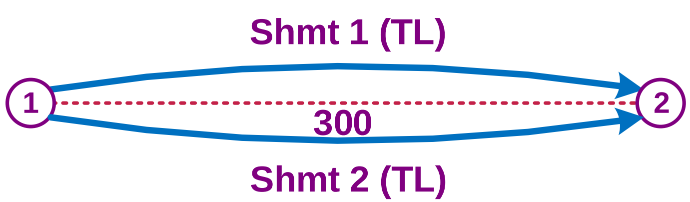
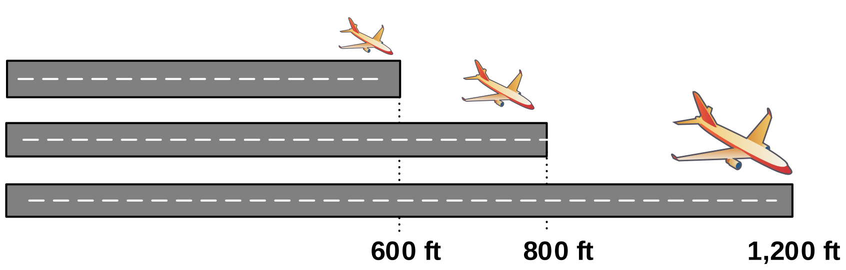
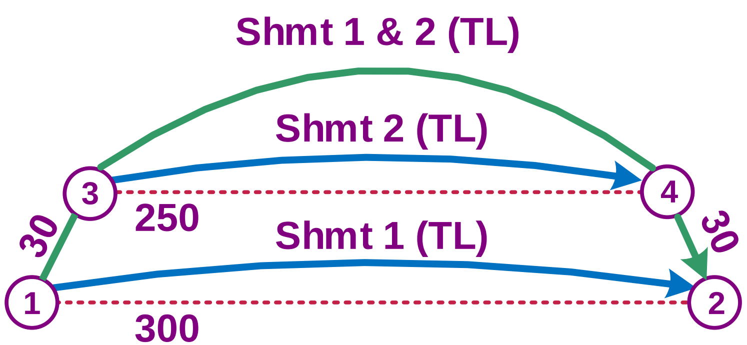
Cost Allocation for Routing

- Allocation Problem: If shipments from different firms are sharing the same vehicle, how much should each shipment contribute to the total cost paid to carrier?
  - What is a “fair” allocation?
  - Allocated cost should not exceed cost as an independent shipment
  - Examples:

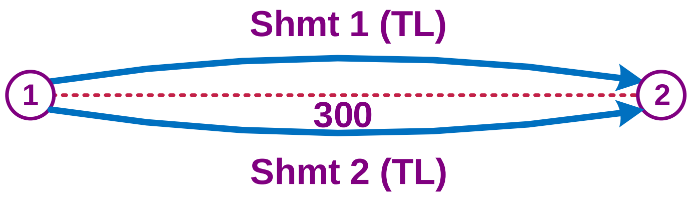




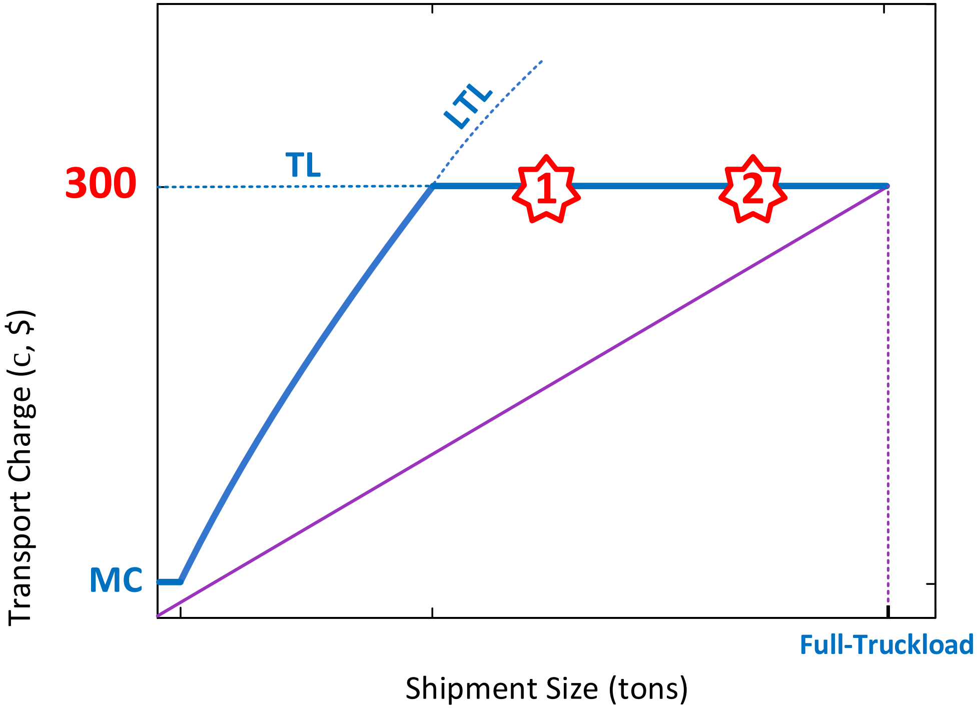


Ex 22: TL + TL Same O/D

- Shipment 1
  - sets r = 1, d = 300, TL, max c = 300
- Shipment 2
  - same O/D, TL, max c = 300



