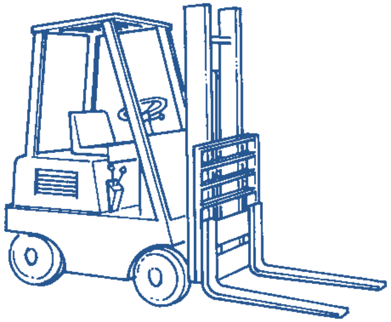
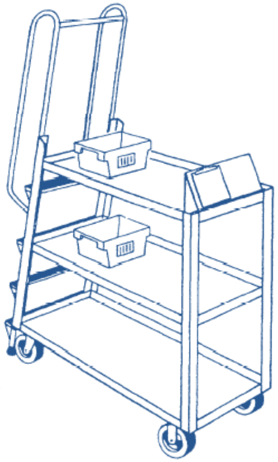
Storage and Retrieval Cycle

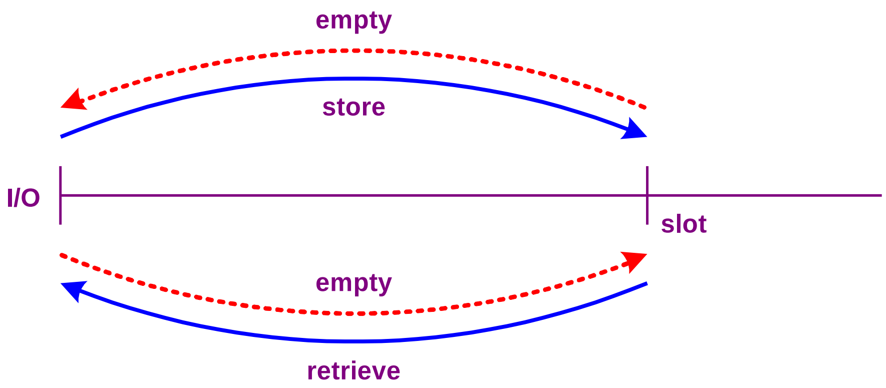
- A storage and retrieval (S/R) cycle is one complete roundtrip from an I/O port to slot(s) and back to the I/O
- Type of cycle depends on load carrying ability:
  - Carrying one load at-a-time (load carried on a pallet):
    - Single command
    - Dual command



  - Carrying multiple loads (order picking of small items):
    - Multiple command



Single-Command S/R Cycle



- Single-command (SC) cycles:
  - Storage: carry one load to slot for storage and return empty back to I/O port, or
  - Retrieval: travel empty to slot to retrieve load and return with it back to I/O port

Expected time for each SC S/R cycle:

$$t_{SC} = \frac{d_{SC}}{v} + t_L + t_U = \frac{d_{SC}}{v} + 2t_{L/U}$$

where

d_{SC} = expected distance per SC cycle

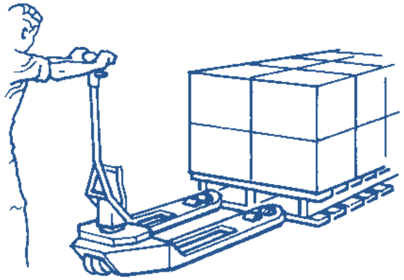
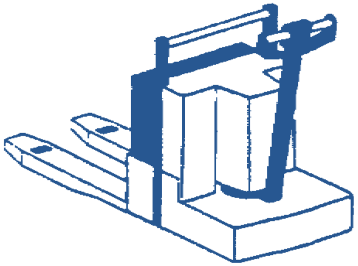
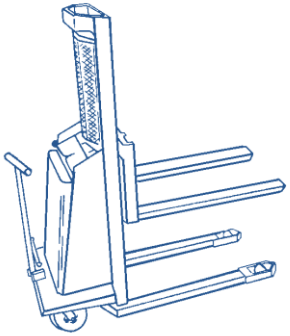
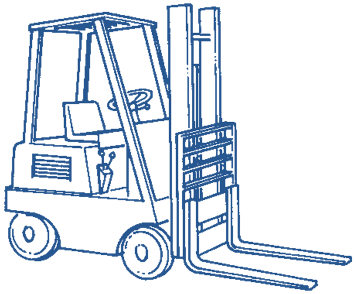
v = average travel speed (e.g.: 2 mph = 176 fpm walking; 7 mph = 616 fpm riding)

t_L = loading time

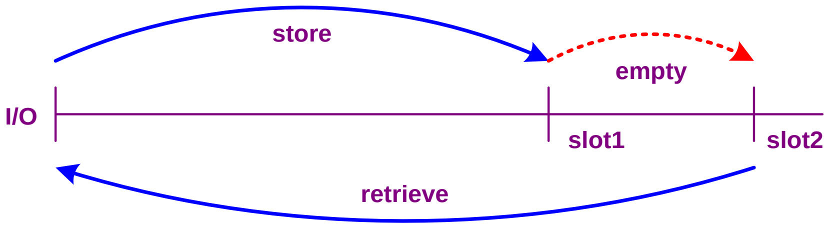
t_U = unloading time

$t_{L/U}$ = loading/unloading time, if same value

Industrial Trucks: Walk vs. Ride

Walk (2 mph = 176 fpm)	**Ride (7 mph = 616 fpm)**
	
Pallet Jack	Pallet Truck
	
Walkie Stacker	Sit-down Counterbalanced Lift Truck

Dual-Command S/R Cycle

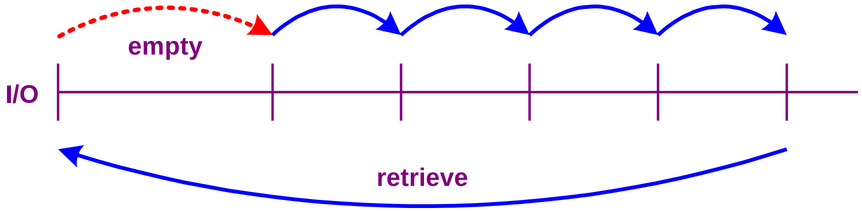


Expected time for each SC S/R cycle:

$$t_{DC} = \frac{d_{DC}}{v} + 2t_L + 2t_U = \frac{d_{DC}}{v} + 4t_{L/U}$$

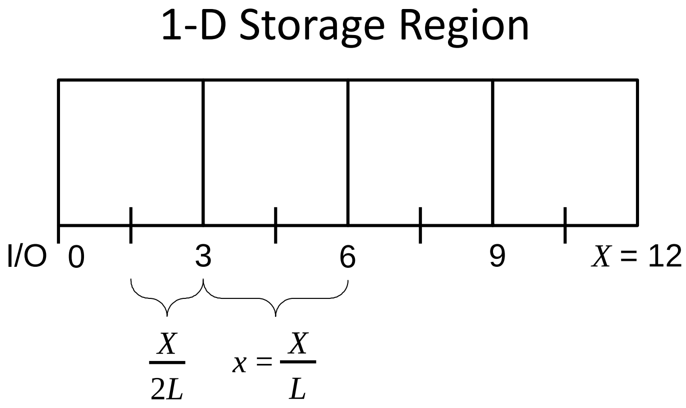
- Dual-command (DC):
- Combine storage with a retrieval:
  - store load in slot 1, travel empty to slot 2 to retrieve load
- Can reduce travel distance by a third, on average
- Also termed task "interleaving"

Multi-Command S/R Cycle



- Multi-command: multiple loads can be carried at the same time
- Used in case and piece order picking
- Picker routed to slots
  - Simple VRP procedures can be used

1-D Expected Distance



$$TD_{1-way} = \sum_{i=1}^{L}\left(i\frac{X}{L}-\frac{X}{2L}\right) = \frac{X}{L}\sum_{i=1}^{L} i - \frac{X}{2L}(1)$$

$$= \frac{X}{L}\left(\frac{L(L+1)}{2}\right) - \frac{X}{2L}(L)$$

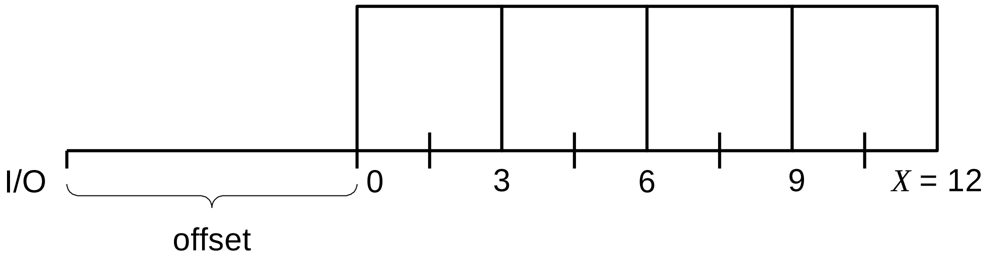
$$= \frac{XL + X - X}{2} = \frac{XL}{2}$$

$$ED_{1-way} = \frac{TD_{1-way}}{L} = \frac{X}{2}$$

$$d_{SC} = 2(ED_{1-way}) = X$$

- Assumptions:
  - All single-command cycles
  - Rectilinear distances
  - Each slot is region used with equal frequency (i.e., randomized storage)
- Expected distance is the average distance from I/O port to midpoint of each slot
  - e.g., [2(1.5) + 2(4.5) + 2(6.5) + 2(10.5)]/4 = 12

Off-set I/O Port



$$d_{SC} = 2(d_{\text{offset}}) + X$$

- If the I/O port is off-set from the storage region, then 2 times the distance of the offset is added the expected distance within the slots

2-D Expected Distances

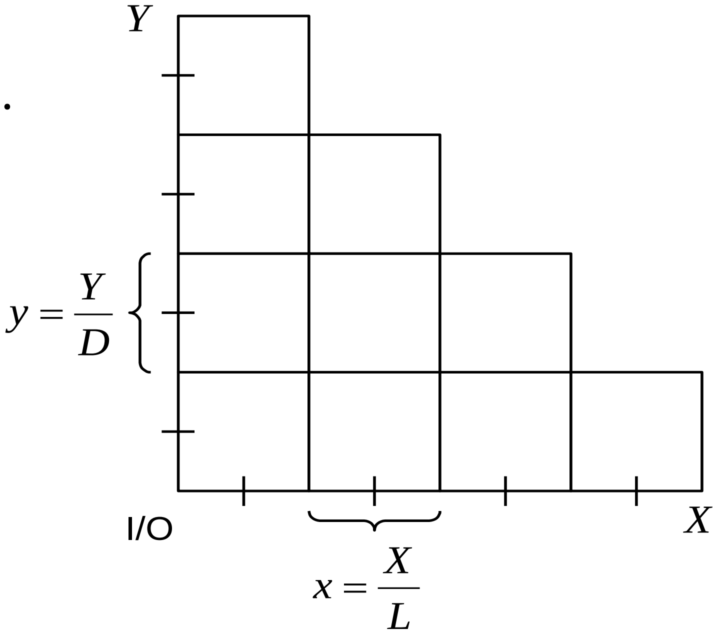
- Since dimensions *X* and *Y* are independent of each other for rectilinear distances, the expected distance for a 2-D rectangular region with the I/O port in a corner is just the sum of the distance in *X* and in *Y*: $d_{SC}^{rect} = X + Y$
- For a triangular region with the I/O port in the corner:

