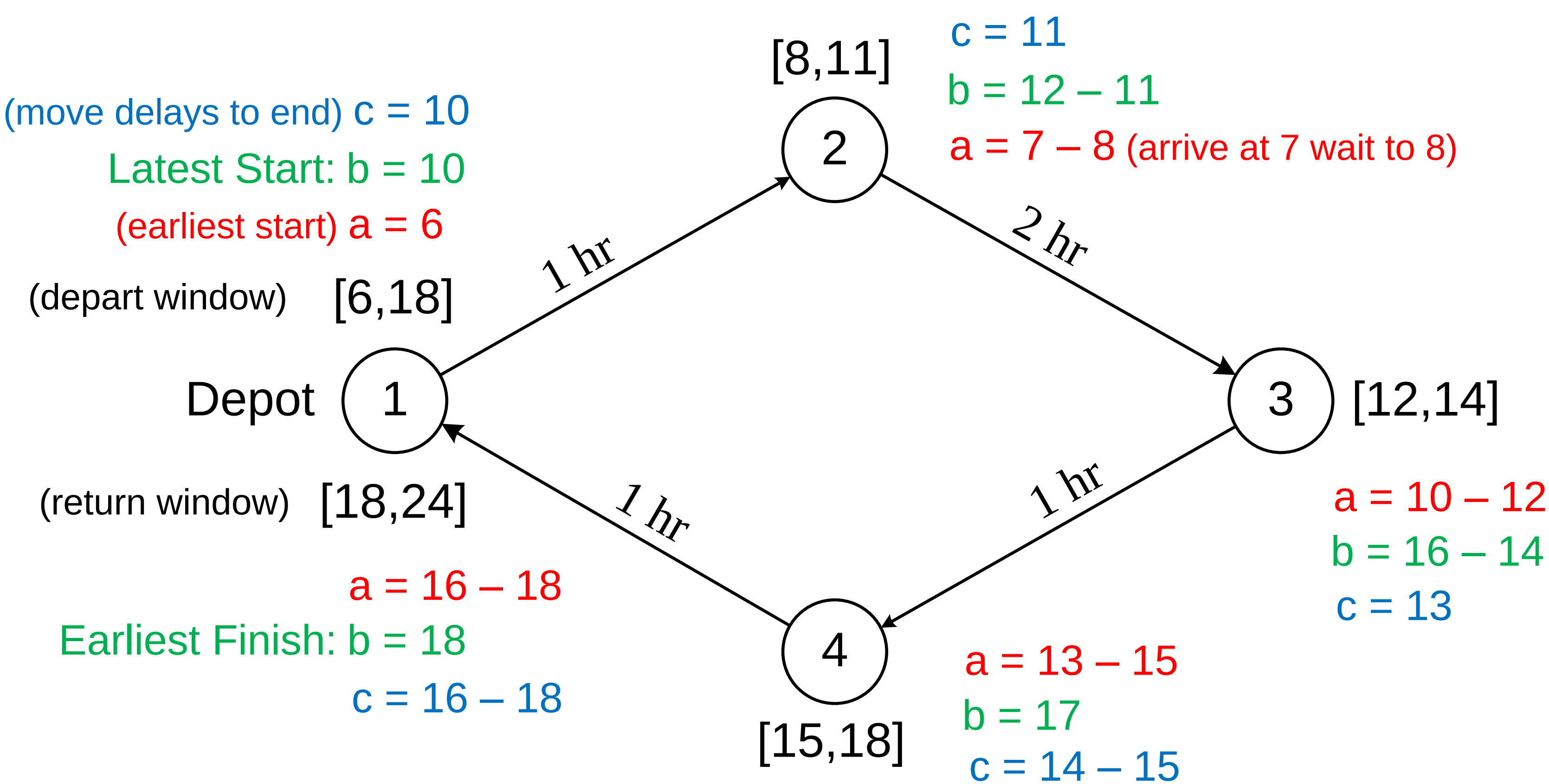
Vehicle Routing Problem

- VRP = TSP + vehicle constraints
- Constraints:
  - Capacity (weight, cube, etc.)
  - Maximum TC (HOS: 11 hr max)
  - Time windows (with/without delay at customer)
    - VRP uses absolute windows that can be checked by simple scanning
    - Project scheduling uses relative windows solved by shortest path with negative arcs
  - Maximum number of routes/vehicles (hard)
- Criteria:
  1. Number of routes/vehicles
  2. TC (time or distance)
- VRP solution can be one time or periodic
  - One time (operational) VRP minimizes TC
  - Periodic (tactical) VRP minimizes TLC (sometimes called a “milk run”)

