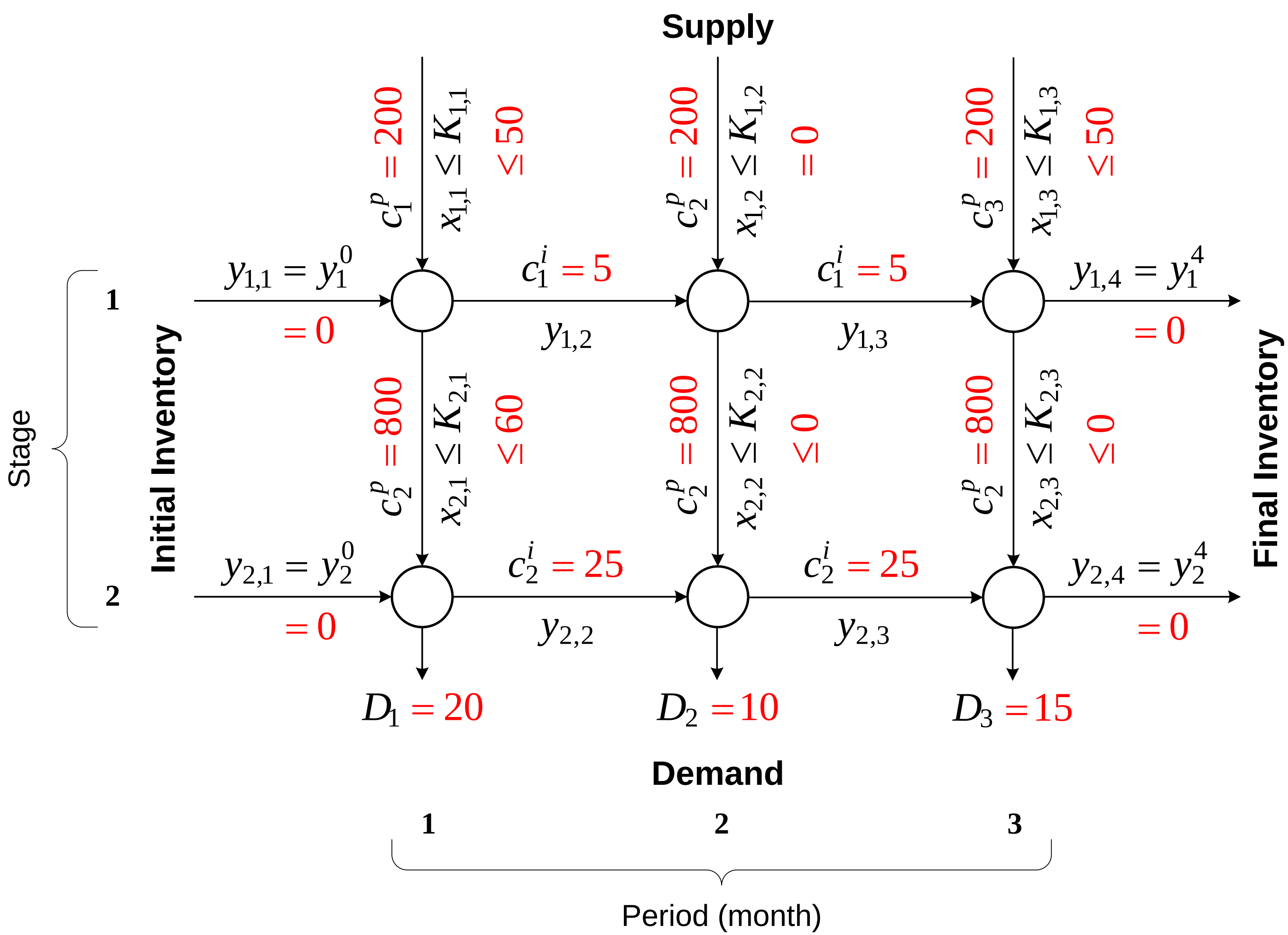
Production and Inventory: One Product



$$h = 0.3 \frac{\$}{\$\text{-yr}} = 0.3$$

$$\frac{h}{T} = \frac{0.3}{12} \frac{\$}{\$\text{-month}} = 0.025$$

$$c_m^i = \frac{h}{T} \sum_{j=1}^{m} c_j^p$$

$$c_1^i = \frac{0.3}{12} 200 = 5$$

$$c_2^i = \frac{0.3}{12} (200 + 800)$$

$$= 25$$

Production and Inventory: One Product

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T} c_m^p x_{mt} + \sum_{m=1}^{M}\sum_{t=1}^{T+1} c_m^i y_{mt}$$

