ISE 754: Logistics Engineering

Michael G. Kay

Fall 2019

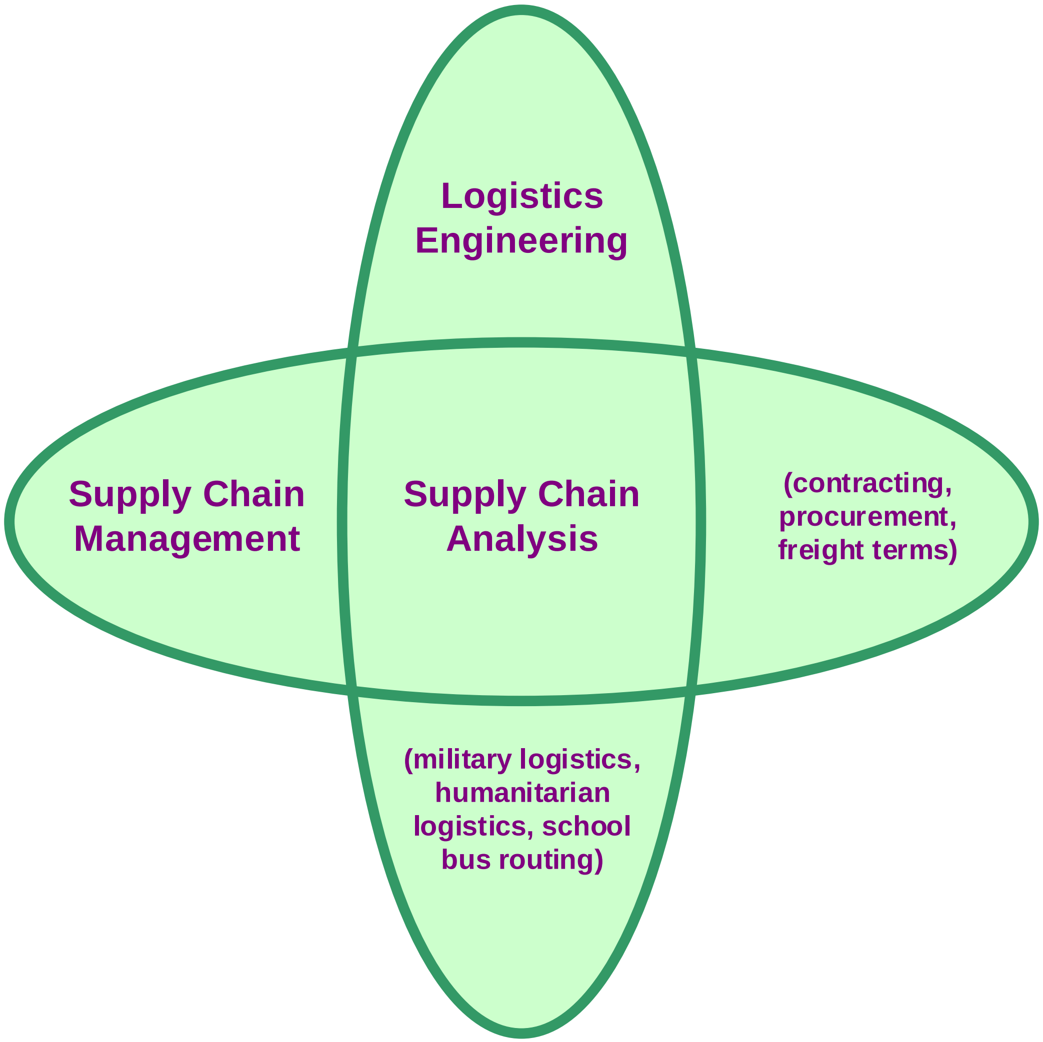
Topics

1. Introduction
2. Facility location
3. Freight transport
   - Exam 1 (take home)
4. Network models
5. Routing
   - Exam 2 (take home)
6. Warehousing
   - Final exam (in class)

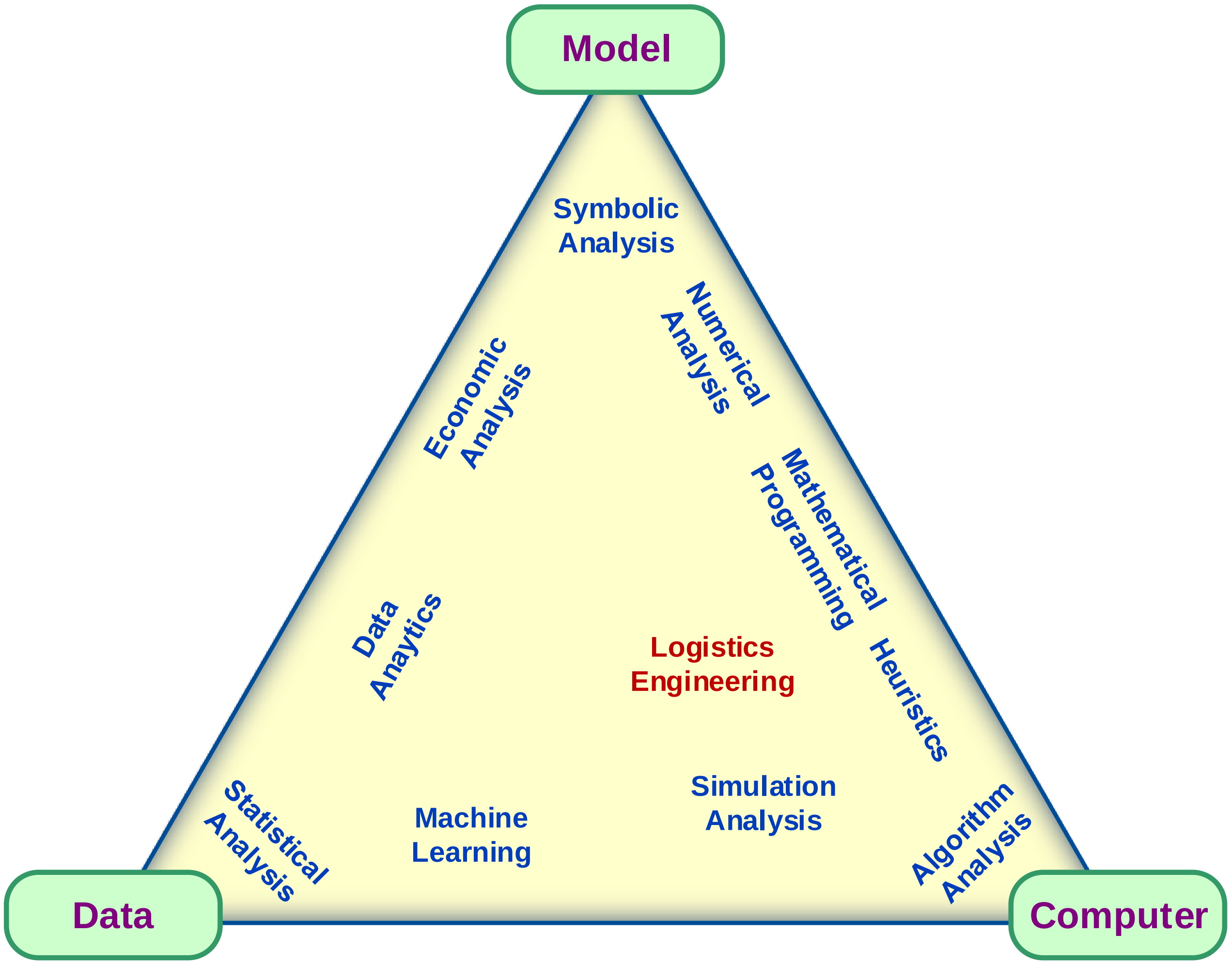
Outside the box

Inside the box

SCM vs Logistics Engineering



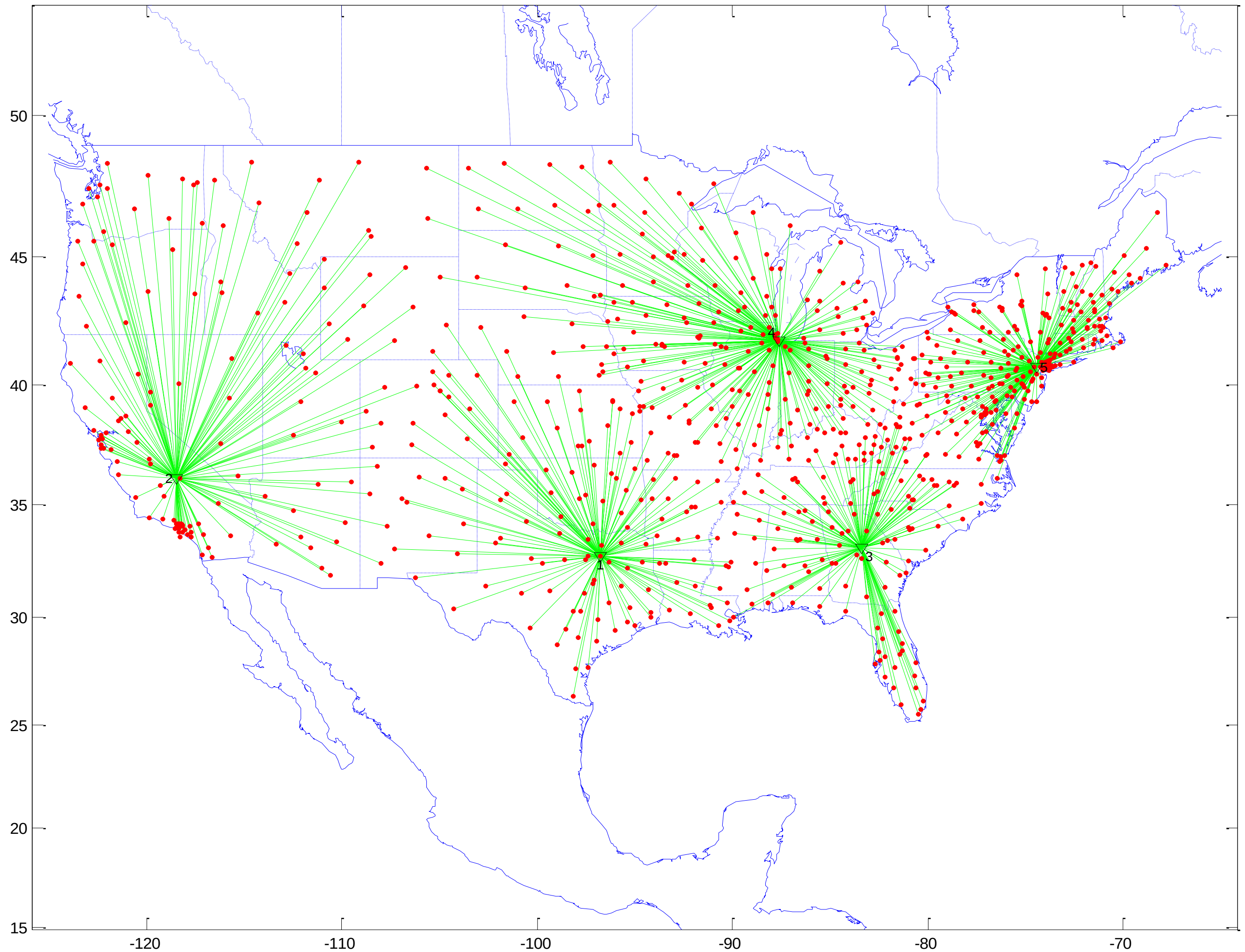
Analysis Triangle



Scope

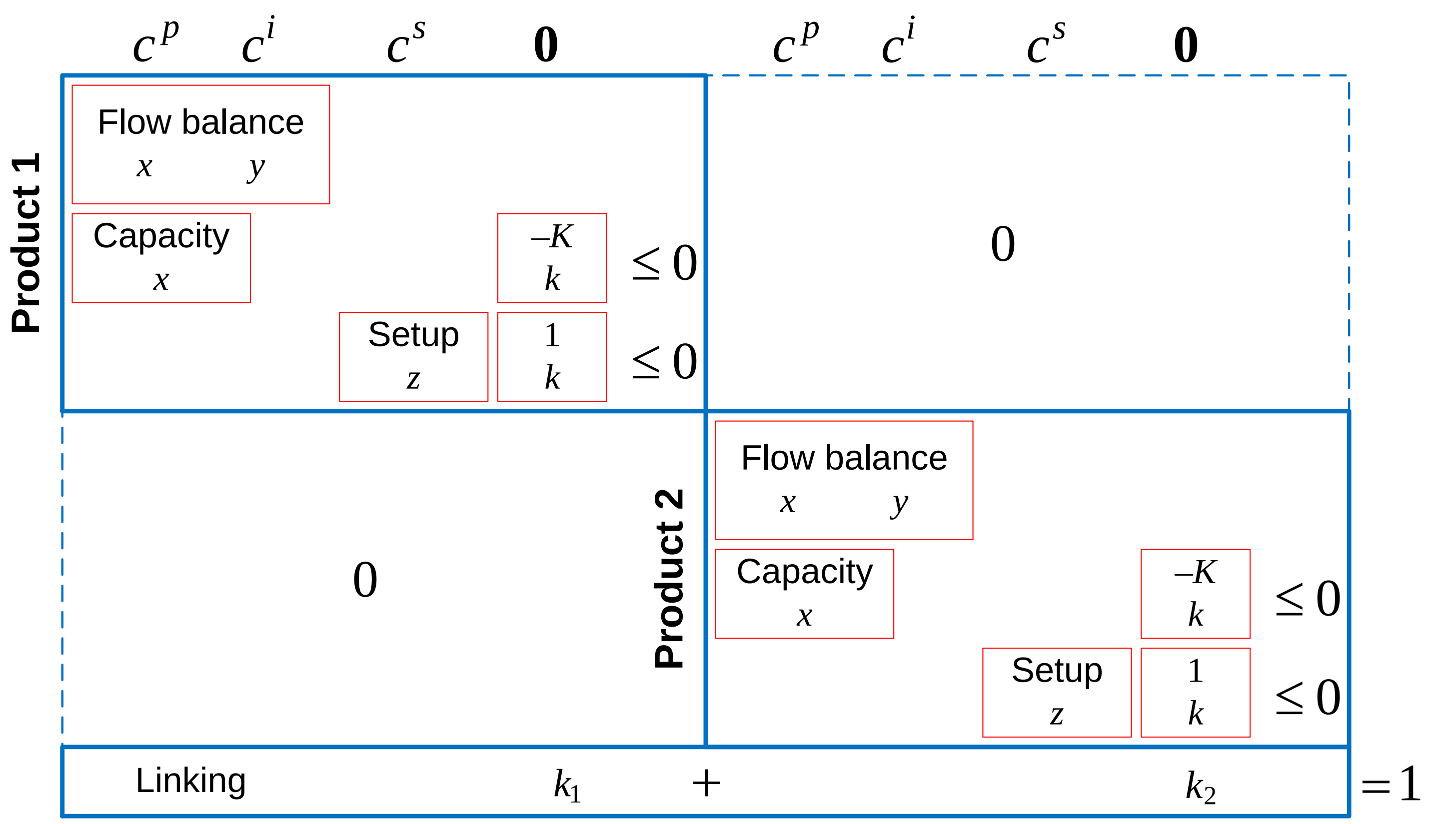
- Strategic (years)
  - Network design
- Tactical (weeks-year)
  - Multi-echelon, multi-period, multi-product production and inventory models
- Operational (minutes-week)
  - Vehicle routing

Strategic: Network Design



Optimal locations for five DCs serving 877 customers throughout the U.S.

Tactical: Production-Inventory Model



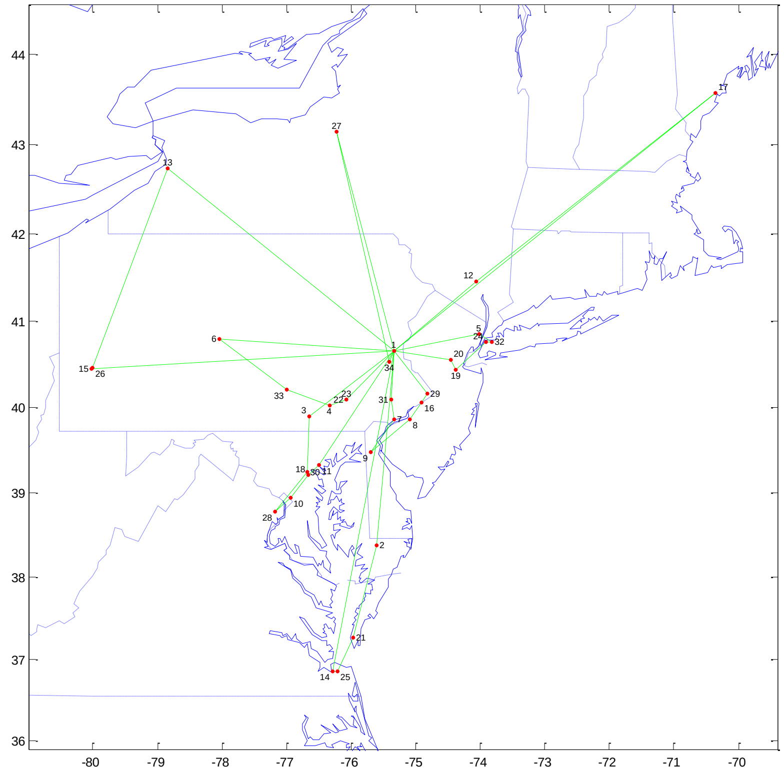
Constraint matrix for a 2-product, multi-period model with setups

Vehicle Routing

Eight routes served from DC in Harrisburg, PA

Route Summary Information

Route	Load Weight	Route Time	Customers in Route	Layover Required
1	12,122	18.36	4	1
2	4,833	16.05	2	1
3	9,642	17.26	3	1
4	25,957	13.77	6	0
5	12,512	9.90	2	0
6	15,156	13.70	5	0
7	29,565	11.30	6	0
8	32,496	8.84	5	0
		109.18		3



Detailed Route Information

						Start	L/D (hr)	Depart	Total Time (hr)	Zip Code
Route		1								
	1					23:29	0	23:29	0	18020
	14	2,328	7.51	7:00	399	7:00	0.59	7:35	8.1	23510
	25	4,697	0.19	7:46	6	7:46	0.68	8:27	0.87	23502
	21	3,682	1.12	9:34	37	9:34	0.64	10:13	1.76	23310
	2	1,415	2.51	12:43	93	12:43	0.56	13:17	3.07	21801
	1	0	4.57	17:51	196	17:51	0	17:51	4.57	18020
Total		12,122	15.89		731		2.47		18.36	

Geometric Mean

- How many people can be crammed into a car?
  - Certainly more than one and less than 100: the average (50) seems to be too high, but the geometric mean (10) is reasonable

$$\text{Geometric Mean:} \quad X = \sqrt{LB \times UB} = \sqrt{1 \times 100} = 10$$

- Often it is difficult to directly estimate input parameter *X*, but is easy to estimate reasonable lower and upper bounds (LB and UB) for the parameter
  - Since the guessed LB and UB are usually orders of magnitude apart, use of the arithmetic mean would give too much weight to UB
  - Geometric mean gives a more reasonable estimate because it is a logarithmic average of LB and UB

Fermi Problems

- Involves "reasonable" (i.e., +/– 10%) *guesstimation* of input parameters needed and back-of-the-envelope type approximations
  - Goal is to have an answer that is within an order of magnitude of the correct answer (or what is termed a *zeroth-order approximation*)
  - Works because over- and under-estimations of each parameter tend to cancel each other out as long as there is no consistent bias
- How many McDonald's restaurants in U.S.? (actual 2013: 14,267)

Parameter	**LB**		**UB**	**Estimate**	
Annual per capita demand	1	1 order/person-day x 350 day/yr =	350	18.71	(order/person-yr)
U.S. population				300,000,000	(person)
Operating hours per day				16	(hr/day)
Orders per store per minute (in-store + drive-thru)				1	(order/store-min)
Analysis					
Annual U.S. demand		(person) x (order/person-yr) =		5,612,486,080	(order/yr)
Daily U.S. demand		(order/yr)/365 day/yr =		15,376,674	(order/day)
Daily demand per store		(hrs/day) x 60 min/hr x (order/store-min) =		960	(order/store-day)
Est. number of U.S. stores		(order/day) / (order/store-day) =		16,017	(store)

System Performance Estimation

- Often easy to estimate performance of a new system if can assume either perfect or no control
- Example: estimate waiting time for a bus
  - 8 min. avg. time (aka "headway") between buses
  - Customers arrive at random
    - assuming no web-based bus tracking
  - Perfect control (LB): wait time = half of headway
  - No control (*practical* UB): wait time = headway
    - assuming buses arrive at random (Poisson process)

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{8}{2} \times 8} = 5.67 \text{ min}$$

  - Bad control can result in higher values than no control

http://www.nextbuzz.gatech.edu/

SELF-COORDINATING BUSES
REDUCE BUNCHING

HOME THE IDEA PROOF OF CONCEPT HOW IT WORKS CONTRIBUTORS

A BUS-HEADWAY CONTROLLER

A software system to coördinate buses on a route, based on an idea by John J. Bartholdi III and Donald D. Eisenstein. The current version of the software was designed and largely written by Loren K. Platzman. Implementation has been led by Russ Clark, Jin Lee, and David Williamson.

THE IDEA

Delaying buses briefly at certain checkpoints equalizes headways

Read more

PROOF OF CONCEPT

Coordinating trolleys on Georgia Tech's busiest route

Read more

HOW IT WORKS

Tablets, GPS, cellular networks, and web-based control

Read more

Levels of Modeling

0. Guesstimation (order of magnitude)
1. Mean value analysis (linear, ±20%)
2. Nonlinear models (incl. variance, ±5%)
3. Simulation models (complex interactions)
4. Prototypes/pilot studies
5. Build/do and then tweak it

Crowdsourcing

- Obtain otherwise hard to get information from a large group of online workers
- Amazon’s Mechanical Turk is best known
  - Jobs posted as HITs (Human Information Tasks) that typically pay $1-2 per hour
  - Main use has been in machine learning to create tagged data sets for training purposes
  - Has been used in logistics engineering to estimate the percentage homes in U.S. that have sidewalks (sidewalk deliveries by Starship robots)

Starship Technologies

- Started by Skype co-founders
- 99% autonomous
- Goal: “deliver ‘two grocery bags’ worth of goods (weighing up to 20lbs) in 5-30 minutes for ‘10-15 times less than the cost of current last-mile delivery alternatives.’”

Ex 1: Geometric Mean

- If, during the morning rush, there are three buses operating on Wolfline Route 13 and it takes them 45 minutes, on average, to complete one circuit of the route, what is the estimated waiting time for a student who does not use TransLoc for real-time bus tracking?

Answer :

$$\text{Frequency (TH)} = \frac{WIP}{CT} = \frac{3\text{ bus/circuit}}{45\text{ min/circuit}} = \frac{1}{15}\text{ bus/min}, \quad \text{Headway} = \frac{1}{\text{Freq.}} = 15\text{ min/bus}$$

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{15}{2} \times 15} = 10.61\,\text{min}$$

Ex 2: Fermi Problem

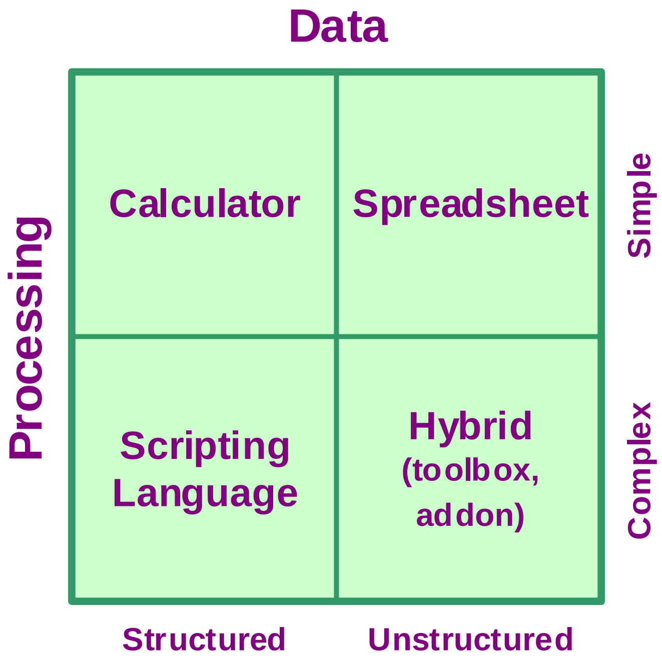
- Estimate the average amount spent per trip to a grocery store. Total U.S. supermarket sales were recently determined to be \$649,087,000,000, but it is not clear whether this number refers to annual sales, or monthly, or weekly sales.

$$\textbf{Answer:} \quad \frac{\$6.5e11}{3e8} \approx \$2,000 \,/\, \text{person-yr}, \quad LB = 1 \text{ trips/wk}, UB = 7 \text{ trips/wk}$$

$$\Rightarrow \sqrt{1(7)} \times 52 \approx 2 \times 52 \approx 100 \text{ trips/yr} \Rightarrow \frac{\$2,000}{100} = \$20 \,/\, \text{person-trip} \Rightarrow \text{Annual}$$

Supermarket / Grocery Store Statistics	Data
Total number of grocery store employees	3,400,000
Total supermarket sales in 2015	\$649,087,000,000
Total supermarket sales in 2012	\$602,609,000,000
Total number of grocery stores / supermarkets	37,053
Median weekly sales per supermarket store	\$384,911
Average grocery store transaction amount	\$27.30
Average number of grocery store trips per week a consumers makes	2.2
Average number of items carried in a supermarket	38,718

Computational Tools



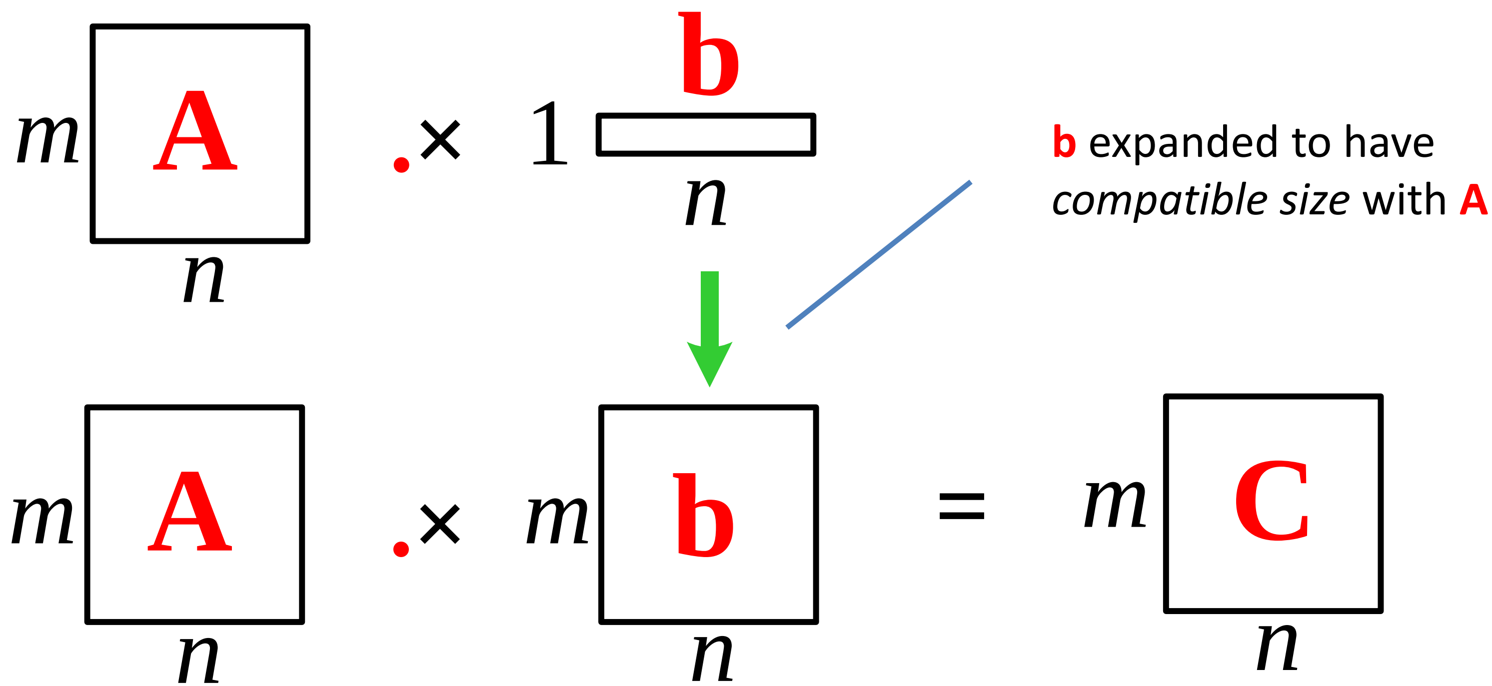
Matrix Multiplication

$$m\underset{n}{\boxed{\mathbf{A}}} \quad \times \quad n\underset{p}{\boxed{\mathbf{B}}} \quad = \quad m\underset{p}{\boxed{\mathbf{C}}}$$

$$(m\times \cancel{n})\times(\cancel{n}\times p) = (m\times p)$$

Arrays must have same
inner dimensions

Element-by-Element Multiplication



$$(m \times n) \times (1 \times n) \quad = \quad (m \times n)$$

Compatible Sizes

- Two arrays have compatible sizes if, for *each respective dimension*, either
  - has the same size, or
  - size of one of arrays is one, in which case it is automatically duplicated so that it matches the size of the other array

$$\mathbf{A}_{m\times n} \;.*\; \mathbf{B}_{m\times n} = \mathbf{C}_{m\times n}$$

$$\mathbf{A}_{m\times n} \;.*\; \mathbf{b}_{1\times n} = \mathbf{C}_{m\times n}$$

$$\mathbf{a}_{m\times 1} \;.*\; \mathbf{B}_{m\times n} = \mathbf{C}_{m\times n}$$

$$\mathbf{a}_{m\times 1} \;.*\; \mathbf{b}_{1\times n} = \mathbf{C}_{m\times n}$$

$$\mathbf{a}_{1\times n} \;.*\; \mathbf{b}_{m\times 1} = \mathbf{C}_{m\times n}$$

