Truck Shipment Example: Periodic

11. Continuing with the example: assuming a constant annual demand for the product of 20 tons, what is the number of full truckloads per year?

$$f = 20 \text{ ton/yr}$$

$$q = q_{\text{max}} = 6.1111 \text{ ton/ TL} \quad (\text{full truckload} \Rightarrow q \equiv q_{\text{max}})$$

$$n = \frac{f}{q} = \frac{20}{6.1111} = 3.2727 \text{ TL/yr}, \quad \text{average shipment frequency}$$

- Why should this number not be rounded to an integer value?

Truck Shipment Example: Periodic

12. What is the shipment interval?

$$t = \frac{1}{n} = \frac{q}{f} = \frac{6.1111}{20} = 0.3056 \text{ yr/TL}, \quad \text{average shipment interval}$$

- How many days are there between shipments?

$$365.25 \text{ day/yr}$$

$$t \times 365.25 = \frac{365.25}{n} = 111.6042 \text{ day/TL}$$

Truck Shipment Example: Periodic

13. What is the annual full-truckload transport cost?

$$d = 532 \text{ mi}, \quad r_{TL} = \$2.5511/\text{mi}$$

$$r_{FTL} = \frac{r_{TL}}{q_{\max}} = \frac{2.5511}{6.1111} = \$0.4175/\text{ton-mi}$$

$$TC_{FTL} = f\, r_{FTL}\, d = n\, r_{TL} d \quad \left(= w\,d,\, w = \text{monetary weight in \$/mi}\right)$$

$$= 3.2727\,(2.5511)\,532 = \$4,441.73/\text{yr}$$

- What would be the cost if the shipments were to be made at least every three months?

$$t_{\max} = \frac{3}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 4 \text{ TL/yr} \quad \Rightarrow \quad q = \frac{f}{\max\left\{n, n_{\min}\right\}}$$

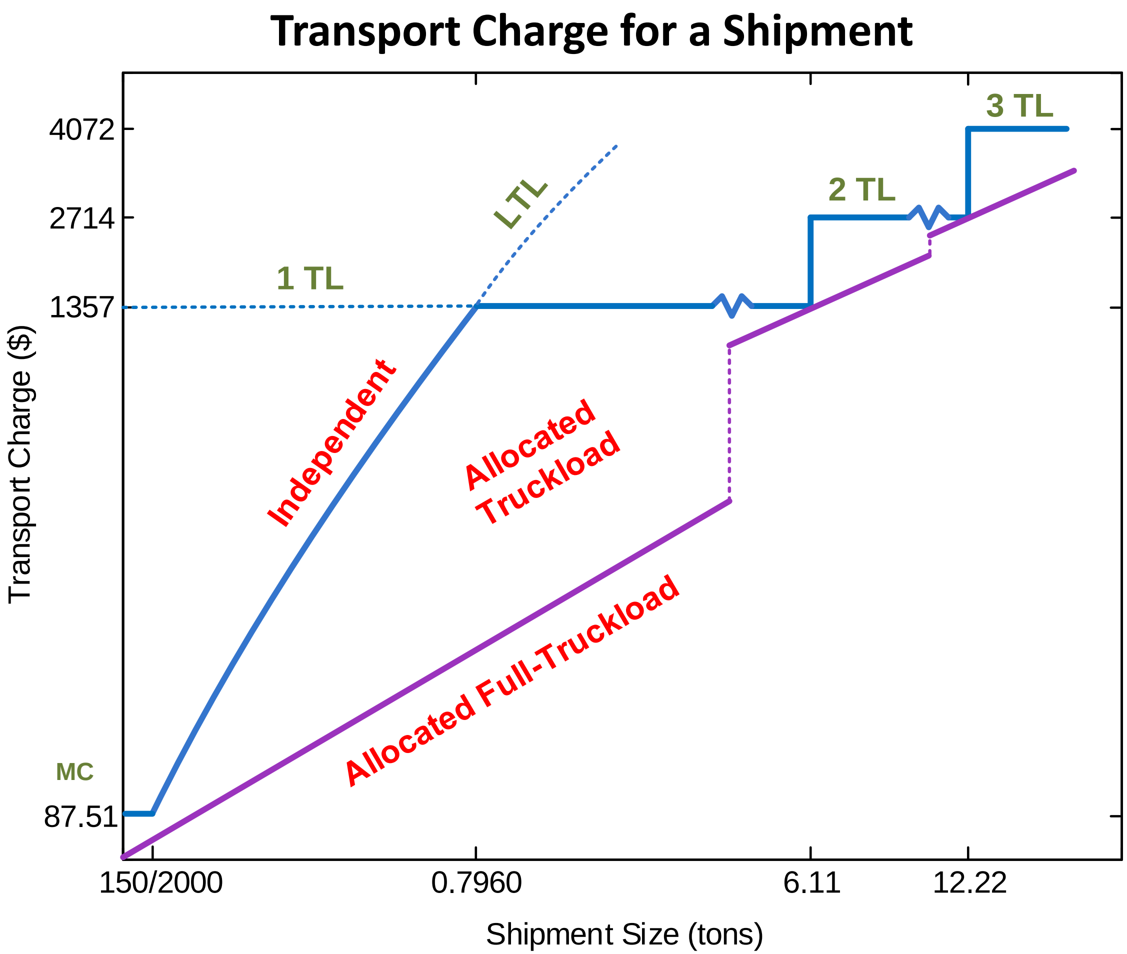
$$TC'_{FTL} = \max\left\{n, n_{\min}\right\} r_{TL} d$$