subject to

Flow balance

$$x_{mt} - x_{(m+1)t} + y_{mt} - y_{m(t+1)} = 0, \qquad m = 1,\ldots,M-1;\ t = 1,\ldots,T$$

$$x_{Mt} + y_{Mt} - y_{M(t+1)} = D_t, \qquad t = 1,\ldots,T$$

Capacity

$$x_{mt} \le K_{mt}, \quad m = 1,\ldots,M;\ t = 1,\ldots,T$$

Initial/Final inventory

$$y_{m1} = y_m^0, \quad m = 1,\ldots,M$$

$$y_{m(T+1)} = y_m^{T+1}, \quad m = 1,\ldots,M$$

$$x, y \ge 0 \text{ and continuous}$$

Use var. LB & UB instead of constraints

where

M = number of production stages

c_m^p = production cost in stage m (\$/ton)

x_{mt} = production at stage m in period t (ton)

c_m^i = inventory cost for stage m (\$/ton)

y_{mt} = stage-m inventory period $t-1$ to t (ton)

T = number of periods of production

D_t = demand in period t (ton)

K_{mt} = capacity of stage m in period t (ton)

y_m^0 = initial inventory of stage m (ton)

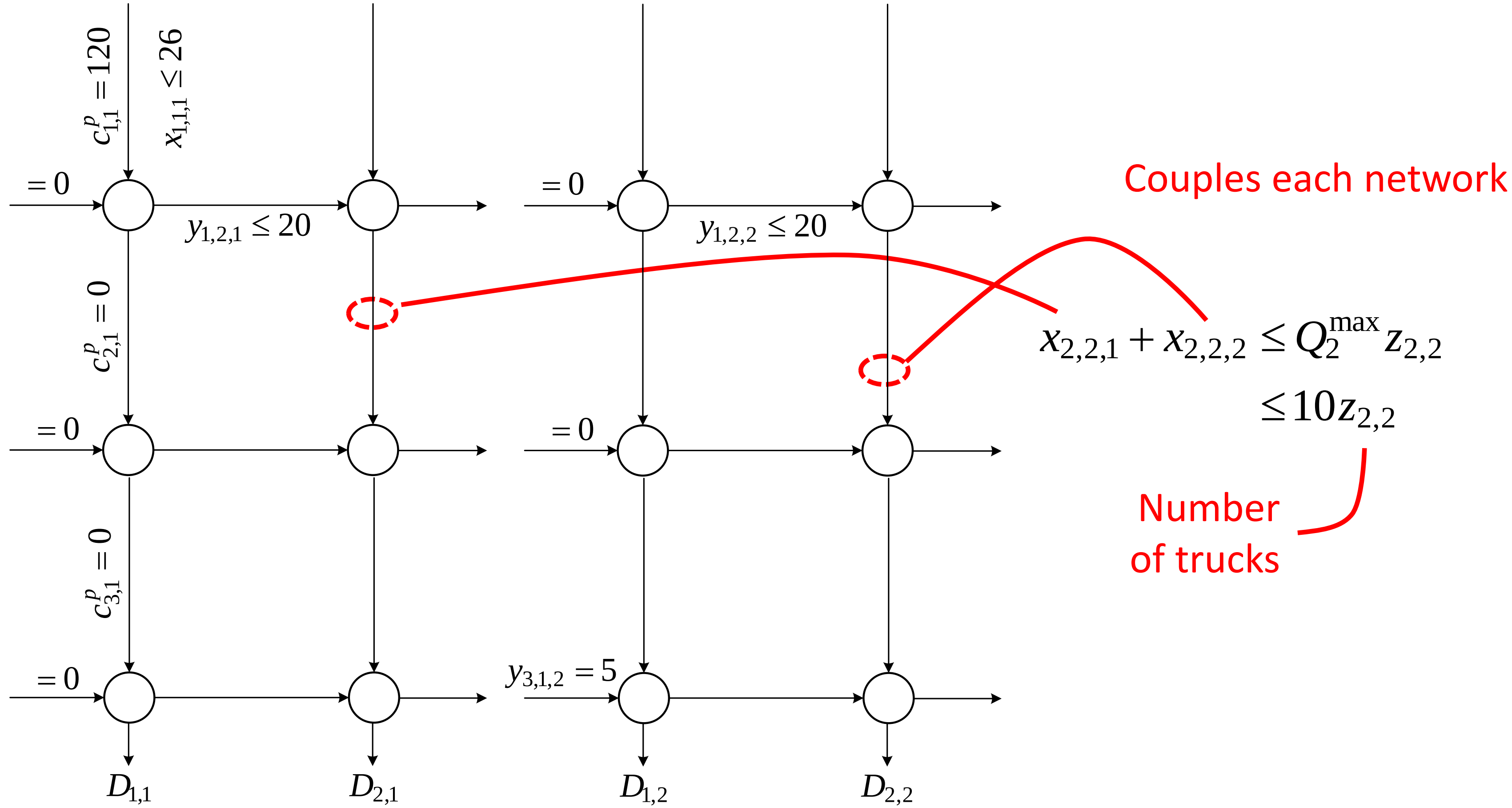
y_m^{T+1} = final inventory of stage m (ton)