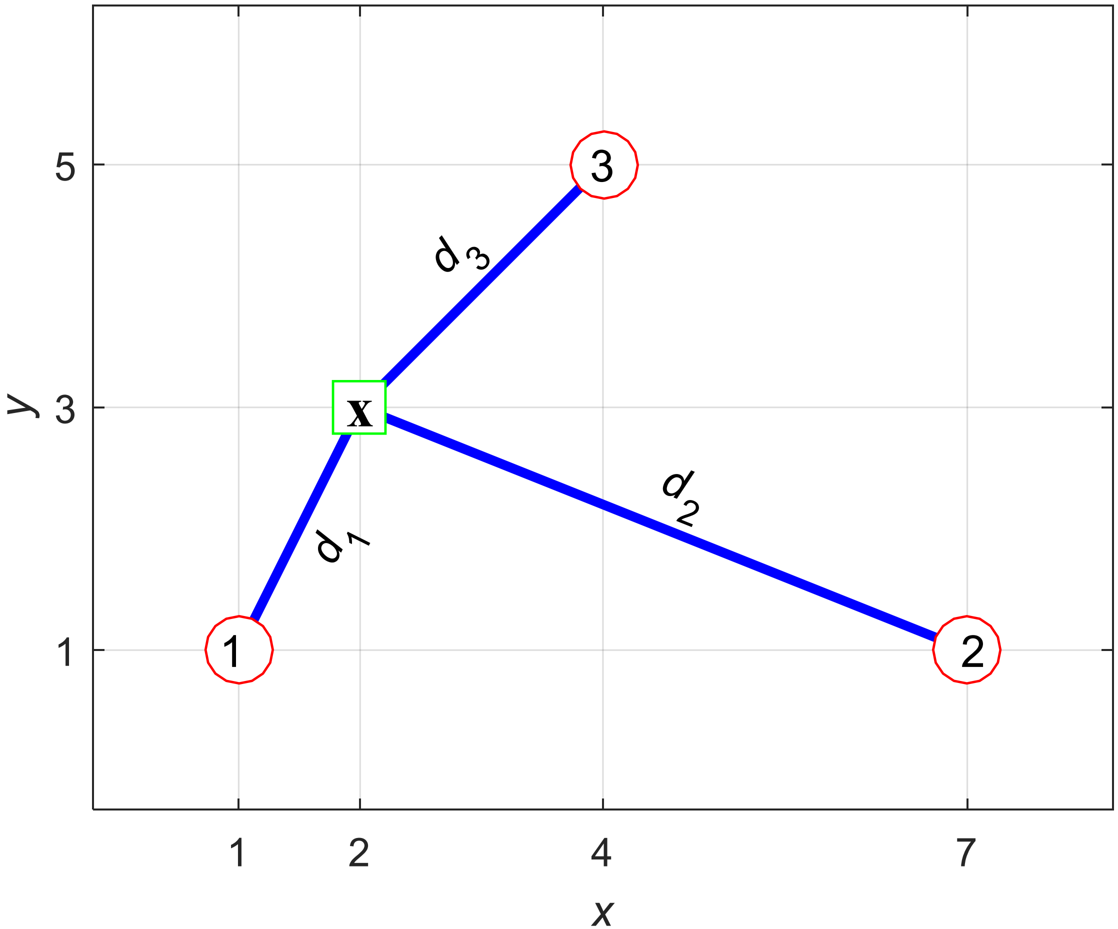
$$\cancel{\mathbf{A}_{m\times n} \;.*\; \mathbf{B}_{m\times p}}$$

$$\cancel{\mathbf{A}_{m\times n} \;.*\; \mathbf{b}_{n\times 1}}$$

$$\cancel{\mathbf{a}_{m\times 1} \;.*\; \mathbf{B}_{p\times n}}$$

$$\cancel{\mathbf{a}_{m\times 1} \;.*\; \mathbf{b}_{n\times 1}}$$

2-D Euclidean Distance

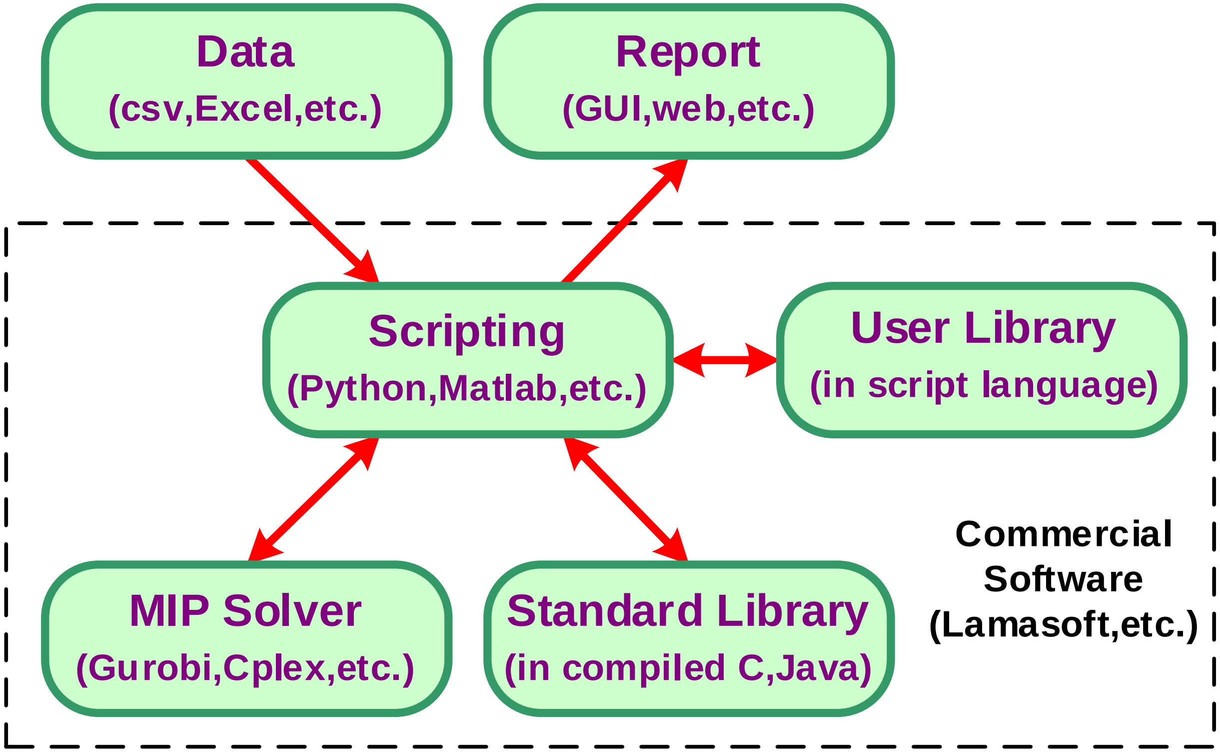


$$\mathbf{x} = \begin{bmatrix} 2 & 3 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} \sqrt{(x_1 - p_{1,1})^2 + (x_2 - p_{1,2})^2} \\ \sqrt{(x_1 - p_{2,1})^2 + (x_2 - p_{2,2})^2} \\ \sqrt{(x_1 - p_{3,1})^2 + (x_2 - p_{3,2})^2} \end{bmatrix}$$

$$\mathbf{d} = \sqrt{\sum \left(\begin{bmatrix} 2 & 3 \\ 2 & 3 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix} \right)^2}$$

Logistics Software Stack

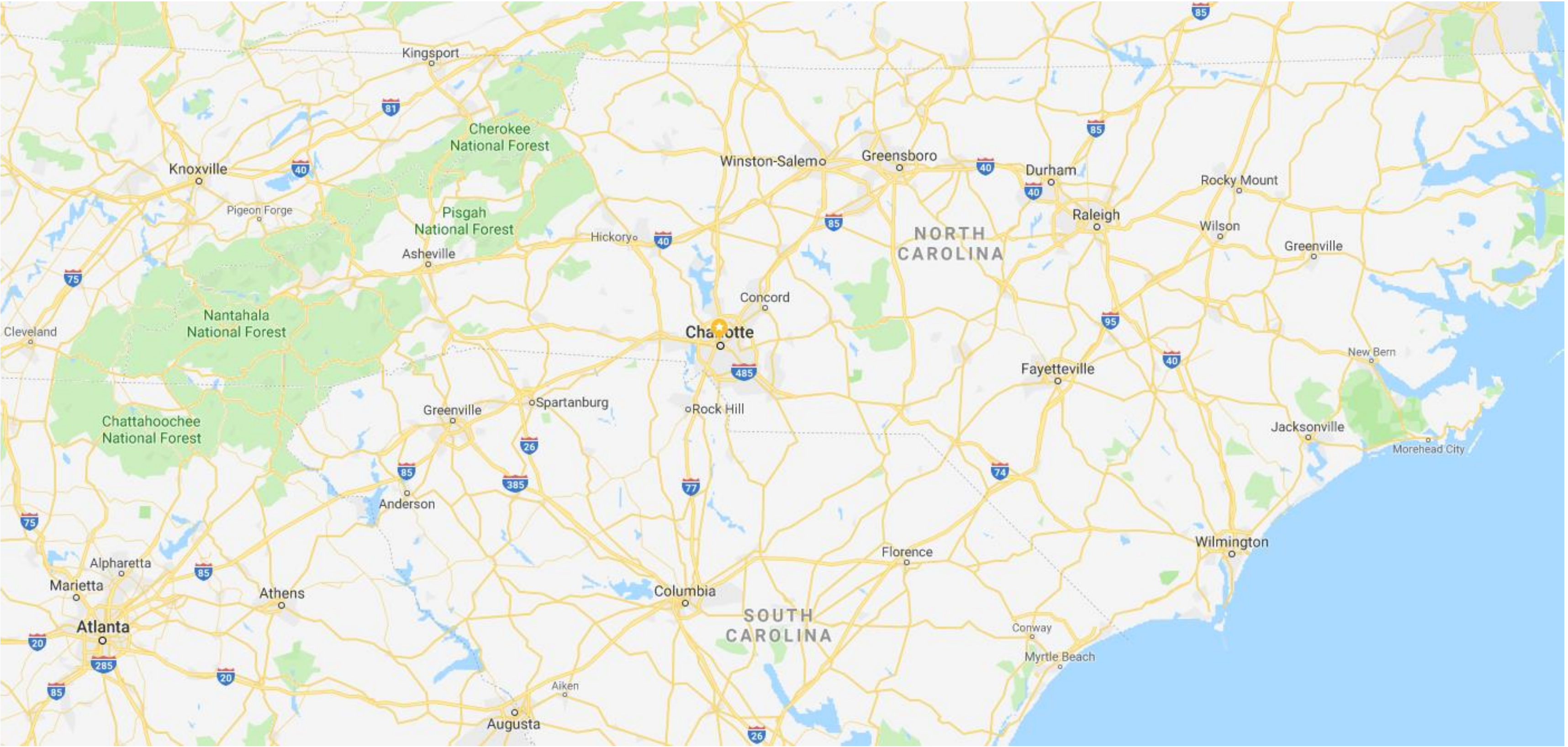


- New *Julia* (1.0) scripting language
  - (almost?) as fast as C and Java (but not FORTRAN)
  - does not require compiled standard library for speed
  - uses *multiple dispatch* to make type-specific versions of functions

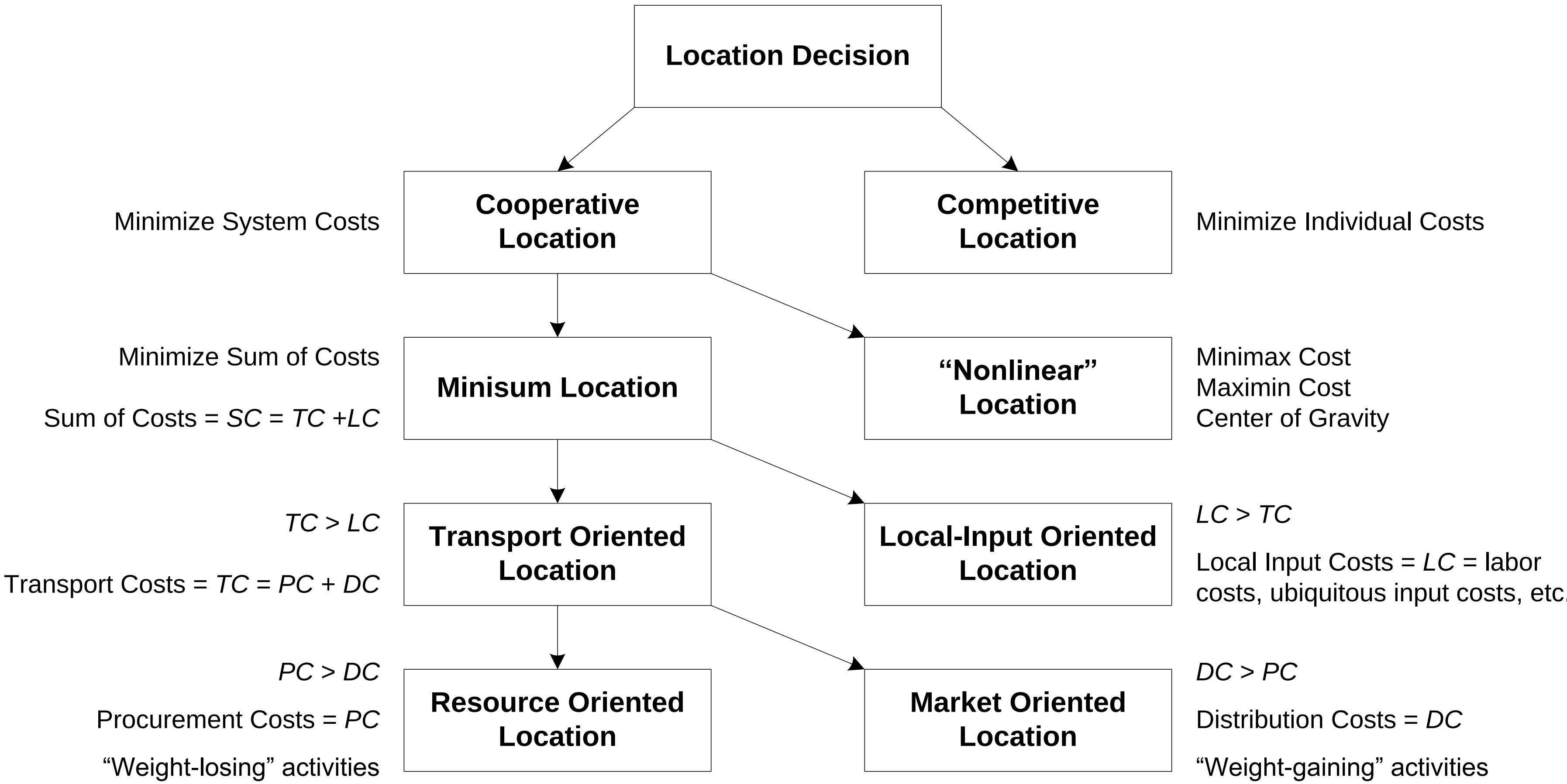
Basic Matlab Workflow

- Given problem to solve:
  1. Test critical steps at Command Window
  2. Copy working critical steps to a cell (`&&`) in script file (myscript.m) along with supporting code (can copy selected lines from Command History)
  – Repeat using new cells for additional problems
- Once all problems solved, report using:
  – `>> diary hw1soln.txt`
  – Evaluate each cell in script:
    - To see code + results: select text then Evaluate Selection on mouse menu (or F9)
    - To see results: position cursor in cell then Evaluate Current Section (Cntl+Enter)
  – `>> diary off`
- Can also report using Publish (see Matlab menu) as html or Word
- Submit all files created, which may include additional
  – Data files (myscript.mat) or spreadsheet files (myexcel.xlsx)
  – Function files (myfun.m) that can allow use to re-use same code used in multiple problems
    - All code inside function isolated from other code except for inputs/outputs: `[out1,out2] = myfun(input1,input2)`

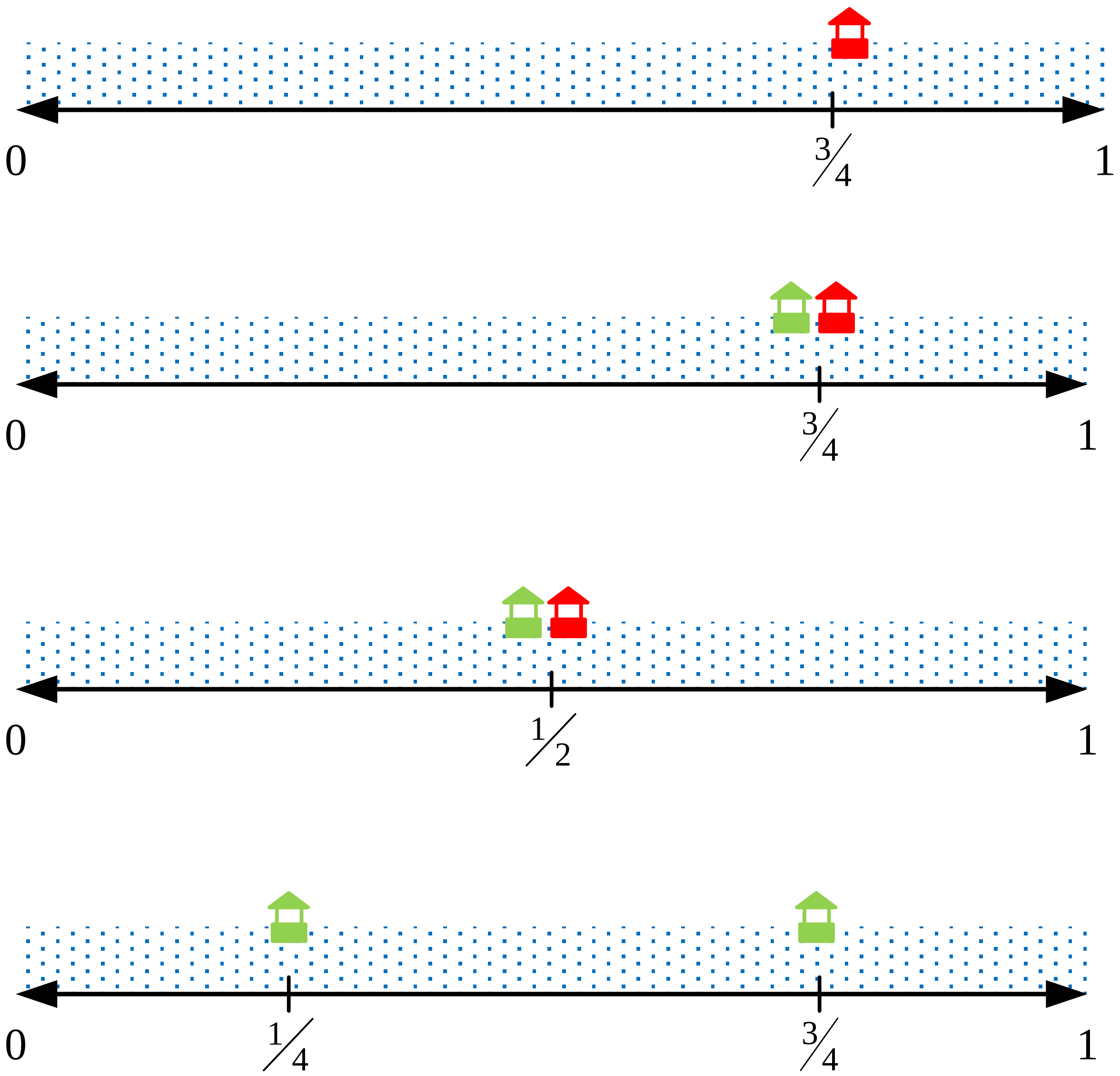
Why Are Cities Located Where They Are?



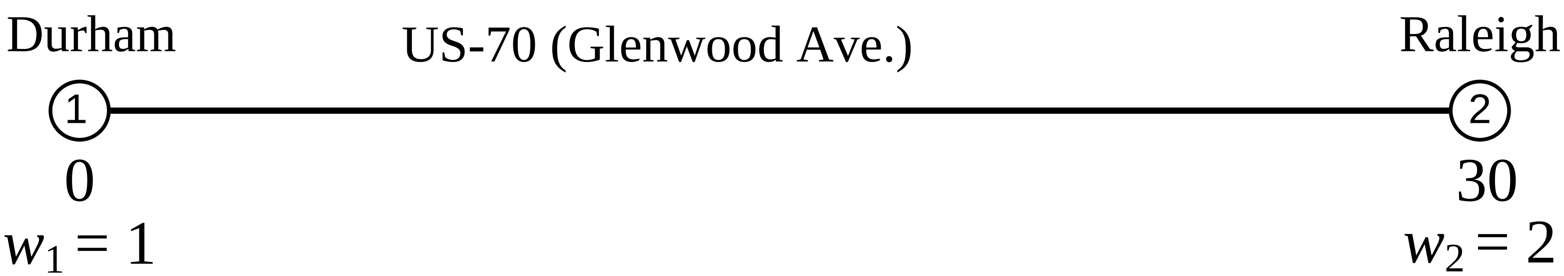
Taxonomy of Location Problems



Hotelling's Law



1-D Cooperative Location



$$\text{Min } TC = \sum w_i \, d_i$$

$$\text{Min } TC = \sum w_i \, d_i^2$$

$$\text{Min } TC = \sum w_i \, d_i^k$$

$$a_1 = 0, \quad a_2 = 30$$

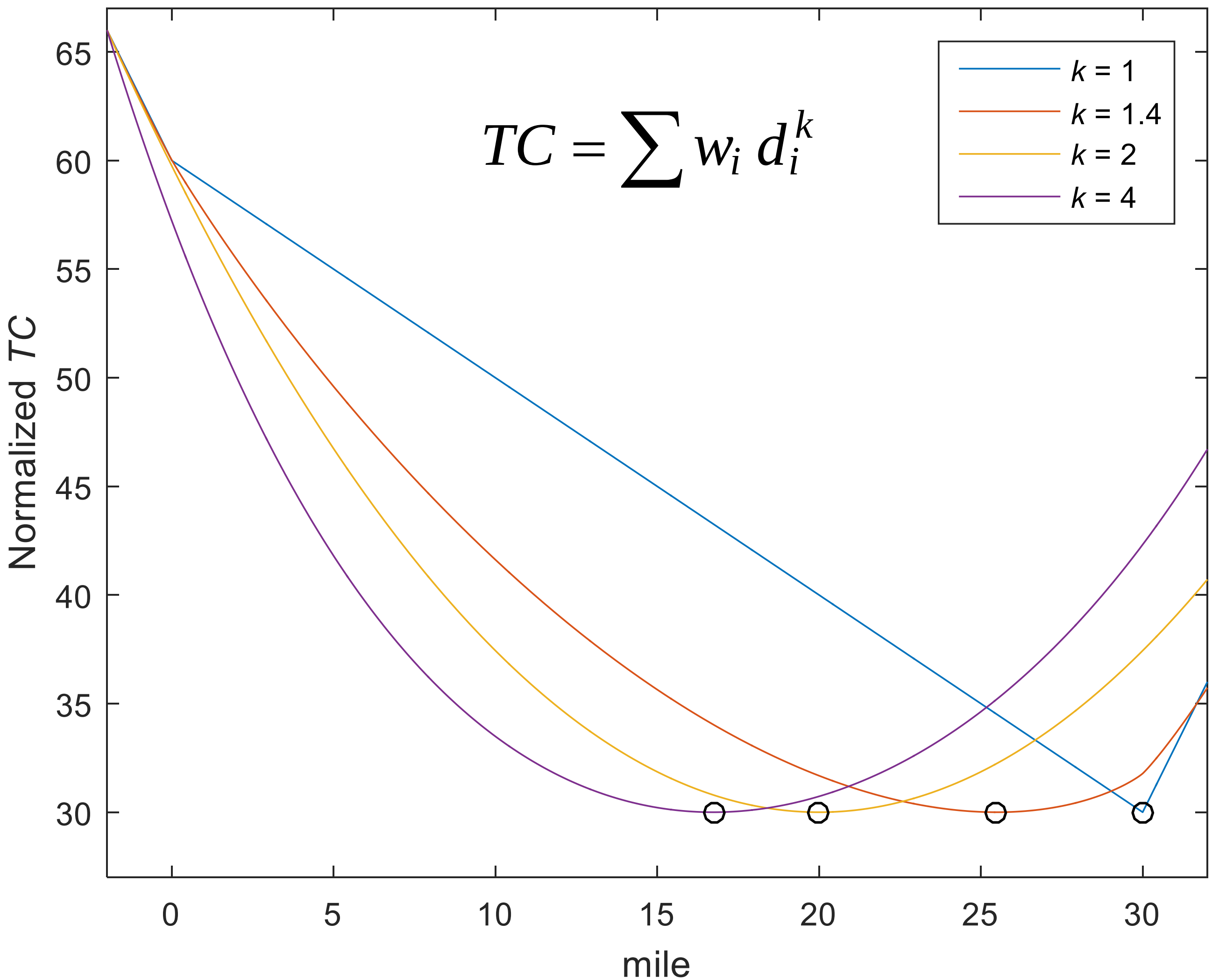
$$TC = \sum w_i \, d_i^2 = \sum w_i \left(x - a_i \right)^2$$

$$\frac{dTC}{dx} = 2 \sum w_i \left(x - a_i \right) = 0 \Rightarrow$$

$$x \sum w_i = \sum w_i \, a_i \Rightarrow$$

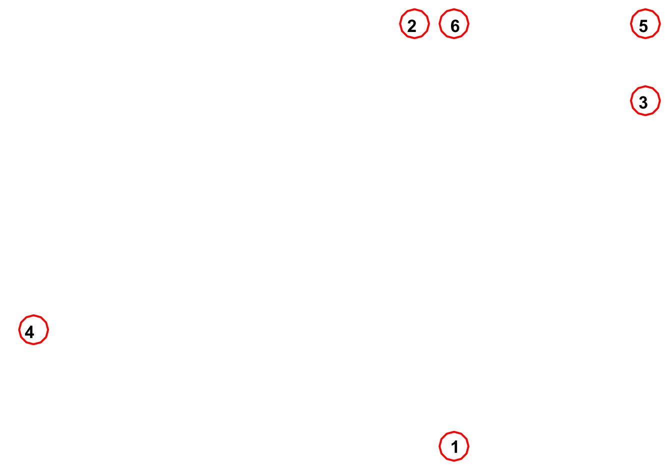
Squared−Euclidean Distance $\Rightarrow$ Center of Gravity: $$x^* = \frac{\sum w_i \, a_i}{\sum w_i} = \frac{1(0) + 2(30)}{1+2} = 20$$

“Nonlinear” Location

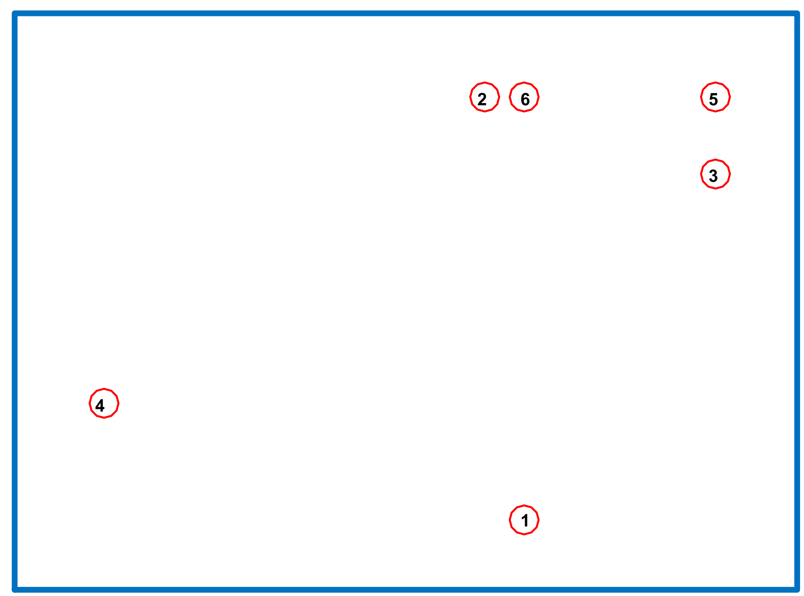


Minimax and Maximin Location

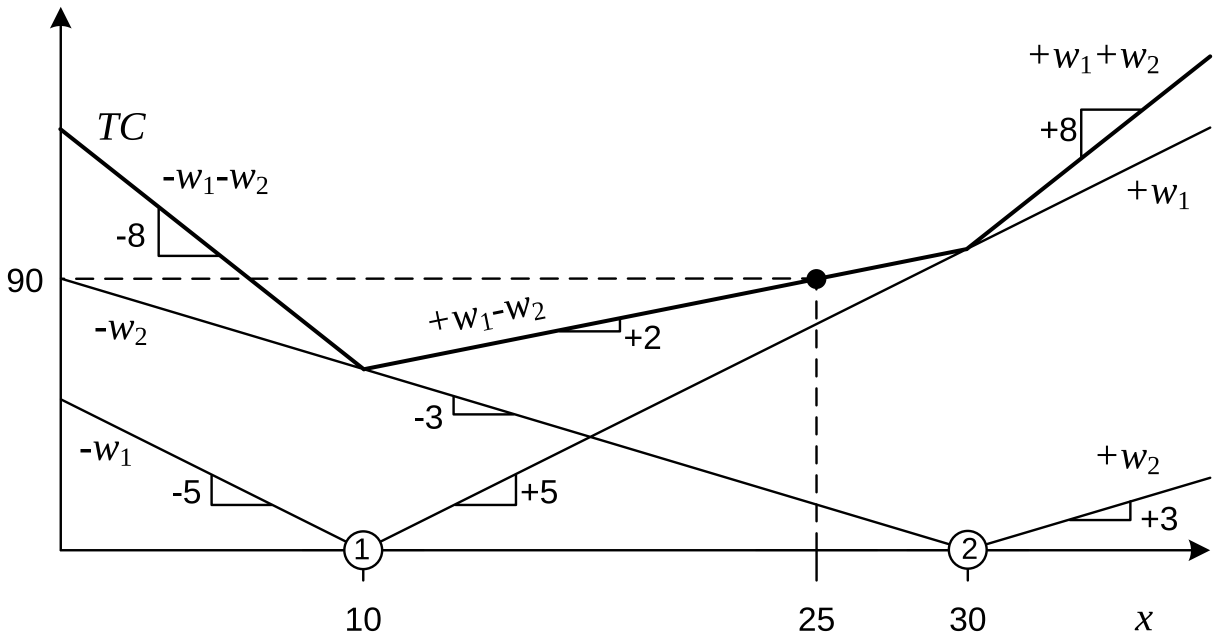
- Minimax
  - Min max distance
  - Set covering problem



