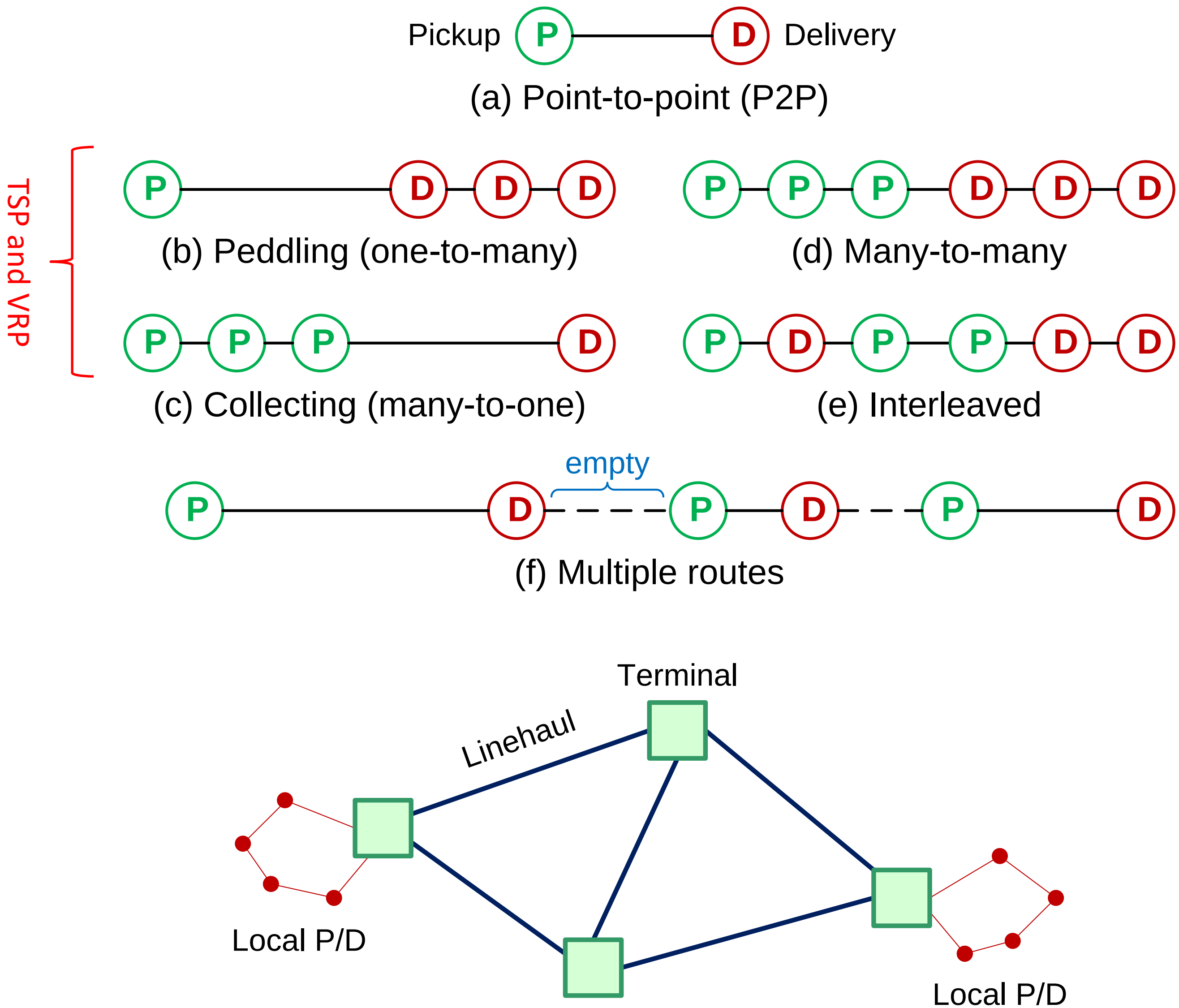
Topics

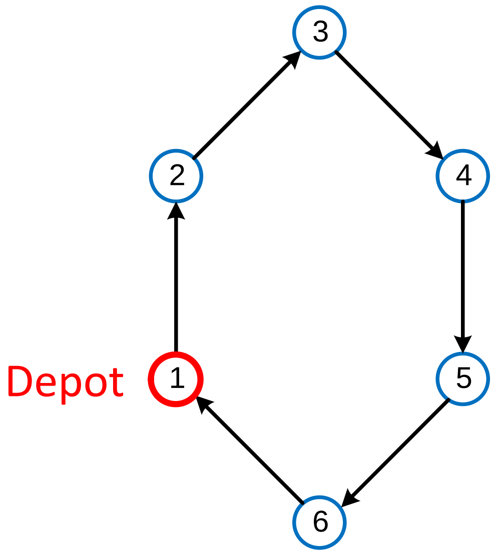
1. Introduction
2. Facility location
3. Freight transport
   – Exam 1 (take home)
4. Network models
5. **Routing**
   – Exam 2 (take home)
6. Warehousing
   – Final exam (in class)