$$= \max\left\{3.2727, 4\right\} 2.5511(532) = \$5,428.78/\text{yr}$$

Truck Shipment Example: Periodic

- Independent and allocated full-truckload charges:

$$q \le q_{\max} \Rightarrow \left[UB, LB\right] = \left[c_0(q), q r_{FTL} d\right]$$



Truck Shipment Example: Periodic

- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$
\begin{aligned}
TLC &= TC + IC + PC \\
TC &= \text{transport cost} \\
IC &= \text{inventory cost} \\
&= IC_{\text{cycle}} + IC_{\text{pipeline}} + IC_{\text{safety}} \\
PC &= \text{purchase cost}
\end{aligned}
$$

- *Cycle inventory*: held to allow cheaper large shipments
- *Pipeline inventory*: goods in transit or awaiting transshipment
- *Safety stock*: held due to transport uncertainty
- *Purchase cost*: can be different for different suppliers

Truck Shipment Example: Periodic

- Same units of inventory can serve multiple roles at each position in a production process

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)
- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)
- *Safety stock:* held to buffer effects of uncertainty
  - (decoupling, MRO (maintenance, repair, and operations))

Truck Shipment Example: Periodic

14. Since demand is constant throughout the year, one half of a shipment is stored at the destination, on average. Assuming that the production rate is also constant, one half of a shipment will also be stored at the origin, on average. Assuming each ton of the product is valued at $25,000, what is a "reasonable estimate" for the total annual cost for this cycle inventory?

$$IC_{\text{cycle}} = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$

$$= (vh)(\alpha q)$$

$$v = \text{unit value of shipment (\$/ton)}$$

$$h = \text{inventory carrying rate, the cost per dollar of inventory per year (1/yr)}$$

$$\alpha = \text{average inter-shipment inventory fraction at Origin and Destination}$$

$$q = \text{shipment size (ton)}$$

Truck Shipment Example: Periodic

- **Inv. Carrying Rate** (h) = interest + warehousing + obsolescence
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate (yr) h = 0.3 $\Rightarrow h_{\text{obs}} \approx$ 0.2 (mfg product)
  - *Low* FGI cost (yr): $h = h_{\text{int}} + h_{\text{wh}} + h_{\text{obs}}$
  - *High* FGI cost (hr): $h \approx h_{\text{obs}}$, can ignore interest & warehousing
    - $(h_{\text{int}}+h_{\text{wh}})/H$ = (0.05+0.06)/2000 = 0.000055 (H = oper. hr/yr)
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h_{obs}

$$h_{\text{obs}} t_h v = x_h v \Rightarrow h_{\text{obs}} t_h = x_h \Rightarrow \boxed{h_{\text{obs}} = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h_{\text{obs}}}$$