- Maximin
  - Max min distance
  - AKA obnoxious facility location



2-EF Minisum Location



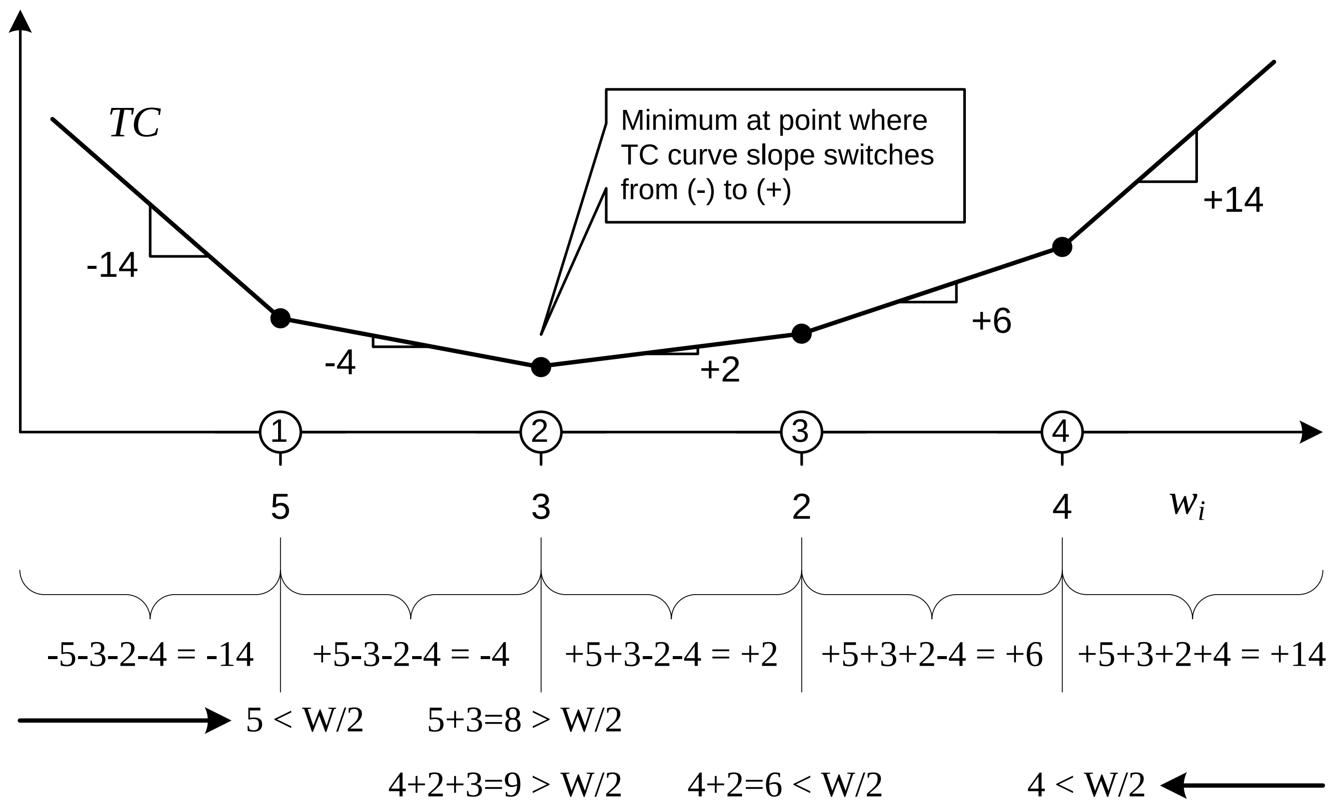
$$TC(x) = \sum w_i\, d_i = \beta_1(x - x_1) + \beta_2(x - x_2), \quad \text{where } \beta_i = \begin{cases} w_i, & \text{if } x \ge x_i \\ -w_i, & \text{if } x < x_i \end{cases}$$

$$\begin{aligned} TC(25) &= w_1(25-10) + (-w_2)(25-30) \\ &= 5(15) + (-3)(-5) = 90 \end{aligned}$$

Median Location: 1-D 4 EFs

Median location: For each dimension x of X:

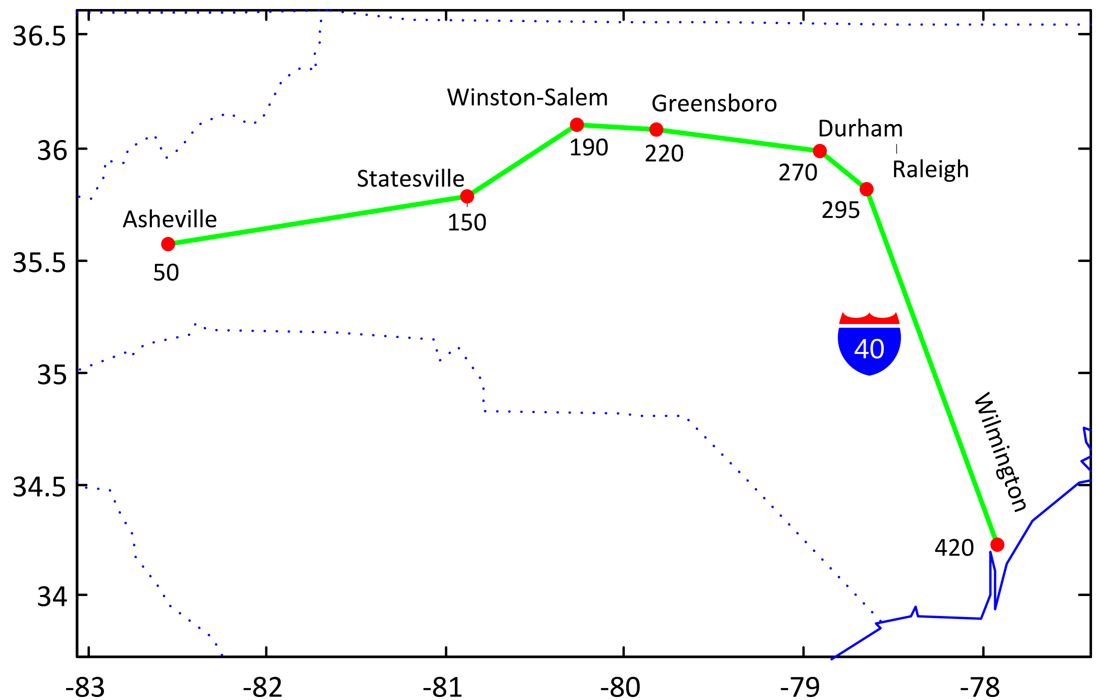
1. Order EFs so that $|x_1| \leq |x_2| \leq \cdots \leq |x_m|$
2. Locate x-dimension of NF at the first EF j where $\sum_{i=1}^{j} w_i \geq \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

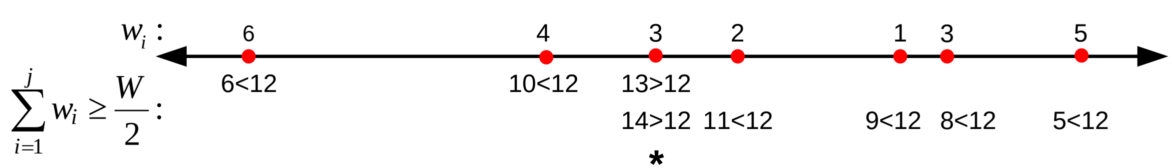


Median Location: 1-D 7 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EF j where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$





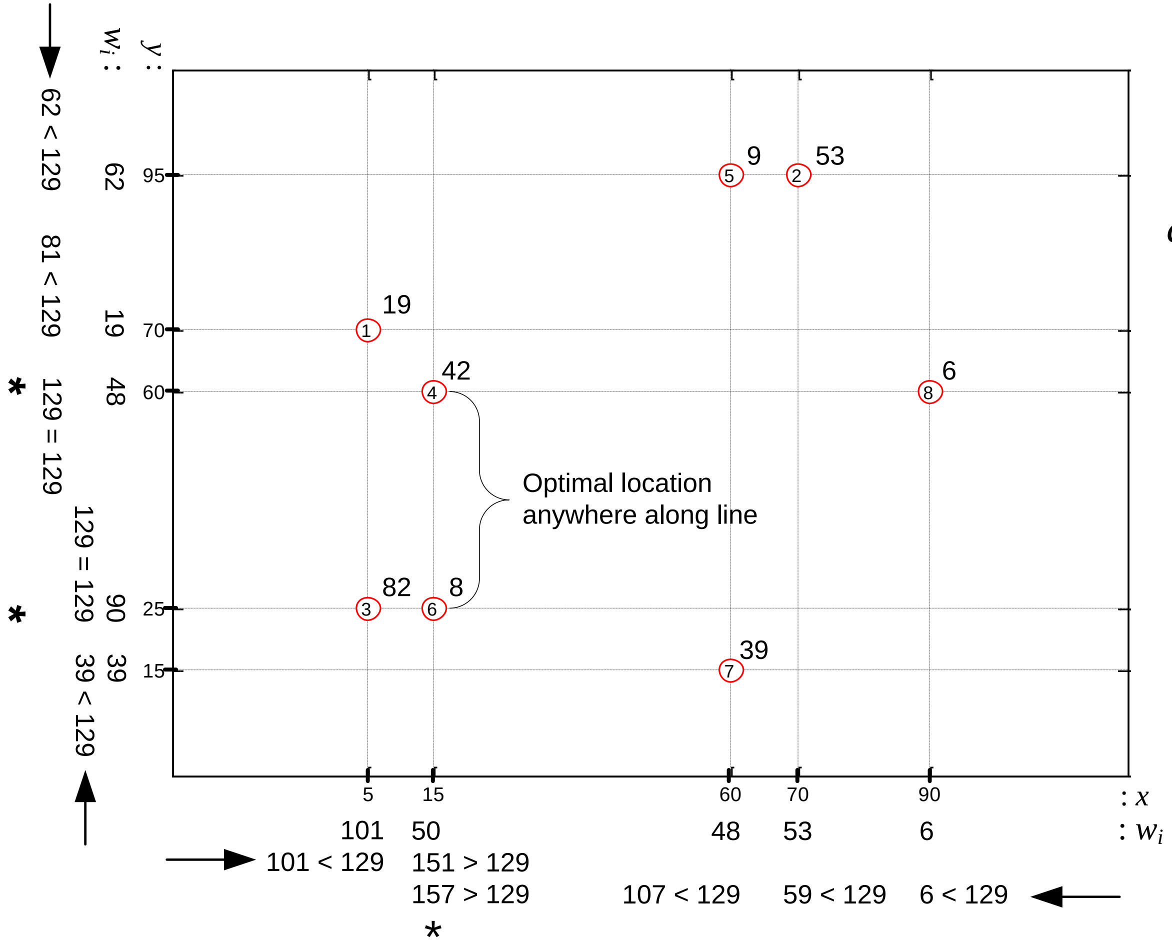
Median Location: 2-D Rectilinear Distance 8 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EF j where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

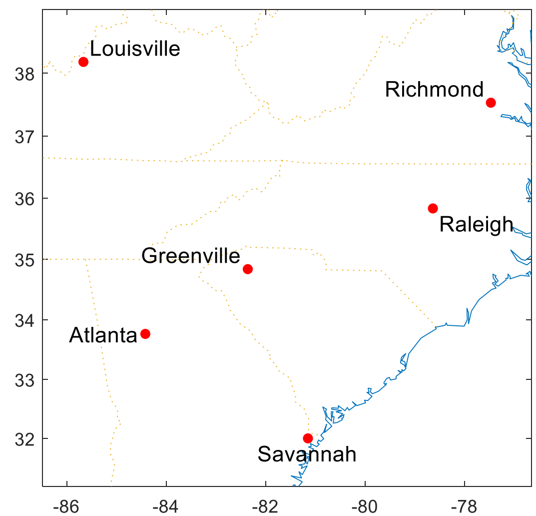
$$d_1(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$$

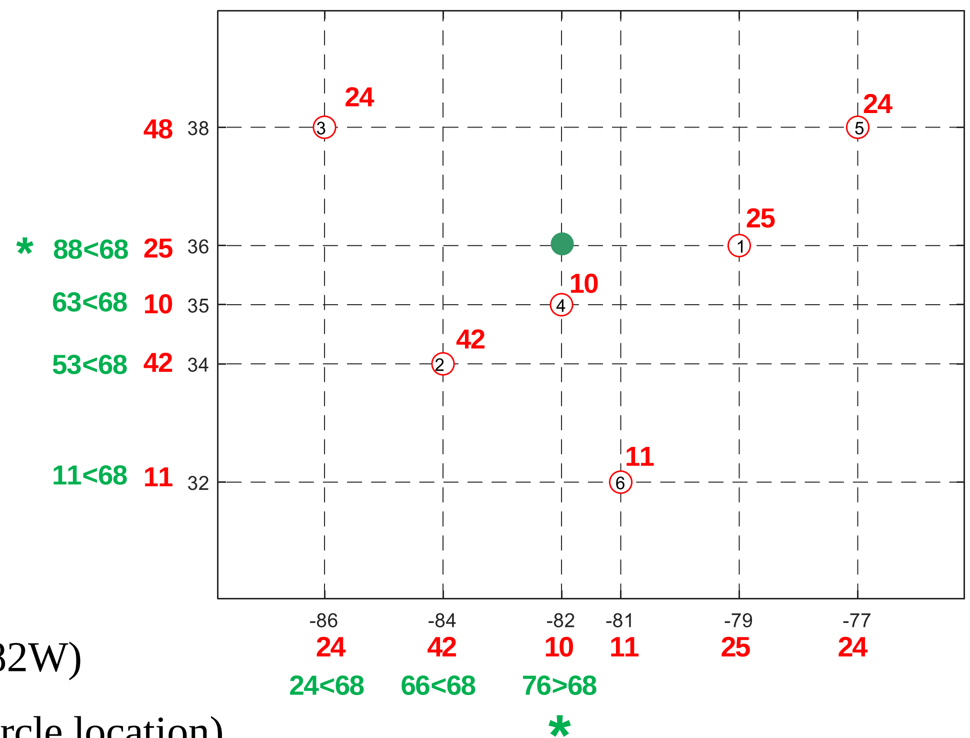
$$d_2(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



Ex 3: 2D Loc with Rect Approx to GC Dist

- It is expected that 25, 42, 24, 10, 24, and 11 truckloads will be shipped each year from your DC to six customers located in Raleigh, NC (36N,79W), Atlanta, GA (34N,84W), Louisville, KY (38N,86W), Greenville, SC (35N, 82W), Richmond, VA (38N,77W), and Savannah, GA (32N,81W). Assuming that all distances are rectilinear, where should the DC be located in order to minimize outbound transportation costs?



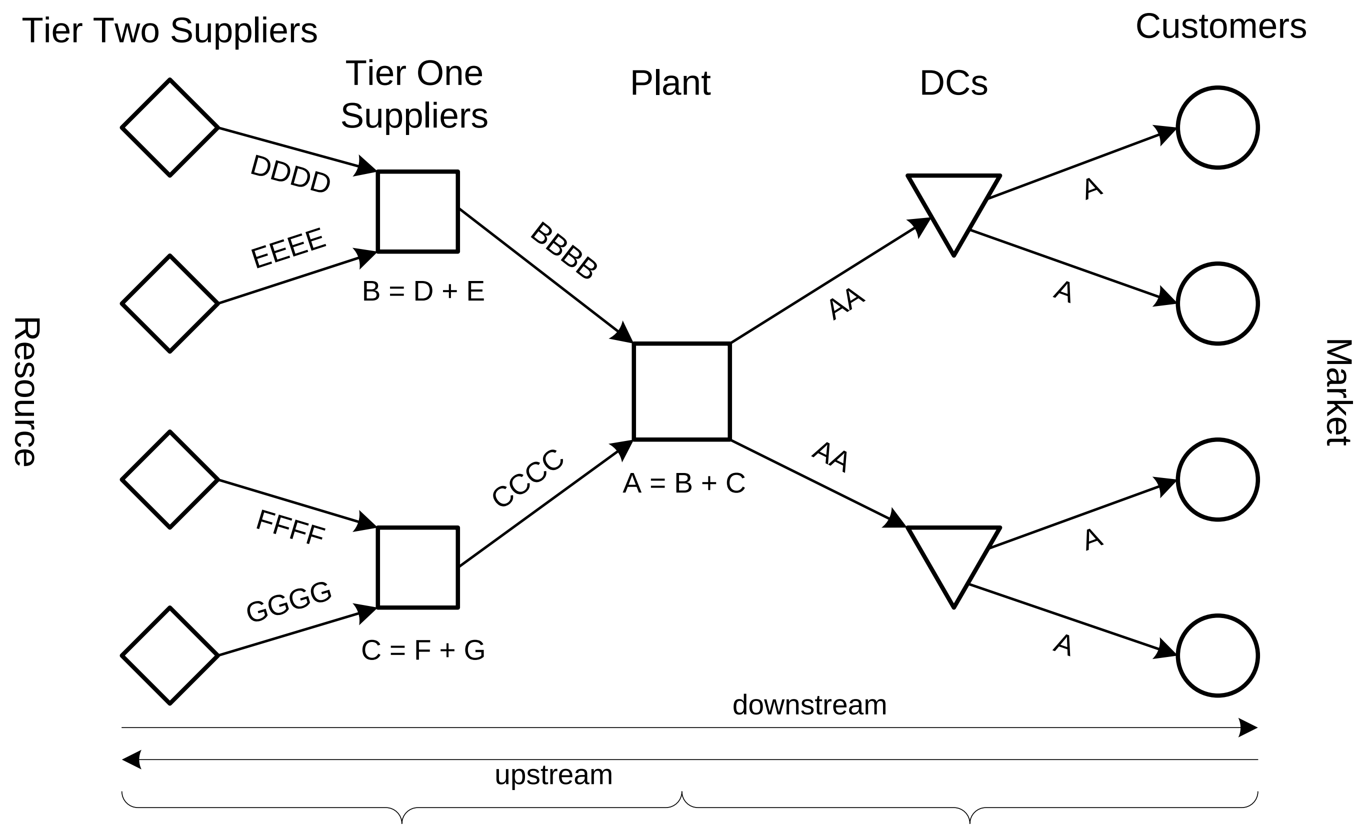


$$W = \sum w_i = 136, \quad \frac{W}{2} = 68$$

Answer : Optimal location (36N,82W)

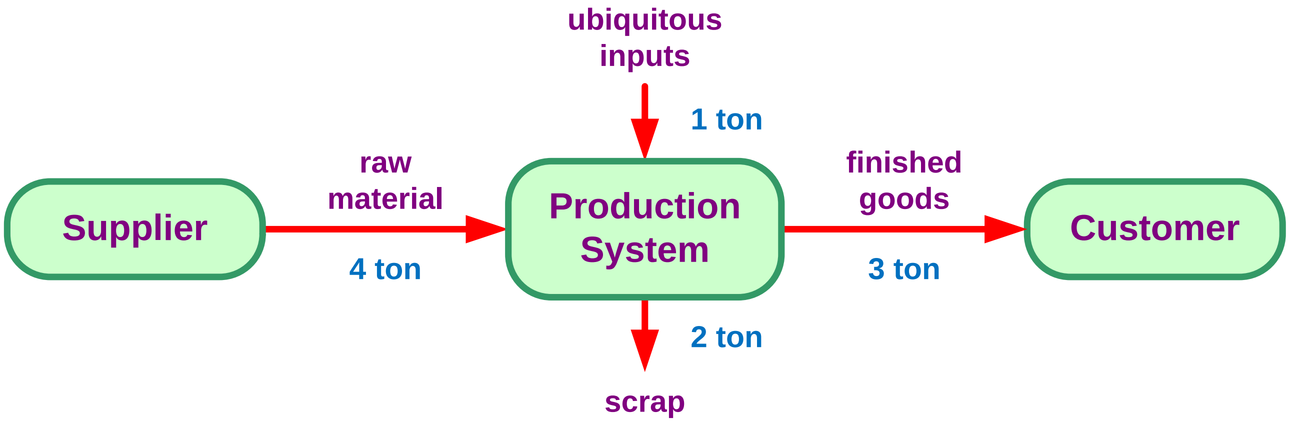
(65 mi from opt great-circle location)

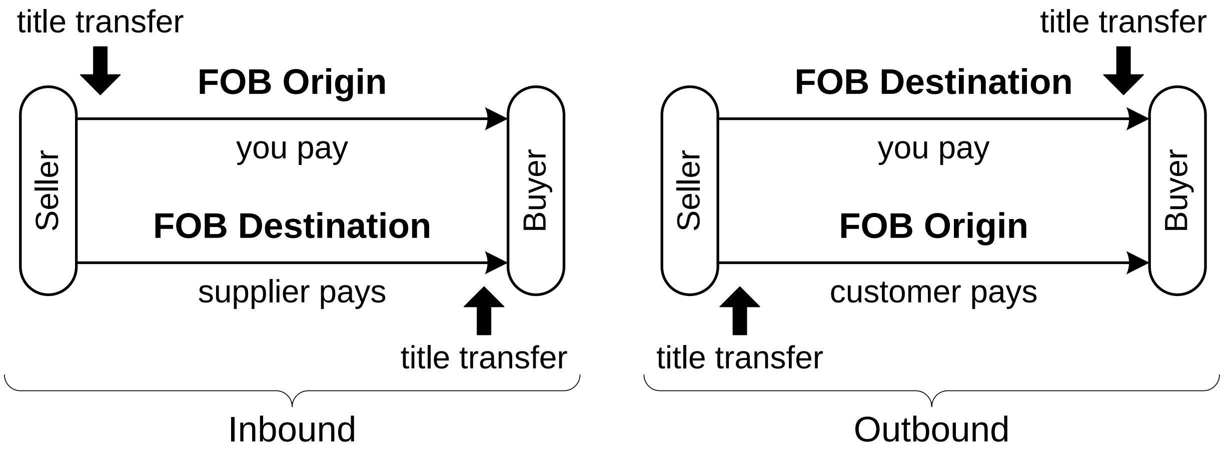
Logistics Network for a Plant



Assembly Network	vs.	Distribution Network
Procurement	vs.	Distribution
Inbound Logistics	vs.	Outbound Logistics
Raw Materials	vs.	Finished Goods

Basic Production System





FOB (free on board)

FOB and Location

- Choice of FOB terms (who directly pays for transport) usually does not impact location decisions:

Procurement cost	=	Landed cost at supplier	+	Inbound transport cost
Production cost	=	Procurement cost	+	Local resource cost (labor, etc.)
Total delivered cost	=	Production cost	+	Outbound transport cost
Transport cost (TC)	=	Inbound transport cost	+	Outbound transport cost

  - *Purchase price* from supplier and *sale price* to customer adjusted to reflect who is paying transport cost
  - Usually determined by who can provide the transport at the lowest cost
    - Savings in lower transport cost allocated (bargained) between parties

Monetary vs. Physical Weight

$$\min TC(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} \, d(X, P_i)$$

where TC = total transport cost ($/yr)

w_i = monetary weight ($/mi-yr)

f_i = physical weight rate (ton/yr)

r_i = transport rate ($/ton-mi)

$d(X, P_i)$ = distance between NF at X and EF$_i$ at P_i (mi)

NF = new facility to be located

EF = existing facility

m = number of EFs

(Montetary) Weight Gaining: $\Sigma w_{\text{in}} < \Sigma w_{\text{out}}$

Physically Weight Losing: $\Sigma f_{\text{in}} > \Sigma f_{\text{out}}$

Minisum Location: TC vs. TD

- Assuming local input costs are
  - same at every location or
  - insignificant as compared to transport costs,

  the minisum transport-oriented single-facility location problem is to locate NF to minimize TC
- Can minimize total distance (TD) if transport rate is same:

$$\min TD(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} \, d(X, P_i)$$

where TD = total transport distance (mi/yr)

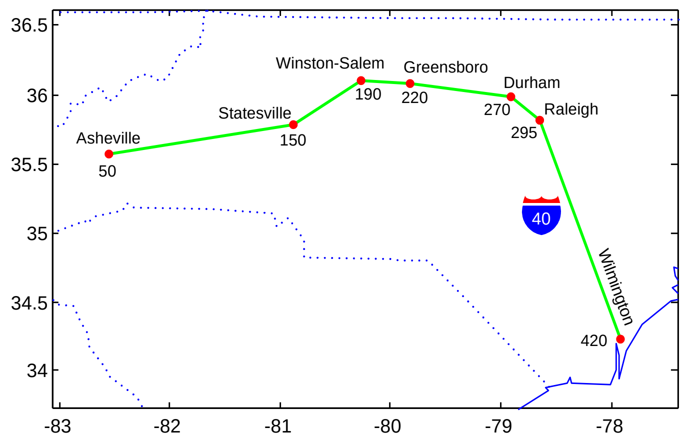
w_i = monetary weight (trip/yr)

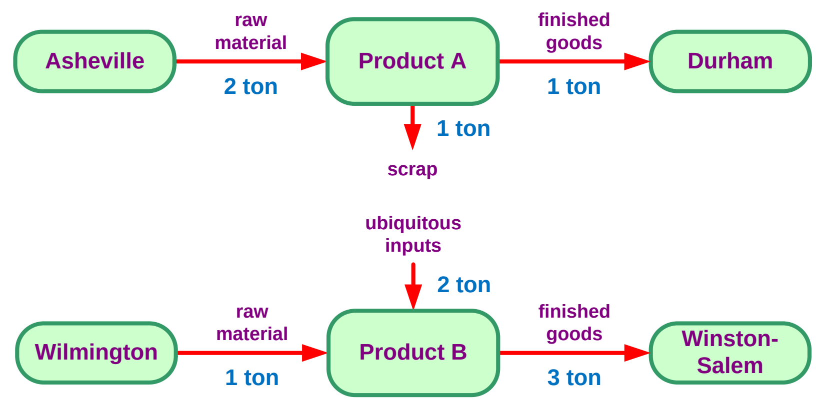
f_i = trips per year (trip/yr)

r_i = transport rate = 1

$d(X, P_i)$ = per-trip distance between NF and EF_i (mi/trip)

Ex 4: Single Supplier and Customer Location

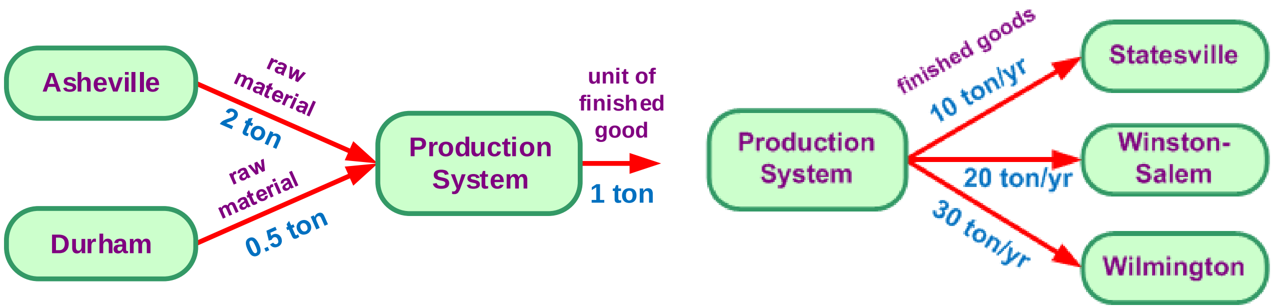




- The cost per ton-mile (i.e., the cost to ship one ton, one mile) for both raw materials and finished goods is the same (e.g., \$0.10).
  1. Where should the plant for each product be located?
  2. How would location decision change if customers paid for distribution costs (FOB Origin) instead of the producer (FOB Destination)?
     - In particular, what would be the impact if there were competitors located along I-40 producing the same product?
  3. Which product is weight gaining and which is weight losing?
  4. If both products were produced in a single shared plant, why is it now necessary to know each product's annual demand (f_i)?

$$TC(X) = \sum_{i=1}^{m} f_i \frac{r_i}{w_i} d(X, P_i)$$

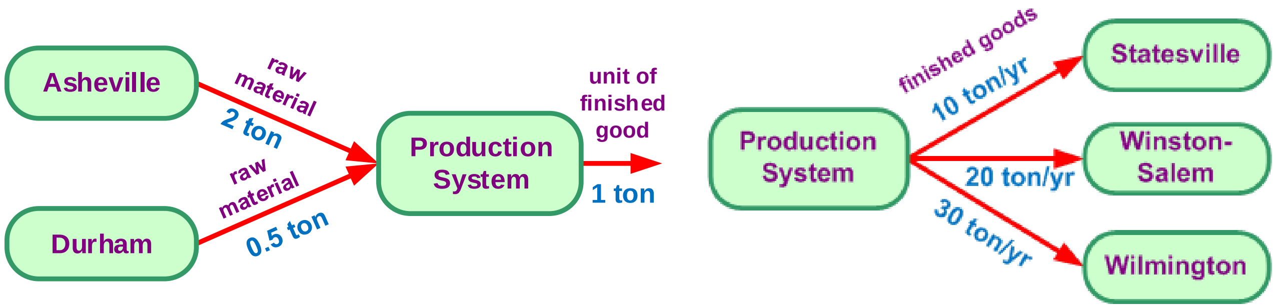
Ex 5: 1-D Location with Procurement and Distribution Costs



Assume: all scrap is disposed of locally

A product is to be produced in a plant that will be located along I-40. Two tons of raw materials from a supplier in Ashville and a half ton of a raw material from a supplier in Durham are used to produce each ton of finished product that is shipped to customers in Statesville, Winston-Salem, and Wilmington. The demand of these customers is 10, 20, and 30 tons, respectively, and it costs $0.33 per ton-mile to ship raw materials to the plant and $1.00 per ton-mile to ship finished goods from the plant. Determine where the plant should be located so that procurement and distribution costs (i.e., transportation costs to and from the plant) are minimized, and whether the plant is weight gaining or weight losing.