Ex 15: Coupled Networks via Truck Capacity

- Facility that extracts two different raw materials for pharmaceuticals
  1. Extracted material to be sent over rough terrain in a truck to a staging station where it is then loaded onto a tractor trailer for transport to its final destination
  2. Facility can extract up to 26 and 15 tons per week of each material, respectively, at a cost of $120 and $200 per ton
  3. Annual inventory carrying rate is 0.15
  4. Facility can store up to 20 tons of each material on site, and unlimited amounts of material can be stored at the staging station and the final destination
  5. Currently, five tons of the second material is in inventory at the final destination and this same amount should be in inventory at the end of the planning period
  6. Costs $200 for a truck to make the roundtrip from the facility to the staging station, and it costs $800 for each truckload transported from the station to the final destination
  7. Each truck and tractor trailer can carry up to 10 and 25 tons of material, respectively, and each load can contain both types of material
- Determine the amount of each material that should be extracted and when it should be transported in order to minimize total costs over the planning horizon
- Separate networks for two products are coupled via sharing truck capacity

Ex 15: Coupled Networks via Truck Capacity

- Separate networks for each raw material are coupled via sharing the same trucks (added as constraint to model)



Ex 15: Coupled Networks via Truck Capacity

- Math programming model:

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{g=1}^{G} c_{mg}^{p}\, x_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T+1}\sum_{g=1}^{G} c_{mg}^{i}\, y_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T} c_{m}^{t} z_{mt}$$

subject to

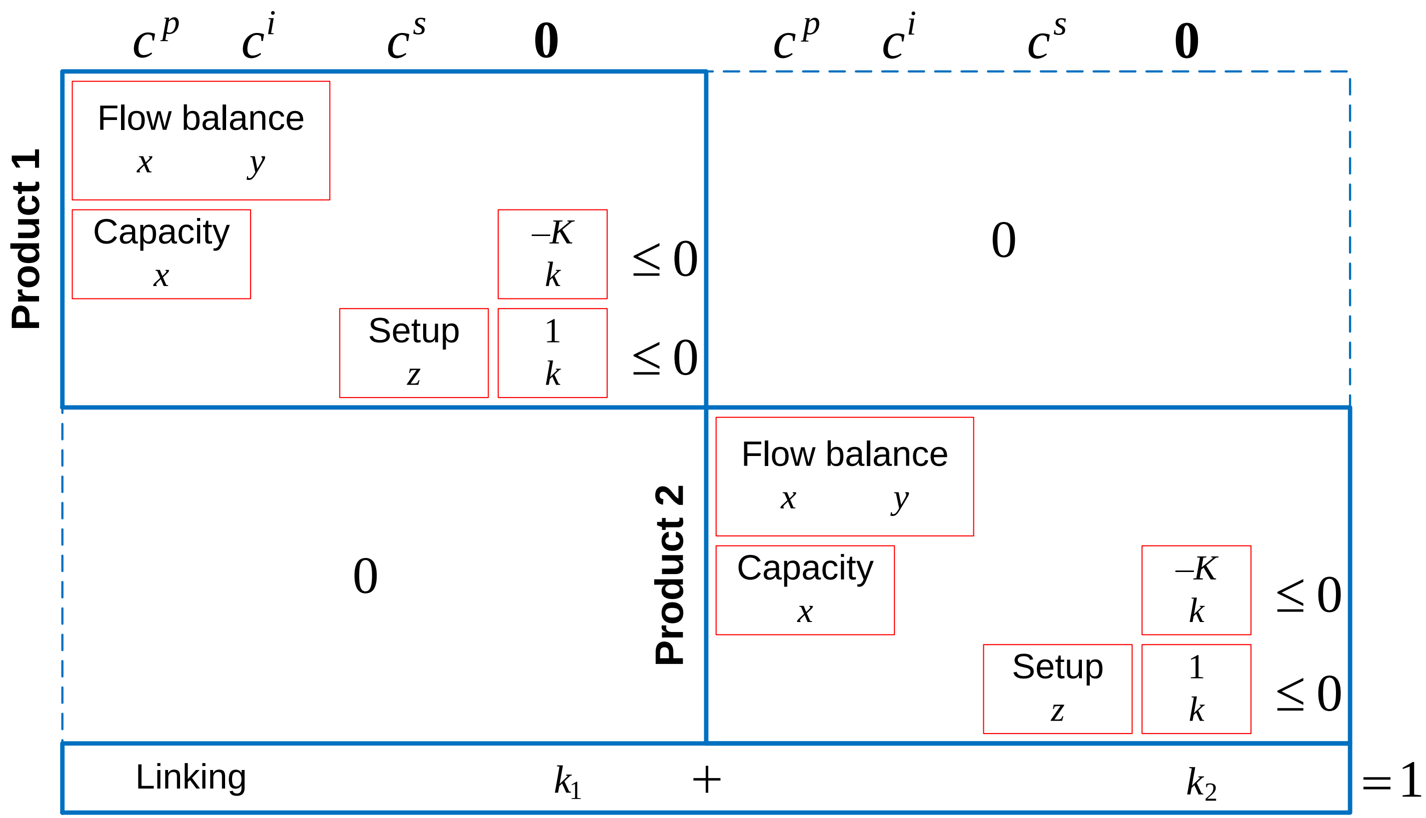
$$x_{mtg} - x_{(m+1)tg} + y_{mtg} - y_{m(t+1)g} = 0, \qquad m = 1,\dots,M-1; \quad t = 1,\dots,T; \; g = 1,\dots,G$$