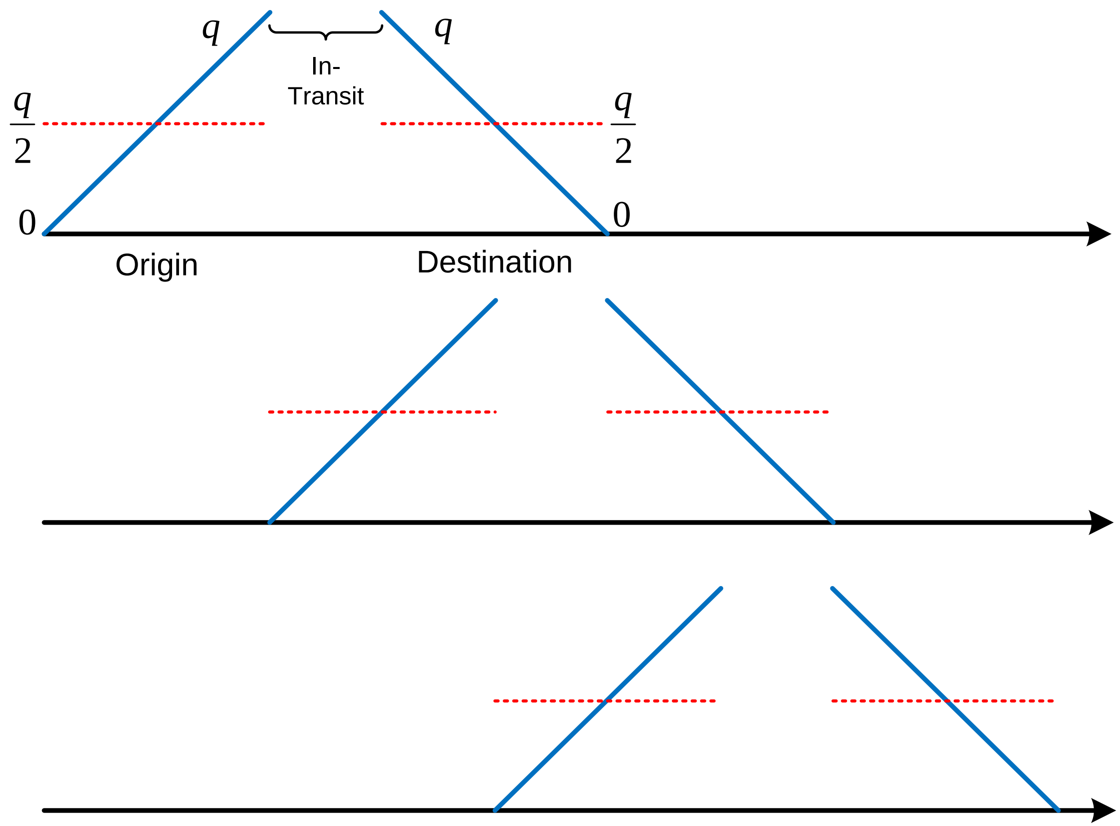
  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as t_{CT}