Ex 5: 1-D Location with Procurement and Distribution Costs



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

monetary weight   physical weight

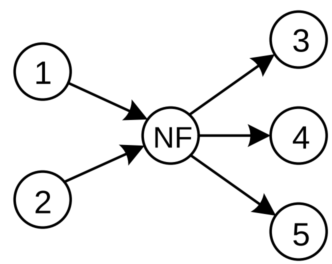
$$\underset{(\$/\text{mi-yr})}{w_i} = \underset{(\text{ton/yr})}{f_i} \times \underset{(\$/\text{ton-mi})}{r_i}$$

Assume: all scrap is disposed of locally

$$r_{\text{in}} = \$0.33/\text{ton-mi} \qquad r_{\text{out}} = \$1.00/\text{ton-mi}$$

$$f_1 = BOM_1 \sum f_{\text{out}} = 2(60) = 120, \quad w_1 = f_1 r_{\text{in}} = 40$$

$$f_2 = BOM_2 \sum f_{\text{out}} = 0.5(60) = 30, \quad w_2 = f_2 r_{\text{in}} = 10$$



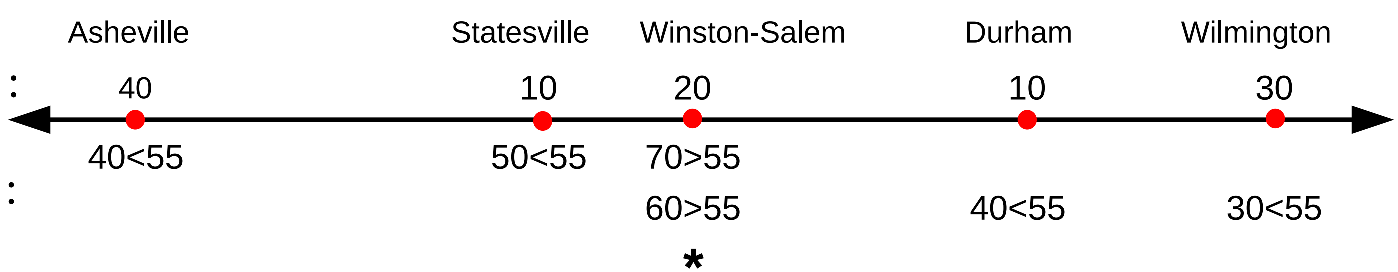
$$f_3 = 10, \quad w_3 = f_3 r_{\text{out}} = 10$$

$$f_4 = 20, \quad w_4 = f_4 r_{\text{out}} = 20$$

$$f_5 = 30, \quad w_5 = f_5 r_{\text{out}} = 30$$

w_i :

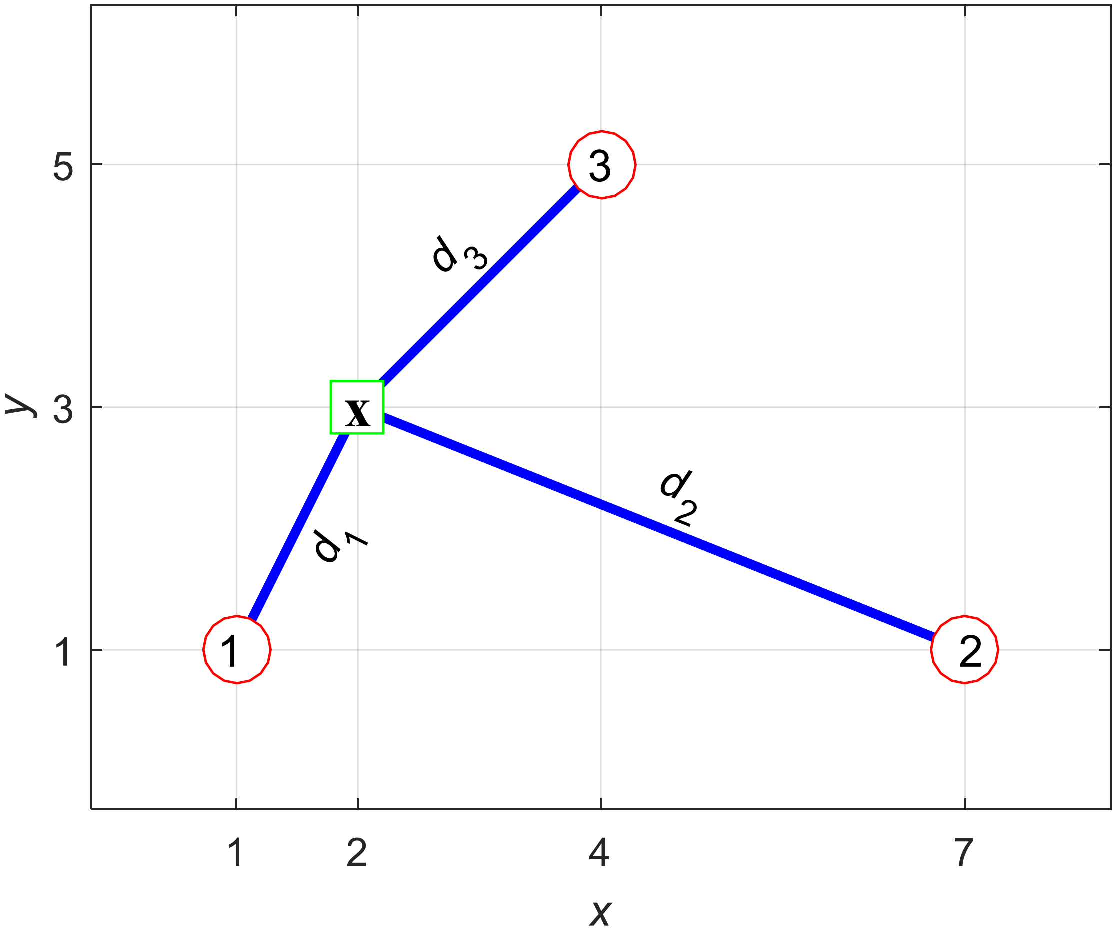
$\sum_{i=1}^{j} w_i \le \frac{W}{2}$:



$$\text{(Montetary) Weight Gaining:} \quad \Sigma w_{\text{in}} = 50 < \Sigma w_{\text{out}} = 60$$

$$\text{Physically Weight Losing:} \quad \Sigma f_{\text{in}} = 150 > \Sigma f_{\text{out}} = 60$$

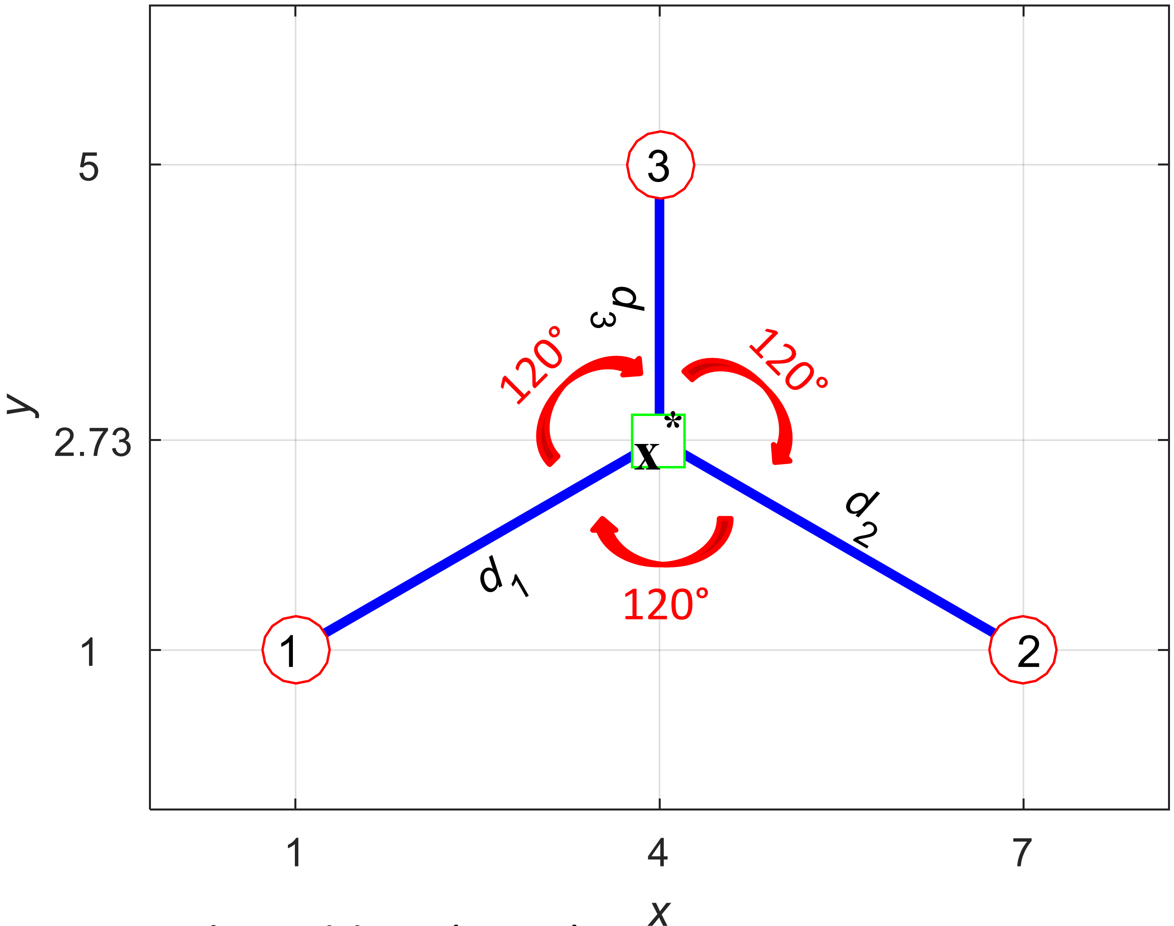
2-D Euclidean Distance



$$\mathbf{x} = \begin{bmatrix} 2 & 3 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_1 - p_{1,1}\right)^2 + \left(x_2 - p_{1,2}\right)^2} \\ \sqrt{\left(x_1 - p_{2,1}\right)^2 + \left(x_2 - p_{2,2}\right)^2} \\ \sqrt{\left(x_1 - p_{3,1}\right)^2 + \left(x_2 - p_{3,2}\right)^2} \end{bmatrix}$$

Minisum Distance Location



$$\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 7 & 1 \\ 4 & 5 \end{bmatrix}$$

$$d_i(\mathbf{x}) = \sqrt{\left(x_1 - p_{i,1}\right)^2 + \left(x_2 - p_{i,2}\right)^2}$$

$$TD(\mathbf{x}) = \sum_{i=1}^{3} d_i(\mathbf{x})$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TD(\mathbf{x})$$

$$TD^* = TD(\mathbf{x}^*)$$

Fermat's Problem (1629):

Given three points, find fourth (Steiner point) such that sum to others is minimized

(Solution: Optimal location corresponds to all angles = 120°)

Minisum Weighted-Distance Location

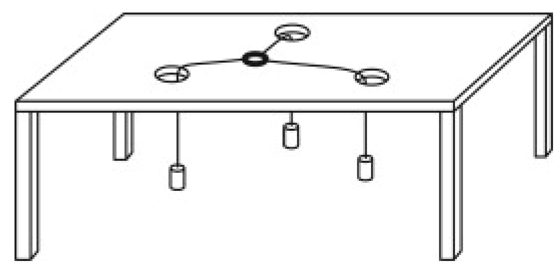
- Solution for 2-D+ and non-rectangular distances:
  - *Majority Theorem:* Locate NF at EF*j* if $w_j \geq \frac{W}{2}, \quad \text{where}\, W = \sum_{i=1}^{m} w_i$
  - Mechanical (Varigon frame)
  - 2-D rectangular approximation
  - Numerical: nonlinear unconstrained optimization
    - Analytical/estimated gradient (quasi-Newton, `fminunc`)
    - Direct, gradient-free (Nelder-Mead, `fminsearch`)

$$m = \text{number of EFs}$$

$$TC(\mathbf{x}) = \sum_{i=1}^{m} w_i d_i(\mathbf{x})$$

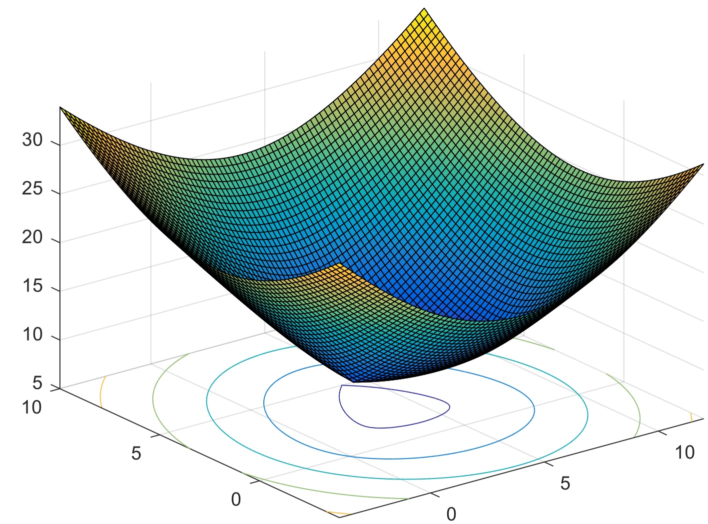
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

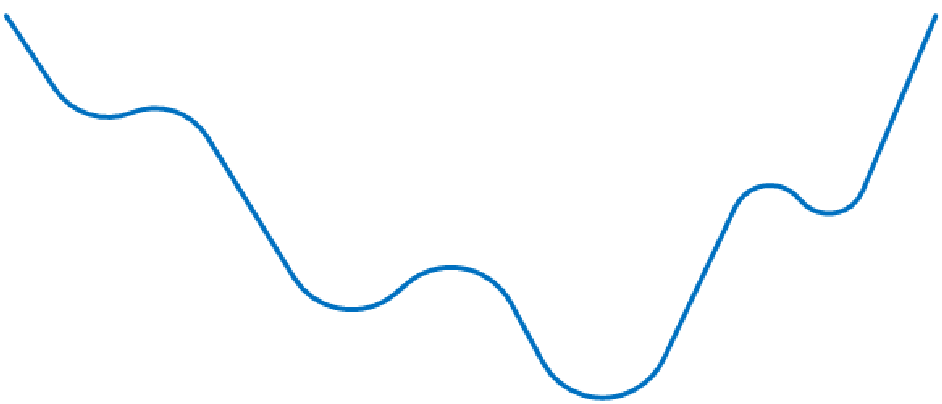
$$TC^* = TC(\mathbf{x}^*)$$



Varignon Frame

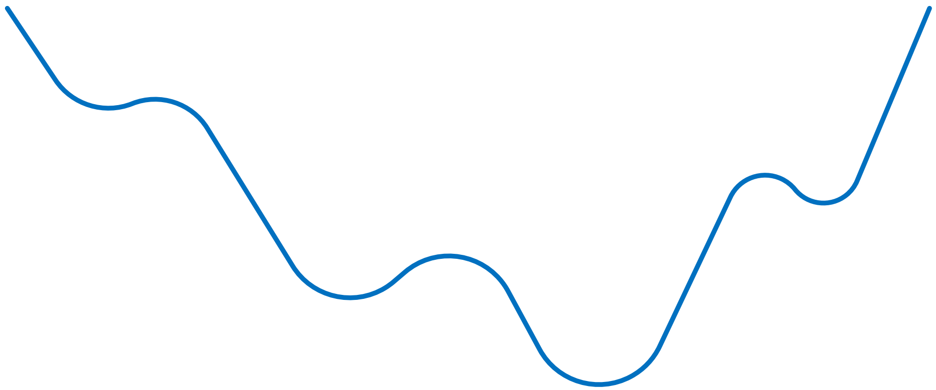
Convex vs Nonconvex Optimization



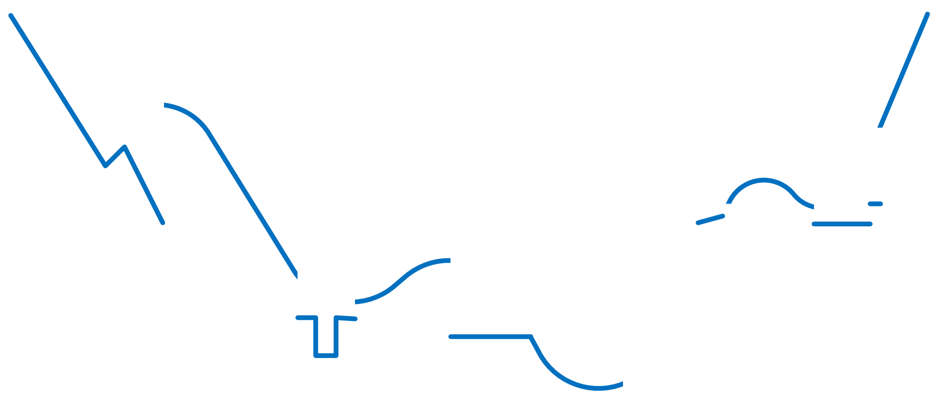


Gradient vs Direct Methods

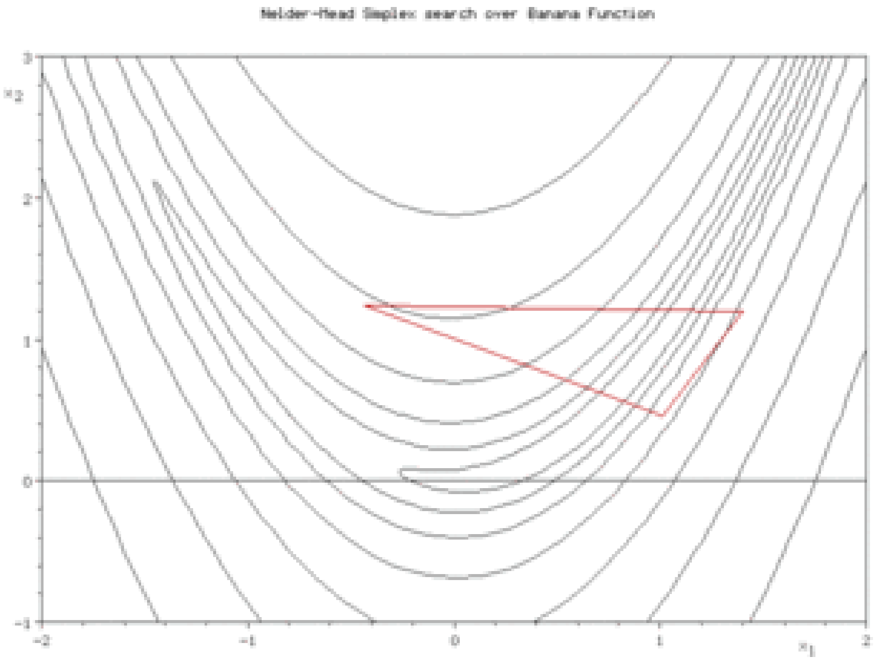
- Numerical nonlinear unconstrained optimization:
  - Analytical/estimated gradient
    - quasi-Newton
    - `fminunc`

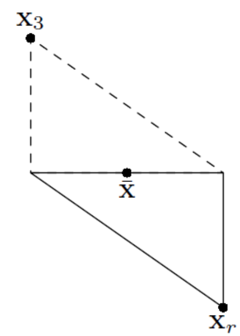


  - Direct, gradient-free
    - Nelder-Mead
    - `fminsearch`

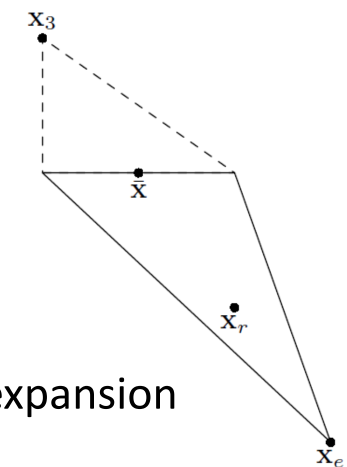


Nelder-Mead Simplex Method



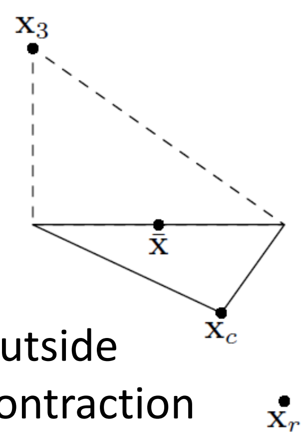


reflection

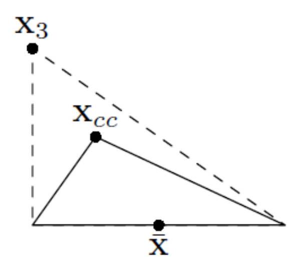


expansion

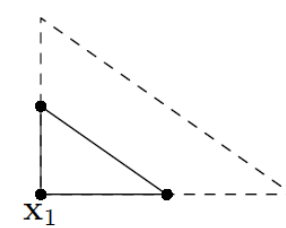
- AKA amoeba method
- Simplex is triangle in 2-D (dashed line in figures)



outside contraction

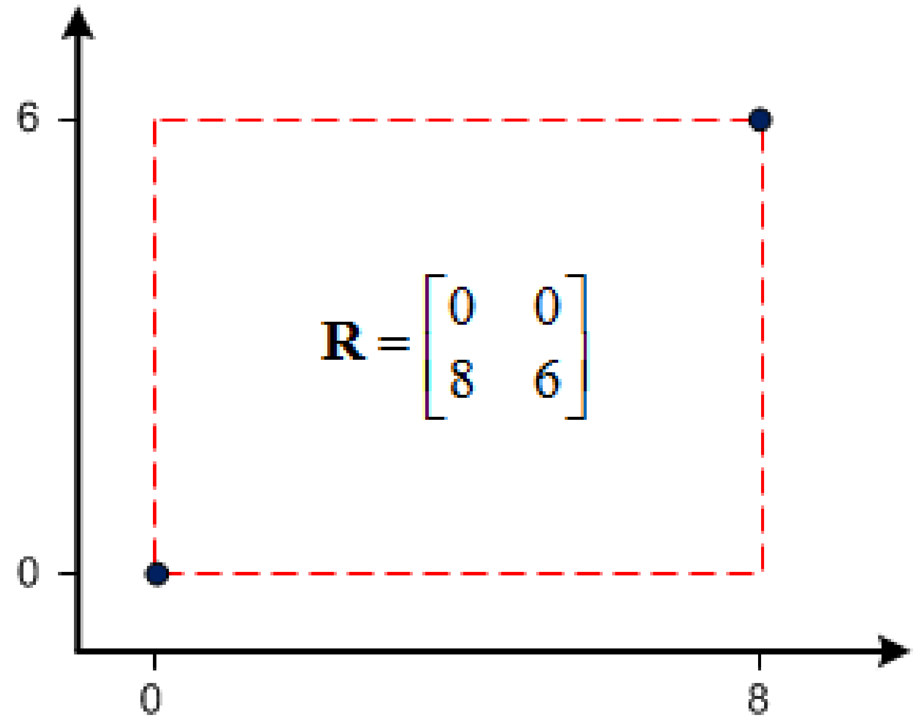


inside contraction



a shrink

Feasible Region

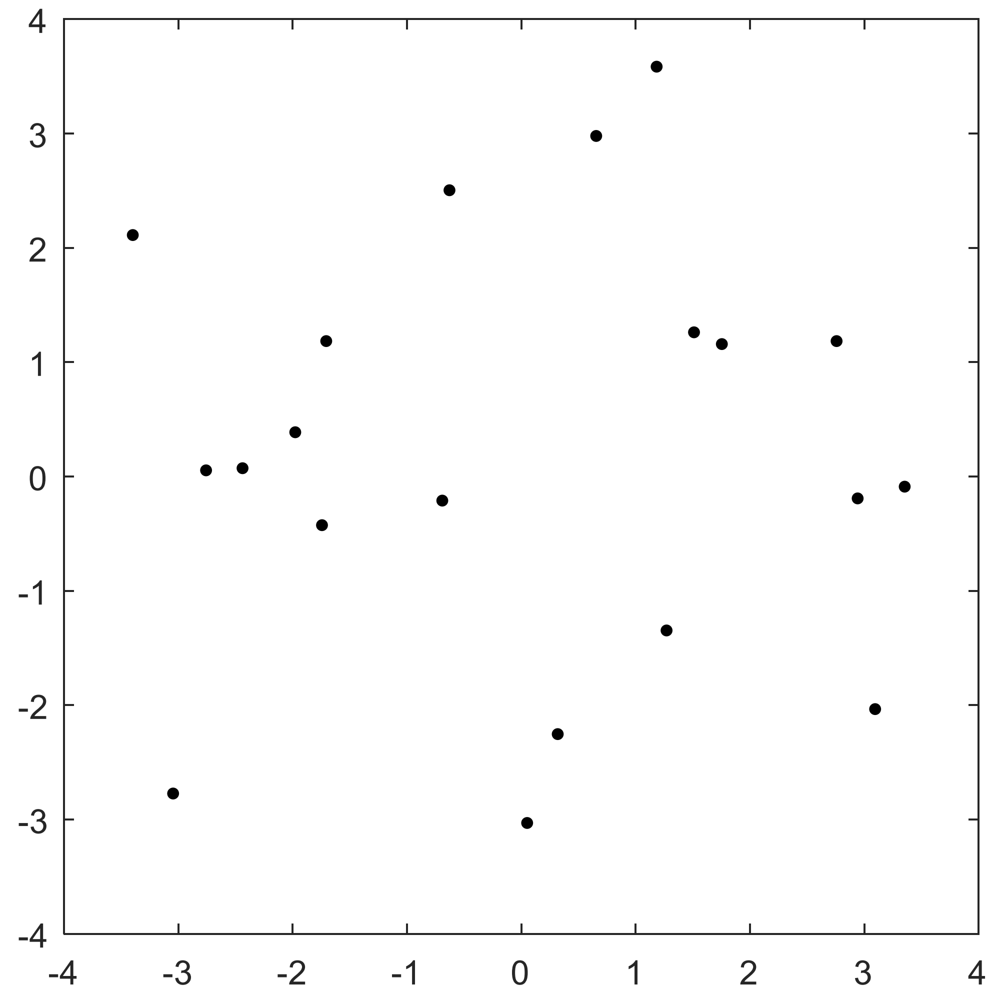



```
if a is true
  return b
else
  return c
end
```

$$= iff\,(a,b,c) = \begin{cases} b, & \text{if } a \text{ is true} \\ c, & \text{otherwise} \end{cases} \Rightarrow \begin{cases} TC(\mathbf{x}), & \text{if } \mathbf{x} \text{ is in } \mathbf{R} \\ \infty, & \text{otherwise} \end{cases}$$

Computational Geometry

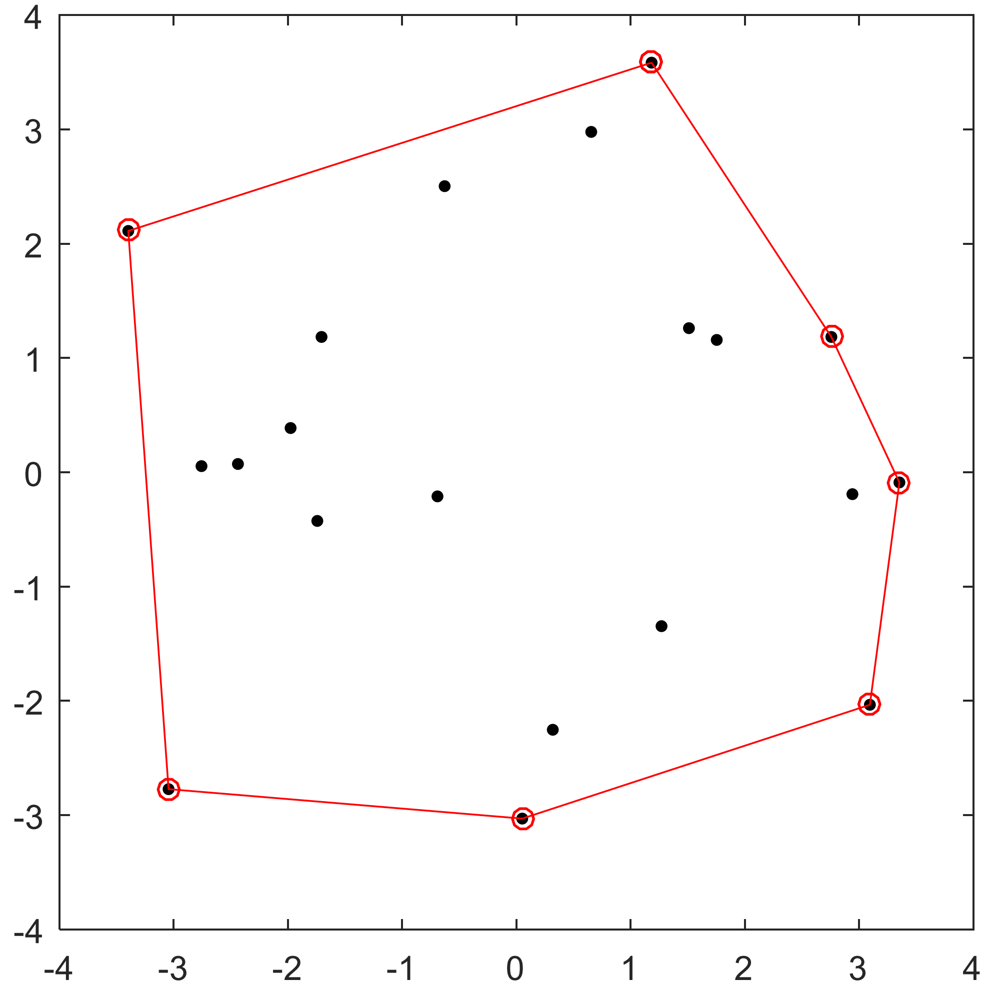
- Design and analysis of algorithms for solving geometric problems
  - Modern study started with Michael Shamos in 1975
- Facility location:
  - geometric data structures used to “simplify” solution procedures



Convex Hull

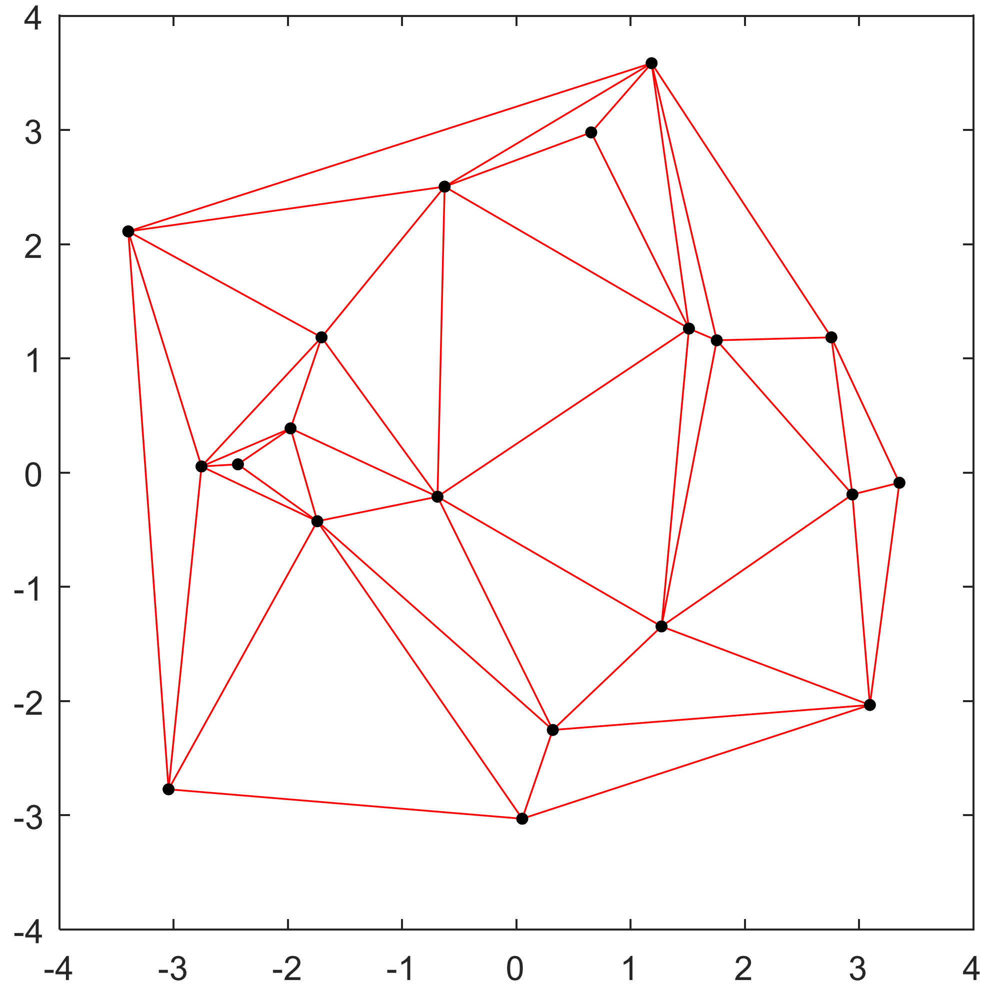
- Find the points that enclose all points
  - Most important data structure
  - Calculated, via Graham's scan in