$$x_{Mtg} + y_{Mtg} - y_{M(t+1)g} = D_{tg}, \qquad t = 1,\dots,T; \; g = 1,\dots,G$$

$$\sum_{g=1}^{G} x_{mtg} \le Q_{m}^{\max} z_{mt}, \quad m = 1,\dots,M; \; t = 1,\dots,T$$

Use LB,UB for capacity constraints

Production and Inventory: Multiple Products



Production and Inventory: Multiple Products

$k_{mtg} \in \{0,1\}$, production indicator

k_{mtg}	1	2	3	4	5	6	7
1	0	1	1	0	1	1	1
2	0	0	0	1	0	0	0

$z_{mtg} \in \{0,1\}$, setup indicator

z_{mtg}	1	2	3	4	5	6	7
1	0	1	0	0	1	0	0
2	0	0	0	1	0	0	0

	$-z_t$	$+\ k_t$	$-\ k_{t-1}$	$\leq\ 0$	
	0	0	0	0	
	0	0	1	–1	
Don't want (not feasible)	0	1	0	1	
	0	1	1	0	
	1	0	0	–1	Feasible, but not min cost
	1	0	1	–2	Feasible, but not min cost
Want (feasible)	1	1	0	0	
	1	1	1	–1	Feasible, but not min cost

Production and Inventory: Multiple Products

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{g=1}^{G} c_{mg}^{p}\, x_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T+1}\sum_{g=1}^{G} c_{mg}^{i}\, y_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{g=1}^{G} c_{mg}^{s}\, z_{mtg} \left(\underbrace{+\mathbf{0}_{MTG}\, k_{mtg}}_{\text{dummy}}\right)$$

subject to

Flow balance

$$x_{mtg} - x_{(m+1)tg} + y_{mtg} - y_{m(t+1)g} = 0, \qquad m=1,\dots,M-1;\quad t=1,\dots,T;\ g=1,\dots,G$$

$$x_{Mtg} + y_{Mtg} - y_{M(t+1)g} = D_{tg}, \qquad t=1,\dots,T;\ g=1,\dots,G$$

Capacity

$$x_{mtg} \le K_{mg} k_{mtg}, \qquad m=1,\dots,M;\ t=1,\dots,T;\ g=1,\dots,G$$

Setup

$$-z_{m1g} + k_{m1g} \le k_{mg}^{0}, \qquad m=1,\dots,M;\ g=1,\dots,G$$

$$-z_{mtg} + k_{mtg} - k_{m(t-1)g} \le 0, \qquad m=1,\dots,M;\ t=2,\dots,T;\ g=1,\dots,G$$