$$TD_{\text{1-way}} = \sum_{i=1}^{L} \sum_{j=1}^{L-i+1} \left[\left(i\frac{X}{L} - \frac{X}{2L} \right) + \left(j\frac{X}{L} - \frac{X}{2L} \right) \right] = \quad \cdots$$

$$= \frac{X}{6}\left(2L^2 + 3L + 1\right)$$

$$ED_{\text{1-way}} = \frac{TD_{\text{1-way}}}{\frac{L(L+1)}{2}} = \frac{2}{3}X + \frac{X}{3L} = \frac{2}{3}X, \quad \text{as } L \to \infty$$

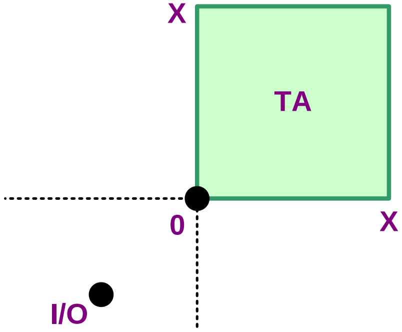
$$d_{SC}^{tri} = 2\left(\frac{2}{3}X\right) = 2\left(\frac{1}{3}X + \frac{1}{3}Y\right) = \frac{2}{3}(X+Y) = \frac{4}{3}X, \quad \text{if } X = Y$$

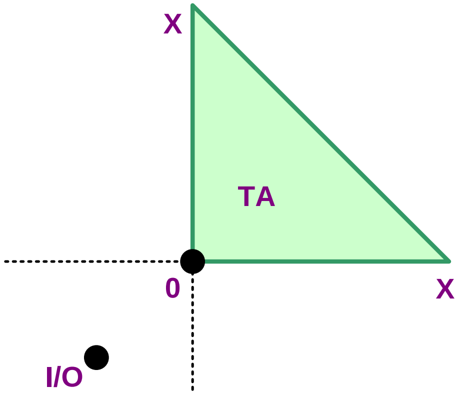


I/O-to-Side Configurations

Rectangular

Triangular





$$TA = X^2$$

$$\Rightarrow X = \sqrt{TA}$$

$$\Rightarrow d_{SC} = 2\sqrt{TA}$$

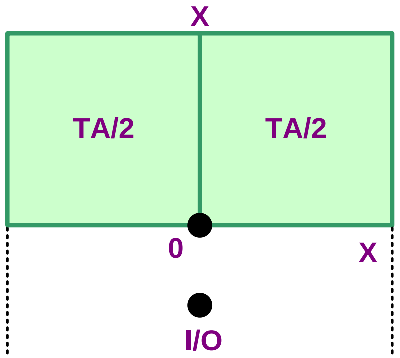
$$TA = \frac{1}{2} X^2$$

$$\Rightarrow X = \sqrt{2TA} = \sqrt{2}\sqrt{TA}$$

$$\Rightarrow d_{SC} = \frac{4}{3}\sqrt{2}\sqrt{TA} = 1.886\sqrt{TA}$$

I/O-at-Middle Configurations

Rectangular

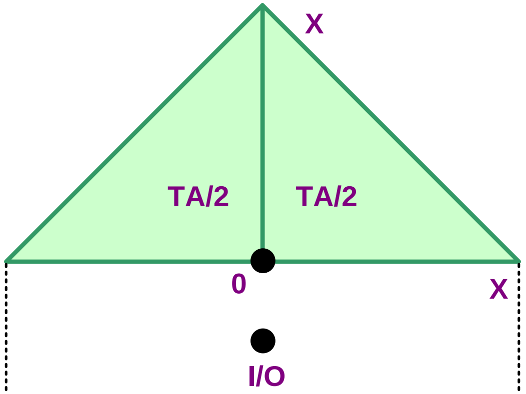


$$\frac{TA}{2} = X^2$$

$$\Rightarrow X = \sqrt{\frac{TA}{2}} = \frac{\sqrt{TA}}{\sqrt{2}}$$

$$\Rightarrow d_{SC} = \sqrt{2}\sqrt{TA} = 1.414\sqrt{TA}$$

Triangular



$$\frac{TA}{2} = \frac{1}{2}X^2$$

$$\Rightarrow X = \sqrt{TA}$$

$$\Rightarrow d_{SC} = \frac{4}{3}\sqrt{TA} = 1.333\sqrt{TA}$$

Ex 28: Handling Requirements

Pallet loads will be unloaded at the receiving dock of a warehouse and placed on the floor. From there, they will be transported 500 feet using a dedicated pallet truck to the in-floor induction conveyor of an AS/RS. Given