    $O(n \log n), \quad n$ points



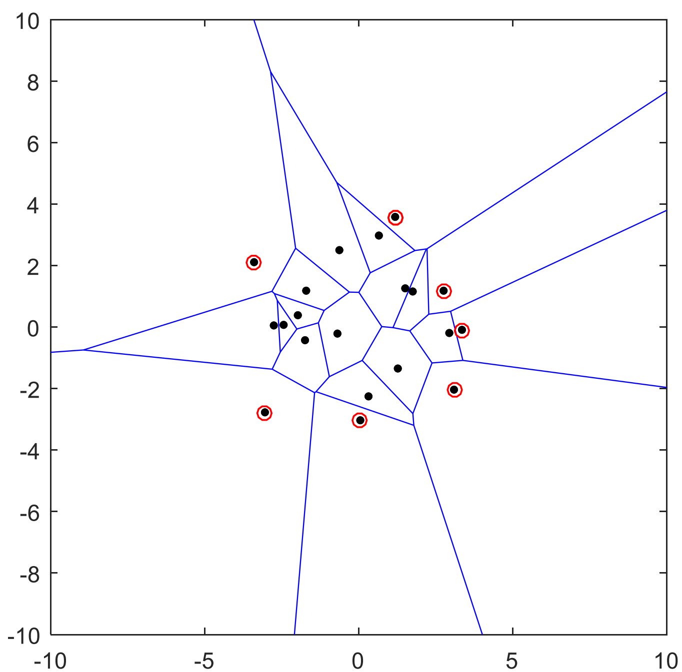
Delaunay Triangulation

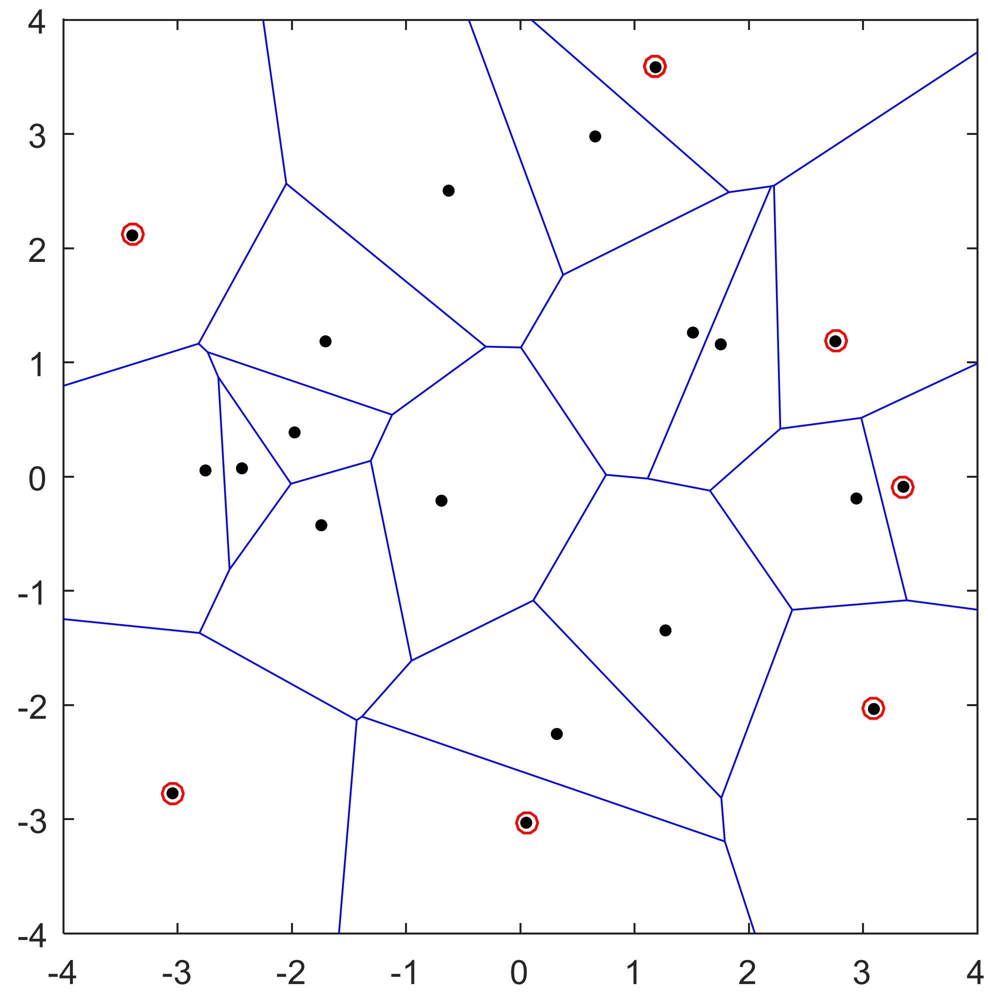
- Find the triangulation of points that maximizes the minimum angle of any triangle
  - Captures proximity relationships
  - Used in 3-D animation
  - Calculated, via divide and conquer, in $O(n \log n)$, n points



Voronoi Diagram

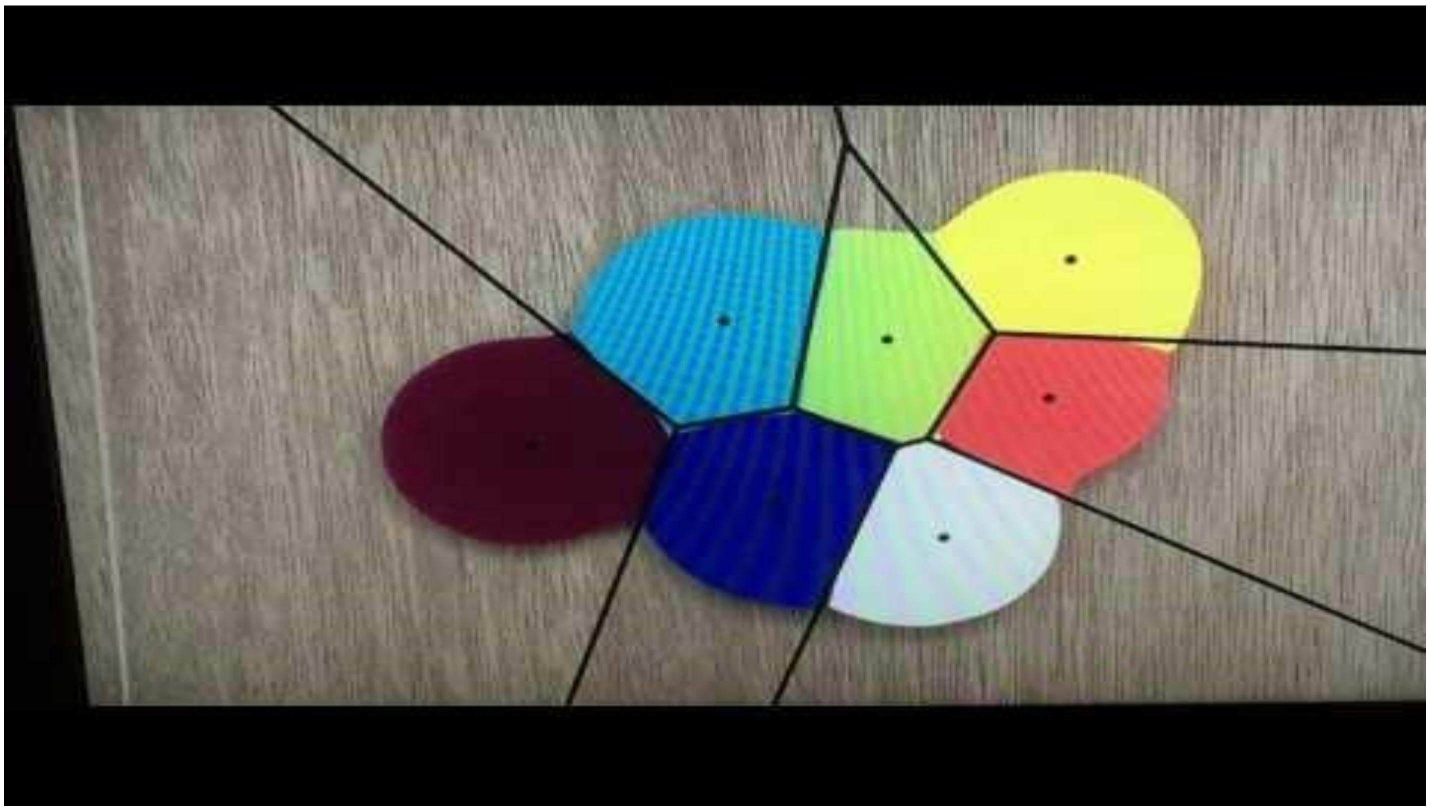
- Each region defines area closest to a point
  - Open face regions indicate points in convex hull





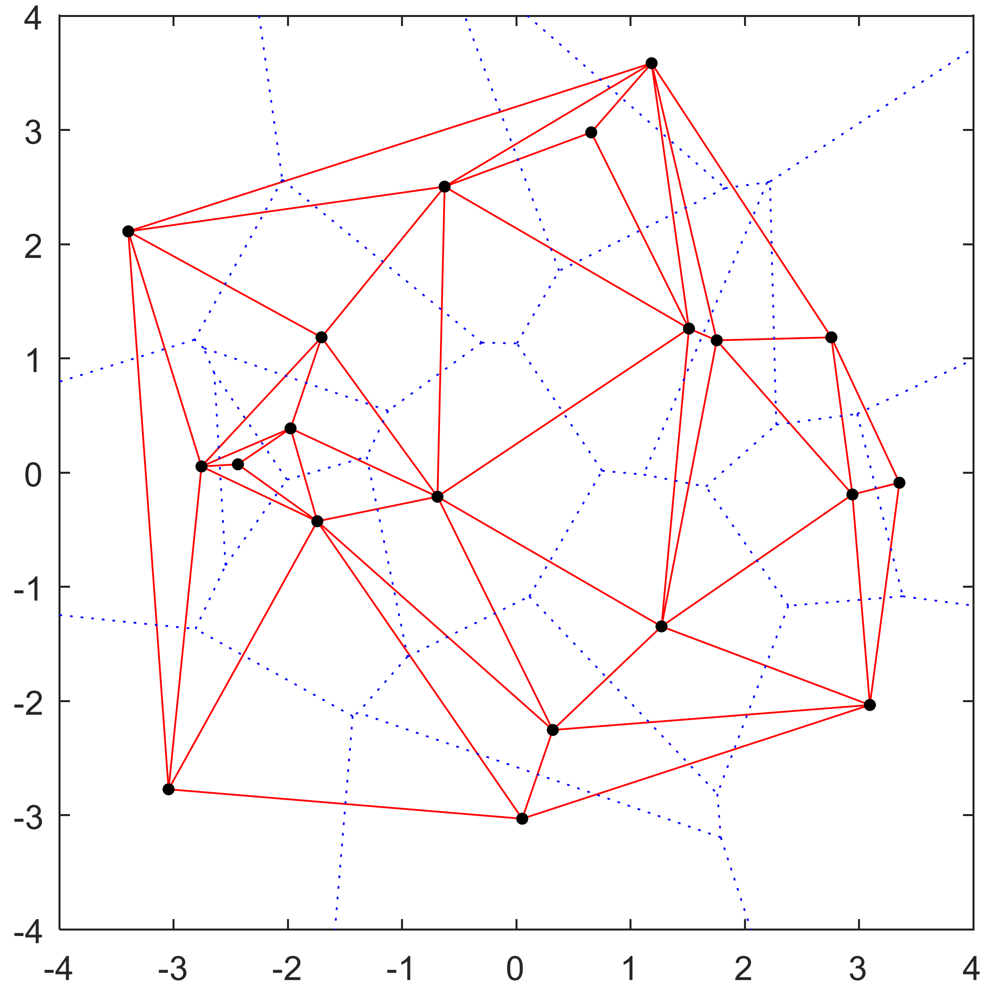
Voronoi Diagram

- Voronoi diagram from smooshing paint between glass
  – https://youtu.be/yDMtGT0b_kg



Delaunay-Voronoi

- Delaunay triangulation is straight-line dual of Voronoi diagram
  - Can easily convert from one to another

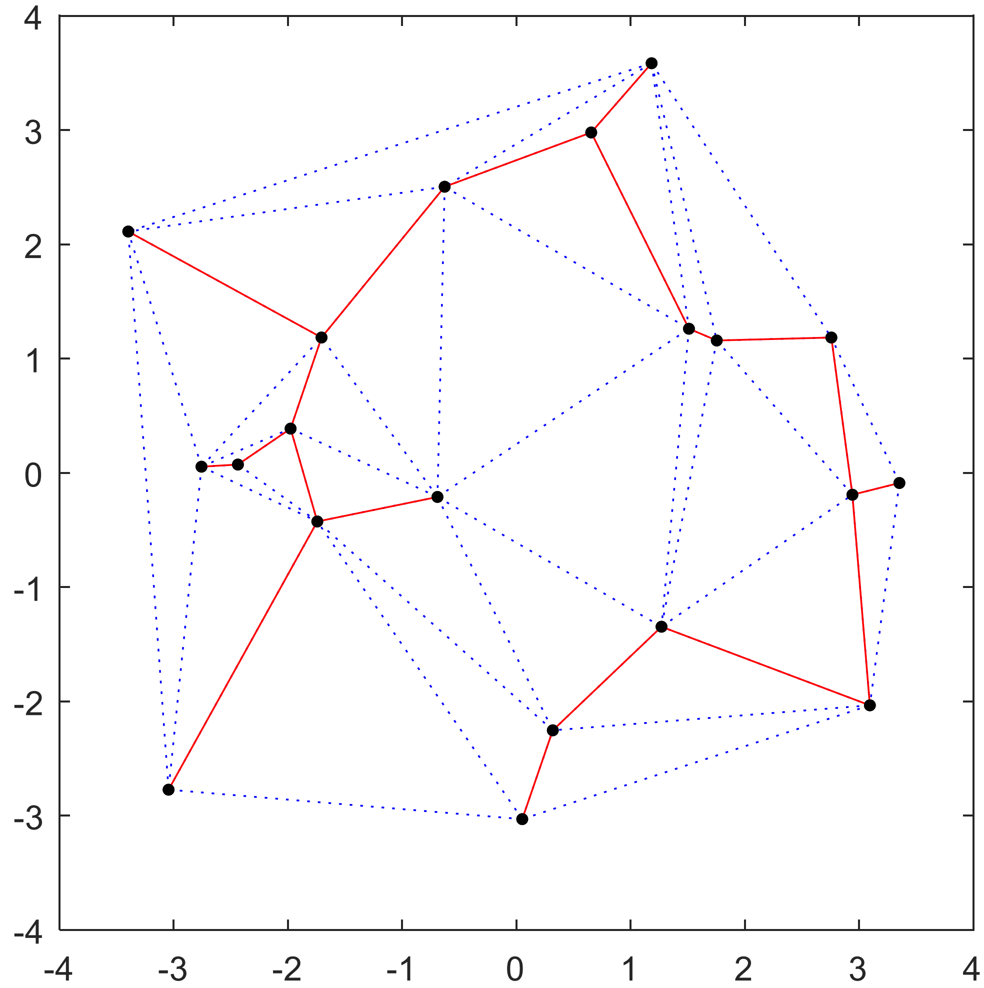


Minimum Spanning Tree

- Find the minimum weight set of arcs that connect all nodes in a graph
  - *Undirected* arcs: calculated, via Kruskal's algorithm,

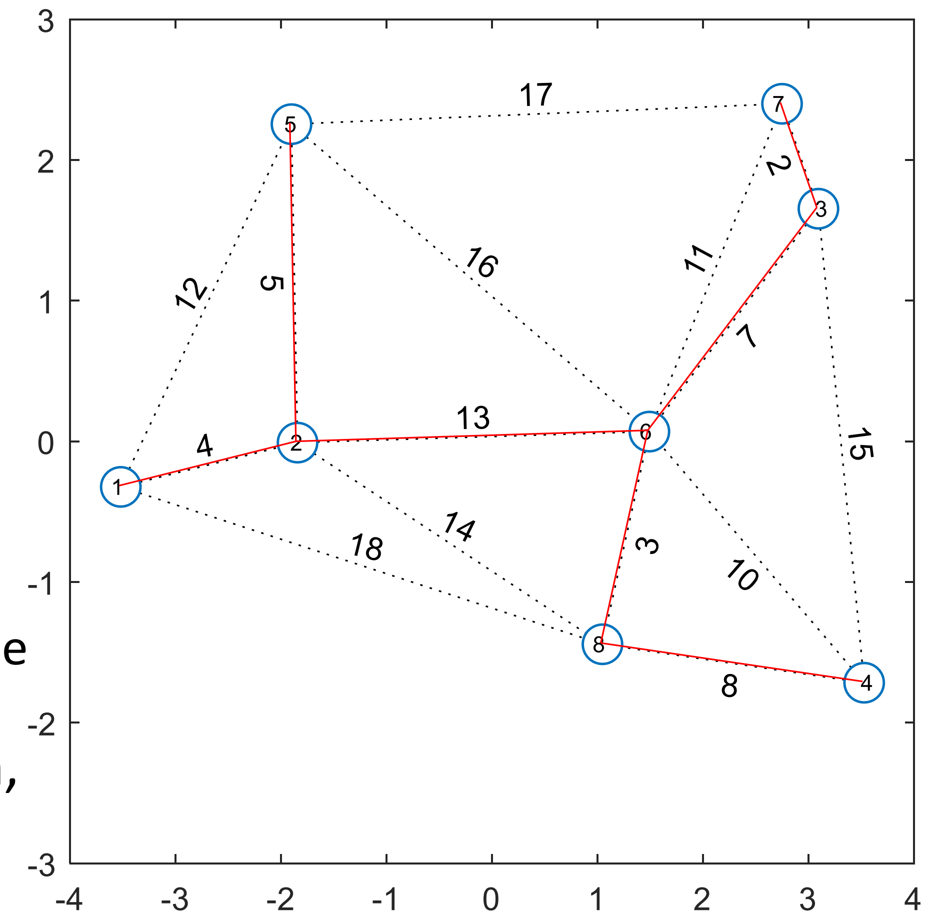
    $O(m \log n)$, m arcs, n nodes
  - *Directed* arcs: calculated, via Edmond's branching algorithm, in

    $O(mn)$, m arcs, n nodes



Kruskal's Algorithm for MST

- **Algorithm:**
  1. Create set *F* of single node trees
  2. Create set *S* of all arcs
  3. While *S* nonempty and *F* is not yet spanning
     4. Remove min arc from *S*
     5. If removed arc connects two different trees, then add to *F*, combining two trees into single tree
  6. If graph connected, *F* forms single MST; otherwise, forms multi-tree min spanning forest
- Optimal "greedy" algorithm, runs in $O(m \log n)$
- If directed arcs, $O(mn)$
  - useful in VRP to min vehicles
  - harder to code



m = 15 arcs, *n* = 8 nodes

Min Spanning vs Steiner Trees

- Steiner point added to reduce distance connecting three existing points compared to min spanning tree

$$\frac{b}{2} = \frac{1}{2}\sqrt{3}a \Rightarrow b = \sqrt{3}a, \quad 30-60-90 \text{ triangle}$$

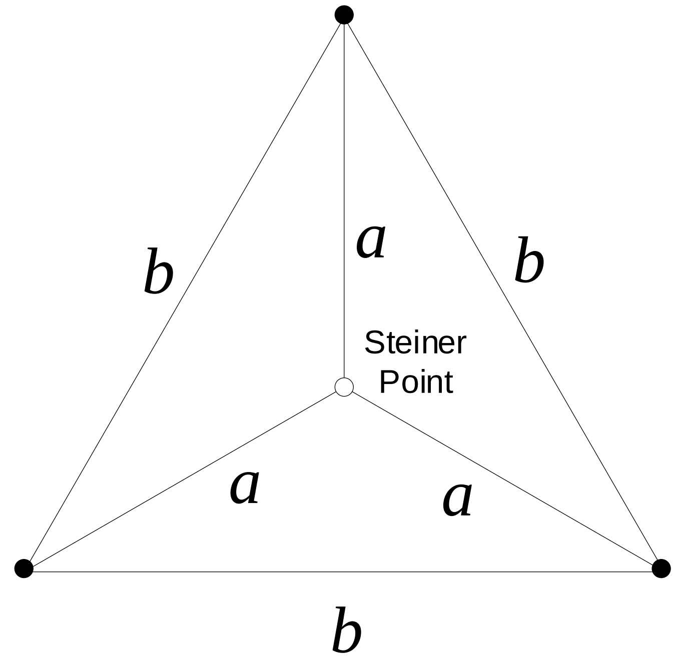
Min spanning tree distance > Steiner tree distance

$$2b > 3a$$

$$2\sqrt{3}a > 3a$$

$$2 > \sqrt{3}$$

$$\sqrt{4} > \sqrt{3}$$



Steiner Network

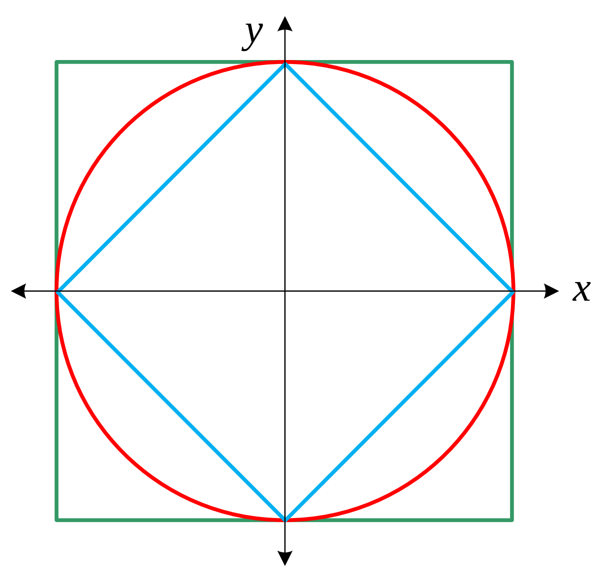
Metric Distances

General l_p: $d_p(P_1, P_2) = \left[|x_1 - x_2|^p + |y_1 - y_2|^p \right]^{\frac{1}{p}}, \quad p \geq 1$

Rectilinear $(p=1)$: $d_1(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$

Euclidean $(p=2)$: $d_2(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Chebychev $(p \to \infty)$: $d_\infty(P_1, P_2) = \max \left\{ |x_1 - x_2|, |y_1 - y_2| \right\}$



Chebychev Distances

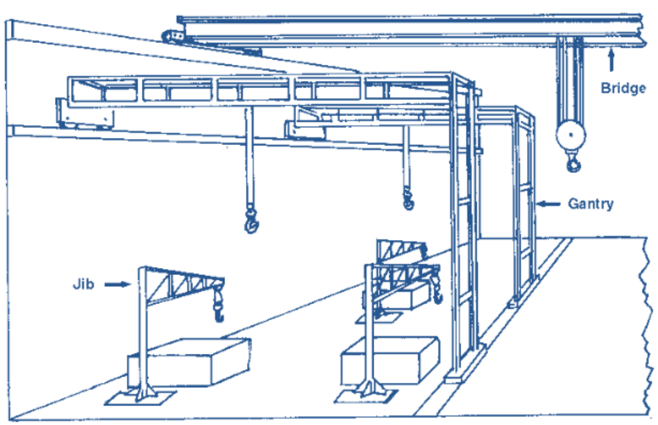
Proof

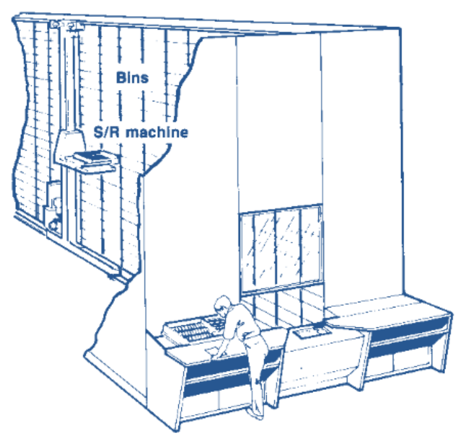
Without loss of generality, let $P_1 = (x, y)$, for $x, y \geq 0$, and $P_2 = (0, 0)$. Then $d_\infty(P_1, P_2) = \max\{x, y\}$ and $d_p(P_1, P_2) = \left[x^p + y^p\right]^{1/p}$.

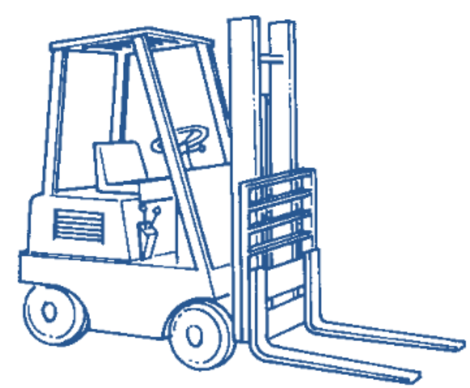
If $x = y$, then $\lim_{p\to\infty}\left[x^p + y^p\right]^{1/p} = \lim_{p\to\infty}\left[2x^p\right]^{1/p} = \lim_{p\to\infty}\left[2^{1/p}x\right] = x$.

If $x < y$, then $\lim_{p\to\infty}\left[x^p + y^p\right]^{1/p} = \lim_{p\to\infty}\left[\left((x/y)^p + 1\right)y^p\right]^{1/p} = \lim_{p\to\infty}\left((x/y)^p + 1\right)^{1/p} y = 1 \cdot y = y$.