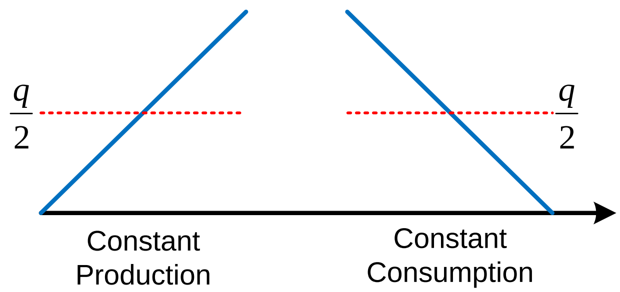
Truck Shipment Example: Periodic

- Average annual inventory level $= \frac{q}{2} + \frac{q}{2} = \left(\frac{1}{2} + \frac{1}{2}\right)q = (1)q \Rightarrow \alpha = 1$

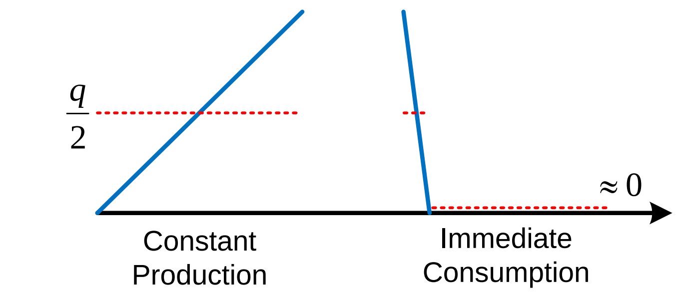


Truck Shipment Example: Periodic

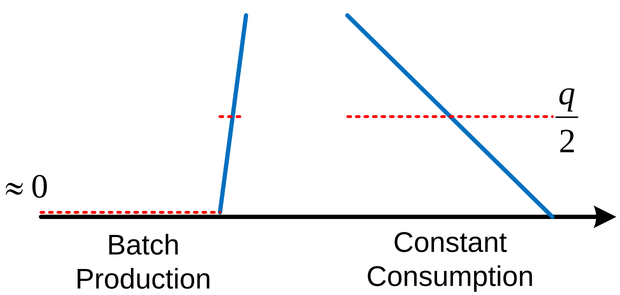
- Inter-shipment inventory fraction alternatives: $\alpha = \alpha_O + \alpha_D$



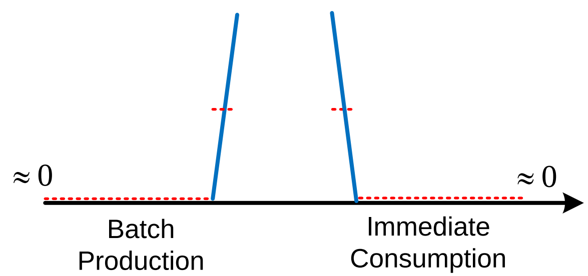
$$\alpha = \frac{1}{2} + \frac{1}{2} = 1$$



$$\alpha = \frac{1}{2} + 0 = \frac{1}{2}$$



$$\alpha = 0 + \frac{1}{2} = \frac{1}{2}$$



$$\alpha = 0 + 0 = 0$$

Truck Shipment Example: Periodic

- "Reasonable estimate" for the total annual cost for the cycle inventory:

$$\begin{aligned} IC_{\text{cycle}} &= \alpha v h q \\ &= (1)(25{,}000)(0.3)6.1111 \\ &= \$45{,}833.33\,/\,\text{yr} \end{aligned}$$

where

$$\alpha = \frac{1}{2}\text{at Origin} + \frac{1}{2}\text{at Destination} = 1$$

$$v = \$25{,}000 = \text{unit value of shipment (\$/ton)}$$

$$h = 0.3 = \text{estimated carrying rate for manufactured products (1/yr)}$$

$$q = q_{\max} = 6.111 = \text{FTL shipment size (ton)}$$

Truck Shipment Example: Periodic

15. What is the annual total logistics cost (TLC) for these full-truckload TL shipments?

$$
\begin{aligned}
TLC_{FTL} &= TC_{FTL} + IC_{\text{cycle}} \\
&= n\, r_{TL} d + \alpha v h q \\
&= 3.2727\,(2.5511)\,532 + (1)(25,000)(0.3)6.1111 \\
&= 4,441.73 + 45,833.33 \\
&= \$50,275.06\,/\,yr
\end{aligned}
$$

Truck Shipment Example: Periodic

16. What is minimum possible annual total logistics cost for TL shipments, where the shipment size can now be less than a full truckload?