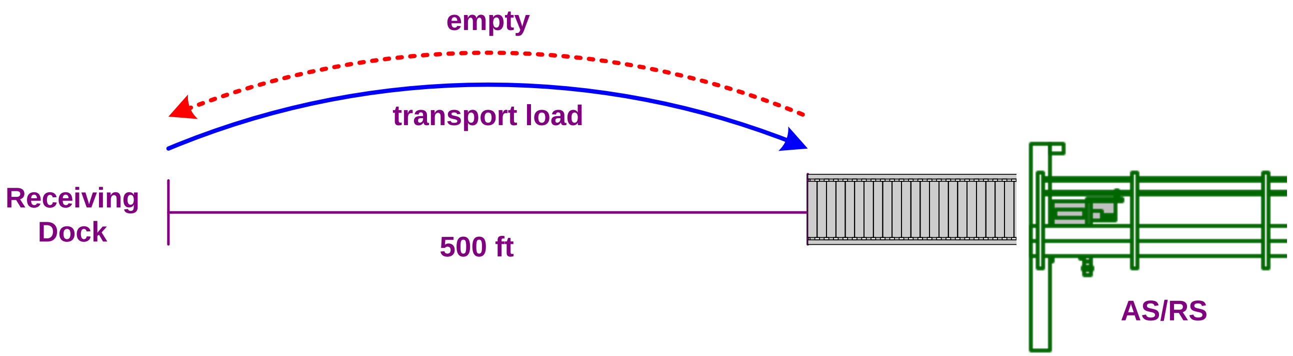
a. It takes 30 sec to load each pallet at the dock

b. 30 sec to unload it at the induction conveyor

c. There will be 80,000 loads per year on average

d. Operator rides on the truck (because a pallet truck)

e. Facility will operate 50 weeks per year, 40 hours per week



Ex 28: Handling Requirements

1. Assuming that it will take 30 seconds to load each pallet at the dock and 30 seconds to unload it at the induction conveyor, what is the expected time required for each single-command S/R cycle?

$$d_{SC} = 2(500) = 1000 \text{ ft/mov}$$

$$t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{1000 \text{ ft/mov}}{616 \text{ ft/min}} + 2\left(\frac{30}{60}\right) \text{min/mov}$$

$$= 2.62 \text{ min/mov} = \frac{2.62}{60} \text{ hr/mov}$$

(616 fpm because operator rides on a pallet truck)

Ex 28: Handling Requirements

2. Assuming that there will be 80,000 loads per year on average and that the facility will operate for 50 weeks per year, 40 hours per week, what is the minimum number of trucks needed?

$$r_{avg} = \frac{80,000 \text{ mov/yr}}{50(40) \text{ hr/yr}} = 40 \text{ mov/hr}$$

$$m = \lfloor r_{avg} t_{SC} + 1 \rfloor$$

$$= \left\lfloor 40\left(\frac{2.62}{60}\right) + 1 \right\rfloor = \lfloor 1.75 + 1 \rfloor$$

$$= 2 \text{ trucks}$$

Ex 28: Handling Requirements

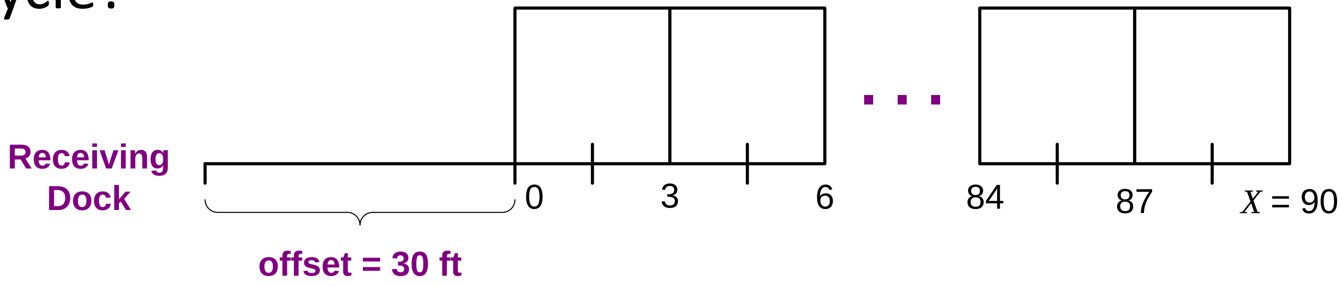
3. How many trucks are needed to handle a peak expected demand of 80 moves per hour?

$$r_{peak} = 80 \text{ mov/hr}$$

$$\begin{aligned} m &= \lfloor r_{peak} t_{SC} + 1 \rfloor \\ &= \left\lfloor 80\left(\frac{2.62}{60}\right) + 1 \right\rfloor = \lfloor 3.50 + 1 \rfloor \\ &= 4 \text{ trucks} \end{aligned}$$

Ex 28: Handling Requirements

4. If, instead of unloading at the conveyor, the 3-foot-wide loads are placed side-by-side in a staging area along one side of 90-foot aisle that begins 30 feet from the dock, what is the expected time required for each single-command S/R cycle?