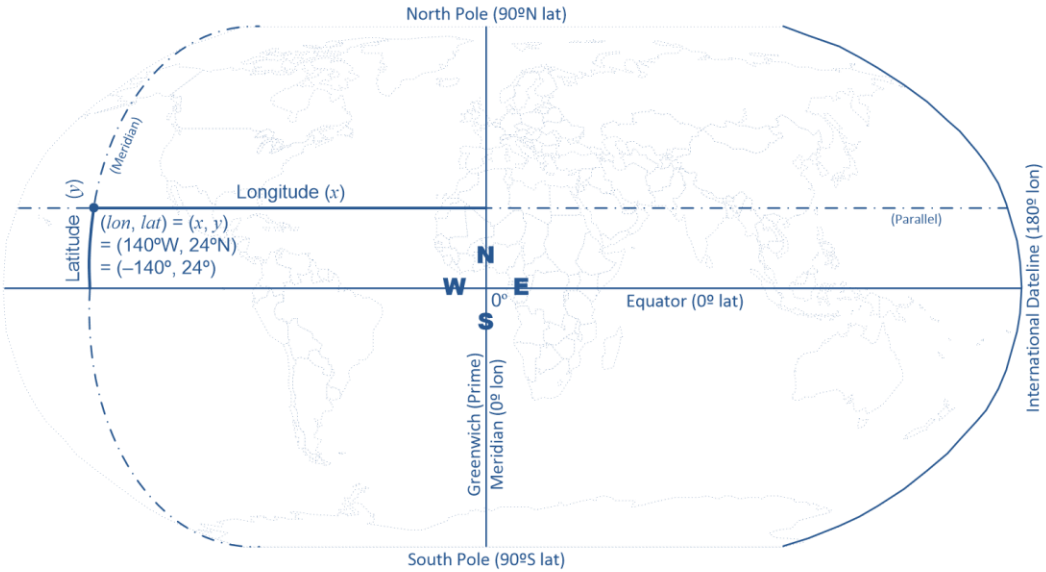
A similar argument can be made if $x > y$. ■

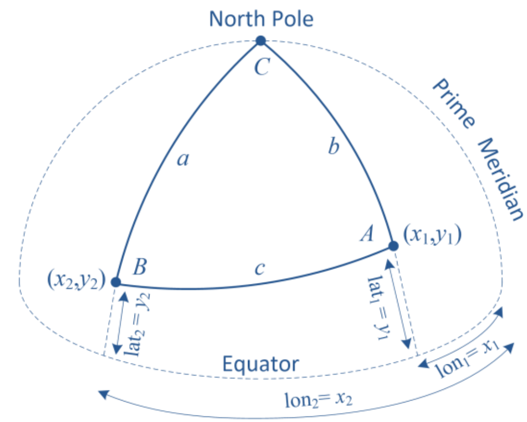


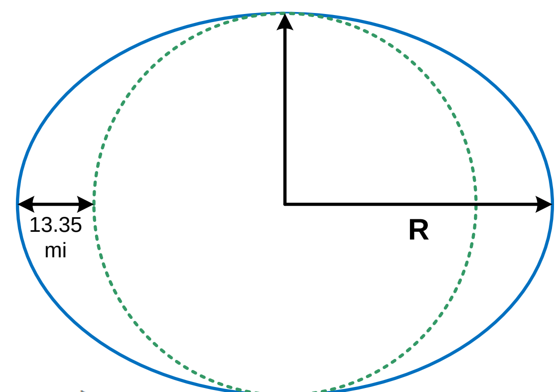




Great Circle Distances







$$(lon_1, lat_1) = (x_1, y_1), \quad (lon_2, lat_2) = (x_2, y_2)$$

$$d_{rad} = (\text{great circle distance in radians of a sphere})$$

$$= \cos^{-1}\left[\sin y_2 \sin y_1 + \cos y_2 \cos y_1 \cos(x_1 - x_2)\right]$$

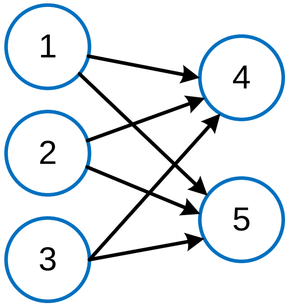
$$R = (\text{radius of earth at equator}) - (\text{bulge from north pole to equator})$$

$$= 3{,}963.34 - 13.35 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ mi}, \quad = 6{,}378.388 - 21.476 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ km}$$

$$d_{\text{GC}} = \text{distance } (x_1, y_1) \text{ to } (x_2, y_2) = \boxed{d_{rad} \cdot R}$$

$$x_{\text{deg}} = \begin{cases} DD + \dfrac{MM}{60} + \dfrac{SS}{3{,}600}, & \text{if } E \text{ or } N \\ -DD - \dfrac{MM}{60} - \dfrac{SS}{3{,}600}, & \text{if } W \text{ or } S \end{cases}$$

Metric Distances using `dists`



$$\underset{\substack{3\times 2\\ n\times m}}{\mathbf{D} = \begin{array}{c|cc} & 4 & 5 \\ \hline 1 & \bullet & \bullet \\ 2 & \bullet & \bullet \\ 3 & \bullet & \bullet \end{array}} \quad = \quad \text{dists}(\underset{\substack{3\times 2\\ n\times d}}{\mathbf{X1}}, \underset{\substack{2\times 2\\ m\times d}}{\mathbf{X2}}, p), \qquad p = \left\{\begin{array}{lll} \text{'mi'} & \text{'km'} & \\ 1 & 2 & \text{Inf} \end{array}\right.$$

***d* = 2**

$$\mathbf{X1} = \begin{bmatrix} \bullet & \bullet \end{bmatrix}, \mathbf{X2} = \begin{bmatrix} \bullet & \bullet \end{bmatrix} \quad \Rightarrow \quad \mathbf{d} = \begin{bmatrix} \bullet \end{bmatrix}$$

$$\mathbf{X1} = \begin{bmatrix} \bullet & \bullet \end{bmatrix}, \mathbf{X2} = \begin{bmatrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{bmatrix} \quad \Rightarrow \quad \mathbf{d} = \begin{bmatrix} \bullet & \bullet & \bullet \end{bmatrix}$$

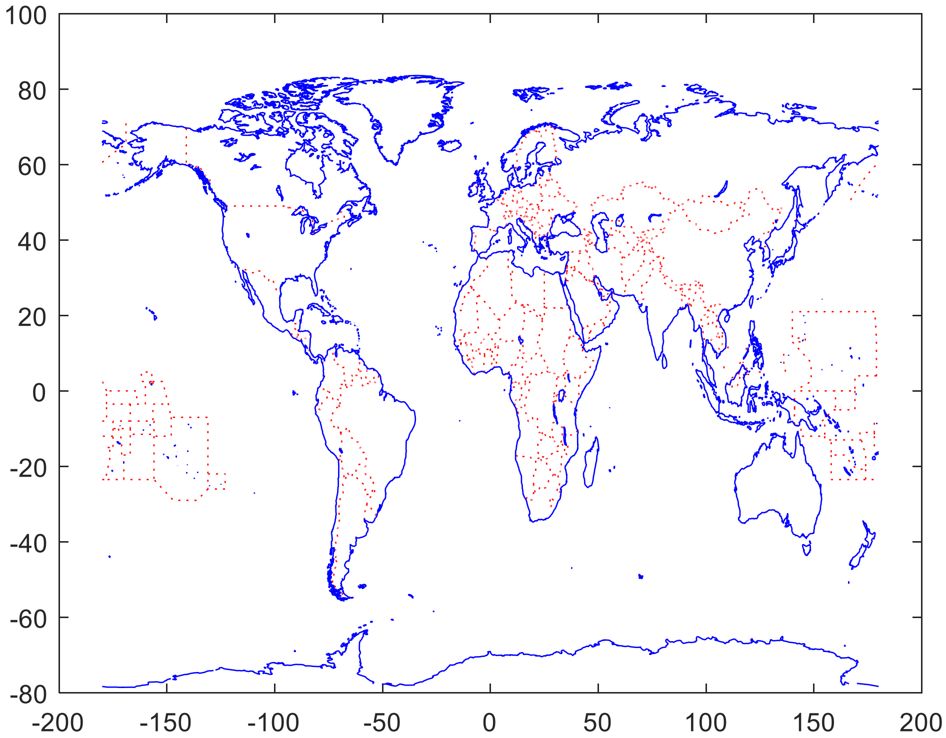
$$\mathbf{X1} = \begin{bmatrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{bmatrix}, \mathbf{X2} = \begin{bmatrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{bmatrix} \quad \Rightarrow \quad \mathbf{D} = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}$$

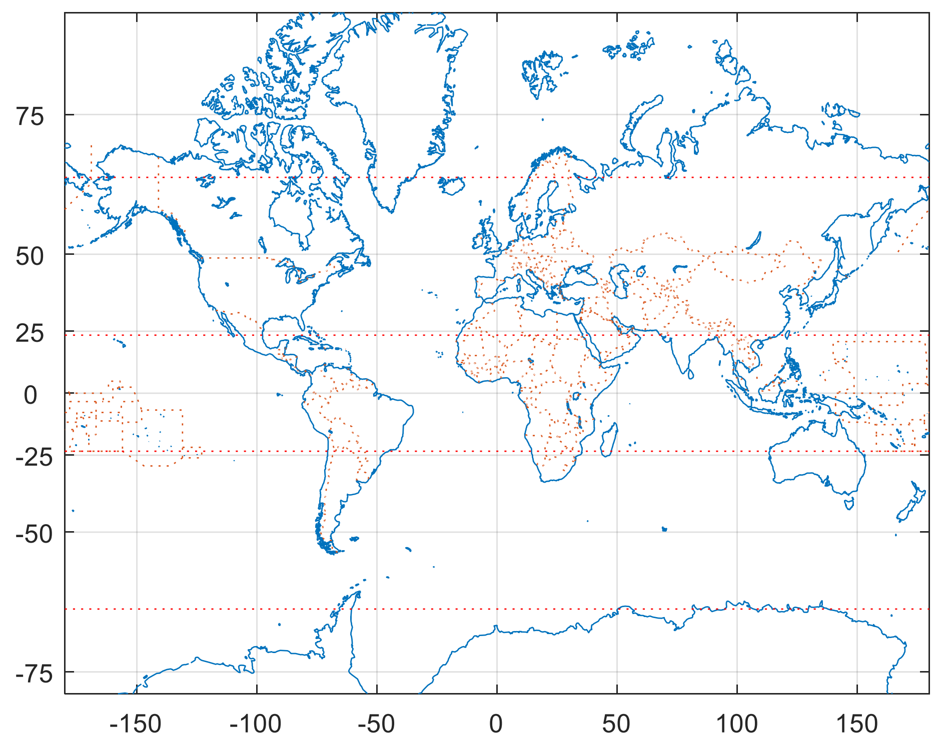
***d* = 1**

$$\mathbf{X1} = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}, \mathbf{X2} = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \quad \Rightarrow \quad \mathbf{D} = \begin{bmatrix} \bullet & \bullet \\ \bullet & \bullet \end{bmatrix}$$

$$\mathbf{X1} = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}, \mathbf{X2} = \begin{bmatrix} \bullet & \bullet \end{bmatrix} \quad \Rightarrow \quad \textit{Error}$$

Mercator Projection





$$x_{\text{proj}} = x$$

$$y_{\text{proj}} = \sinh^{-1}(\tan y)$$

$$y = \tan^{-1}(\sinh y_{\text{proj}})$$

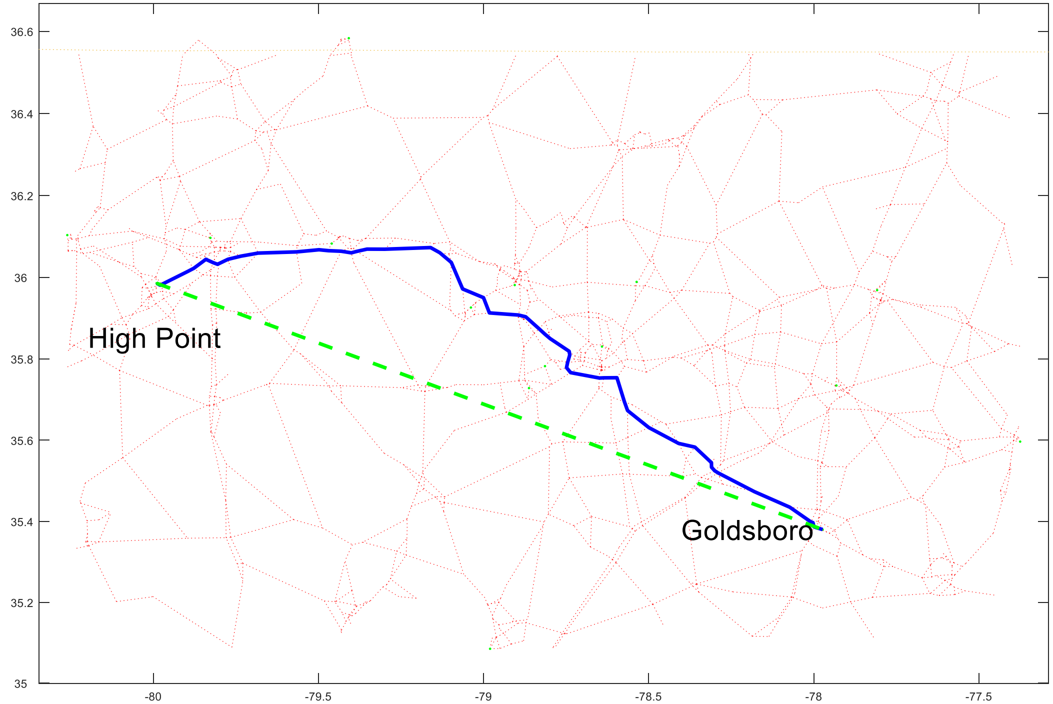
$$x_{\text{rad}} = \frac{x_{\text{deg}}}{180}\pi \quad \text{and} \quad x_{\text{deg}} = \frac{x_{\text{rad}} \cdot 180}{\pi}$$

Circuity Factor

$$Circuity\ Factor: \quad g = \sum v_i \frac{d_{\text{road}_i}}{d_{GC_i}}, \quad \text{where usually } 1.15 \le g \le 1.5,\ v_i \text{ weight of sample } i$$

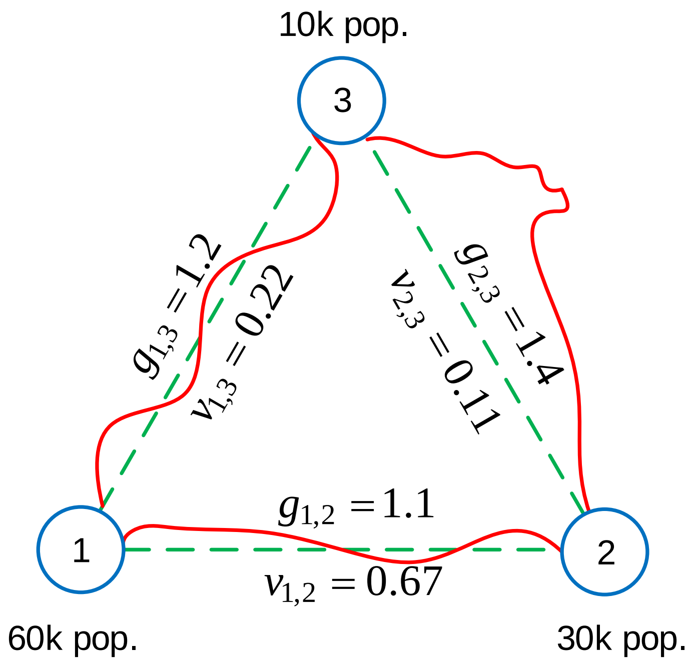
$$d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2), \quad \text{estimated road distance from } P_1 \text{ to } P_2$$

From High Point to Goldsboro: Road = 143 mi, Great Circle = 121 mi, Circuity = 1.19



Estimating Circuity Factors

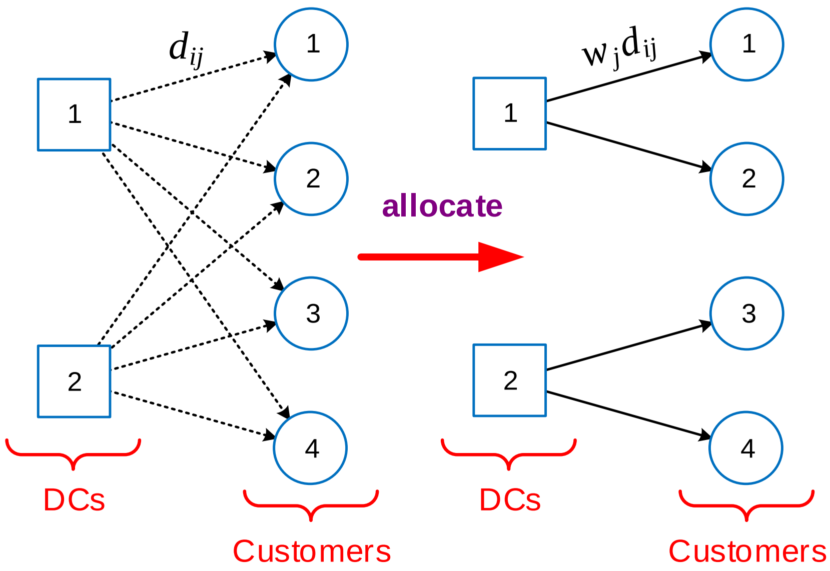
- Circuity factor depends on both the trip density and directness of travel network
  - Circuity of high trip density areas should be given more weight when estimating overall factor for a region
  - Obstacles (water, mountains) limit direct road travel




```
v = [.6 .3 .1];
V = v'*v
  = 0.3600    0.1800    0.0600
    0.1800    0.0900    0.0300
    0.0600    0.0300    0.0100
V = triu(V,1)
  =      0    0.1800    0.0600
         0         0    0.0300
         0         0         0
V = V/sum(sum(V))
  =      0    0.6667    0.2222
         0         0    0.1111
         0         0         0
```

Allocation

- **Example:** given n DCs and m customers, with customer j receiving w_j TLs per week, determine the total distance per week assuming each customer is served by its *closest* DC



$$w = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$$

$$D = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 45 & 35 & 25 & 15 \end{bmatrix}$$

$$\begin{aligned} TD &= 2(10) + 4(20) + 6(25) + 8(15) \\ &= 370 \end{aligned}$$

Pseudocode

- Different ways of representing how allocation and TD can be calculated
  - High-level pseudocode most concise, but leaves out many implementation details (sets don't specify order, initial starting points)
  - Low-level pseudocode includes more implementation details, which can hide/obscure the core idea, and are usually not essential

Low-level Pseudocode

```
TD = 0
for j = 1:m
   dj = D(1,j)
   for i = 2:n
      if D(i,j) < dj
         dj = D(i,j)
      end
   end
   TD = TD + w(j)*dj
end
```

High-level Pseudocode

$$N = \{1,\ldots,n\}, \quad n = |N|$$

$$M = \{1,\ldots,m\}, \quad m = |M|$$

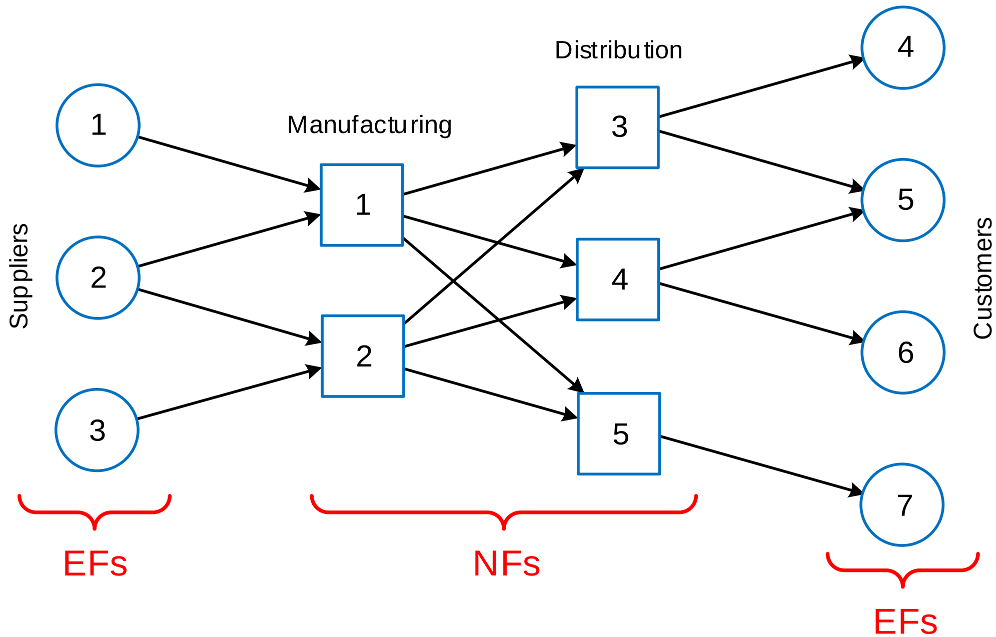
$$\alpha = [\alpha_j] = \arg\min_{i \in N} d_{ij}$$

$$TD = \sum_{j \in M} w_j d_{\alpha_j, j}$$

Matlab/Matlog

```
a = argmin(D);
W = sparse(a,1:m,w,n,m)
TD = sum(sum(W.*D))
```

Minisum Multifacility Location



$n =$ no. of NFs, $\quad m =$ no. of EFs

$\mathbf{X}_{n\times d} =$ NF locations, $\quad \mathbf{P}_{m\times d} =$ EF locations

$\mathbf{V}_{n\times n} =$

NF-NF	1	2	3	4	5
1	0	0	+	+	+
2	0	0	+	+	+
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0

$\mathbf{W}_{n\times m} =$

NF-EF	1	2	3	4	5	6	7
1	+	+	0	0	0	0	0
2	0	+	+	0	0	0	0
3	0	0	0	+	+	0	0
4	0	0	0	0	+	+	0
5	0	0	0	0	0	0	+

$$TC(\mathbf{X}) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk}\, d(\mathbf{X}_j, \mathbf{X}_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji}\, d(\mathbf{X}_j, \mathbf{P}_i)$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} TC(\mathbf{X})$$

$$TC^* = TC(\mathbf{X}^*)$$

Majority Theorem for Minisum Location

- Single-facility: Locate NF at EFj if $w_j \ge \dfrac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$
- Multifacility: can be used to reduce and sometimes solve

Given m EF and n NF, let $\mathbf{V} \leftarrow \mathbf{V} + \mathbf{V}'$

1. While any $v_{ik} \ge \dfrac{1}{2}\left(\sum_{j=1}^{m} w_{ij} + \sum_{j=1}^{n} v_{ij} \right)$, co-locate NF$i$ and NFk and

   (a) add row k to row i of $\mathbf{W}$, remove row k from $\mathbf{W}$

   (b) add row k to row i and column k to column i of $\mathbf{V}$

   (c) remove row k and column k from $\mathbf{V}$, and set $0 \leftarrow v_{ii}$

2. Locate all NFi at EFk if $w_{ik} \ge \dfrac{1}{2}\left(\sum_{j=1}^{m} w_{ij} + \sum_{j=1}^{n} v_{ij} \right)$,

   where any NFj co-located with NFi are also located at EFk.

Ex 6: Multifacility Majority Theorem

3 EF, 2 NF, $\mathbf{V}=\begin{bmatrix}0 & 2\\ 0 & 0\end{bmatrix} \Rightarrow \mathbf{V}=\begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix}$:

1. No solutions or reductions possible

$$[\mathbf{W}\ \mathbf{V}]=\left[\begin{array}{ccc:cc}2 & 1 & 0 & 0 & 2\\ 4 & 0 & 5 & 2 & 0\end{array}\right]\begin{array}{l}\sum=5\\ \sum=11\end{array}$$

2. Modified $\mathbf{V} \Rightarrow$ solution

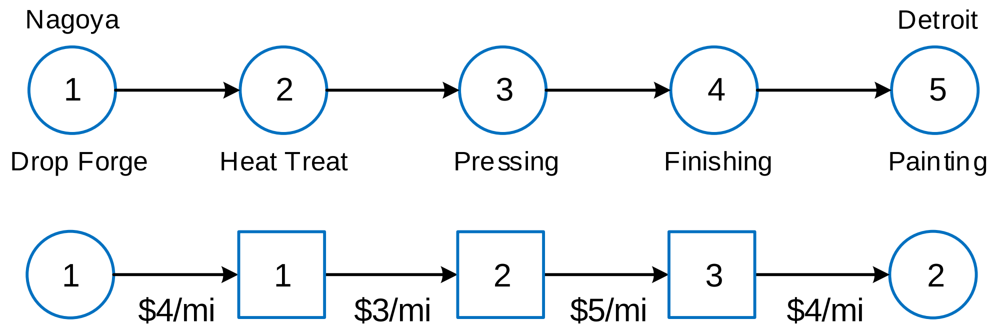
$$[\mathbf{W}\ \mathbf{V}]=\left[\begin{array}{ccc:cc}2 & 1 & 0 & 0 & 0.5\\ 4 & 0 & 5 & 0.5 & 0\end{array}\right]\begin{array}{l}\sum=3.5 \Rightarrow w_{1,1}=2>1.75 \Rightarrow \text{ NF1 at EF1}\\ \sum=9.5 \Rightarrow w_{2,3}=5>4.75 \Rightarrow \text{ NF2 at EF3}\end{array}$$

3. Modified $\mathbf{V} \Rightarrow$ reduction $\Rightarrow$ solution

$$[\mathbf{W}\ \mathbf{V}]=\left[\begin{array}{ccc:cc}2 & 1 & 0 & 0 & 4\\ 4 & 0 & 5 & 4 & 0\end{array}\right]\begin{array}{l}\sum=7 \Rightarrow v_{1,2}=4>3.5 \Rightarrow \text{ NF1 and NF2 co-located}\\ \sum=13 \Rightarrow \text{all } w_{ij}, v_{ij}<6.5\end{array}$$

Reduced $\mathbf{W}$, no $\mathbf{V}$: $\mathbf{W}=\begin{bmatrix}6 & 1 & 5\end{bmatrix} \sum=12 \Rightarrow w_1=6 \geq 6 \Rightarrow$ NF1 (and NF2) at EF1

Ex 7: Location of Production Processes

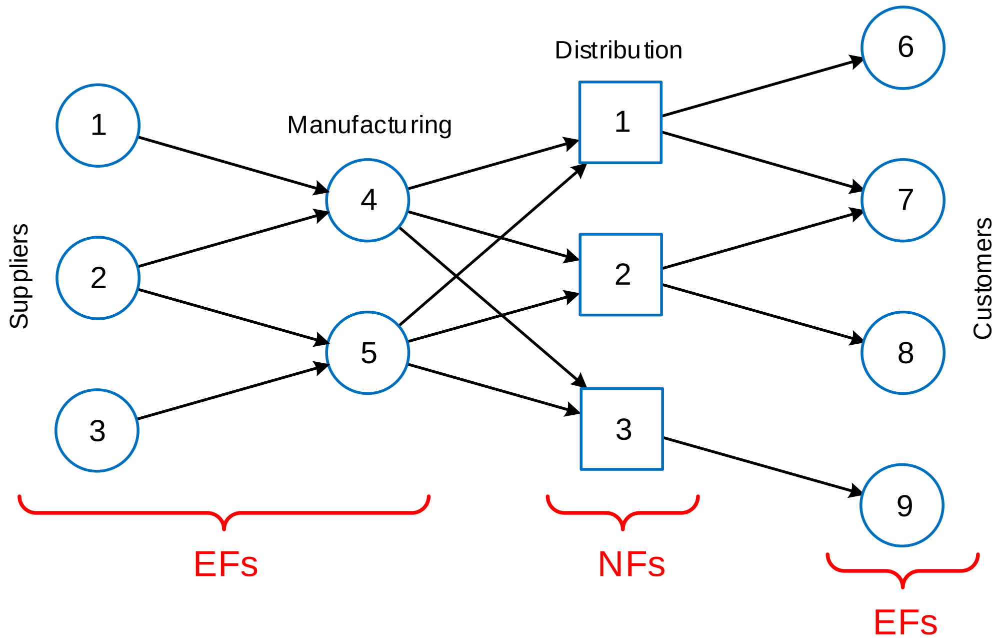


$$2 \text{ EF, } 3 \text{ NF, } \mathbf{W} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \\ 0 & 4 \end{bmatrix}, \mathbf{V} = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \mathbf{V} = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 5 \\ 0 & 5 & 0 \end{bmatrix}$$

$$1. \ [\mathbf{W}\ \mathbf{V}] = \left[\begin{array}{cc:ccc} 4 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 5 \\ 0 & 4 & 0 & 5 & 0 \end{array}\right] \sum = 8 \Rightarrow v_{2,3} = 5 > 4 \Rightarrow \text{NF2 and NF3 co-located}$$

$$2. \ [\mathbf{W}\ \mathbf{V}] = \left[\begin{array}{cc:cc} 4 & 0 & 0 & 3 \\ 0 & 4 & 3 & 0 \end{array}\right] \begin{array}{l} \sum = 7 \Rightarrow w_{1,1} = 4 > 3.5 \Rightarrow \text{NF1 at EF1} \\ \sum = 7 \Rightarrow w_{2,2} = 4 > 3.5 \Rightarrow \text{NF2 (and NF3) at EF2} \end{array}$$

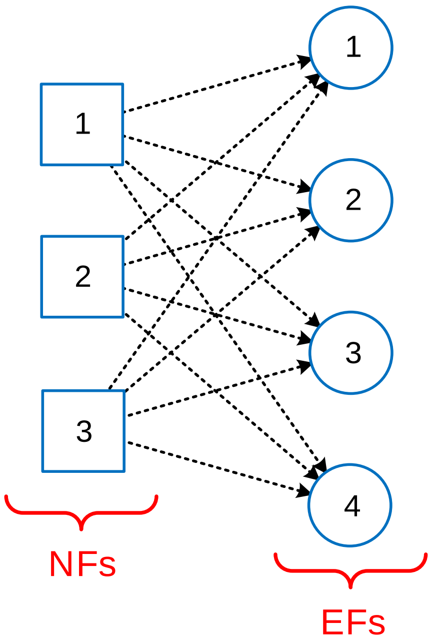
Multiple Single-Facility Location



$$TC(\mathbf{X}) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(\mathbf{X}_j, \mathbf{X}_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$= \sum_{j=1}^{n} TC(\mathbf{X}_j)$$

Facility Location–Allocation Problem



- Determine both the location of *n* NFs and the allocation of flow requirements of *m* EFs that minimize TC

$$w_{ji} = r_{ji} f_{ji} = (1) f_{ji} = \text{flow between NF}j \text{ and EF}i$$

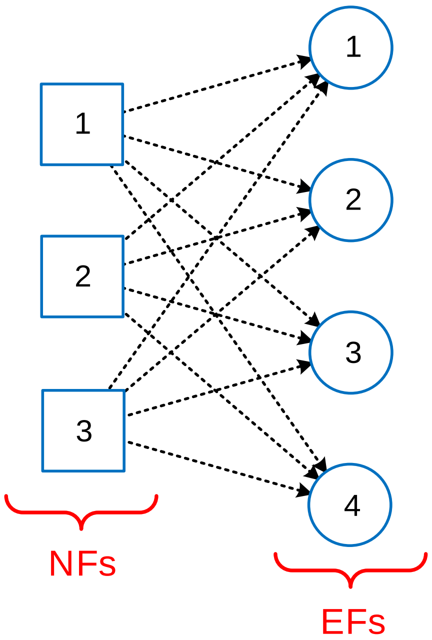
$$w_i = \text{total flow requirememts of EF}i$$

$$TC(\mathbf{X}, \mathbf{W}) = \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$\mathbf{X}^*, \mathbf{W}^* = \arg\min_{\mathbf{X},\mathbf{W}} \left\{ TC(\mathbf{X}, \mathbf{W}) : \sum_{j=1}^{n} w_{ji} = w_i,\ w_{ji} \geq 0 \right\}$$

$$TC^* = TC(\mathbf{X}^*, \mathbf{W}^*)$$

Integrated Formulation



- If there are no capacity constraints on NFs, it is optimal to always satisfy all the flow requirements of an EF from its closest NF
- Requires search of (n x d)-dimensional TC that combines location with allocation

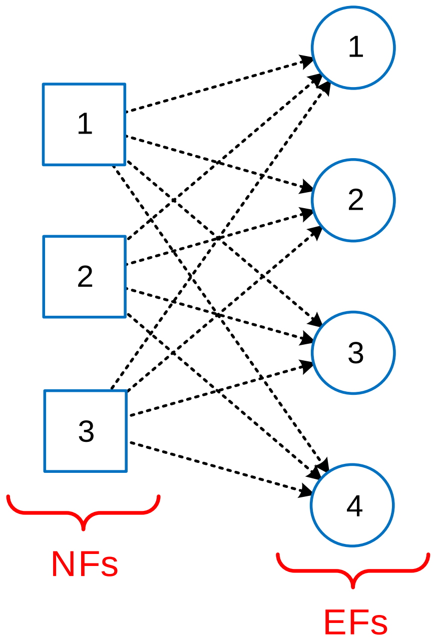
$$\alpha_i(\mathbf{X}) = \arg\min_j d(\mathbf{X}_j, \mathbf{P}_i)$$

$$TC(\mathbf{X}) = \sum_{i=1}^{m} w_i d(\mathbf{X}_{\alpha_i(\mathbf{X})}, \mathbf{P}_i)$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} TC(\mathbf{X})$$

$$TC^* = TC(\mathbf{X}^*)$$

Alternating Formulation



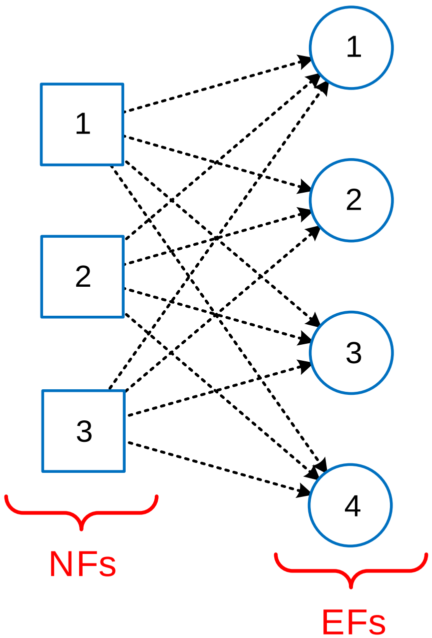
- Alternate between finding locations and finding allocations until no further TC improvement
- Requires *n* *d*-dimensional location searches together with separate allocation procedure
- Separating location from allocation allows other types of location and/or allocation procedures to be used:
  - Allocation with NF with capacity constraints (solved as minimum cost network flow problem)
  - Location with some NFs at fixed locations

$$allocate(\mathbf{X}) = \left[w_{ji} \right] = \begin{cases} w_i, & \text{if } \arg\min_k d(\mathbf{X}_k, \mathbf{P}_i) = j \\ 0, & \text{otherwise} \end{cases}$$

$$TC(\mathbf{X}, \mathbf{W}) = \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$locate(\mathbf{W}, \mathbf{X}) = \arg\min_{\mathbf{X}} TC(\mathbf{X}, \mathbf{W})$$