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha vhq = \frac{f}{q} rd + \alpha vhq$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q_{TL}^{*} = \sqrt{\frac{f r_{TL} d}{\alpha vh}} = \sqrt{\frac{20(2.5511)532}{(1)25000(0.3)}} = 1.9024 \text{ ton}$$

$$TLC_{TL}(q_{TL}^{*}) = \frac{f}{q_{TL}^{*}} r_{TL} d + \alpha vh q_{TL}^{*}$$

$$= \frac{20}{1.8553}(2.5511)532 + (1)25000(0.3)1.8553$$

$$= 14,268.12 + 14,268.12$$

$$= \$28,536.25 / \text{yr}$$

Truck Shipment Example: Periodic

- Including the minimum charge and maximum payload restrictions:

$$q_{TL}^{*} = \min\left\{\sqrt{\frac{f\ \max\left\{r_{TL}d, MC_{TL}\right\}}{\alpha vh}}, q_{\max}\right\} \approx \sqrt{\frac{f r_{TL} d}{\alpha vh}}$$

- What is the TLC if this size shipment could be made as an allocated full-truckload?

$$TLC_{AllocFTL}(q_{TL}^{*}) = \frac{f}{q_{TL}^{*}}\left(q_{TL}^{*} r_{FTL} d\right) + \alpha vh q_{TL}^{*} = f\ \frac{r_{TL}}{q_{\max}} d + \alpha vh q_{TL}^{*}$$

$$= 20\,\frac{2.5511}{6.1111}532 + (1)25000(0.3)1.9024$$

$$= 4,441.73 + 14,268.12$$

$$= \$18,709.85 / \text{yr} \quad \left(\text{vs. } \$28,536.25 \text{ as independent P2P TL}\right)$$

Truck Shipment Example: Periodic

17. What is the optimal LTL shipment size?

$$TLC_{LTL}(q) = TC_{LTL}(q) + IC(q) = \frac{f}{q} c_{LTL}(q) + \alpha vhq$$

$$q_{LTL}^{*} = \arg\min_{q} TLC_{LTL}(q) = 0.7622 \text{ ton}$$

- Must be careful in picking starting point for optimization since LTL formula only valid for limited range of values:

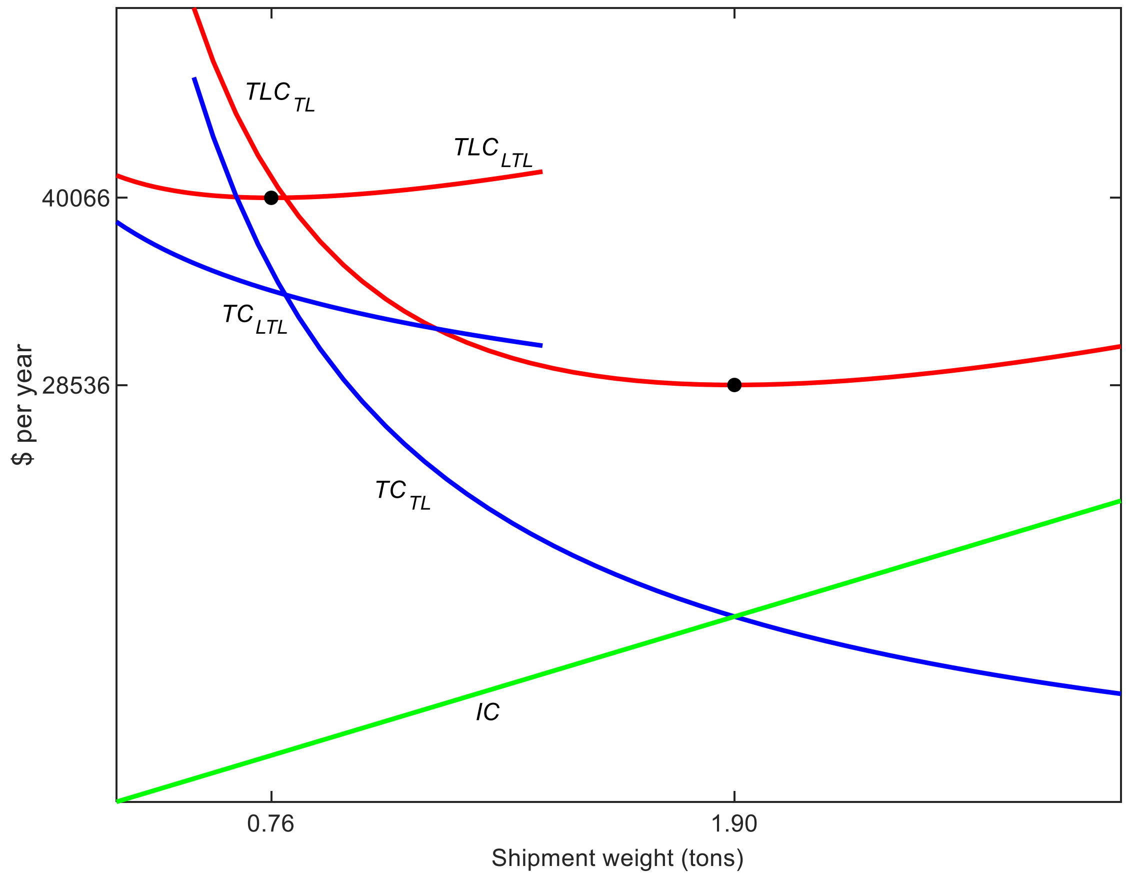
$$r_{LTL} = PPI_{LTL}\left[\frac{\frac{s^2}{8}+14}{\left(q^{\frac{1}{7}}d^{\frac{15}{29}}-\frac{7}{2}\right)\left(s^2+2s+14\right)}\right], \quad \begin{cases} 37 \le d \le 3354 \text{ (dist)} \\ \frac{150}{2,000} \le q \le \frac{10,000}{2,000} \text{ (wt)} \\ 2000\frac{q}{s} \le 650 \text{ ft}^3 \text{ (cube)} \end{cases}$$

$$\frac{150}{2000} \le q \le \min\left\{\frac{10,000}{2,000}, \frac{650s}{2000}\right\} \Rightarrow 0.075 \le q \le 1.44$$

Truck Shipment Example: Periodic

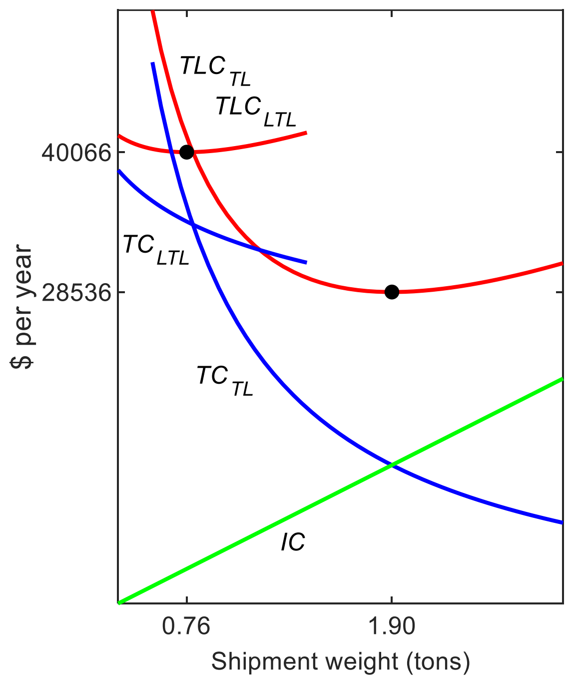
18. Should the product be shipped TL or LTL?