$$c = \max\left\{\min\left\{c_{LTL}, c_{TL}\right\}, MC\right\}$$

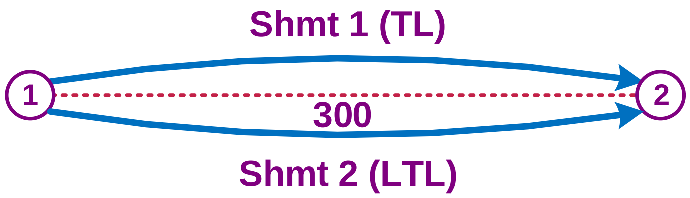


$$c = c_1 = 300$$

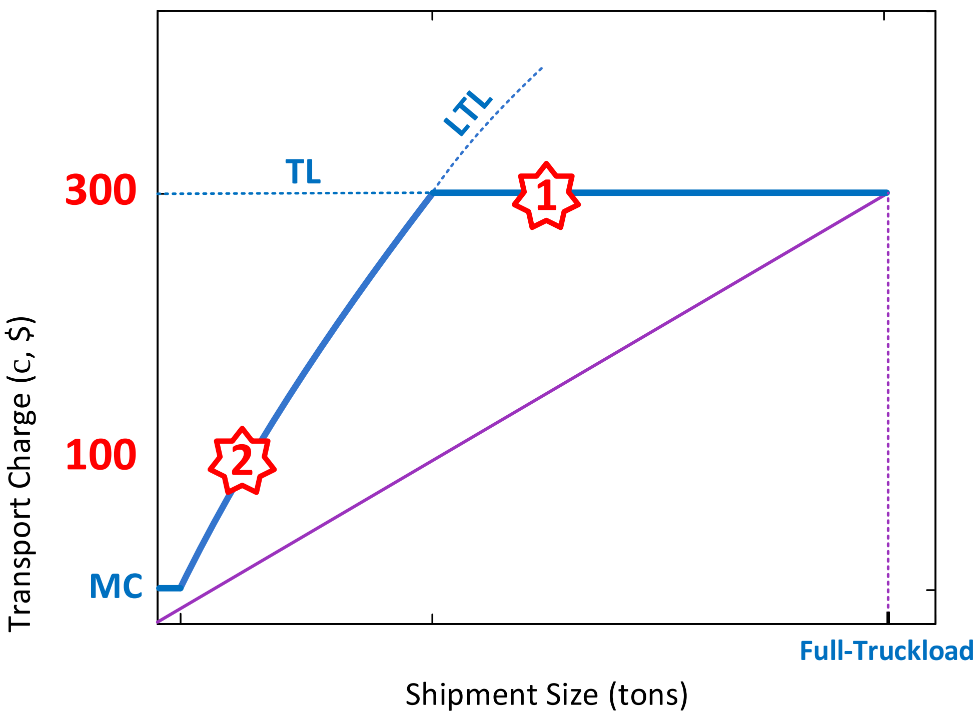
$$c = 300 = c_1 + c_2, \quad c_1 = c_2 = \frac{c}{2} = 150$$

Ex 23: TL + LTL Same O/D

- Shipment 1
  - sets r = 1, d = 300, TL, max c = 300
- Shipment 2
  - same O/D, LTL, max c = 100