Routing Alternatives



TSP

- Problem: find connected sequence through all nodes of a graph that minimizes total arc cost
  - Subroutine in most vehicle routing problems
  - Node sequence can represent a route only if all pickups and/or deliveries occur at a single node (depot)



Node sequence = permutation + start node

1	2	3	4	5	6	1

$n = 6 \Rightarrow (n-1)! = 120$ possible solutions

TSP

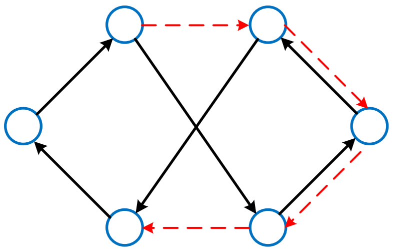
- TSP can be solved by a mix of *construction* and *improvement* procedures
  - BIP formulation has an exponential number of constraints to eliminate subtours ($\Rightarrow$ column generation techniques)
- Asymmetric: only best-known solutions for large *n*

$$(n-1)! \quad n=13 \Rightarrow \approx \frac{1}{2}\text{billion solutions}$$

- Symmetric: solved to optimal using BIP

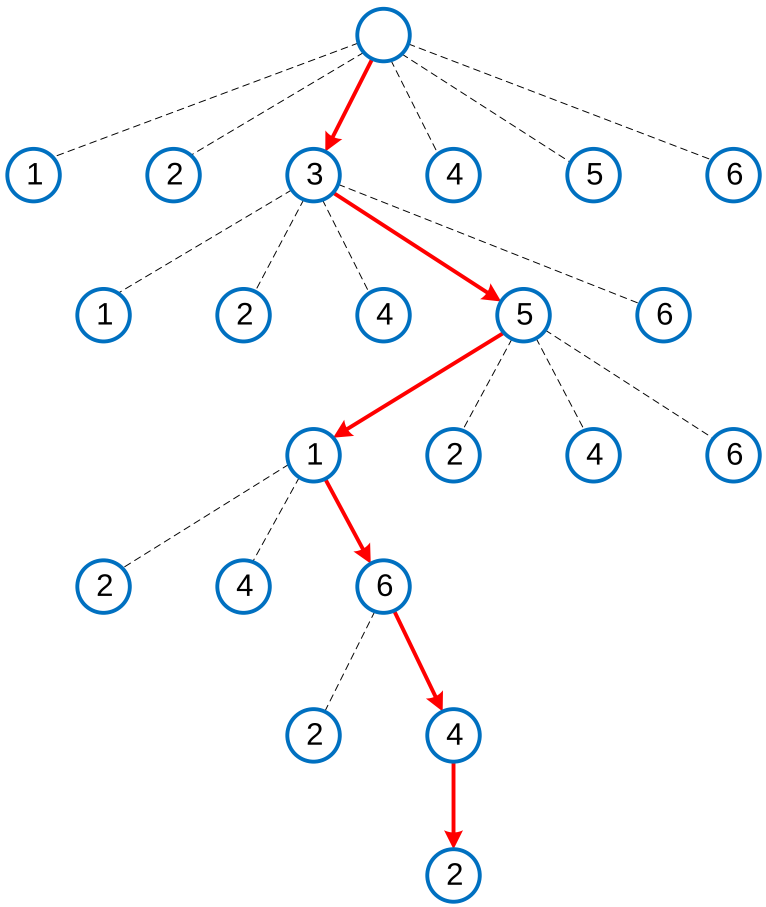
$$c_{ij} = c_{ji} \Rightarrow \frac{(n-1)!}{2} \quad \text{solutions}$$

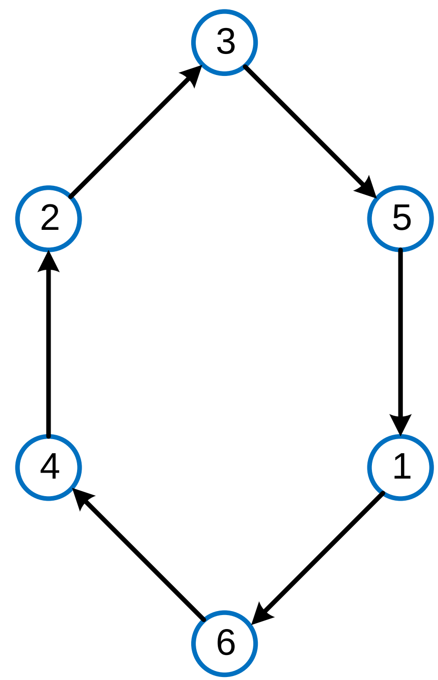
- Euclidean: arcs costs = distance between nodes