ALA: Alternate Location–Allocation



procedure $ala(\mathbf{X})$

$TC \leftarrow \infty$, $done \leftarrow$ false

repeat

$\quad \mathbf{W}' \leftarrow allocate(\mathbf{X})$

$\quad \mathbf{X}' \leftarrow locate(\mathbf{W}', \mathbf{X})$

$\quad TC' \leftarrow TC(\mathbf{X}', \mathbf{W}')$

$\quad$ **if** $TC' < TC$

$\quad\quad TC \leftarrow TC', \mathbf{X} \leftarrow \mathbf{X}', \mathbf{W} \leftarrow \mathbf{W}'$

$\quad$ **else**

$\quad\quad done \leftarrow$ true

$\quad$ **endif**

until $done =$ true

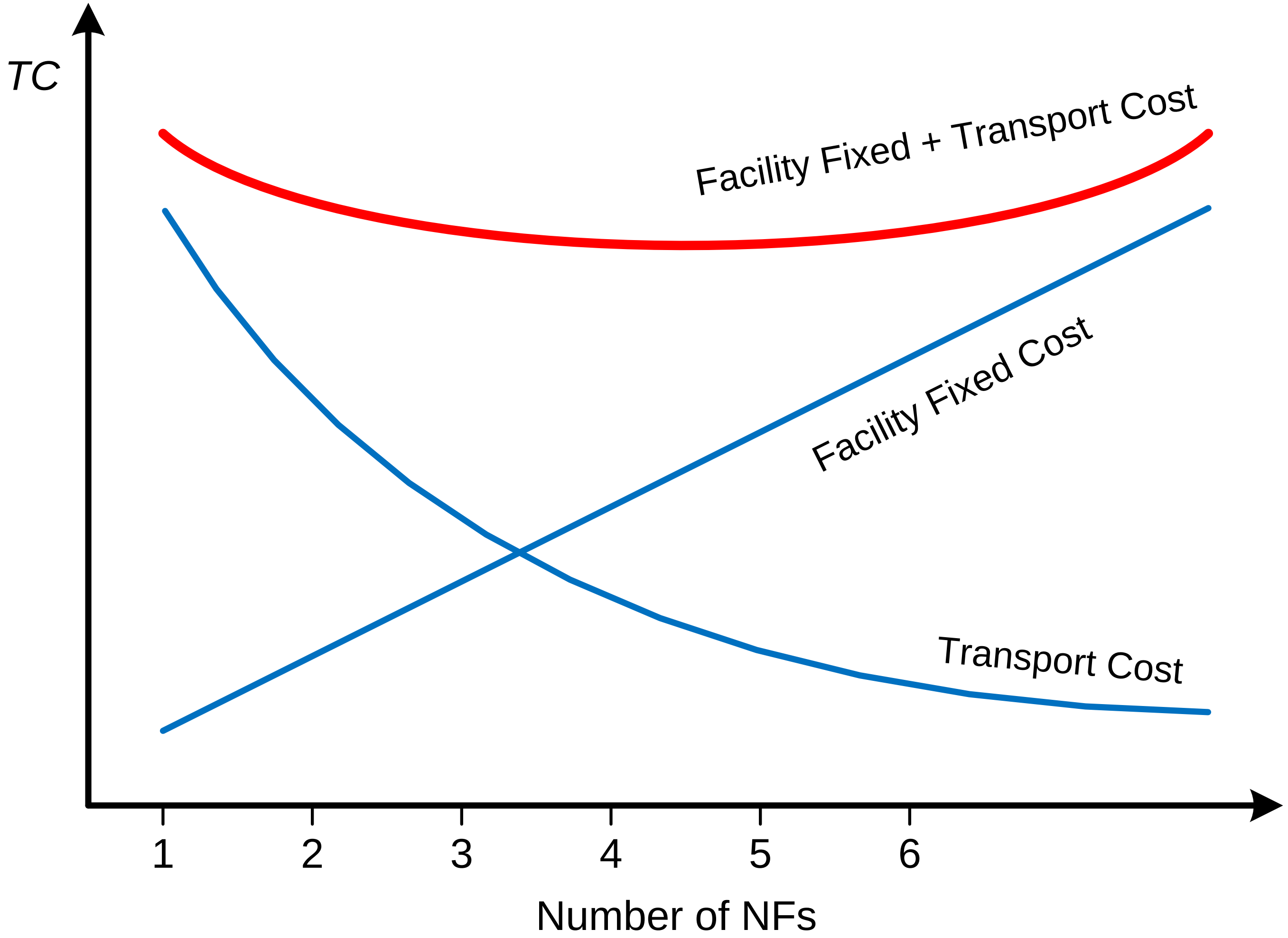
return $\mathbf{X}, \mathbf{W}$

```
%% ALA Matlab Code
X = randX(P,n);
TC = Inf; done = false;
while ~done
   Wi = alloc_h(X);
   Xi = loc_h(Wi,X);
   TCi = TCh(Wi,Xi);
   if TCi < TC
      TC = TCi; X = Xi; W = Wi;
   else
      done = true;
   end
end
X, W
```

Best Retail Warehouse Locations

Number of Locations	Average Transit Time (days)	Warehouse Location		
1	2.20	Bloomington, IN		
2	1.48	Ashland, KY	Palmdale, CA	
3	1.29	Allentown, PA	Palmdale, CA	McKenzie, TN
4	1.20	Edison, NJ Meridian, MS	Palmdale; CA	Chicago, IL
5	1.13	Madison, NJ Dallas, TX	Palmdale, CA Macon, GA	Chicago, IL
6	1.08	Madison, NJ Dallas, TX	Pasadena, CA Macon, GA	Chicago, IL Tacoma, WA
7	1.07	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA	Chicago, IL Tacoma, WA
8	1.05	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA
9	1.04	Madison, NJ Dallas, TX Lakeland. FL	Alhambra, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA Oakland, CA
10	1.04	Newark, NJ Palistine, TX Lakeland, FL Mansfield, OH	Alhambra, CA Gainesville, GA Denver, CO	Rockford, IL Tacoma, WA Oakland. CA

Optimal Number of NFs



Uncapacitated Facility Location (UFL)

- NFs can only be located at discrete set of sites
  - Allows inclusion of fixed cost of locating NF at site $\Rightarrow$ opt number NFs
  - Variable costs are usually transport cost from NF to/from EF
  - Total of $2^n - 1$ potential solutions (all nonempty subsets of sites)

$$M = \{1,\ldots,m\}, \quad \text{existing facilites (EFs)}$$

$$N = \{1,\ldots,n\}, \quad \text{sites available to locate NFs}$$

$$M_i \subseteq M, \quad \text{set of EFs served by NF at site } i$$

$$c_{ij} = \text{variable cost to serve EF } j \text{ from NF at site } i$$

$$k_i = \text{fixed cost of locating NF at site } i$$

$$Y \subseteq N, \quad \text{sites at which NFs are located}$$

$$Y^* = \arg\min_Y \left\{ \sum_{i \in Y} k_i + \sum_{i \in Y} \sum_{j \in M_i} c_{ij} : \bigcup_{i \in Y} M_i = M \right\}$$

$$= \text{min cost set of sites where NFs located}$$

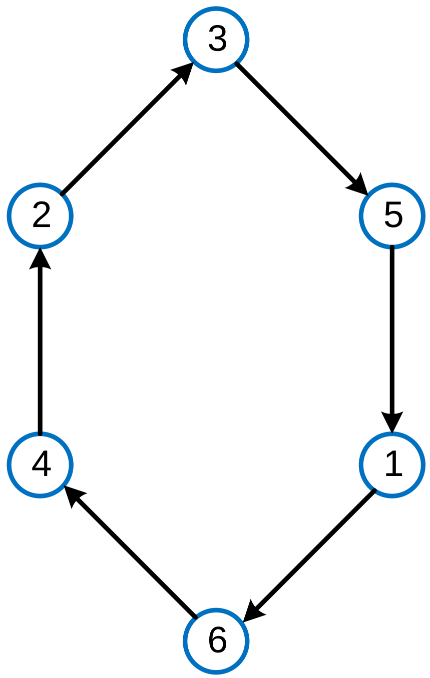
$$\left| Y^* \right| = \text{number of NFs located}$$

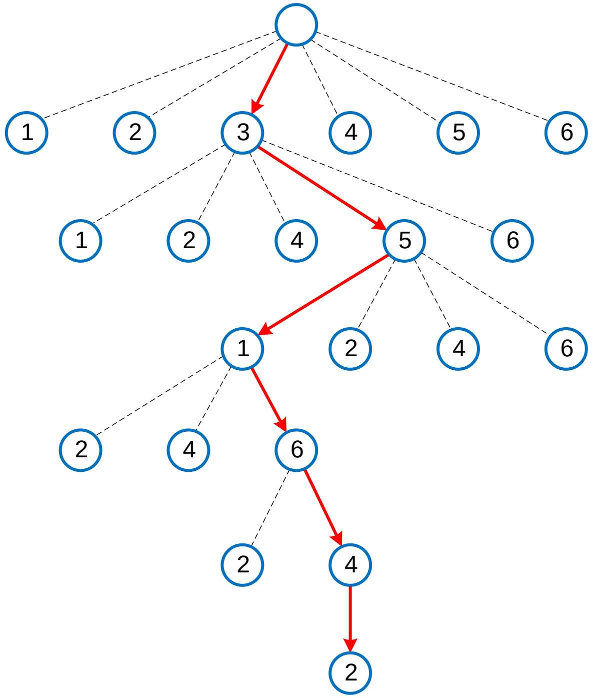
Heuristic Solutions

- Most problems in logistics engineering don't admit optimal solutions, only
  - Within some bound of optimal (provable bound, opt. gap)
  - Best known solution (stop when need to have solution)
- Heuristics - computational effort split between
  - Construction: construct a feasible solution
  - Improvement: find a better feasible solution
- Easy construction:
  - any random point or permutation is feasible
  - can then be improved $\Rightarrow$ *construct-then-improve* multiple times
- Hard construction:
  - almost no chance of generating a random feasible solution due to constraints on what is a feasible solution
  - need to include randomness at decision points as solution is generated in order to construct multiple different solutions (which "might" then be able to be improved)

Heuristic Construction Procedures

- Easy construction:
  - any random permutation is feasible and can then be improved
- Hard construction:
  - almost no chance of generating a random solution in a single step that is feasible, need to include randomness at decision points as solution is constructed

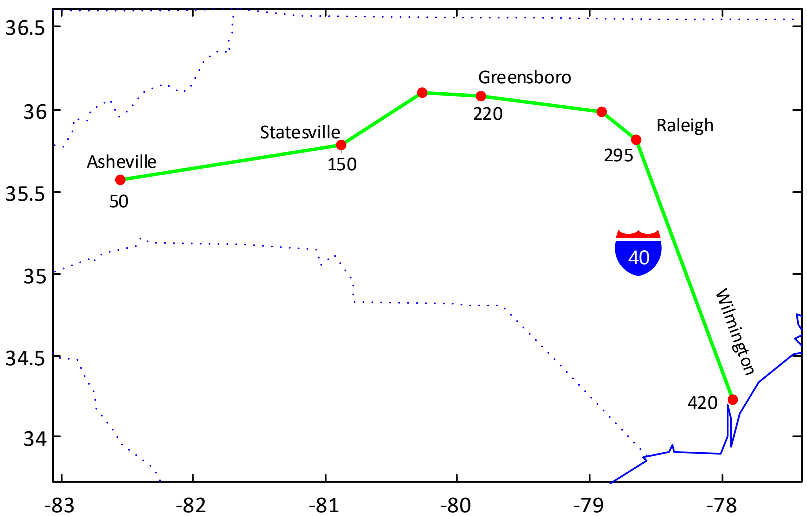




UFL Solution Techniques

- Being uncapacitated allows simple heuristics to be used to solve
  - ADD construction: add one NF at a time
  - DROP construction: drop one NF at a time
  - XCHG improvement: move one NF at a time to unoccupied sites
  - HYBRID algorithm combination of ADD and DROP construction with XCHG improvement, repeating until no change in Y
    - Use as default heuristic for UFL
    - See Daskin [2013] for more details
- UFL can be solved as a MILP
  - Easy MILP, LP relaxation usually optimal (for strong formulation)
  - MILP formulation allows constraints to easily be added
    - e.g., capacitated facility location, fixed number of NFs, some NF at fixed location
  - Will model UFL as MILP mainly to introduce MILP, will use UFL HYBRID algorithm to solve most problems

Ex 8: UFL ADD



$$k = \begin{bmatrix} 150 & 200 & 150 & 150 & 200 \end{bmatrix}$$

$$c_{ij} = w_j d_{ij} = f_j r d_{ij} = (1)(1) d_{ij} = d_{ij}$$

	c_{ij}	1	2	3	4	5
Asheville:	1	0	100	170	245	370
Statesville:	2	100	0	70	145	270
Greensboro:	3	170	70	0	75	200
Raleigh:	4	245	145	75	0	125
Wilmington:	5	370	270	200	125	0

$Y = \{\ \}$

Y	1	2	3	4	5	c_{Yj}	k_Y	$c_{Yj} + k_Y$
1	0	100	170	245	370	885	150	1,035
2	100	0	70	145	270	585	200	785
3	170	70	0	75	200	515	150	665
4	245	145	75	0	125	590	150	740
5	370	270	200	125	0	965	200	1,165

$Y = \{3\}$

Y	1	2	3	4	5	c_{Yj}	k_Y	$c_{Yj} + k_Y$
3,1	0	70	0	75	200	345	300	645
3,2	100	0	0	75	200	375	350	725
3,4	170	70	0	0	125	365	300	665
3,5	170	70	0	75	0	315	350	665

$Y = \{3,1\}$

Y	1	2	3	4	5	c_{Yj}	k_Y	$c_{Yj} + k_Y$
3,1,2	0	0	0	75	200	275	500	775
3,1,4	0	70	0	0	125	195	450	645
3,1,5	0	70	0	75	0	145	500	645

$Y^* = \{3,1\}$

UFLADD: Add Construction Procedure

procedure $ufladd(\mathbf{k},\mathbf{C})$

$Y \leftarrow \{\}$

$TC \leftarrow \infty$, $done \leftarrow$ false

repeat

$\quad TC' \leftarrow \infty$

$\quad$ **for** $i' \in \{1,\ldots,n\} \setminus Y$

$$TC'' \leftarrow \sum_{h \in Y \cup i'} k_h + \sum_{j=1}^{m} \min_{h \in Y \cup i'} c_{hj}$$

$\quad\quad$ **if** $TC'' < TC'$

$\quad\quad\quad TC' \leftarrow TC''$, $i \leftarrow i'$

$\quad\quad$ **endif**

$\quad$ **endfor**

$\quad$ **if** $TC' < TC$

$\quad\quad TC \leftarrow TC'$, $Y \leftarrow Y \cup i$

$\quad$ **else**

$\quad\quad done \leftarrow$ true

$\quad$ **endif**

until $done =$ true

return Y, TC

```
%% UFLADD Matlab code, given k and C as inputs
y = [];
TC = Inf; done = false;
while ~done
   TC1 = Inf;                          % Stops if y = all NF,
   for i1 = setdiff(1:size(C,1),y)  % since i1 = []
      TC2 = sum(k([y i1])) + sum(min(C([y i1],:),[],1));
      if TC2 < TC1
         TC1 = TC2; i = i1;
      end
   end
   if TC1 < TC % not true if y = all NF, since TC1 = Inf
      TC = TC1; y = [y i];
   else
      done = true;
   end
end
y,TC
```

UFLXCHG: Exchange Improvement Procedure

procedure $uflxchg(\mathbf{k}, \mathbf{C}, Y)$

$TC \leftarrow \sum_{i \in Y} k_i + \sum_{j=1}^{m} \min_{i \in Y} c_{ij}$

$TC' \leftarrow \infty$, $done \leftarrow$ false

while $|y| > 1$ and $done =$ false

  for $i' \in y$

    for $j' \in \{1, \ldots, n\} \setminus Y$

      $Y' \leftarrow Y \setminus i' \cup j'$

      $TC'' \leftarrow \sum_{i \in Y'} k_i + \sum_{j=1}^{m} \min_{i \in Y'} c_{ij}$

      if $TC'' < TC'$

        $TC' \leftarrow TC''$, $i \leftarrow i'$, $j \leftarrow j'$

      endif

    endfor

  endfor

  if $TC' < TC$

    $TC \leftarrow TC', Y \leftarrow Y \setminus i \cup j$

  else

    $done \leftarrow$ true

  endif

endwhile

return Y, TC

```matlab
%% UFLXCHG Matlab code, given k, C, and y as inputs
TC = sum(k(y)) + sum(min(C(y,:),[],1));
TC1 = Inf; done = false;
while length(y) > 1 && ~done
   for i1 = y
      for j1 = setdiff(1:size(C,1),y)
         y1 = [setdiff(y,i1) j1];
         TC2 = sum(k(y1)) + sum(min(C(y1,:),[],1));
         if TC2 < TC1
            TC1 = TC2; i = i1; j = j1;
         end
      end
   end
   if TC1 < TC
      TC = TC1; y = [setdiff(y,i) j];
   else
      done = true;
   end
end
y, TC
```

Modified UFLADD

$$
\begin{array}{l}
\textbf{procedure } \textit{ufladd}(\mathbf{k},\mathbf{C},\boxed{Y,p}) \\
\cancel{Y \leftarrow \{\}} \\
TC \leftarrow \infty,\ done \leftarrow \text{false} \\
\textbf{repeat} \\
\quad TC' \leftarrow \infty \\
\quad \textbf{for } i' \in \{1,\dots,n\} \setminus Y \\
\qquad TC'' \leftarrow \sum_{h \in Y \cup i'} k_h + \sum_{j=1}^{m} \min_{h \in Y \cup i'} c_{hj} \\
\qquad \textbf{if } TC'' < TC' \\
\qquad\quad TC' \leftarrow TC'',\ i \leftarrow i' \\
\qquad \textbf{endif} \\
\quad \textbf{endfor} \\
\quad \textbf{if } \boxed{\left(p = \{\} \text{ and } TC' < TC\right) \text{ or } \left(p \neq \{\} \text{ and } |Y| < p\right)} \\
\qquad TC \leftarrow TC',\ Y \leftarrow Y \cup i \\
\quad \textbf{else} \\
\qquad done \leftarrow \text{true} \\
\quad \textbf{endif} \\
\textbf{until } done = \text{true} \\
\textbf{return } Y, TC
\end{array}
$$

- *Y* input can be used to start UFLADD with *Y* NFs
  - Used in hybrid heuristic
- *p* input can be used to keep adding until number of NFs = *p*
  - Used in p-median heuristic

UFL: Hybrid Algorithm

procedure $ufl(\mathbf{k},\mathbf{C})$
$Y', TC' \leftarrow ufladd(\mathbf{k},\mathbf{C})$
$done \leftarrow \text{false}$
repeat
  $Y, TC \leftarrow uflxchg(\mathbf{k},\mathbf{C},Y')$
  if $Y \neq Y'$
    $Y', TC' \leftarrow ufladd(\mathbf{k},\mathbf{C},Y)$
    $Y'', TC'' \leftarrow ufldrop(\mathbf{k},\mathbf{C},Y)$
    if $TC'' < TC'$
      $TC' \leftarrow TC'', Y' \leftarrow Y''$
    endif
    if $TC' \geq TC$
      $done \leftarrow \text{true}$
    endif
  else
    $done \leftarrow \text{true}$
  endif
until $done = \text{true}$
return Y, TC

```
%% UFL Matlab code, given k and C
[y1,TC1] = ufladd(k,C);
done = false;
while ~done
   [y,TC] = uflxchg(k,C,y1);
   if ~isequal(y,y1)
      [y1,TC1] = ufladd(k,C,y);
      [y2,TC2] = ufldrop(k,C,y);
      if TC2 < TC1
         TC1 = TC2; y1 = y2;
      end
      if TC1 >= TC
         done = true;
      end
   else
      done = true;
   end
end
y, TC
```

P-Median Location Problem

- Similar to UFL, except
  - Number of NF has to equal *p* (discrete version of ALA)
  - No fixed costs

$$p = \text{number of NFs}$$

$$Y^* = \arg\min_Y \left\{ \sum_{i \in Y} \sum_{j \in M_i} c_{ij} : \bigcup_{i \in Y} M_i = M, |Y| = p \right\}$$

$$
\begin{array}{l}
\textbf{procedure } \ pmedian(p, \mathbf{C}) \\
Y \leftarrow ufladd(0, \mathbf{C}, \{\}, p) \\
Y, TC \leftarrow uflxchg(0, \mathbf{C}, Y) \\
Y' \leftarrow ufldrop(0, \mathbf{C}, \{\}, p) \\
Y', TC' \leftarrow uflxchg(0, \mathbf{C}, Y) \\
\textbf{if } \ TC' < TC \\
\quad TC \leftarrow TC', Y \leftarrow Y' \\
\textbf{endif} \\
\textbf{return } \ Y, TC
\end{array}
$$

Bottom-Up vs Top-Down Analysis

- Bottom-Up: HW3 Q3

$$\boxed{r \begin{array}{l} \nearrow TC_{\mathrm{NEW}_1} \\ \searrow TC_{\mathrm{NEW}_2} \end{array}}$$

$$\mathbf{P}_{3\times 2} = \text{lon-lat of EFs}$$

$$\mathbf{f} = \left[48, 24, 35\right] \quad \text{(TL/yr)}$$

$$r = 2 \quad \text{(\$/TL-mi)}$$

$$g = \frac{1}{3}\left[\frac{d_{RD}(\mathbf{P}_1, \mathbf{P}_2)}{d_{GC}(\mathbf{P}_1, \mathbf{P}_2)} + \frac{d_{RD}(\mathbf{P}_2, \mathbf{P}_3)}{d_{GC}(\mathbf{P}_2, \mathbf{P}_3)} + \frac{d_{RD}(\mathbf{P}_3, \mathbf{P}_1)}{d_{GC}(\mathbf{P}_3, \mathbf{P}_1)}\right]$$

$$TC(\mathbf{x}) = \sum_{i=1}^{3} f_i\, r\, g\, d_{GC}(\mathbf{x}, \mathbf{P}_i) \quad \text{(outbound trans. costs)}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

$$TC^* = TC(\mathbf{x}^*)$$

$$\mathbf{x}^{\text{cary}} = \text{lon-lat of Cary}$$

$$TC^{\text{cary}} = TC(\mathbf{x}^{\text{cary}})$$

$$\Delta TC = TC^{\text{cary}} - TC^*$$

- Top-Down: estimate *r* (circuity factor cancels, so not needed, HW4 Q6)

$$\boxed{TC_{\mathrm{OLD}} \rightarrow r_{\mathrm{nom}} \rightarrow TC_{\mathrm{NEW}}}$$

$$TC^{\text{cary}} = \text{current known } TC$$

$$10 \text{ ton/TL} = \text{known tons per truckload}$$

$$\mathbf{f} = \left[480, 240, 350\right] \quad \text{(ton/yr)}$$

$$r_{\mathrm{nom}} = \frac{TC^{\text{cary}}}{\sum_{i=1}^{3} f_i\, d_{GC}(\mathbf{x}^{\text{cary}}, \mathbf{P}_i)} \quad \text{(\$/ton-mi)}$$

$$TC(\mathbf{x}) = \sum_{i=1}^{3} f_i\, r_{\mathrm{nom}} d_{GC}(\mathbf{x}, \mathbf{P}_i)$$

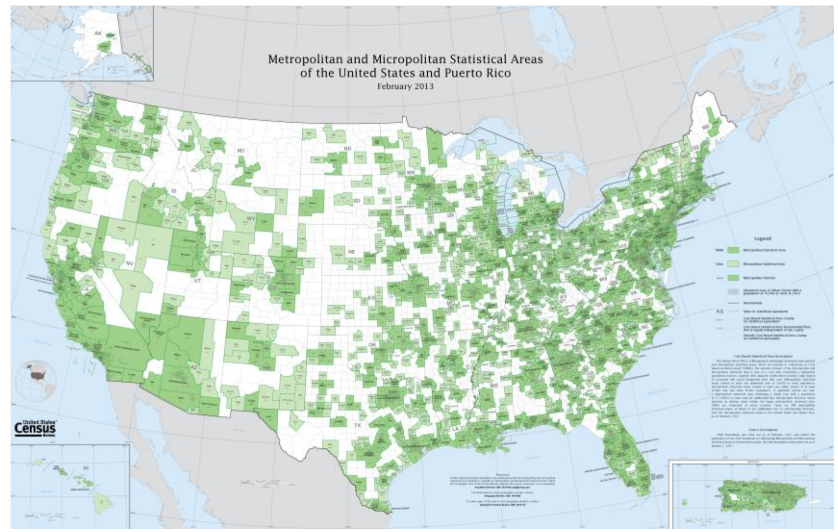
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

$$TC^* = TC(\mathbf{x}^*)$$

$$\Delta TC = TC^{\text{cary}} - TC^*$$

U.S. Geographic Statistical Areas

- Defined by Office of Management and Budget (OMB)
  - Each consists of one or more counties
- Top-to-bottom:
  1. Metropolitan divisions
  2. Combined statistical areas (CSAs)
  3. Core-based statistical areas (CBSAs)
  4. Metropolitian/ micropolitan statistical areas (MSAs)
  5. County (rural)



Aggregate Demand Point Data Sources

- Aggregate demand point: centroid of population + area + population
- Good rule of thumb: use at least 10x number of NFs (≈ 100 pts provides minimum coverage for locating ≈ 10 NFs)

1. City data: ONLY USE FOR LABELING!, not as aggregate demand points
2. 3-digit ZIP codes: ≈ 1000 pts covering U.S., = 20 pts NC
3. County data: ≈ 3000 pts covering U.S., = 100 pts NC
   - Grouped by state or CSA (Combined Statistical Area)
   - CSA = defined by set of counties (174 CSAs in U.S.)
   - FIPS code = 5-digit state-county FIPS code
= 2-digit state code + 3-digit county code
= 37183 = 37 NC FIPS + 183 Wake FIPS
   - CSA List: www2.census.gov/programs-surveys/metro-micro/geographies/reference-files/2017/delineation-files/list1.xls
4. 5-digit ZIP codes: > 35K pts U.S., ≈ 1000 pts NC
5. Census Block Group: > 220K pts U.S., ≈ 1000 pts Raleigh-Durham-Chapel Hill, NC CSA
   - Grouped by state, county, or CSA
   - Finest resolution aggregate demand data source

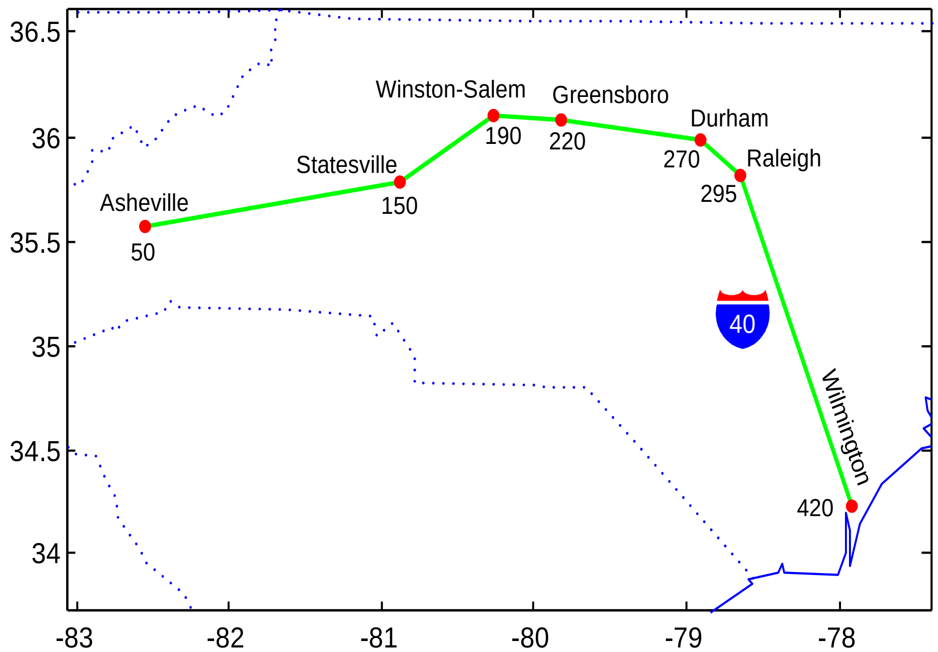
City vs CSA Population Data

Rank	City; State	2010 population	2012 population
1	New York City; New York	8,175,133	**8,336,697**
2	Los Angeles; California	3,792,621	**3,857,799**
3	Chicago; Illinois	2,695,598	**2,714,856**
4	Houston; Texas	2,099,451	**2,160,821**
5	Philadelphia; Pennsylvania	1,526,006	**1,547,607**
6	Phoenix; Arizona	1,445,632	**1,488,750**
7	San Antonio; Texas	1,327,407	**1,382,951**
8	San Diego; California	1,307,402	**1,338,348**
9	Dallas; Texas	1,197,816	**1,241,162**
10	San Jose; California	945,942	**982,765**
11	Austin; Texas	790,390	**842,592**
12	Jacksonville; Florida	821,784	**836,507**
13	Indianapolis; Indiana	820,445	**834,852**
14	San Francisco; California	805,235	**825,863**
15	Columbus; Ohio	787,033	**809,798**
16	Fort Worth; Texas	741,206	**777,992**
17	Charlotte; North Carolina	731,424	**775,202**
18	Detroit; Michigan	713,777	**701,475**
19	El Paso; Texas	649,121	**672,538**
20	Memphis; Tennessee	646,889	**655,155**

