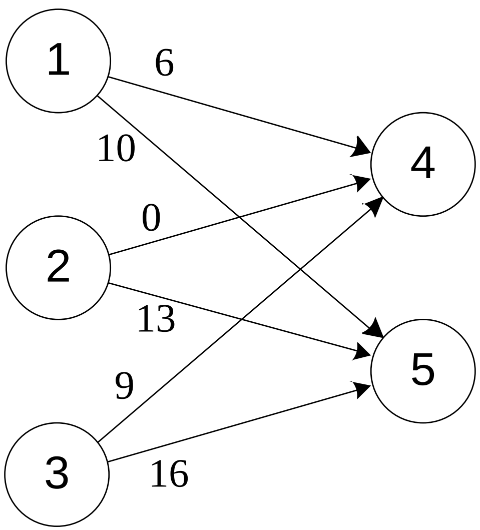
# Topics

1. Introduction
2. Facility location
3. Freight transport
   - Exam 1 (take home)
4. **Network models**
5. Routing
   - Exam 2 (take home)
6. Warehousing
   - Final exam (in class)

# Graph Representations

- Complete bipartite directed (or digraph):
  - Suppliers to multiple DCs, single mode of transport




```
C:   1    2
-:-------
1:   6   10
2:   0   13
3:   9   16
```

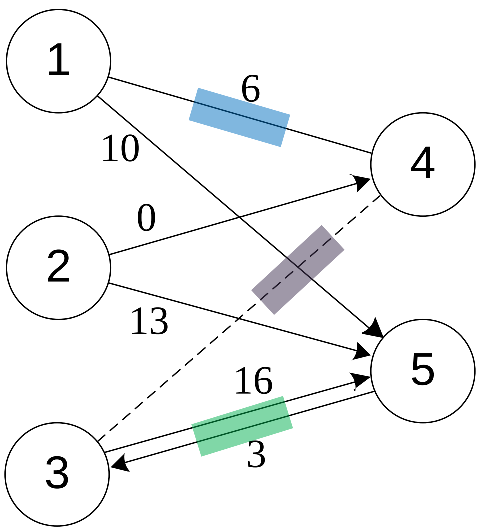
Interlevel matrix

```
W =     0      0      0      6     10
        0      0      0    NaN     13
        0      0      0      9     16
        0      0      0      0      0
        0      0      0      0      0
```

Weighted adjacency matrix

# Graph Representations

- Bipartite:
  - One- or two-way connections between nodes in two groups



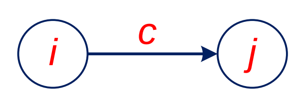

W =

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 6 | 10 |
| 0 | 0 | 0 | NaN | 13 |
| 0 | 0 | 0 | 0 | 16 |
| 6 | 0 | 0 | 0 | 0 |
| 0 | 0 | 3 | 0 | 0 |

IJC =

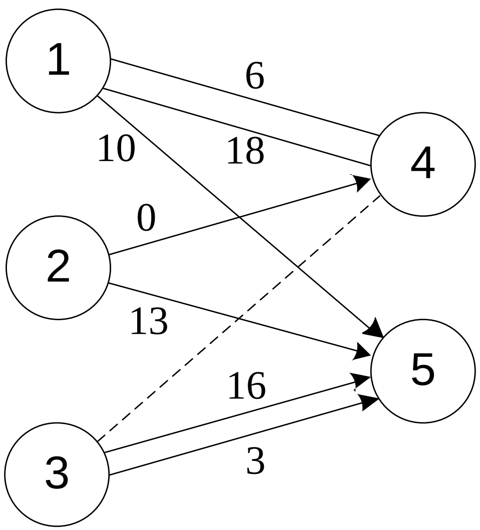
| | | |
|---|---|---|
| 4 | 1 | 6 |
| 5 | 3 | 3 |
| 1 | 4 | 6 |
| 2 | 4 | 0 |
| 1 | 5 | 10 |
| 2 | 5 | 13 |
| 3 | 5 | 16 |