TSP Construction

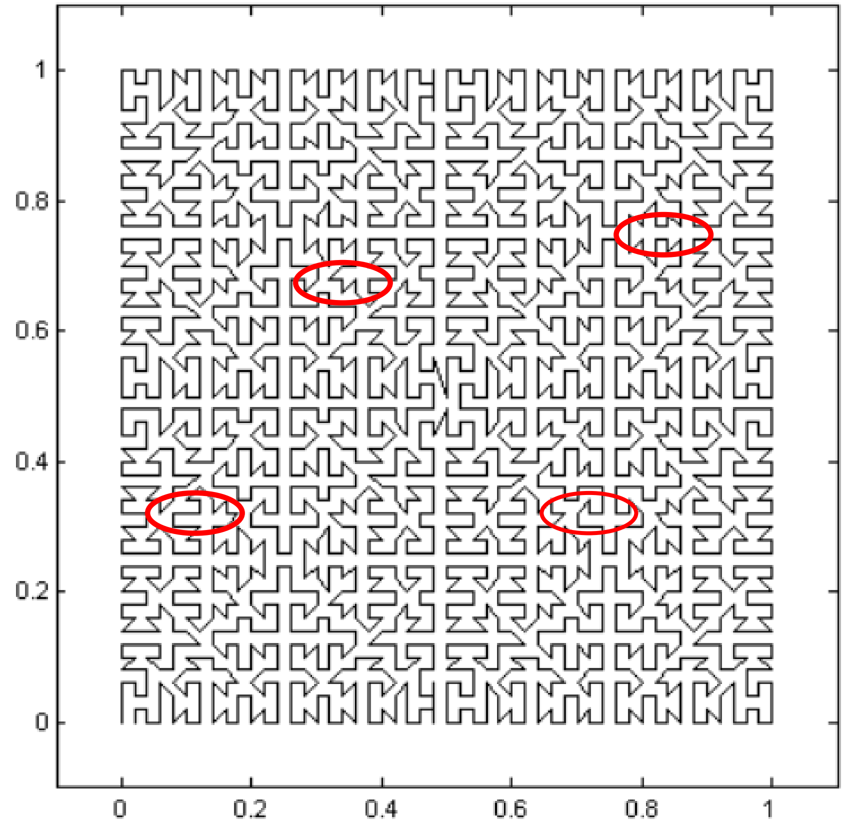
- Construction easy since any permutation is feasible and can then be improved