Ex 18: VRP with Time Windows

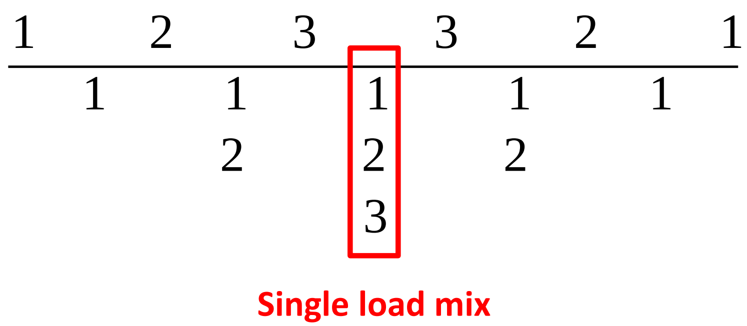
[0,24] hr; Loading/unloading time = 0; Capacity = ∞; LB = 5 hr

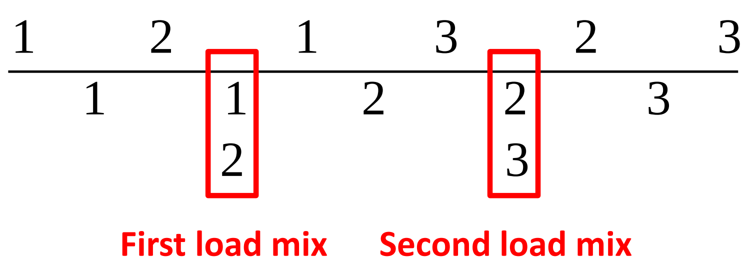


Earliest Finish – Latest Start = 18 – 10 = 8 hr = 5 travel + 3 delay

Periodic Multi-Stop Routing

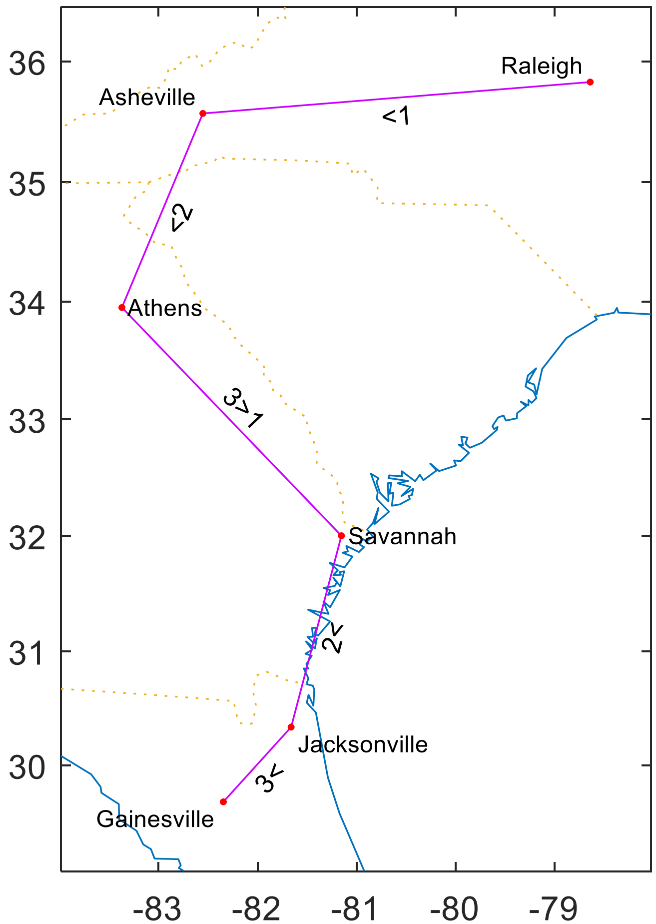
- Periodic consolidated shipments that have the same frequency/interval
- Min TLC of aggregate shipment may not be feasible
  - Different combinations of shipments (*load mix*) may be on board during each segment of route
  - Minimum TLC of unconstrained aggregate of all shipments first determined
  - If needed, all shipment sizes reduced in proportion to load mix with the minimum max payload (to keep common frequency)