Arc list matrix



# Graph Representations

- Multigraph:
  – Multiple connections, multiple modes of transport

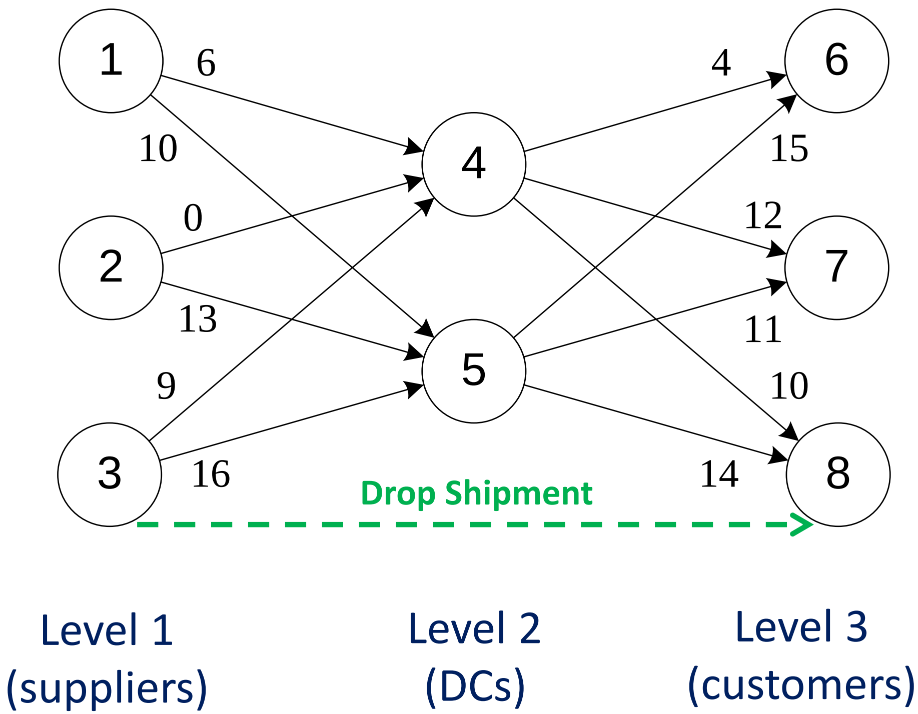



```
IJC = 1     -4     6
      1     -4    18
      1      5    10
      2      4     0
      2      5    13
      3      5    16
      3      5     3
no_W =
      0      0     0    24    10
      0      0     0   NaN    13
      0      0     0     0    19
     24      0     0     0     0
      0      0     0     0     0
```

Can't represent using adjacency matrix

# Graph Representations

- Complete multipartite directed:
  - Typical supply chain (no drop shipments)



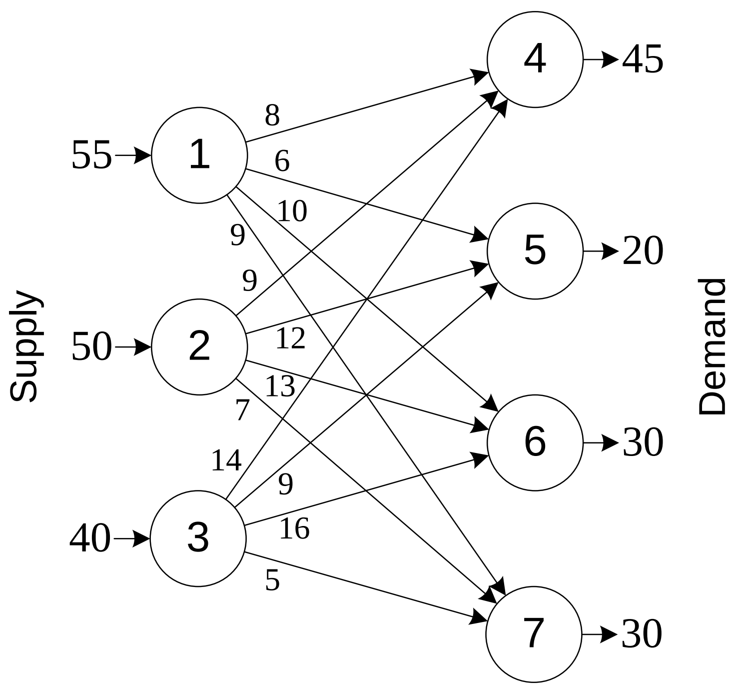

| C12: | 1 | 2 |
|---|---|---|
| 1: | 6 | 10 |
| 2: | 0 | 13 |
| 3: | 9 | 16 |

| C23: | 1 | 2 | 3 |
|---|---|---|---|
| 1: | 4 | 12 | 10 |
| 2: | 15 | 11 | 14 |

| W: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1: | 0 | 0 | 0 | 6 | 10 | 0 | 0 | 0 |
| 2: | 0 | 0 | 0 | | 13 | 0 | 0 | 0 |
| 3: | 0 | 0 | 0 | 9 | 16 | 0 | 0 | 0 |
| 4: | 0 | 0 | 0 | 0 | 0 | 4 | 12 | 10 |
| 5: | 0 | 0 | 0 | 0 | 0 | 15 | 11 | 14 |
| 6: | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7: | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8: | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

