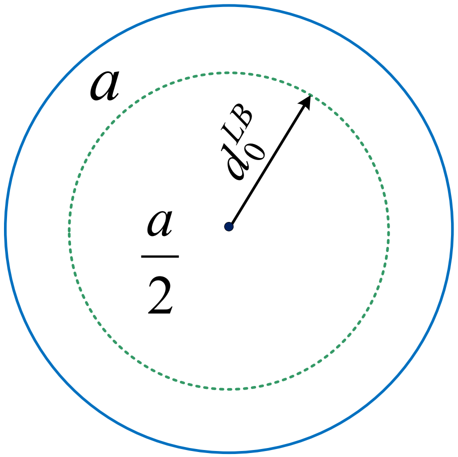
$$x_{\text{agg}} = \sqrt{\frac{w_1 x_1^2 + w_2 x_2^2}{w_1 + w_2}} \Rightarrow \quad \text{not centroid}$$

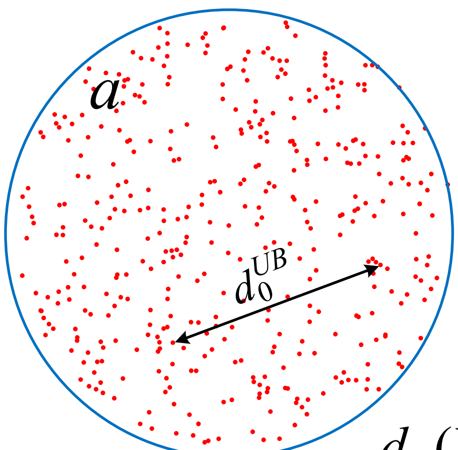
Transport Cost if NF at every EF



$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

Area Adjustment for Aggregate Data Distances





- *LB:* avg. dist. from center to all points in area
- *UB:* avg. dist. between all random pairs of points
- Local circuity factor = 1.5, regular non-local = 1.2

$$\frac{a}{2} = \pi \left(d_0^{LB} \right)^2$$

$$d_0^{LB} = \sqrt{\frac{a}{2\pi}} \approx 0.40\sqrt{a}$$

$$d_0^{UB} = \frac{32}{15} \frac{\sqrt{a}}{\pi \Gamma\left(5/2 \right)} \approx 0.51\sqrt{a}$$

Mathai, A.M., *An Intro to Geo Prob*, p. 207 (2.3.68)

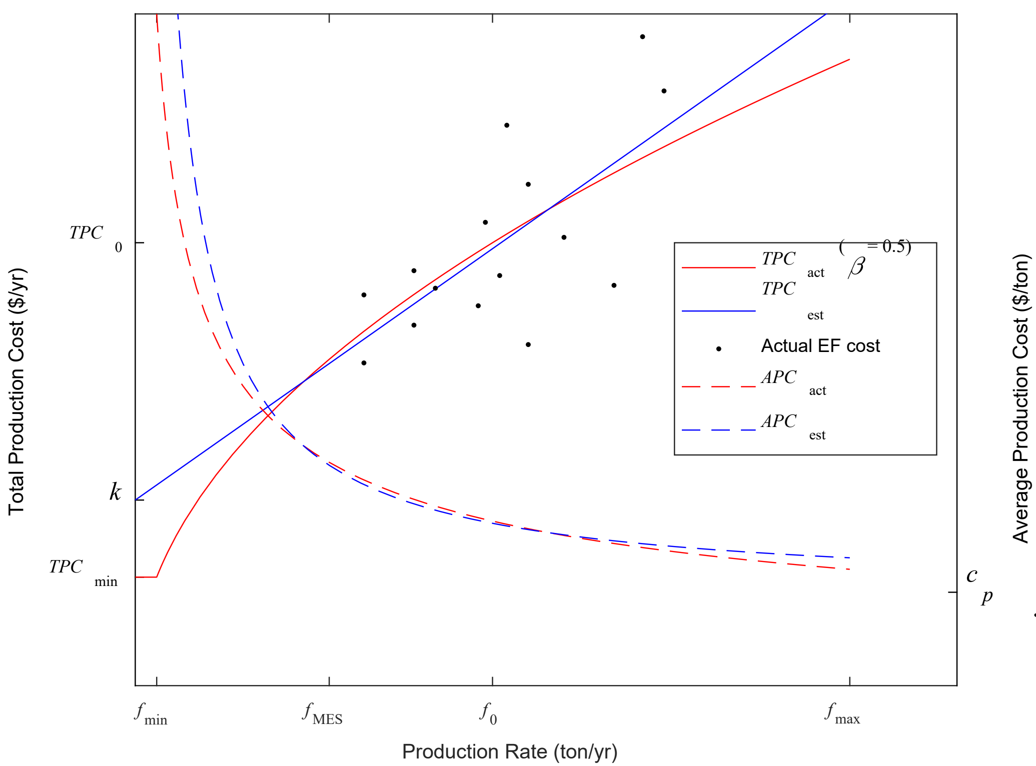
$$d_0 = \sqrt{d_0^{LB} d_0^{UB}} \approx 0.45\sqrt{a}$$