$$TLC_{LTL}(q^*_{LTL}) = TC_{LTL}(q^*_{LTL}) + IC(q^*_{LTL}) = 34,349.19 + 5,716.40 = \$40,065.59\,/\,\text{yr}$$

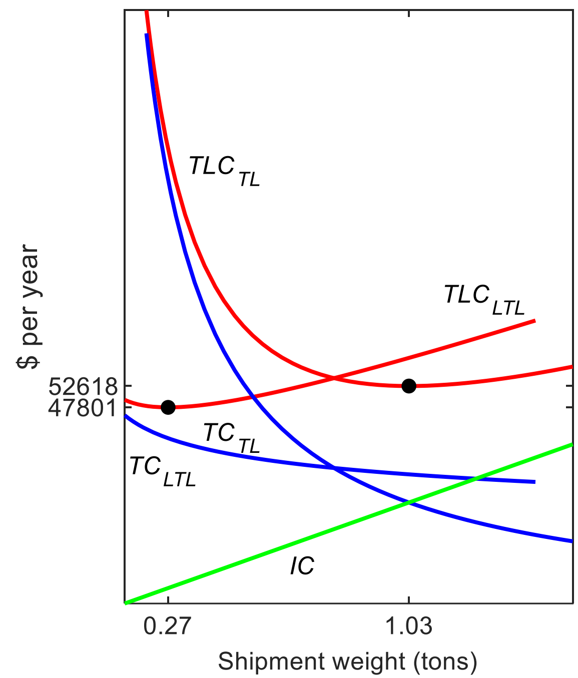


Truck Shipment Example: Periodic

19. If the value of the product increased to $85,000 per ton, should the product be shipped TL or LTL?



(a) $25000 value per ton

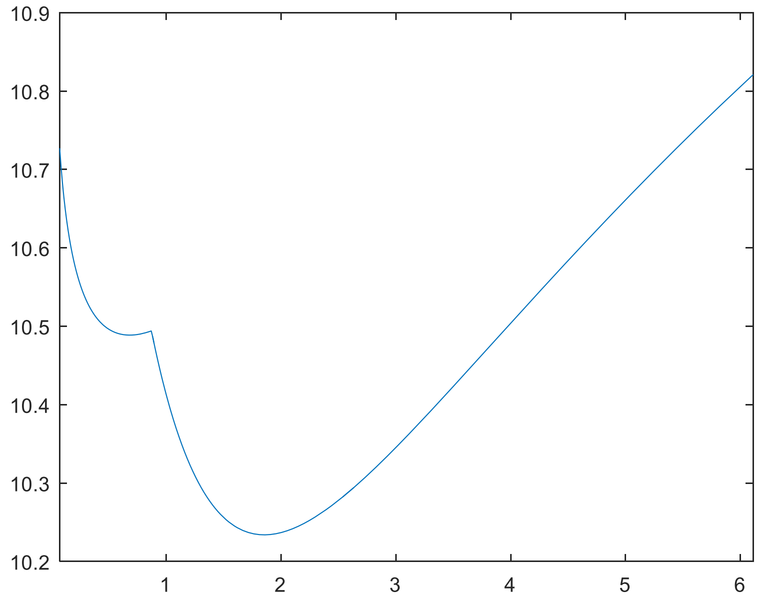


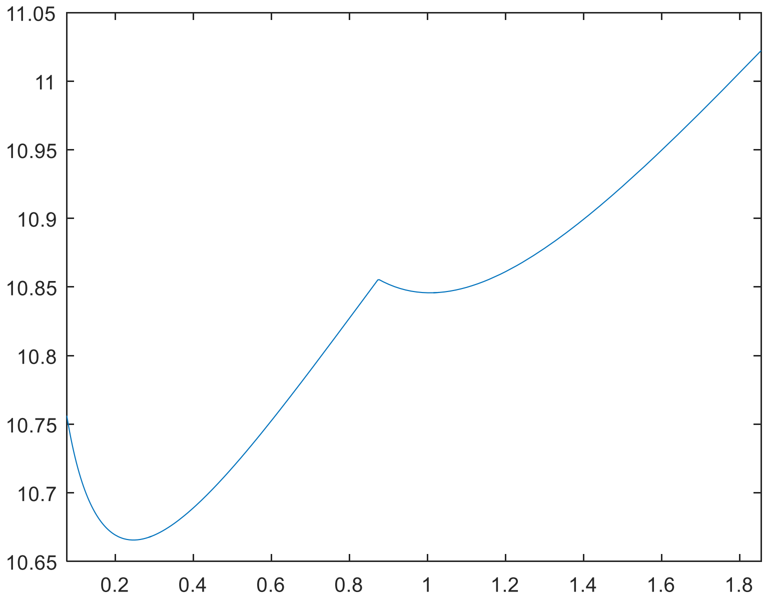
(b) $85000 value per ton

Truck Shipment Example: Periodic

- Better to pick from separate optimal TL and LTL because independent charge has two local minima:

$$q_0^* = \arg\min_q \{TLC_{TL}(q), TLC_{LTL}(q)\} \qquad q_0^* \stackrel{!}{=} \arg\min_q \left\{ \frac{f}{q} c_0(q) + \alpha v h q \right\}$$





Truck Shipment Example: Periodic

20. What is optimal independent shipment size to ship 80 tons per year of a Class 60 product valued at $5000 per ton between Raleigh and Gainesville?

$$s = 32.16 \text{ lb/ft}^3$$

$$q_0^* = \arg\min_q \left\{ TLC_{TL}(q), TLC_{LTL}(q) \right\} = 8.5079 \text{ ton}$$

$$TLC_{TL}(q_0^*) = \$25,523.60 \,/\, \text{yr} < TLC_{LTL}(q_0^*)$$