$$c = \max\left\{\min\left\{c_{LTL}, c_{TL}\right\}, MC\right\}$$



$$c = 300 = c_1 + c_2$$

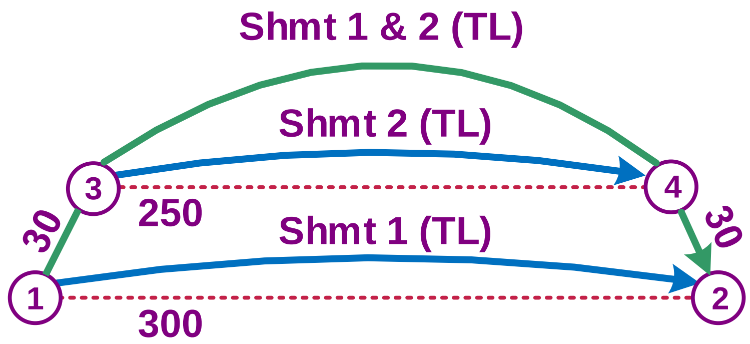
	1	2
1 2	300	0
2 1	200	100
	250	50

$$c_1 = 250, \quad c_2 = 50$$

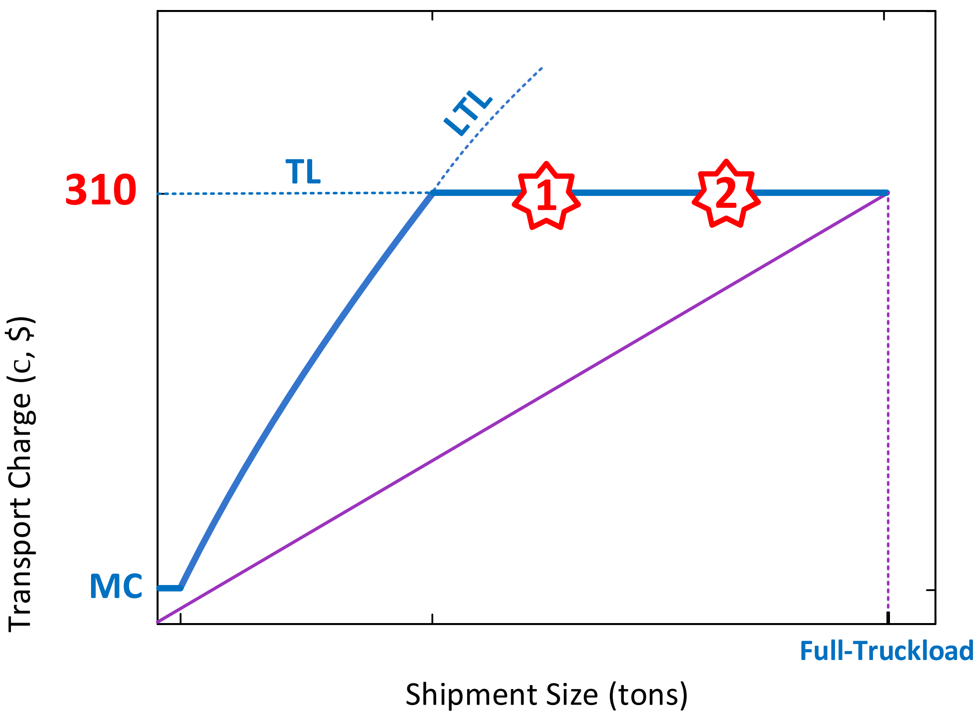
(Shapley value allocation)

Ex 24: TL + TL Different O/D

- Shipment 1
  - sets r = 1, d = 300, TL, max c = 300
- Shipment 2
  - different O/D, TL, max c = 250



$$c = \max\left\{\min\left\{c_{LTL}, c_{TL}\right\}, MC\right\}$$



$$c = 310 = c_1 + c_2$$

	1	2
1 2	300	10
2 1	60	250
	180	130

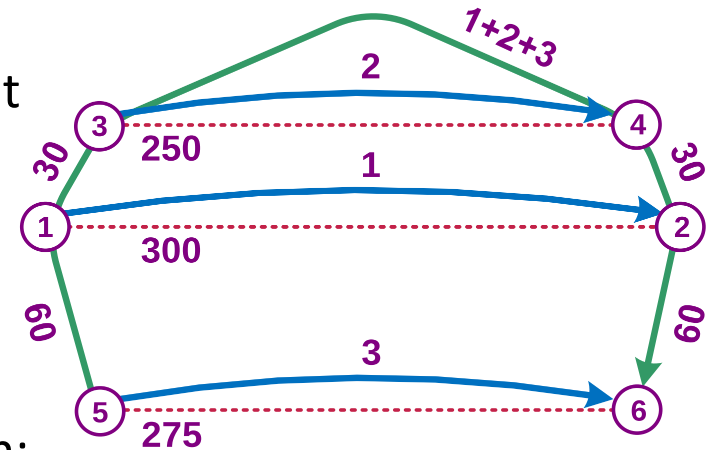
$$c_1 = 180, \quad c_2 = 130$$

Shapley Value Approximation

- Shapley value
  - Average additional cost each shipment imposes by joining route
  - Exact value requires $n!$

$$\alpha_i = \sum_{0 \le m \le n-1} \frac{m!(n-m-1)!}{n!} \sum_{\substack{M \subset N \setminus i \\ |M|=m}} \left(\sigma_{M \cup \{i\}} - \sigma_M \right)$$