Spacefilling Curve

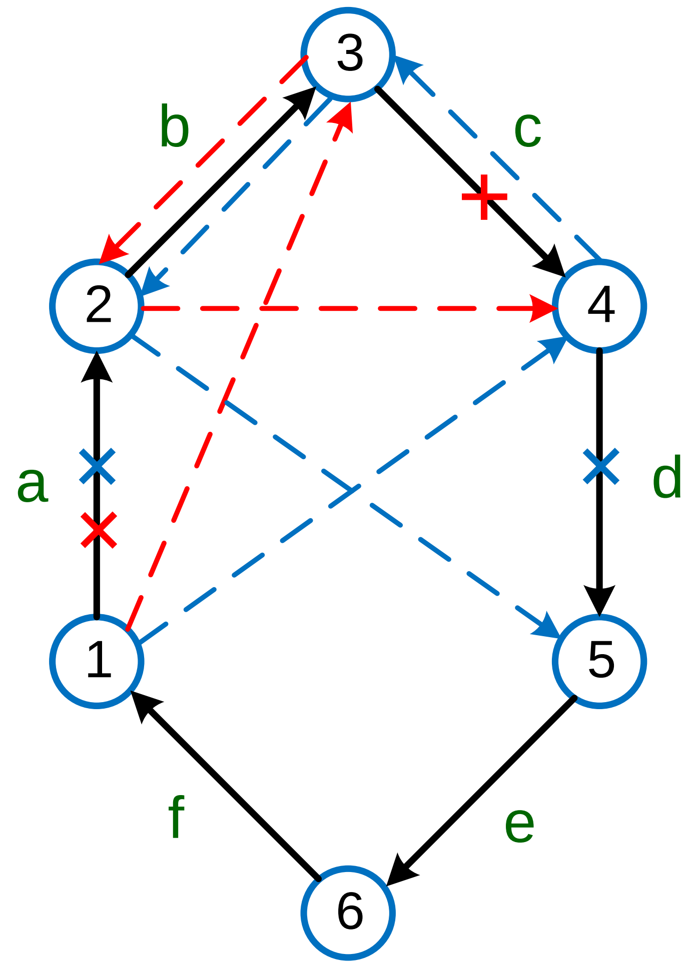
1.0	0.250	0.254	0.265	0.298	0.309	0.438	0.441	0.452	0.485	0.496	0.500
0.9	0.246	0.257	0.271	0.292	0.305	0.434	0.445	0.458	0.479	0.493	0.504
0.8	0.235	0.229	0.279	0.283	0.333	0.423	0.417	0.467	0.471	0.521	0.515
0.7	0.202	0.208	0.158	0.154	0.354	0.390	0.396	0.596	0.592	0.542	0.548
0.6	0.191	0.180	0.167	0.146	0.132	0.379	0.618	0.604	0.583	0.570	0.559
0.5	0.188	0.184	0.173	0.140	0.129	0.375	0.621	0.610	0.577	0.566	0.563
0.4	0.059	0.070	0.083	0.104	0.118	0.871	0.632	0.646	0.667	0.680	0.691
0.3	0.048	0.042	0.092	0.096	0.896	0.860	0.854	0.654	0.658	0.708	0.702
0.2	0.015	0.021	0.971	0.967	0.917	0.827	0.833	0.783	0.779	0.729	0.735
0.1	0.004	0.993	0.979	0.958	0.945	0.816	0.805	0.792	0.771	0.757	0.746
0.0	0.000	0.996	0.985	0.952	0.941	0.813	0.809	0.798	0.765	0.754	0.750
θ	**0.0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**



Sequence determined by sorting position along 1-D line covering 2-D space

2: 0.021
3: 0.154
1: 0.471
4: 0.783

Two-Opt Improvement



	1	2	3	4	5	6	1
(arcs)	a	b	c	d	e	f	
a-c	1	3	2	4	5	6	1
a-d	1	4	3	2	5	6	1
a-e	1	5	4	3	2	6	1
b-d							
b-e							
b-f							
c-e							
c-f							
d-f							

Sequences considered at end to verify local optimum: n nodes $\Rightarrow \overset{\text{first arc } a}{\sum_{j=3}^{n-1}(1)} + \overbrace{\sum_{i=2}^{n-2}\sum_{j=i+2}^{n}}^{\text{arcs } b \text{ to } n-2}(1) = \frac{n(n-3)}{2} = 9 \text{ for } n=6$

Example: Two-Opt Improvement

- Order in which *twoopt* considers each sequence:

1:	1	2	3	4	5	6	1	38
2:	1	3	2	4	5	6	1	39
3:	1	4	3	2	5	6	1	32
4:	1	3	4	2	5	6	1	31
5:	1	4	3	2	5	6	1	32
6:	1	2	4	3	5	6	1	31
7:	1	5	2	4	3	6	1	21
8:	1	2	5	4	3	6	1	21
9:	1	4	2	5	3	6	1	32
10:	1	3	4	2	5	6	1	31
11:	1	5	4	2	3	6	1	12
12:	1	4	5	2	3	6	1	34
13:	1	2	4	5	3	6	1	40
14:	1	3	2	4	5	6	1	39
15:	1	5	2	4	3	6	1	21
16:	1	5	3	2	4	6	1	30
17:	1	5	6	3	2	4	1	31
18:	1	5	4	3	2	6	1	13
19:	1	5	4	6	3	2	1	18
20:	1	5	4	2	6	3	1	20

Local optimal sequence

Sequences considered at end to verify local optimum: n nodes $\Rightarrow$

$$\sum_{j=3}^{n-1}(1)+\sum_{i=2}^{n-2}\sum_{j=i+2}^{n}(1)=\frac{n(n-3)}{2}=9 \text{ for } n=6$$

D:	1	2	3	4	5	6
1:	0	8	6	9	1	5
2:	3	0	1	5	4	2
3:	9	2	0	3	1	1
4:	8	2	1	0	10	6
5:	6	7	10	1	0	10
6:	6	2	5	2	1	0

Note: Not symmetric

TSP Comparison

	TSP Procedure	Total Cost
1	Spacefilling curve	482.7110
2	1 + 2-opt	456
3	Convex hull insert + 2-opt	452
4	Nearest neighbor + 2-opt	439.6
5	Random construction + 2-opt	450, 456
6	Eil51 in TSPLIB	426* optimal