$$d_{SC} = 2(d_{\text{offset}}) + X = 2(30) + 90 = 150 \text{ ft}$$

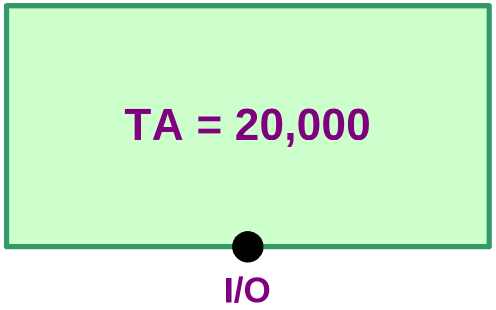
$$t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{150 \text{ ft/mov}}{616 \text{ ft/min}} + 2\left(\frac{30}{60}\right) \text{min/mov}$$

$$= 1.24 \text{ min/mov} = \frac{1.24}{60} \text{ hr/mov}$$

Estimating Handling Costs

- Warehouse design involves the trade-off between building and handling cost.
- Maximizing the cube utilization of a storage region will help minimize building costs.
- Handling costs can be estimated by determining:
  1. Expected time required for each move based on an average of the time required to reach each slot in the region.
  2. Number of vehicles needed to handle a target *peak demand* for moves, e.g., moves per hour.
  3. *Operating costs per hour of vehicle operation*, e.g., labor, fuel (assuming the operators can perform other productive tasks when not operating a truck)
  4. Annual operating costs based on *annual demand* for moves.
  5. Total handling costs as the sum of the annual capital recovery costs for the vehicles and the annual operating costs.

Ex 29: Estimating Handing Cost



$$\Uparrow$$

Add 20% Cross aisle:

$$TA = TA' \times 1.2$$

$$= 20,000 \text{ ft}^2$$

$$\Uparrow$$

Total Storage Area:

$$D^* \Rightarrow L(D^*) \Rightarrow TA'$$

Expected Distance: $d_{SC} = \sqrt{2}\sqrt{TA} = \sqrt{2}\sqrt{20,000} = 200 \text{ ft}$

Expected Time: $t_{SC} = \dfrac{d_{SC}}{v} + 2t_{L/U}$

$$= \frac{200 \text{ ft}}{200 \text{ fpm}} + 2(0.5 \text{ min}) = 2 \text{ min per move}$$

Peak Demand: $r_{\text{peak}} = 75 \text{ moves per hour}$

Annual Demand: $r_{\text{year}} = 100,000 \text{ moves per year}$

Number of Trucks: $m = \left\lfloor r_{\text{peak}} \dfrac{t_{SC}}{60} + 1 \right\rfloor = \lfloor 3.5 \rfloor = 3 \text{ trucks}$

Handling Cost: $TC_{\text{hand}} = mK_{\text{truck}} + r_{\text{year}} \dfrac{t_{SC}}{60} C_{\text{labor}}$

$$= 3(\$2,500 / \text{tr-yr}) + 100,000 \frac{2}{60} (\$10 / \text{hr})$$

$$= \$7,500 + \$33,333 = \$40,833 \text{ per year}$$

Dedicated Storage Assignment (DSAP)

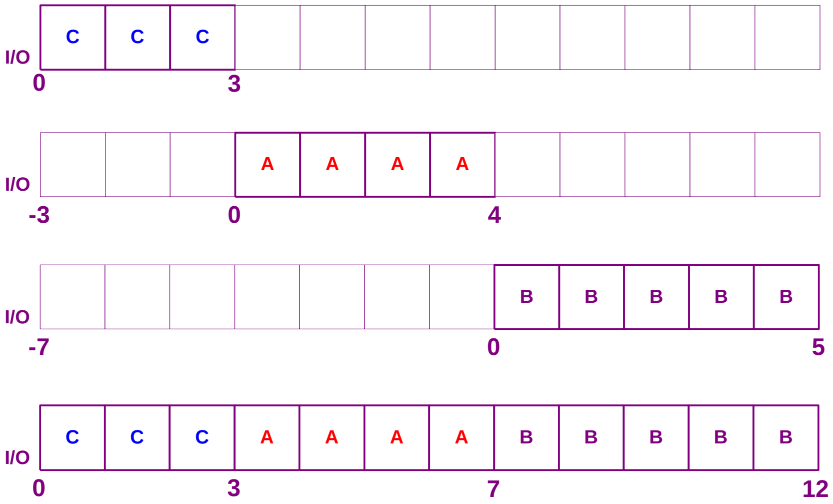
- The assignment of items to slots is termed *slotting*
  - With randomized storage, all items are assigned to all slots
- DSAP (dedicated storage assignment problem):
  - Assign *N* items to slots to minimize total cost of material flow
- DSAP solution procedure:
  1. *Order Slots:* Compute the expected cost for each slot and then put into nondecreasing order
  2. *Order Items:* Put the flow density (flow per unit of volume) for each item *i* into nonincreasing order

$$\frac{f_{[1]}}{M_{[1]}s_{[1]}} \geq \frac{f_{[2]}}{M_{[2]}s_{[2]}} \geq \cdots \geq \frac{f_{[N]}}{M_{[N]}s_{[N]}}$$

  3. *Assign Items to Slots:* For $i = 1, \ldots, N$, assign item [*i*] to the first slots with a total volume of at least $M_{[i]}s_{[i]}$

Ex 30: 1-D Slotting

		A	B	C
Max units	M	4	5	3
Space/unit	s	1	1	1
Flow	f	24	7	21
Flow Density	f/(M x s)	6.00	1.40	7.00