Linking

$$\sum_{g=1}^{G} k_{mtg} = 1, \qquad m=1,\dots,M;\ t=1,\dots,T$$

$$y_{m1g} = y_{mg}^{0}, \qquad m=1,\dots,M;\ g=1,\dots,G$$

$$y_{m(T+1)g} = y_{mg}^{T+1}, \qquad m=1,\dots,M;\ g=1,\dots,G$$

$$x, y \ge 0 \text{ and continuous; } k, z \text{ binary}$$ MILP

Production and Inventory: Multiple Products

where $\mathbf{0}_{MTG}$ is a matrix of zeroes and

M = number of production stages

T = number of periods of production

G = number of products produced

c^p_{mg} = production cost of product g at stage m (\$/ton)

x_{mtg} = production at stage m in period t of product g (ton)

c^i_{mg} = inventory cost of product g for stage m (\$/ton)

y_{mtg} = inventory at stage m between periods $t-1$ and t of product g (ton)

c^s_{mg} = stage-m product-g setup cost (\$)

z_{mtg} = setup indicator at stage m in period t for product g

k_{mtg} = production indicator at stage m in period t for product g

D_{tg} = demand for product g in period t (ton)

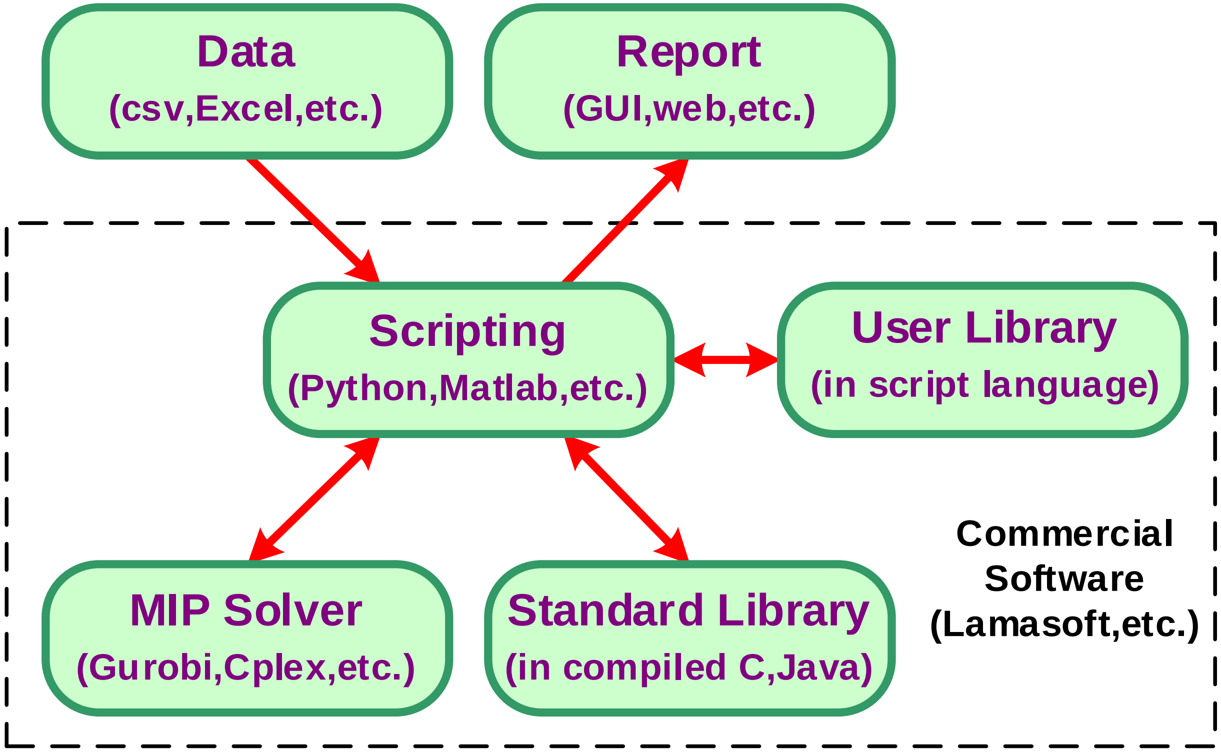
K_{mg} = capacity for product g in stage m (ton)

k^0_{mg} = initial setup at stage m for product g

y^0_{mg} = initial product g inventory at stage m (ton)

y^{T+1}_{mg} = final product g inventory at stage m (ton)

Example of Logistics Software Stack



- **Flow:** *Data → Model → Solver → Output → Report*
  - reports are run on a regular period-to-period, *rolling-horizon* basis as part of normal operations management
  - model only changed when logistics network changes