$$d_a(\mathbf{X}_1, \mathbf{X}_2) = \max\left\{ g\, d_{GC}(\mathbf{X}_1, \mathbf{X}_2),\ g_{\text{local}} 0.45 \max\left\{ \sqrt{a_1}, \sqrt{a_2} \right\} \right\}$$

$$= \max\left\{ 1.2 d_{GC}(\mathbf{X}_1, \mathbf{X}_2),\ 0.675 \max\left\{ \sqrt{a_1}, \sqrt{a_2} \right\} \right\}$$

Fixed Cost and Economies of Scale

- Cost data from existing facilities can be used to fit linear estimate
  - Economies of scale in production $\Rightarrow k > 0$ and $\beta < 1$



$$TPC_{\text{act}} = \max_{f<f_{\max}} \left\{ TPC_{\min}, TPC_0 \left(\frac{f}{f_0} \right)^{\beta} \right\}$$

$$\beta = \begin{cases} 0.62, & \text{Hand tool mfg.} \\ 0.48, & \text{Construction} \\ 0.41, & \text{Chemical processing} \\ 0.23, & \text{Medical centers} \end{cases}$$

$$TPC_{\text{est}} = k + c_p f$$

$$APC_{\text{act}} = \frac{TPC_{\text{act}}}{f} = \frac{TPC_0}{f_0^{\beta}} f^{\beta-1}$$

$$APC_{\text{est}} = \frac{k}{f} + c_p$$

$$k = \text{fixed cost}$$

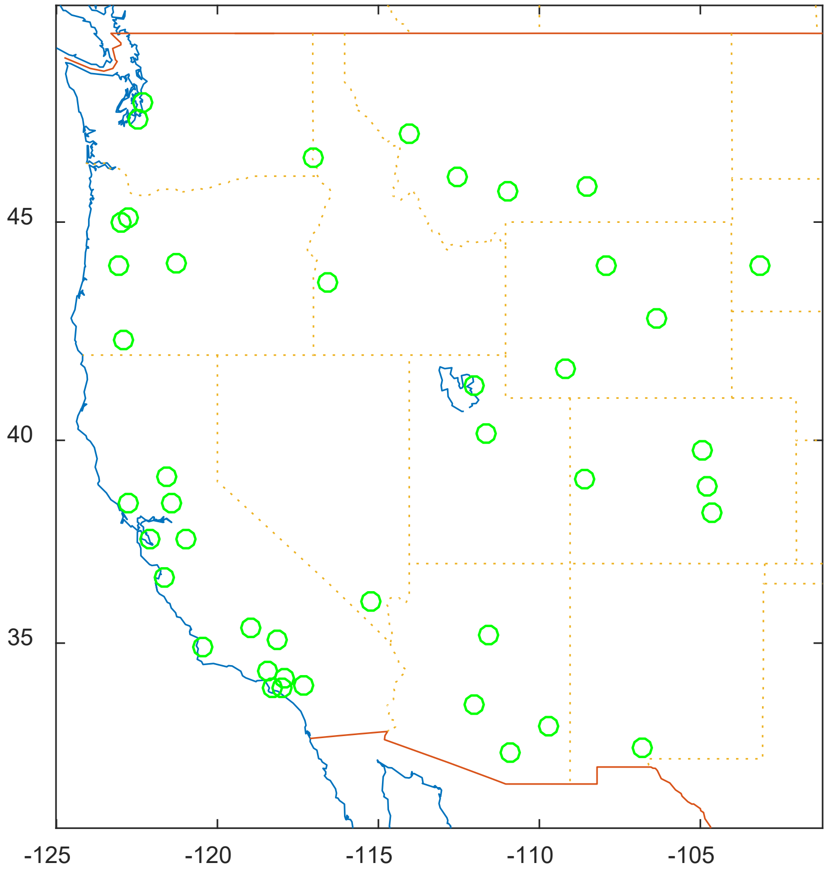
$$c_p = \text{constant unit production cost}$$

$$f_{\min}/f_{\max} = \text{min/max feasible scale}$$

$$f_{\text{MES}} = \textit{Minimum Efficient Scale}$$

$$TPC_0/f_0 = \text{base cost/rate}$$

Ex 9: Popco Bottling Company



- **Problem:** Popco currently has 42 bottling plants across the western U.S. and wants to know if they should consider reducing or adding plants to improve their profitability.
- **Solution:** Formulate as an UFL to determine the number of plants that minimize Popco's production, procurement, and distribution costs.