Flow Density	1-D Slot Assignments	Expected Distance	Flow	Total Distance
$\frac{21}{3}=7.00$	I/O C C C 0 3	2(0) + 3 = 3 ×	21 =	63
$\frac{24}{4}=6.00$	I/O A A A A -3 0 4	2(3) + 4 = 10 ×	24 =	240
$\frac{7}{5}=1.40$	I/O B B B B B -7 0 5	2(7) + 5 = 19 ×	7 =	133
	I/O C C C A A A A B B B B B 0 3 7 12			436

Ex 30: 1-D Slotting

		Dedicated			Random	Class-Based		
		A	B	C	ABC	AB	AC	BC
Max units	M	4	5	3	9	7	7	8
Space/unit	s	1	1	1	1	1	1	1
Flow	f	24	7	21	52	31	45	28
Flow Density	f/(M x s)	6.00	1.40	7.00	5.78	4.43	6.43	3.50

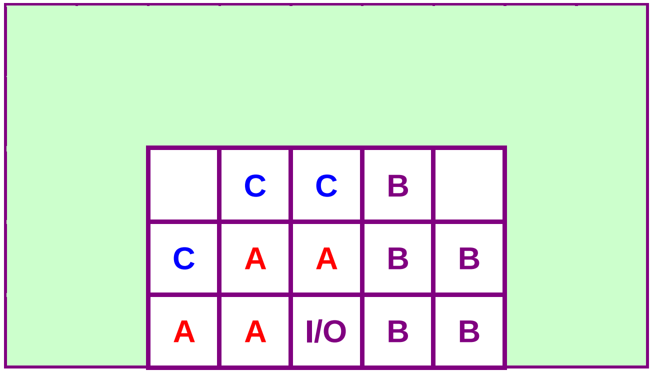
		1-D Slot Assignments												Total Distance	Total Space
Dedicated (flow density)	I/O	C	C	C	A	A	A	A	B	B	B	B	B	436	12
Dedicated (flow only)	I/O	A	A	A	A	C	C	C	B	B	B	B	B	460	12
Class-based	I/O	C	C	C	AB	AB	AB	AB	AB	AB	AB			466	10
Randomized	I/O	ABC	ABC	ABC	ABC	ABC	ABC	ABC	ABC	ABC				468	9

Ex 31: 2-D Slotting

		A	B	C
Max units	M	4	5	3
Space/unit	s	1	1	1
Flow	f	24	7	21
Flow Density	f/(M x s)	6.00	1.40	7.00

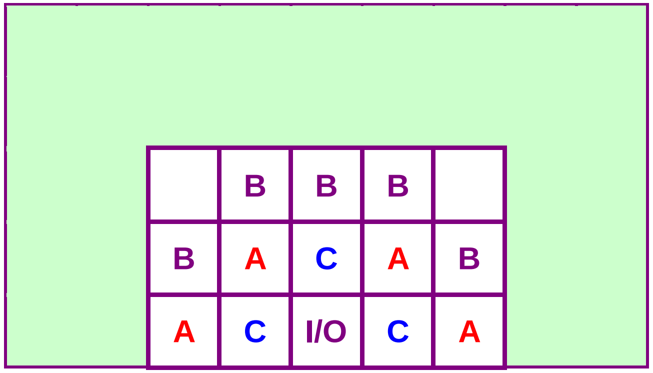
8	7	6	5	4	5	6	7	8
7	6	5	4	3	4	5	6	7
6	5	4	3	2	3	4	5	6
5	4	3	2	1	2	3	4	5
4	3	2	1	0	1	2	3	4

Distance from I/O to Slot



	C	C	B	
C	A	A	B	B
A	A	I/O	B	B

Original Assignment (TD = 215)



	B	B	B	
B	A	C	A	B
A	C	I/O	C	A

Optimal Assignment (TD = 177)

DSAP Assumptions

1. All SC S/R moves
2. For item *i*, probability of move to/from each slot assigned to item is the same
3. The *factoring assumption*:
   a. Handling cost and distances (or times) for each slot are identical for all items
   b. Percent of S/R moves of item stored at slot *j* to/from I/O port *k* is identical for all items

- Depending of which assumptions not valid, can determine assignment using other procedures

$$\left[\left(\frac{f_i}{M_i}\cdot d_j\right)x_{ij}\right] DSAP \subset \underset{\left(c_{ij}x_{ij}\right)}{LAP} \subset LP \subset \underset{\underset{TSP}{\cup}}{QAP}\left(c_{ijkl}x_{ij}x_{kl}\right)$$

Ex 32: 1-D DSAP

- What is the change in the minimum expected total distance traveled if dedicated, as compared to randomized, block stacking is used, where
  a. Slots located on one side of 10-foot-wide down aisle
  b. All single-command S/R operations
  c. Each lane is three-deep, four-high
  d. 40 × 36 in. two-way pallet used for all loads
  e. Max inventory levels of SKUs A, B, C are 94, 64, and 50
  f. Inventory levels are uncorrelated and retrievals occur at a constant rate
  g. Throughput requirements of A, B, C are 160, 140, 130
  h. Single I/O port is located at the end of the aisle

Ex 32: 1-D DSAP

- Randomized:

$$M = \left\lfloor \frac{M_A + M_B + M_C}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{94+64+50}{2} + \frac{1}{2} \right\rfloor = 104$$

$$L_{rand} = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{104 + 3(4)\left(\frac{3-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 11 \text{ lanes}$$