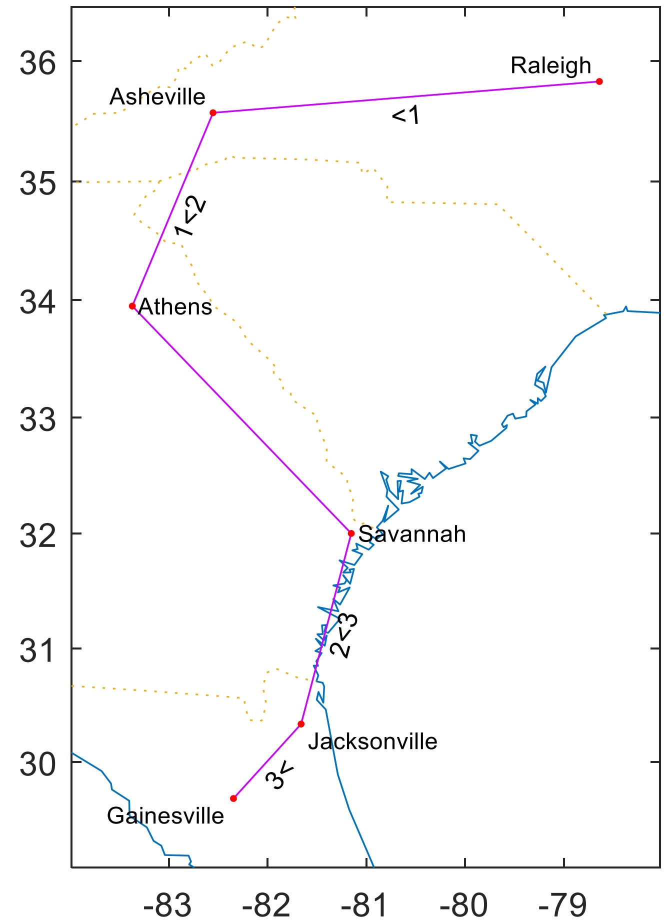


Load Mix Example

Single Load-Mix Instance



Two Load-Mix Instance



TLC Calculation for Multi-Stop Route

- How *minTLC* determines TLC for a route:

$$q_{agg} = \sqrt{\frac{f_{agg} \max\left\{ r_{TL} d_{agg}, MC \right\}}{\alpha v_{agg} h}}$$ (no truck capacity constraints, only min charge)

$$q_i = q_{agg} \frac{f_i}{f_{agg}}$$ (allocate based on demand)

$$s_{L_j} = \sum_{i \in L_j} f_i \Big/ \sum_{i \in L_j} \frac{f_i}{s_i}$$ (aggregate density of shipments in load-mix L_j)

$$k = \min_{L_j} \left\{ 1, \quad \frac{\min\left\{ K_{wt}, \frac{s_{L_j} K_{cu}}{2000} \right\}}{\sum_{i \in L_j} q_i} \right\}$$ (min ratio of max payload to size of shipments in load-mix)

$$q_i^* = k q_i$$ (apply truck capacity deduction factor)

$$TLC^* = \frac{f_{agg}}{\sum q_i^*} r_{TL} d_{agg} + \alpha v_{agg} h \sum q_i^*$$ (d_{agg} = distance of entire route)

Ex 19: Periodic Two Load-Mix Instance

D:	Raleigh	Athens	Asheville	Jax	Savannah	Gville
Raleigh:	0	357	264	501	361	571
Athens:	357	0	145	322	223	361
Asheville:	264	145	0	438	311	488
Jacksonville:	501	322	438	0	143	73
Savannah:	361	223	311	143	0	210
Gainesville:	571	361	488	73	210	0

sh:	f	s	b	e	v	a	h
1:	100	10	1	2	500	1	0.3
2:	200	15	3	4	200	1	0.3
3:	50	7	5	6	800	1	0.3

$$q_{agg} = \sqrt{\frac{f_{agg} \max\left\{r_{TL}\, d_{agg}, MC\right\}}{\alpha v_{agg} h}} = \sqrt{\frac{350 \max\left\{2(848), 135\right\}}{1\,(371.4286)\,0.3}} = 72.9875, \quad q_i = q_{agg}\frac{f_i}{f_{agg}} = 20.85, 41.71, 10.43$$

$$L_1 = \{1,2\}: \quad s_{L_1} = \sum_{i \in L_1} f_i \Big/ \sum_{i \in L_j} \frac{f_i}{s_i} = 12.8571, \quad k = 1$$

$$k = \min\left\{k, \frac{\min\left\{K_{wt}, \dfrac{s_{L_1} K_{cu}}{2000}\right\}}{\sum_{i \in L_1} q_i}\right\} = \min\left\{1, \frac{\min\left\{25, \dfrac{12.8571(2750)}{2000}\right\}}{62.5607}\right\} = \min\{1, 0.2826\} = 0.2826$$