Ex 9: Popco Bottling Company

- Following representative information is available for each of *N* current plants (DC) *i*:

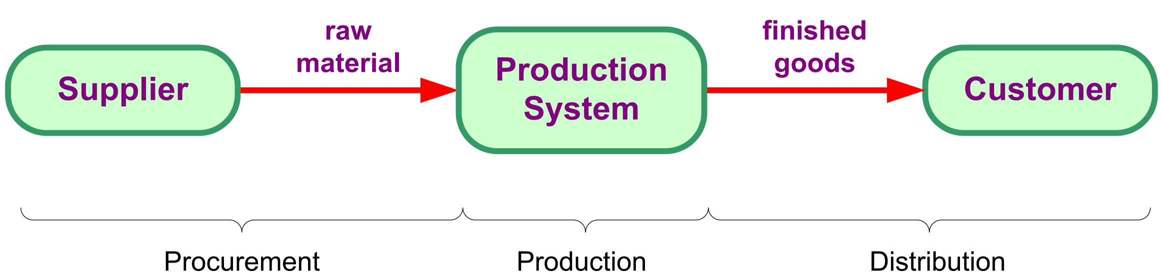
$$xy_i = \text{location}$$

$$f_i^{DC} = \text{aggregate production (tons)}$$

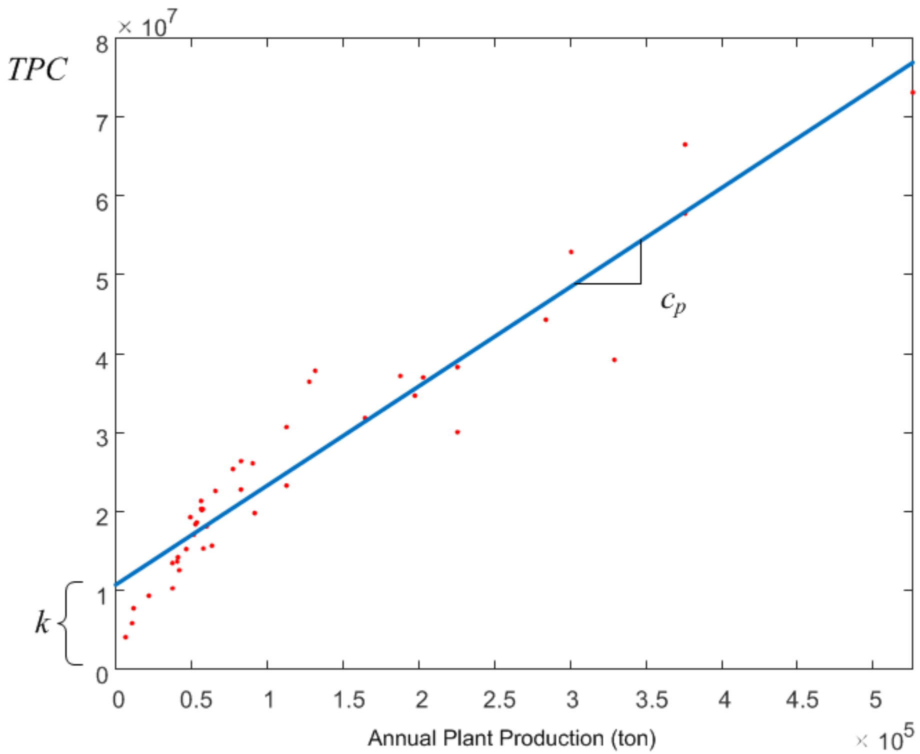
$$TPC_i = \text{total production and procurement cost}$$

$$TDC_i = \text{total distribution cost}$$

- Assuming plants are (monetarily) weight gaining since they are bottling plants, so UFL can ignore inbound procurement costs related to location