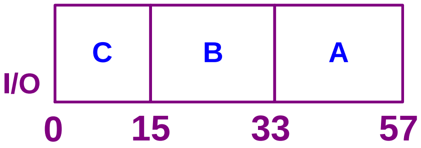
$$X = xL_{rand} = 3(11) = 33 \text{ ft}$$

$$d_{SC} = X = 33 \text{ ft}$$

$$TD_{rand} = \left(f_A + f_B + f_C \right) X = \left(160+140+130\right)33 = 14{,}190 \text{ ft}$$

Ex 32: 1-D DSAP

- Dedicated:



$$\frac{f_A}{M_A} = \frac{160}{94} = 1.7,\ \frac{f_B}{M_B} = \frac{140}{64} = 2.19,\ \frac{f_C}{M_C} = \frac{130}{50} = 2.6 \quad \Rightarrow \quad C > B > A$$

$$L_A = \left\lceil \frac{M_A}{DH} \right\rceil = \left\lceil \frac{94}{3(4)} \right\rceil = 8,\ L_B = \left\lceil \frac{M_B}{DH} \right\rceil = \left\lceil \frac{64}{3(4)} \right\rceil = 6,\ L_C = \left\lceil \frac{M_C}{DH} \right\rceil = \left\lceil \frac{50}{3(4)} \right\rceil = 5$$

$$X_C = xL_C = 3(5) = 15,\ X_B = xL_B = 3(6) = 18,\ X_A = xL_A = 3(8) = 24$$

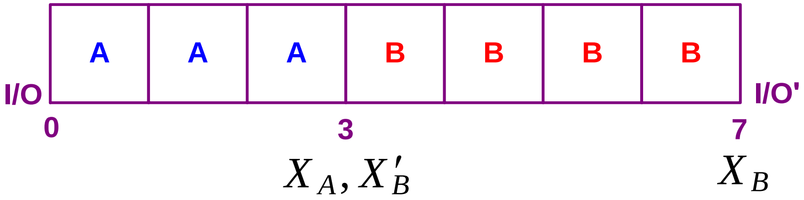
$$d_{SC}^{C} = X_C = 3(5) = 15 \text{ ft}$$

$$d_{SC}^{B} = 2(X_C) + X_B = 2(15) + 18 = 48 \text{ ft}$$

$$d_{SC}^{A} = 2(X_C + X_B) + X_A = 2(15+18) + 24 = 90 \text{ ft}$$

$$TD_{ded} = f_A d_{SC}^{A} + f_B d_{SC}^{B} + f_C d_{SC}^{C} = 160(90) + 140(48) + 130(15) = 23,070 \text{ ft}$$

1-D Multiple Region Expected Distance



- In 1-D, easy to determine the offset
- In 2-D, no single offset value for each region

$$d_{SC}^{A} = d_A = X_A = 3$$

$$d_B = 2d_{offset} + \left(X_B - X_A\right) = 2X_A + \left(X_B - X_A\right) = X_A + X_B = 10$$

$$= 2\left(d_{I/\mathrm{O\ to}\ I/\mathrm{O}'}\right) - X'_B = 2\left(7\right) - 4 = 10$$

$$d_{AB} = 7$$

$$TA_A = X_A = 3, \quad TA_B = X_B - X_A = 4, \quad TA_{AB} = TA_A + TA_B = 7$$

$$TM_A = TA_A d_A, \quad TM_B = TA_B d_B$$

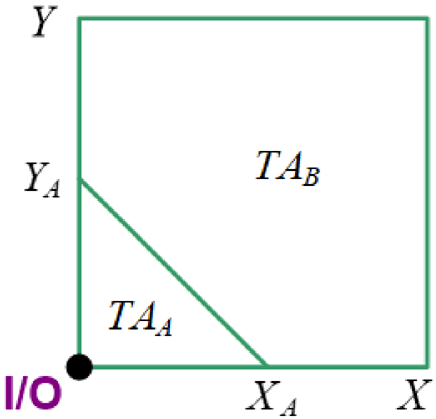
$$TM_{AB} = TA_{AB} d_{AB} = X_B^2 = \left(X_A + X_B - X_A\right)^2 = X_A^2 + \left(X_B - X_A\right)\left(2X_A + X_B - X_A\right)$$

$$= TA_A d_A + TA_B d_B = TM_A + TM_B$$

$$d_B = \frac{TM_B}{TA_B} = \frac{TM_{AB} - TM_A}{TA_B} = \frac{TA_{AB} d_{AB} - TA_A d_A}{TA_B} = \frac{7(7) - 3(3)}{4} = 10$$

2-D Multiple Region Expected Distance

Case: $\boxed{TA_A \leq \frac{1}{2} TA_{AB}}$



Let $X = Y, X_A = Y_A, d = d_{sc}$

$$d_A = \frac{2}{3}(X_A + Y_A) = \frac{4}{3} X_A$$

$$d_B = \frac{TM_B}{TA_B} = \frac{TM_{AB} - TM_A}{TA_B} = \frac{TA_{AB}\, d_{AB} - TA_A d_A}{TA_{AB} - TA_A} = \frac{XY(X+Y) - \frac{1}{2} X_A Y_A \frac{2}{3}(X_A + Y_A)}{XY - \frac{1}{2} X_A Y_A}$$

$$= \frac{2}{3} \frac{3X^2Y + 3XY^2 - X_A^2 Y_A - X_A Y_A^2}{2XY - X_A Y_A} = \frac{4}{3} \frac{3X^3 - X_A^3}{2X^2 - X_A^2}$$