# Transportation Problem

- Satisfy node demand from supply nodes
  - Can be used for allocation in ALA when NFs have capacity constraints
  - Min cost/distance allocation = infinite supply at each node




| Trans | 4 | 5 | 6 | 7 | Supply |
|---|---|---|---|---|---|
| 1 | 8 | 6 | 10 | 9 | 55 |
| 2 | 9 | 12 | 13 | 7 | 50 |
| 3 | 14 | 9 | 16 | 5 | 40 |
| Demand | 45 | 20 | 30 | 30 | |

# Greedy Solution Procedure

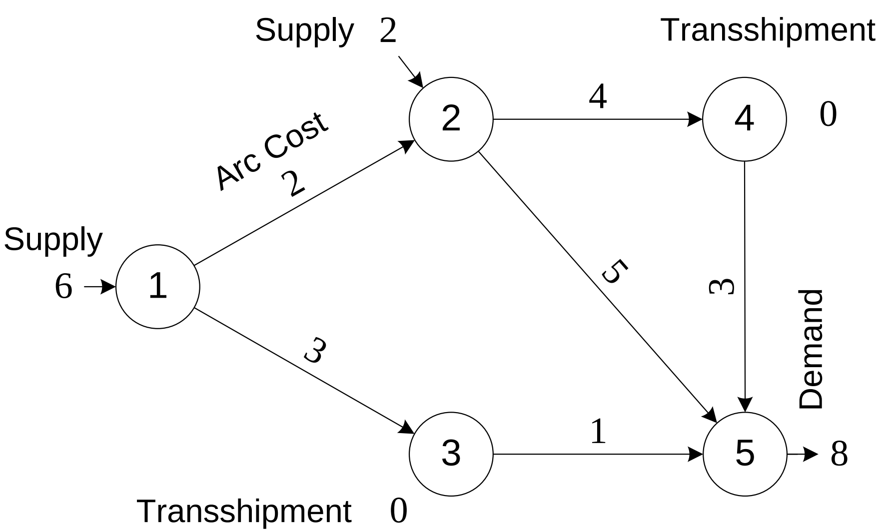
- Procedure for transportation problem: *Continue to select lowest cost supply until all demand is satisfied*
  - Fast, but not always optimal for transportation problem
  - Dijkstra's shortest path and simplex method for LP are optimal greedy procedures

| Trans | 4 | 5 | 6 | 7 | Supply | |
|---|---|---|---|---|---|---|
| 1 | 8 | 6 | 10 | 9 | 55 | -20 = 35 -35 = 0 |
| 2 | 9 | 12 | 13 | 7 | 50 | -10 = 40 -30 = 10 |
| 3 | 14 | 9 | 16 | 5 | 40 | -30 = 10 |
| Demand | ~~45~~ | ~~20~~ | ~~30~~ | ~~30~~ | | |
| | ~~10~~ 0 | 0 | 0 | 0 | | |

$$TC = 5(30) + 6(20) + 8(35) + 9(10) + 13(30) = 1,030 \quad \text{(vs 970 optimal)}$$

# Min Cost Network Flow (MCNF) Problem

- Most general network problem, can solve using any type of graph representation




$$s_i = \text{net supply of node } i$$

$$= \begin{cases} > 0, & \text{supply node} \\ < 0, & \text{demand node} \\ = 0, & \text{transshipment node} \end{cases}$$

$$\text{Arc cost: } \mathbf{c} = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 & 3 \end{bmatrix}'$$

$$\text{Net node supply: } \mathbf{s} = \begin{bmatrix} 6 & 2 & 0 & 0 & -8 \end{bmatrix}'$$

$$\text{Incidence Matrix: } \mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