Truck Shipment Example: Periodic

21. What is the optimal shipment size if both shipments will always be shipped together on the same truck (with same shipment interval)?

$$d_1 = d_2, \quad h_1 = h_2, \quad \alpha_1 = \alpha_2$$

$$f_{\text{agg}} = f_1 + f_2 = 20 + 80 = 100 \text{ ton}$$

$$s_{\text{agg}} = \frac{(\text{aggregate weight, in lb})}{(\text{aggregate cube, in ft}^3)} = \frac{f_{\text{agg}}}{\frac{f_1}{s_1} + \frac{f_2}{s_2}} = \frac{100}{\frac{20}{4.44} + \frac{80}{32.16}} = 14.31 \text{ lb/ft}^3$$

$$v_{\text{agg}} = \frac{f_1}{f_{\text{agg}}} v_1 + \frac{f_2}{f_{\text{agg}}} v_2 = \frac{20}{100} 85,000 + \frac{80}{100} 5000 = \$21,000 / \text{ton}$$

$$q_{TL}^* = \sqrt{\frac{f_{\text{agg}} r d}{\alpha v_{\text{agg}} h}} = \sqrt{\frac{100(2.5511)532}{(1)21000(0.3)}} = 4.6414 \text{ ton}$$

Truck Shipment Example: Periodic

- Summary of results:

	f	s	v	qmax	TLC	q	t
1:	20	4.44	85,000	6.11	47,801.01	0.27	5.00
2:	80	32.16	5,000	25.00	25,523.60	8.51	38.84
1+2:					73,324.60		
Aggregate:	100	14.31	21,000	19.68	58,481.90	4.64	16.95