  - Use n^2 pairwise savings approximation:



$$c_i^{\text{sav}} = \frac{c_L^{\text{sav}}}{n} + \frac{1}{n-1} \sum_{j=1}^{n} \frac{c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}}}{2} - \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk}^{\text{sav}}$$

$$c_{ij}^{\text{sav}} = c_i^0 + c_j^0 - c_{(i,j)}, \quad c_L^{\text{sav}} = \sum_{i=1}^{n} c_i^0 - c_L$$

$$c_L^{\text{sav}} = \sum_{i=1}^{n} c_i^0 - c_L$$

$$= 300 + 250 + 275 - 430$$

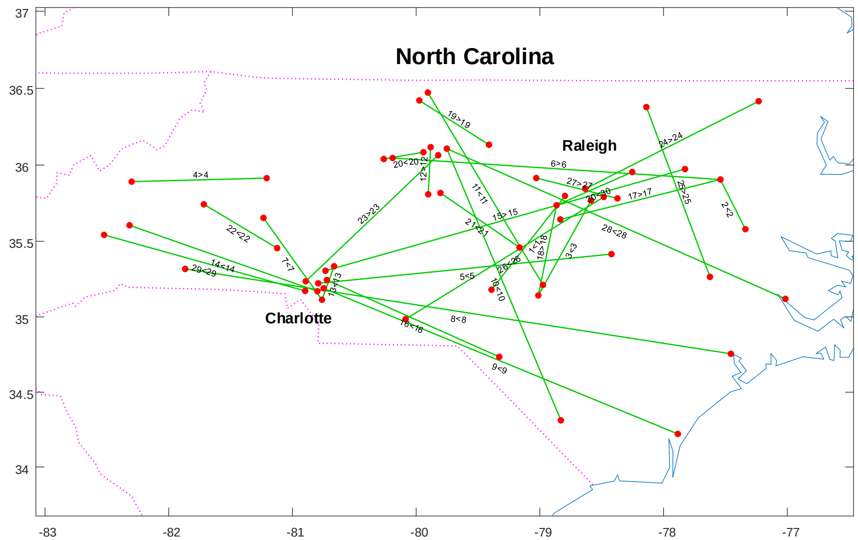
$$= 825 - 430 = \$395 \text{ savings for load}$$

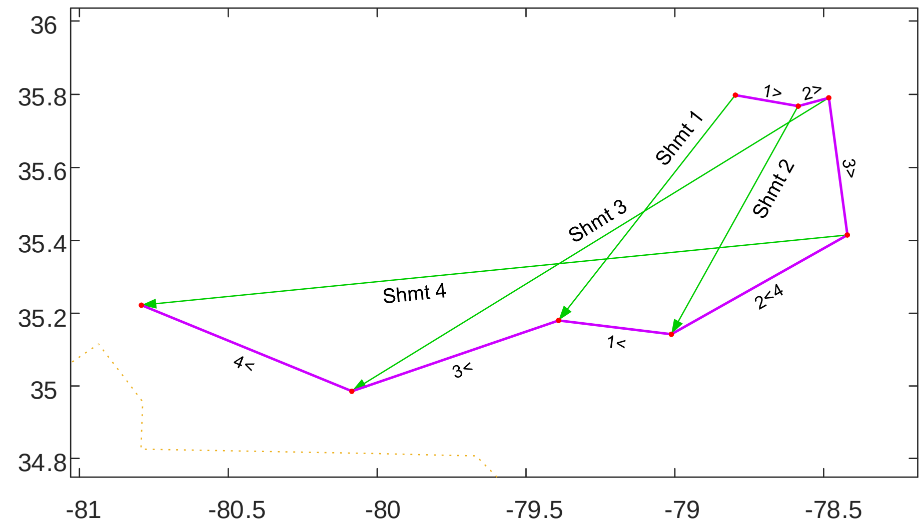
:	c0	c_equal	c_eq_sav	c_Shap_exact	c_Shap_approx
1:	300	143.33	168.33	130.00	130.00
2:	250	143.33	118.33	122.50	122.50
3:	275	143.33	143.33	177.50	177.50
Total:	825	430.00	430.00	430.00	430.00
Avg:	275	143.33	143.33	143.33	143.33

C:	1	2	3
123:	300	10	120
132:	300	35	95
213:	60	250	120
231:	0	250	180
312:	120	35	275
321:	0	155	275

Ex 25: Intercity Trucking

- 4 out 30 available shipments form consolidated load
  - Savings of 824.81 – 452.47 = 372.34 from consolidation
  - Pairwise approximation differs from exact Shapley value





Shmt :	c0	c_equal	(%)	c_eq_sav	(%)	c_Shap_exact	(%)	c_Shap_approx
1:	130	113	13	37	72	62	52	52
2:	119	113	5	25	79	53	55	50
3:	254	113	56	161	37	117	54	123
4:	322	113	65	229	29	220	32	227
Total:	825	452		452		452		452