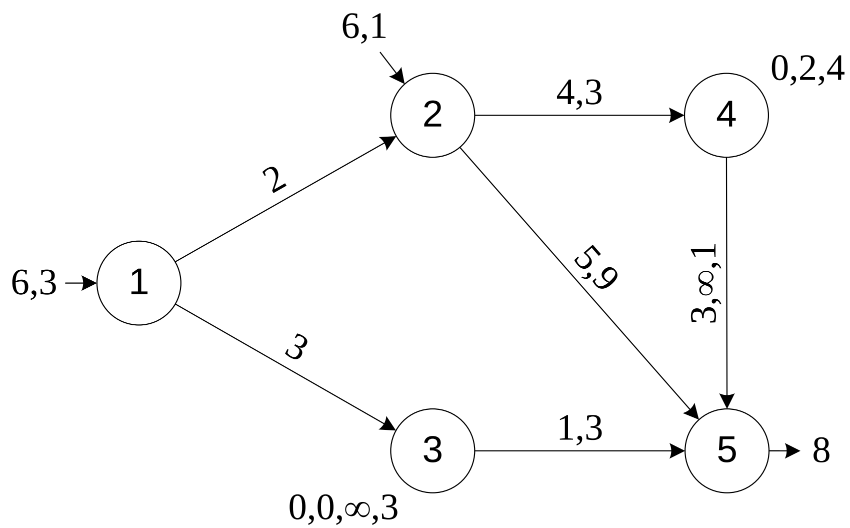
```
MCNF: lhs   C    C    C    C    C    C   rhs
----:-----------------------------------------
 Min:        2    3    4    5    1    3
   1:  6     1    1    0    0    0    0   6
   2:  2    -1    0    1    1    0    0   2
   3:  0     0   -1    0    0    1    0   0
   4:  0     0    0   -1    0    0    1   0
  lb:        0    0    0    0    0    0
  ub:      Inf  Inf  Inf  Inf  Inf  Inf
```

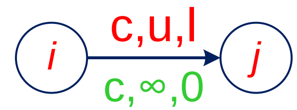
Row for node 5 is redundant

$$\text{MCNF:} \quad \max \ \mathbf{c}'\mathbf{x}$$
$$\text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{s}$$
$$\mathbf{x} \geq 0$$

# MCNF with Arc/Node Bounds and Node Costs

- Bounds on arcs/nodes can represent capacity constraints in a logistic network
- Node cost can represent production cost or intersection delay

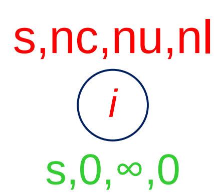



| IJCUL: | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1: | 1 | 2 | 2 | Inf | 0 |
| 2: | 1 | 3 | 3 | Inf | 0 |
| 3: | 2 | 4 | 4 | 3 | 0 |
| 4: | 2 | 5 | 5 | 9 | 0 |
| 5: | 3 | 5 | 1 | 3 | 0 |
| 6: | 4 | 5 | 3 | Inf | 1 |

$$s_i = \text{net supply of node } i$$

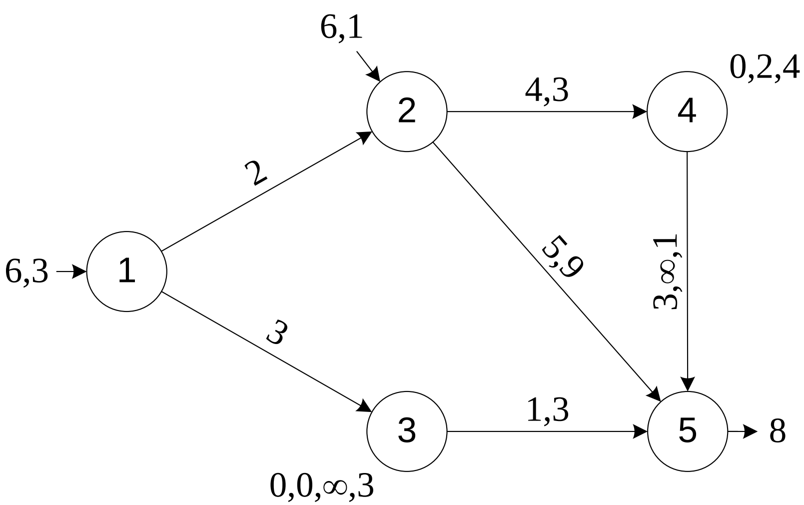
$$= \begin{cases} > 0, & \text{supply node} \\ < 0, & \text{demand node} \\ = 0, & \text{transshipment node} \end{cases}$$