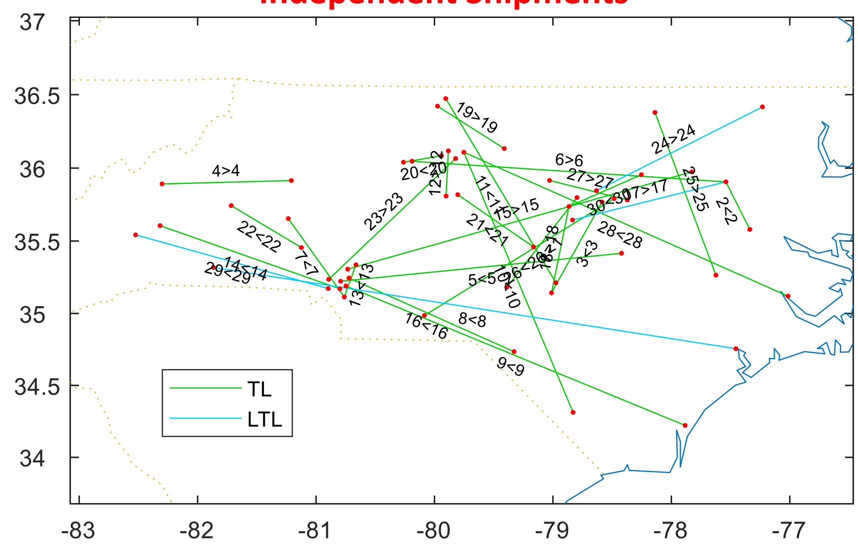
$$L_2 = \{2,3\}: \quad s_{L_2} = 12.2093, \quad k = \min\left\{0.2826, \frac{\min\left\{25, \dfrac{12.2093(2750)}{2000}\right\}}{52.1339}\right\} = \min\{0.2826,\ 0.3220\} = 0.2826$$

$$q_i^* = k\, q_i = 5.8929, 11.7857, 2.9464, \quad TLC^* = \frac{f_{agg}}{\sum q_i^*} r_{TL}\, d_{agg} + \alpha v_{agg} h \sum q_i^* = \$31,079$$

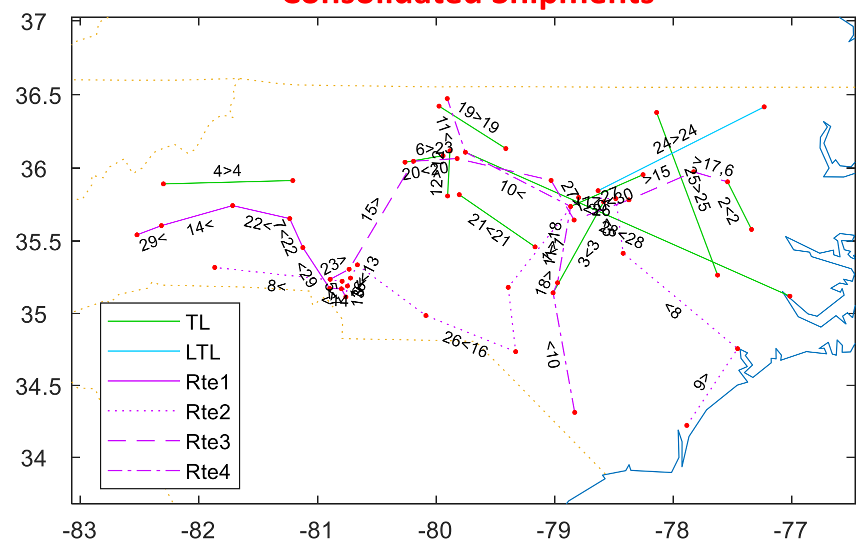
Ex 20: 30 Periodic NC Shipments

sh:	b	e	f	s	v	a	h	d	TLC1	q1	t1	isLTL	qmax
1:	15	42	2.13	1.17	683.19	0.5	0.3	64.97	337.20	1.61	0.76	0	1.61
2:	26	27	4.22	5.46	691.30	0.5	0.3	30.24	325.51	1.57	0.37	0	7.51
3:	23	40	6.27	15.23	5,843.73	0.5	0.3	59.27	1,614.46	0.92	0.15	0	20.94
4:	53	51	6.41	5.71	383.71	0.5	0.3	73.29	464.93	4.04	0.63	0	7.85
5:	17	32	6.32	13.82	2,776.55	0.5	0.3	161.07	1,842.23	2.21	0.35	0	19.00
28:	48	10	1.36	5.22	152.43	0.5	0.3	201.70	224.11	4.90	3.60	0	7.17
29:	37	54	4.92	26.52	8,327.51	0.5	0.3	114.03	1,599.38	0.24	0.05	1	25.00
30:	14	13	2.46	13.09	2,164.49	0.5	0.3	44.83	535.60	0.82	0.33	0	18.00

Independent Shipments



Consolidated Shipments



Ex 21: Minimize Number of Trucks

- Given begin-end times for 10 routes, determine minimum number of trucks needed
  - Trucks begin and end at the depot
  - Optimal solution via direct minimum spanning tree, greedy works fine