Ex 9: Popco Bottling Company



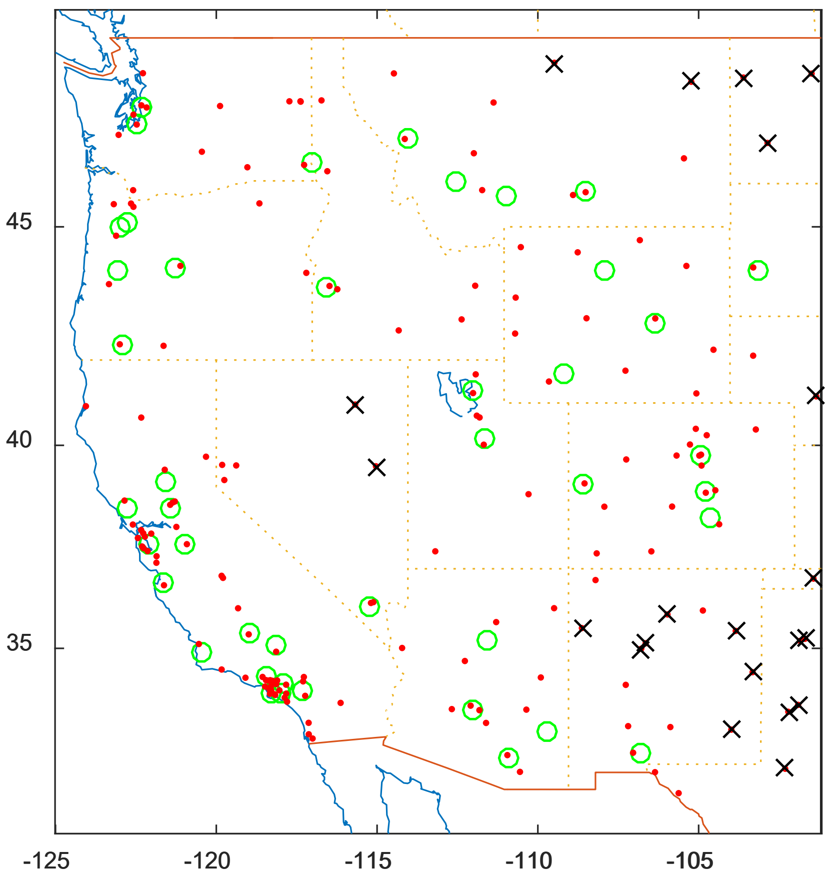
- Difficult to estimate fixed cost of each new facility because this cost must not include any cost related to quantity of product produced at facility.

1. Use plant (DC) production costs to find UFL fixed costs via linear regression
   – variable production costs c_p do not change and can be cut

$$TPC = \sum_{i \in N} TPC_i = \sum_{i \in N} \left(k + c_p f_i^{DC} \right)$$

$$\left(\text{only keep } k \text{ for UFL} \right)$$

Ex 9: Popco Bottling Company



2. Allocate all 3-digit ZIP codes to closest plant (up to 200 mi max) to serve as aggregate customer demand points.

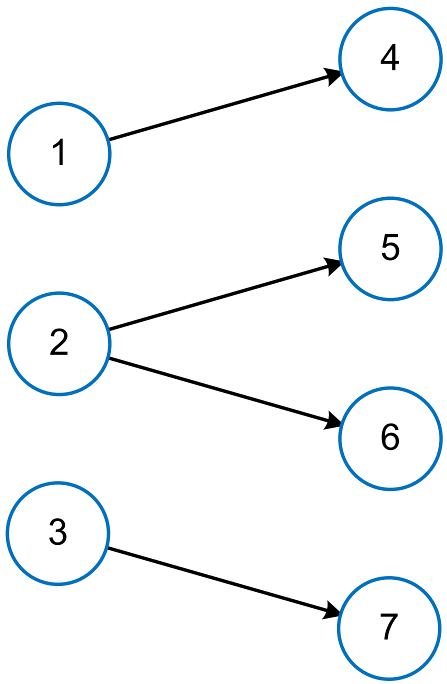
$$M_i = \left\{ j : \arg\min_h d_{hj}^a = i \text{ and } d_{ij}^a \le d_{\max} \right\}$$

$$d_{\max} = 200 \text{ mi}$$

$$M = \bigcup_{i \in N} M_i$$

Ex 9: Popco Bottling Company

3. Allocate each plant's demand (tons of product) to each of its customers based on its population.



$$f_{j \in M_i} = f_i^{DC} \frac{q_j}{\sum_{h \in M_i} q_h}$$

$$q_j = \text{population of EF}j$$

$$f_5 = f_2^{DC} \frac{q_5}{q_5 + q_6}$$

Ex 9: Popco Bottling Company

4. Estimate a nominal transport rate (\$/ton-mi) using the ratio of total distribution cost (\$) to the sum of the product of the demand (ton) at each customer and its distance to its plant (mi).

$$r_{\text{nom}} = \frac{\sum_{i \in N} TDC_i}{\sum_{i \in N} \sum_{j \in M_j} f_j \, d_{ij}^a}$$

Ex 9: Popco Bottling Company

5. Calculate UFL variable transportation cost c_{ij} ($) for each possible NF site *i* (all customer and plant locations) and EF site *j* (all customer locations) as the product of customer *j* demand (ton), distance from site *i* to *j* (mi), and the nominal transport rate ($/ton-mi).

$$\mathbf{C} = \left[c_{ij} \right]_{\substack{i \in M \cup N \\ j \in M}} = \left[r_{\text{nom}} f_j \, d_{ij}^{a} \right]_{\substack{i \in M \cup N \\ j \in M}}$$

6. Solve as UFL, where *TC* returned includes all new distribution costs and the fixed portion of production costs.

$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

$$n = \left| M \cup N \right|, \quad \text{number of potential NF sites}$$

$$m = \left| M \right|, \quad \text{number of EF sites}$$