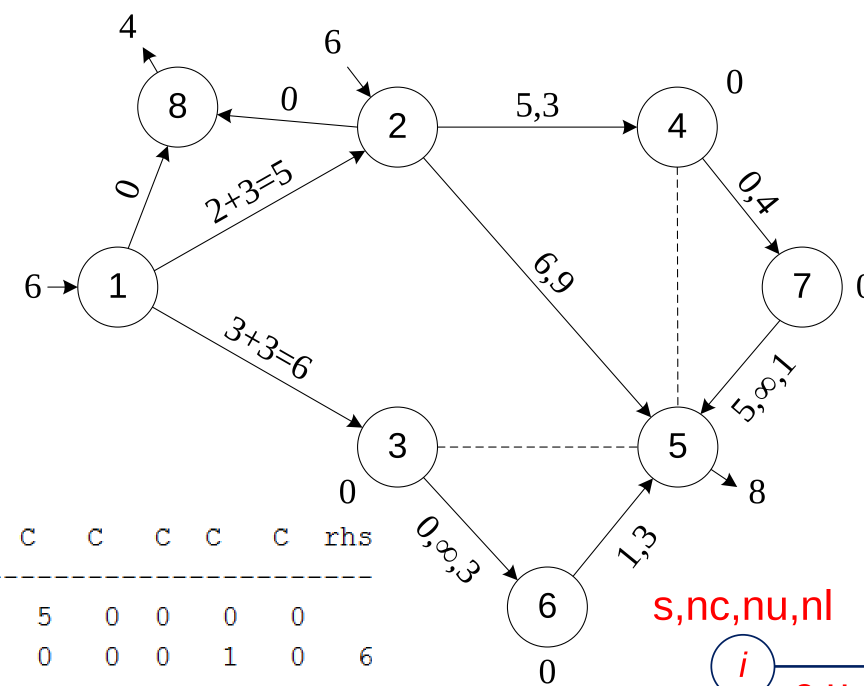


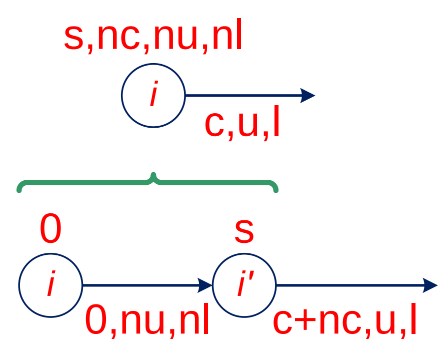

| SCUL: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1: | 6 | 3 | Inf | 0 |
| 2: | 6 | 1 | Inf | 0 |
| 3: | 0 | 0 | Inf | 3 |
| 4: | 0 | 2 | 4 | 0 |
| 5: | -8 | 0 | Inf | 0 |

# Expanded-Node Formulation of MCNF

- Node cost/constraints converted to arc cost/constraints
  - Dummy node (8) added so that supply = demand




| MCNF: | lhs | C | C | C | C | C | C | C | C | C | C | rhs |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Min: | | 5 | 6 | 5 | 6 | 1 | 5 | 0 | 0 | 0 | 0 | |
| 1: | 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 6 |
| 2: | 6 | -1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 6 |
| 3: | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4: | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 5: | -8 | 0 | 0 | 0 | -1 | -1 | -1 | 0 | 0 | 0 | 0 | -8 |
| 6: | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 0 |
| 7: | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 |
| lb: | | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 0 | 0 | 0 | |
| ub: | | Inf | Inf | 3 | 9 | 3 | Inf | Inf | 4 | Inf | Inf | |

# Solving an MCNF as an LP

- Special procedures more efficient than LP were developed to solve MCNF and Transportation problems
  - e.g., Network simplex algorithm (MCNF)
  - e.g., Hungarian method (Transportation and *Transshipment*)
- Now usually easier to transform into LP since solvers are so good, with MCNF just aiding in formulation of problem:
  - Trans $\Rightarrow$ MCNF $\Rightarrow$ LP
  - Special, very efficient procedures only used for shortest path problem (Dijkstra)