If $X = X_A \Rightarrow d_B = \frac{4}{3} \frac{X^2(2X)}{X^2} = \frac{8}{3} X$

2-D Multiple Region Expected Distance

Case: $\boxed{TA_A \geq \frac{1}{2} TA_{AB}}$

Let $X'_B = X - X_B, Y'_B = Y - Y_B$

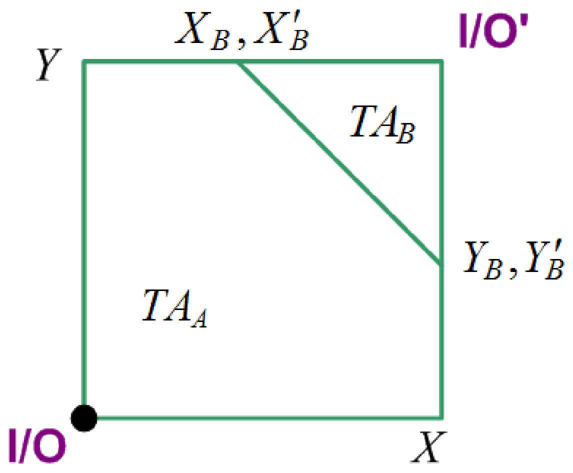
$$d_B = 2\left(d_{I/\text{O to } I/O'}\right) - \frac{2}{3}\left(X'_B + Y'_B\right)$$

$$= 2\left(X + Y\right) - \frac{2}{3}\left[\left(X - X_B\right) + \left(Y - Y_B\right)\right]$$

$$= \frac{4}{3}\left(X + Y\right) + \frac{2}{3}\left(X_B + Y_B\right)$$

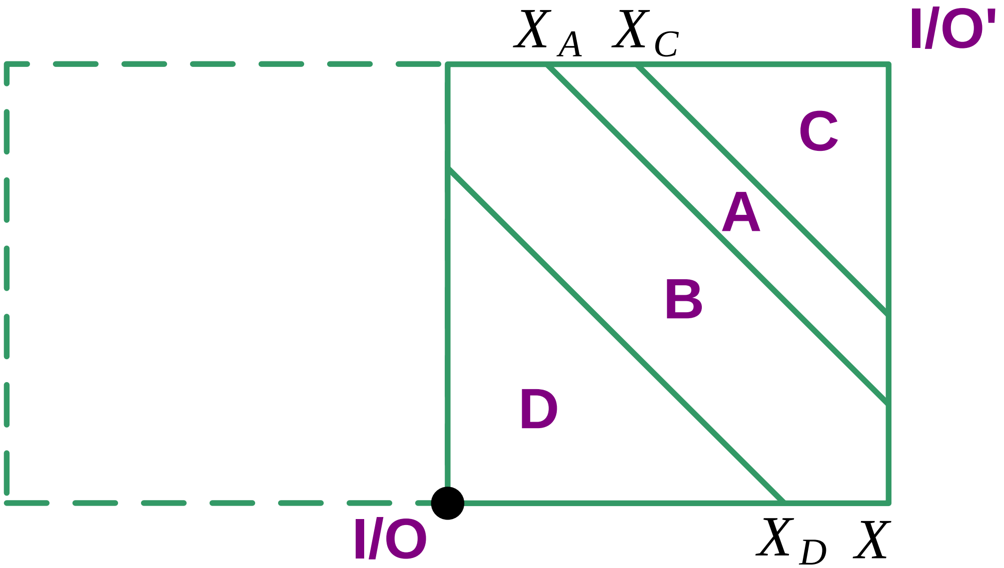
$$= \frac{8}{3} X + \frac{4}{3} X_B, \text{ where } X = Y$$

If $X_B = 0 \Rightarrow d_B = \frac{8}{3} X$



2-D Multiple Region Expected Distance

- If more than two regions:
  - For regions below diagonal (D), start with region closest to I/O
  - For regions above diagonal (A+C), start with regions closest to I/O' (C)
  - For region in the middle (B), solve using whole area less other regions



$$\text{Given } TA_i',\ f_i \Rightarrow \frac{f_i}{TA_i} \Rightarrow D\text{-}B\text{-}A\text{-}C$$

$$TA_i = \frac{TA_i'}{2} \Rightarrow TA = \sum TA_i \Rightarrow X = \sqrt{TA}$$

$$X_D = \sqrt{2}\sqrt{TA_D} \Rightarrow d_D = \frac{4}{3}X_D$$

$$X_C' = \sqrt{2}\sqrt{TA_C} \Rightarrow d_C = 4X - \frac{4}{3}X_C'$$

$$= 4X - \frac{4}{3}\left(X - X_C\right), \quad X_C' = X - X_C$$

$$X_A' = \sqrt{2}\sqrt{TA_{AC}} \Rightarrow d_{AC} = 4X - \frac{4}{3}X_A'$$

$$\Rightarrow d_A = \frac{TM_A}{TA_A} = \frac{TM_{AC} - TM_C}{TA_A}$$

$$= \frac{TA_{AC}d_{AC} - TA_C d_C}{TA_A}$$

$$d_B = \frac{TM_B}{TA_B} = \frac{TM - TM_{AC} - TM_D}{TA_B}$$