Metropolitan Area	2010 Population	City
New York-Northern NJ-Long Island, NY-NJ-PA	18,897,109	New York
Los Angeles-Long Beach-Santa Ana, CA	12,828,837	Los Angeles
Chicago-Joliet-Naperville, IL-IN-WI	9,461,105	Chicago
Dallas-Fort Worth-Arlington, TX	6,371,773	Dallas
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	5,965,343	Philadelphia
Houston-Sugar Land-Baytown, TX	5,946,800	Houston
Washington-Arlington-Alexandria, DC-VA-MD-WV	5,582,170	Washington
Miami-Fort Lauderdale-Pompano Beach, FL	5,564,635	Miami
Atlanta-Sandy Springs-Marietta, GA	5,268,860	Atlanta
Boston-Cambridge-Quincy, MA-NH	4,552,402	Boston
San Francisco-Oakland-Fremont, CA	4,335,391	San Francisco
Detroit-Warren-Livonia, MI	4,296,250	Detroit
Riverside-San Bernardino-Ontario, CA	4,224,851	Riverside
Phoenix-Mesa-Glendale, AZ	4,192,887	Phoenix
Seattle-Tacoma-Bellevue, WA	3,439,809	Seattle
Minneapolis-St. Paul-Bloomington, MN-WI	3,279,833	Minneapolis
San Diego-Carlsbad-San Marcos, CA	3,095,313	San Diego
St. Louis, MO-IL	2,812,896	St. Louis
Tampa-St. Petersburg-Clearwater, FL	2,783,243	Tampa
Baltimore-Towson, MD	2,710,489	Baltimore

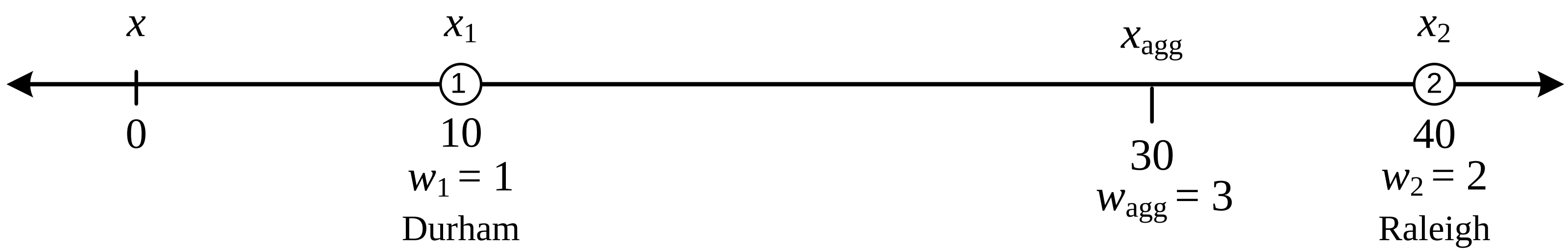
Demand Point Aggregation

- *Existing facility* (EF): actual physical location of demand source
- *Aggregate demand point*: single location representing multiple demand sources



Demand Point Aggregation

- Calculation of aggregate point depends on objective



- For minisum location, would like for any location x:

$$(w_1 + w_2)\, d(x, x_{\text{agg}}) = w_1 d(x, x_1) + w_2 d(x, x_2), \quad \text{let } x = 0,\ x_1, x_2 > 0$$

$$(w_1 + w_2)\, x_{\text{agg}} = w_1 x_1 + w_2 x_2$$

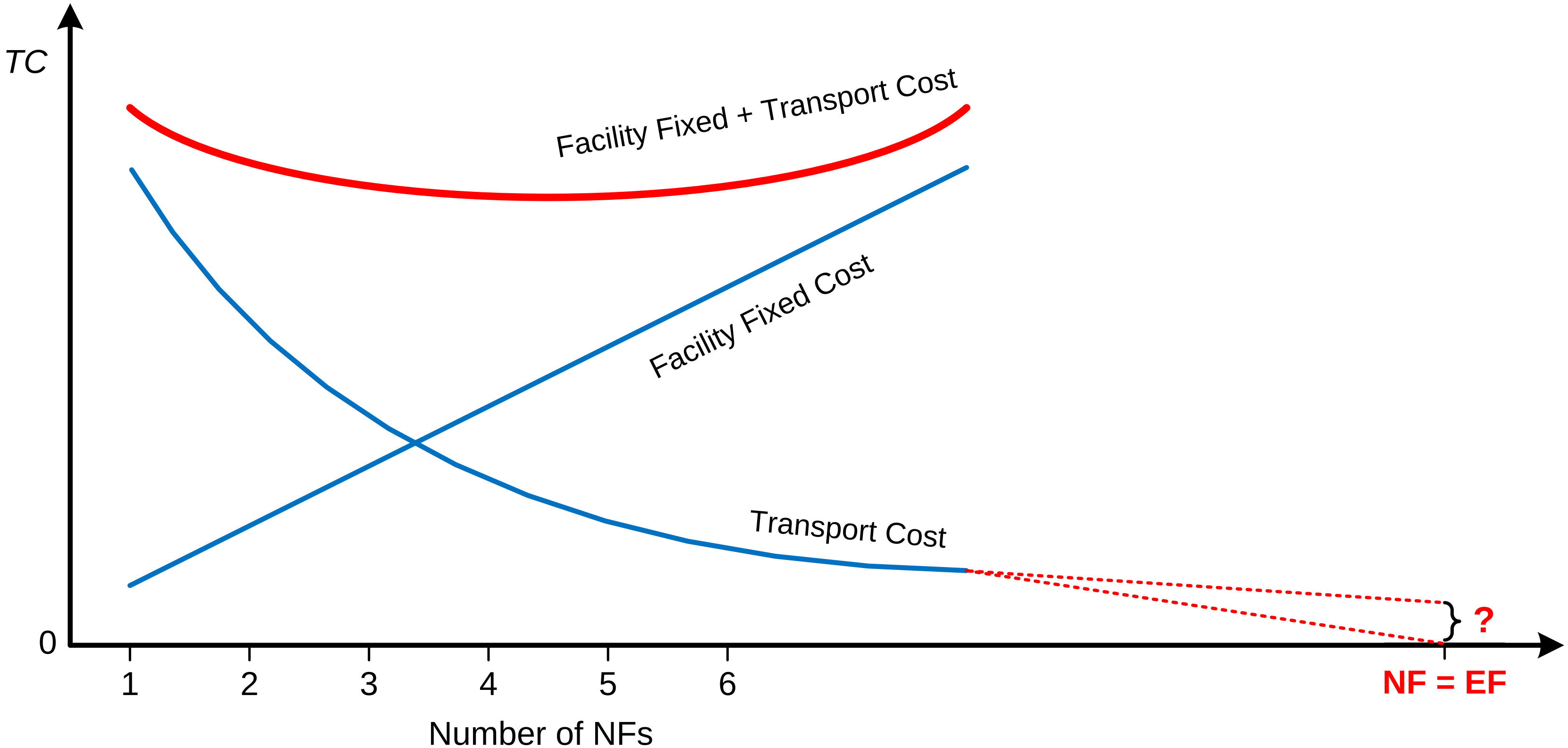
$$x_{\text{agg}} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2} \Rightarrow \quad \text{centroid}$$

$$\text{Note: if } x_1 < x < x_2,\ \text{ then } \mathrm{x}_{\text{agg}} \text{ not centroid}$$

- For squared distance: $(w_1 + w_2)\, x_{\text{agg}}^2 = w_1 x_1^2 + w_2 x_2^2$

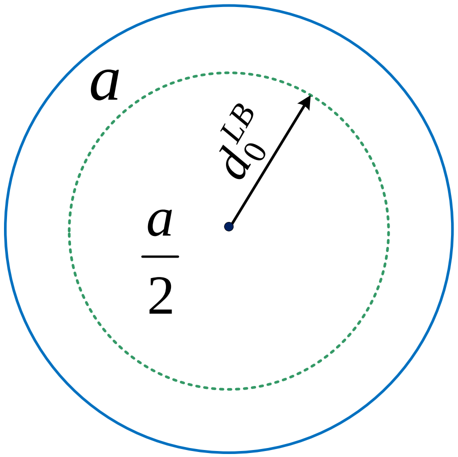
$$x_{\text{agg}} = \sqrt{\frac{w_1 x_1^2 + w_2 x_2^2}{w_1 + w_2}} \Rightarrow \quad \text{not centroid}$$

Transport Cost if NF at every EF



$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

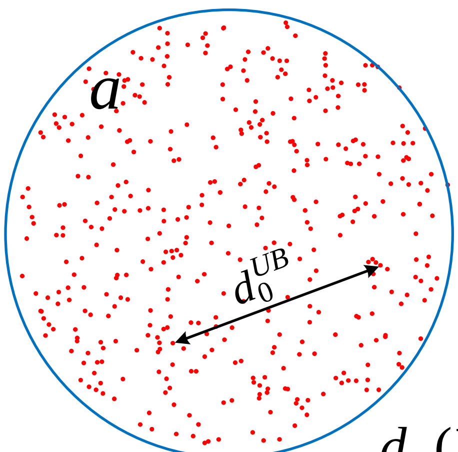
Area Adjustment for Aggregate Data Distances



- *LB:* avg. dist. from center to all points in area
- *UB:* avg. dist. between all random pairs of points
- Local circuity factor = 1.5, regular non-local = 1.2

$$\frac{a}{2} = \pi \left(d_0^{LB} \right)^2$$

$$d_0^{LB} = \sqrt{\frac{a}{2\pi}} \approx 0.40\sqrt{a}$$



$$d_0^{UB} = \frac{32}{15} \frac{\sqrt{a}}{\pi \Gamma \left(5/2 \right)} \approx 0.51\sqrt{a}$$

Mathai, A.M.,
An Intro to Geo Prob, p. 207 (2.3.68)

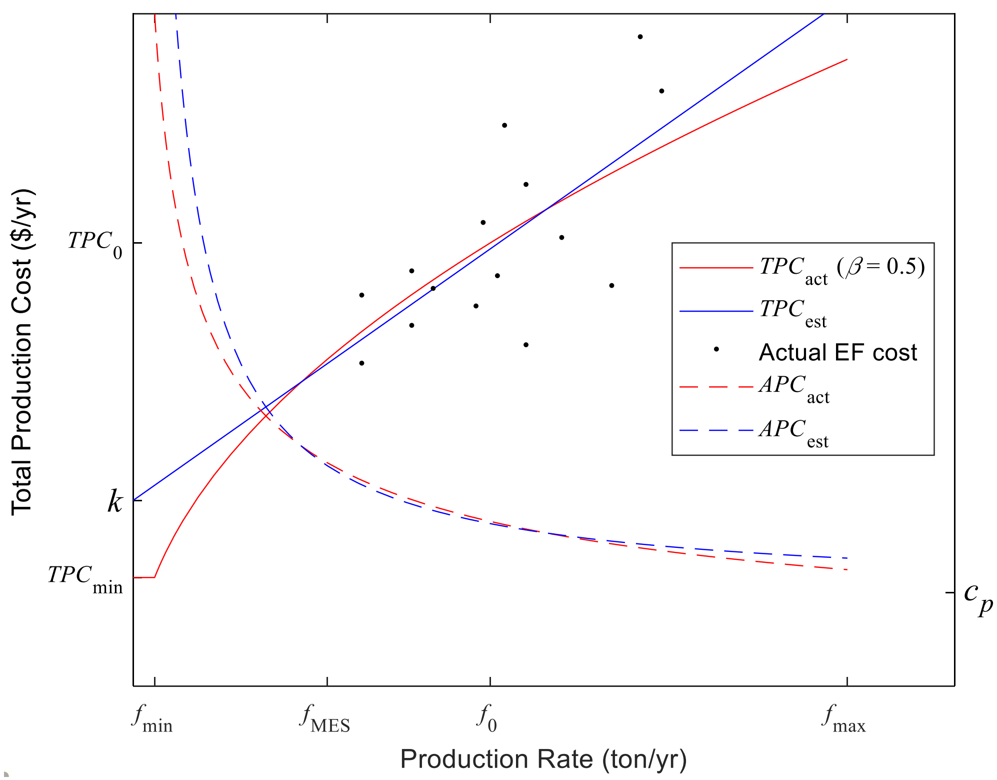
$$d_0 = \sqrt{d_0^{LB} d_0^{UB}} \approx 0.45\sqrt{a}$$

$$d_a(\mathbf{X}_1, \mathbf{X}_2) = \max \left\{ g\, d_{GC}(\mathbf{X}_1, \mathbf{X}_2),\ g_{\text{local}} 0.45 \max \left\{ \sqrt{a_1}, \sqrt{a_2} \right\} \right\}$$

$$= \max \left\{ 1.2 d_{GC}(\mathbf{X}_1, \mathbf{X}_2),\ 0.675 \max \left\{ \sqrt{a_1}, \sqrt{a_2} \right\} \right\}$$

Fixed Cost and Economies of Scale

- Cost data from existing facilities can be used to fit linear estimate
  - Economies of scale in production $\Rightarrow k > 0$ and $\beta < 1$



$$TPC_{\text{act}} = \max_{f < f_{\max}} \left\{ TPC_{\min}, TPC_0 \left(\frac{f}{f_0} \right)^{\beta} \right\}$$

$$\beta = \begin{cases} 0.62, & \text{Hand tool mfg.} \\ 0.48, & \text{Construction} \\ 0.41, & \text{Chemical processing} \\ 0.23, & \text{Medical centers} \end{cases}$$

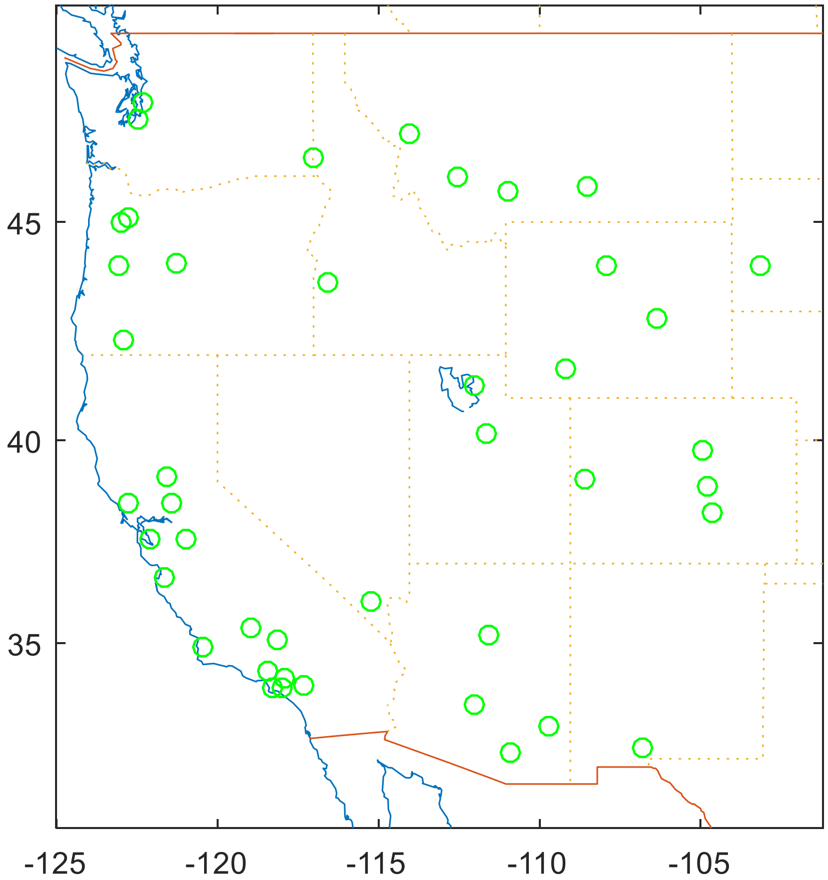
$$TPC_{\text{est}} = k + c_p f$$

$$APC_{\text{act}} = \frac{TPC_{\text{act}}}{f} = \frac{TPC_0}{f_0^{\beta}} f^{\beta - 1}$$

$$APC_{\text{est}} = \frac{k}{f} + c_p$$

$$\begin{aligned} k &= \text{fixed cost} \\ c_p &= \text{constant unit production cost} \\ f_{\min}/f_{\max} &= \text{min/max feasible scale} \\ f_{\text{MES}} &= \textit{Minimum Efficient Scale} \\ TPC_0/f_0 &= \text{base cost/rate} \end{aligned}$$

Ex 9: Popco Bottling Company



- **Problem:** Popco currently has 42 bottling plants across the western U.S. and wants to know if they should consider reducing or adding plants to improve their profitability.
- **Solution:** Formulate as an UFL to determine the number of plants that minimize Popco's production, procurement, and distribution costs.

Ex 9: Popco Bottling Company

- Following representative information is available for each of *N* current plants (DC) *i*:

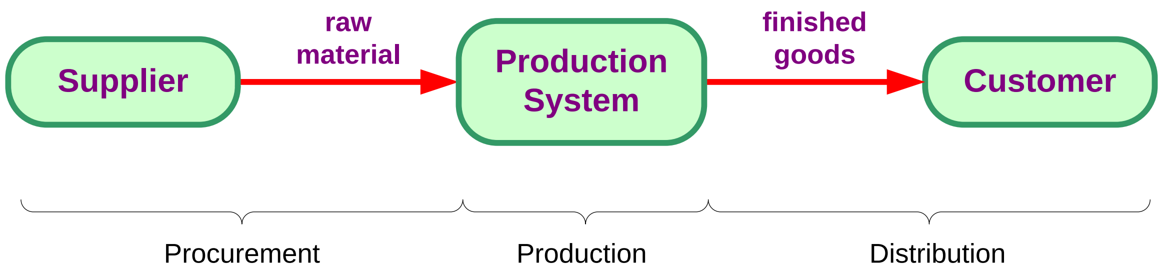
$$xy_i = \text{location}$$

$$f_i^{DC} = \text{aggregate production (tons)}$$

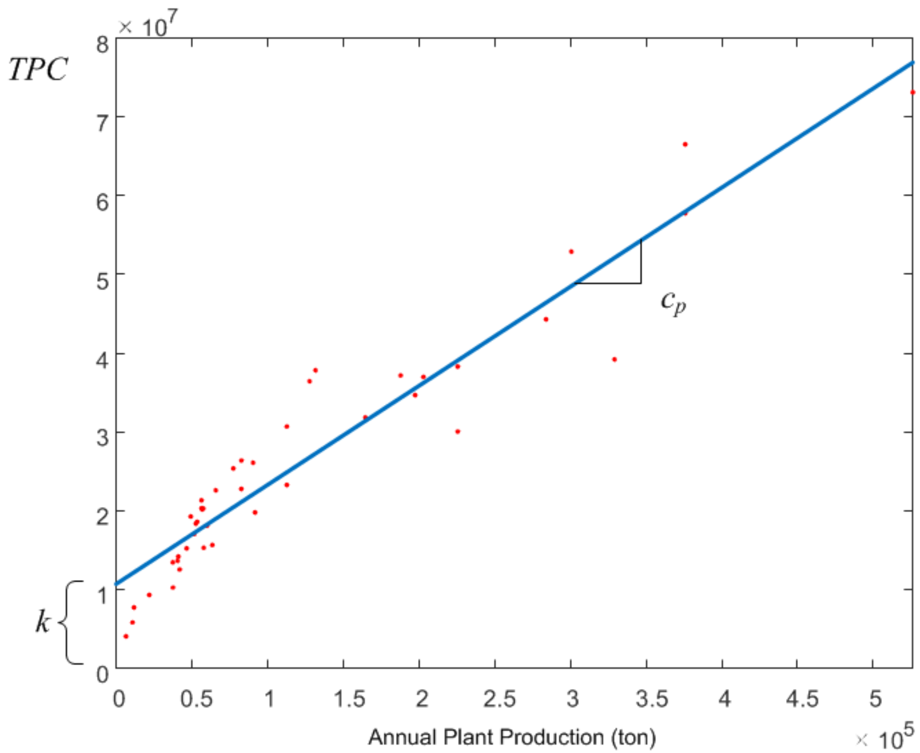
$$TPC_i = \text{total production and procurement cost}$$

$$TDC_i = \text{total distribution cost}$$

- Assuming plants are (monetarily) weight gaining since they are bottling plants, so UFL can ignore inbound procurement costs related to location



Ex 9: Popco Bottling Company



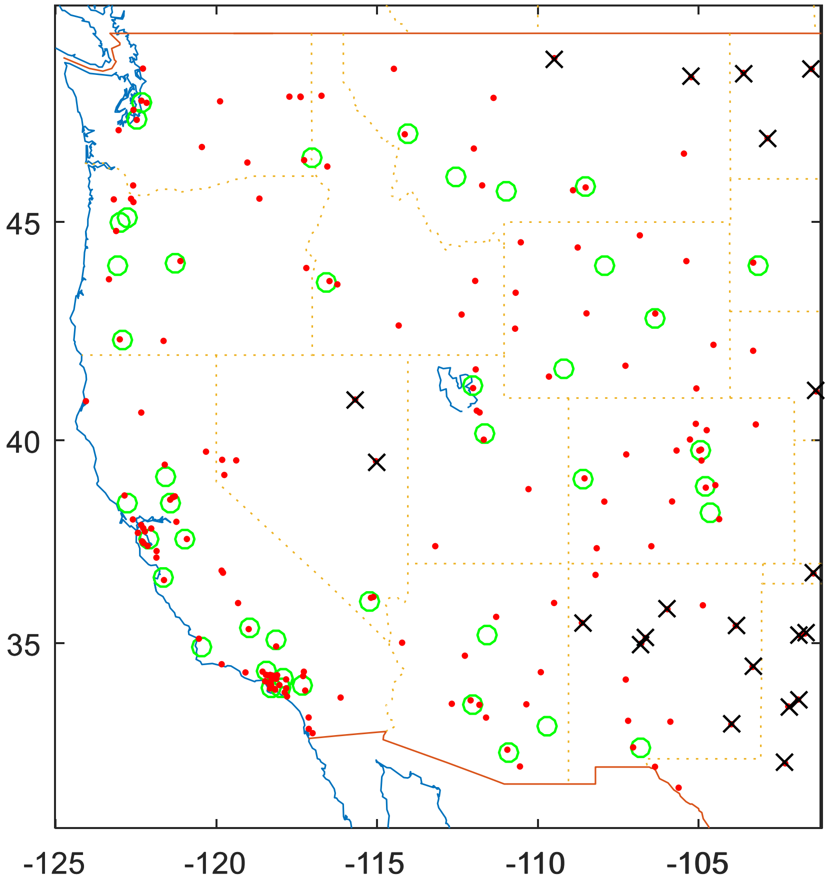
- Difficult to estimate fixed cost of each new facility because this cost must not include any cost related to quantity of product produced at facility.

1. Use plant (DC) production costs to find UFL fixed costs via linear regression
   - variable production costs c_p do not change and can be cut

$$TPC = \sum_{i \in N} TPC_i = \sum_{i \in N} \left(k + c_p f_i^{DC} \right)$$

$$\left(\text{only keep } k \text{ for UFL}\right)$$

Ex 9: Popco Bottling Company



2. Allocate all 3-digit ZIP codes to closest plant (up to 200 mi max) to serve as aggregate customer demand points.

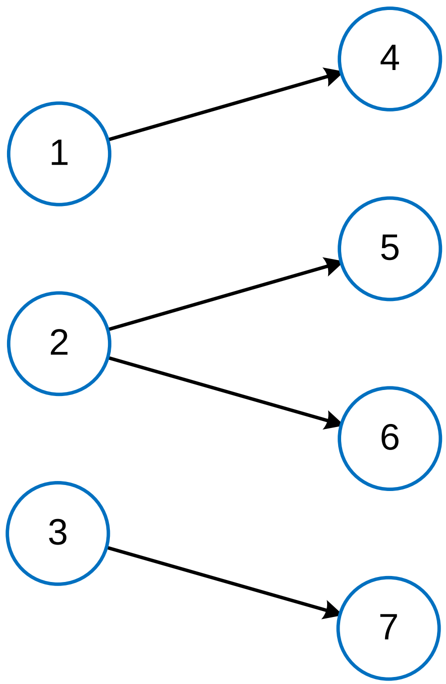
$$M_i = \left\{ j : \arg\min_h d^a_{hj} = i \text{ and } d^a_{ij} \le d_{\max} \right\}$$

$$d_{\max} = 200 \text{ mi}$$

$$M = \bigcup_{i \in N} M_i$$

Ex 9: Popco Bottling Company

3. Allocate each plant's demand (tons of product) to each of its customers based on its population.



$$f_{j \in M_i} = f_i^{DC} \frac{q_j}{\sum_{h \in M_i} q_h}$$

$$q_j = \text{population of EF}j$$

$$f_5 = f_2^{DC} \frac{q_5}{q_5 + q_6}$$

Ex 9: Popco Bottling Company

4. Estimate a nominal transport rate (\$/ton-mi) using the ratio of total distribution cost (\$) to the sum of the product of the demand (ton) at each customer and its distance to its plant (mi).

$$r_{\text{nom}} = \frac{\sum_{i \in N} TDC_i}{\sum_{i \in N} \sum_{j \in M_j} f_j \, d_{ij}^a}$$

Ex 9: Popco Bottling Company

5. Calculate UFL variable transportation cost c_{ij} ($) for each possible NF site *i* (all customer and plant locations) and EF site *j* (all customer locations) as the product of customer *j* demand (ton), distance from site *i* to *j* (mi), and the nominal transport rate ($/ton-mi).

$$\mathbf{C} = \left[c_{ij} \right]_{\substack{i \in M \cup N \\ j \in M}} = \left[r_{\text{nom}} \, f_j \, d_{ij}^{a} \right]_{\substack{i \in M \cup N \\ j \in M}}$$

6. Solve as UFL, where *TC* returned includes all new distribution costs and the fixed portion of production costs.

$$TC = \overset{\text{fixed cost}}{\sum_{i \in Y} k_i} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

$$n = \left| M \cup N \right|, \quad \text{number of potential NF sites}$$

$$m = \left| M \right|, \quad \text{number of EF sites}$$

MILP

LP: $\max \ \mathbf{c}'\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} \le \mathbf{b}$

$\mathbf{x} \ge 0$

MILP: some x_i integer

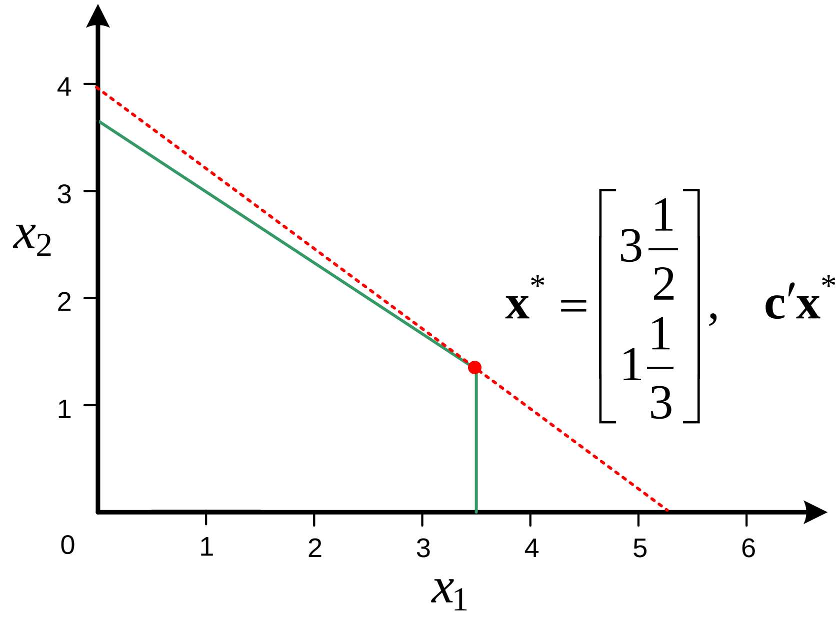
ILP: $\mathbf{x}$ integer

BLP: $\mathbf{x} \in \{0,1\}$

$$\begin{aligned} \max \quad & 6x_1 + 8x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \le 7 \\ & x_1, x_2 \ge 0 \end{aligned}$$

$$\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$



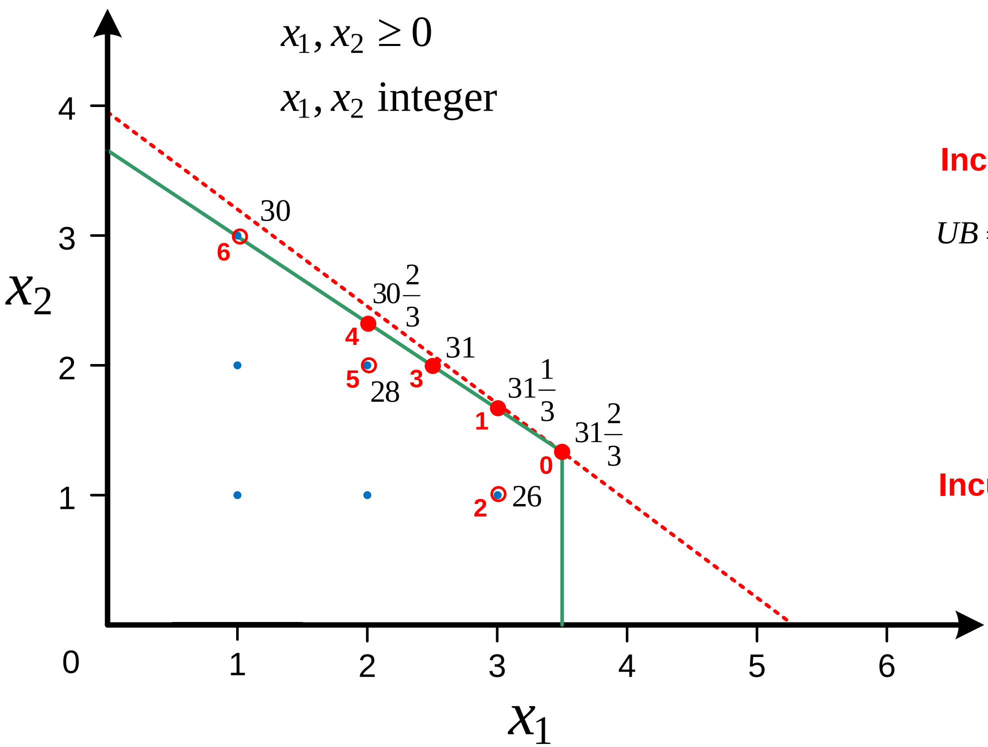
$$\mathbf{x}^* = \begin{bmatrix} 3\frac{1}{2} \\ 1\frac{1}{3} \end{bmatrix}, \quad \mathbf{c}'\mathbf{x}^* = 31\frac{2}{3}$$

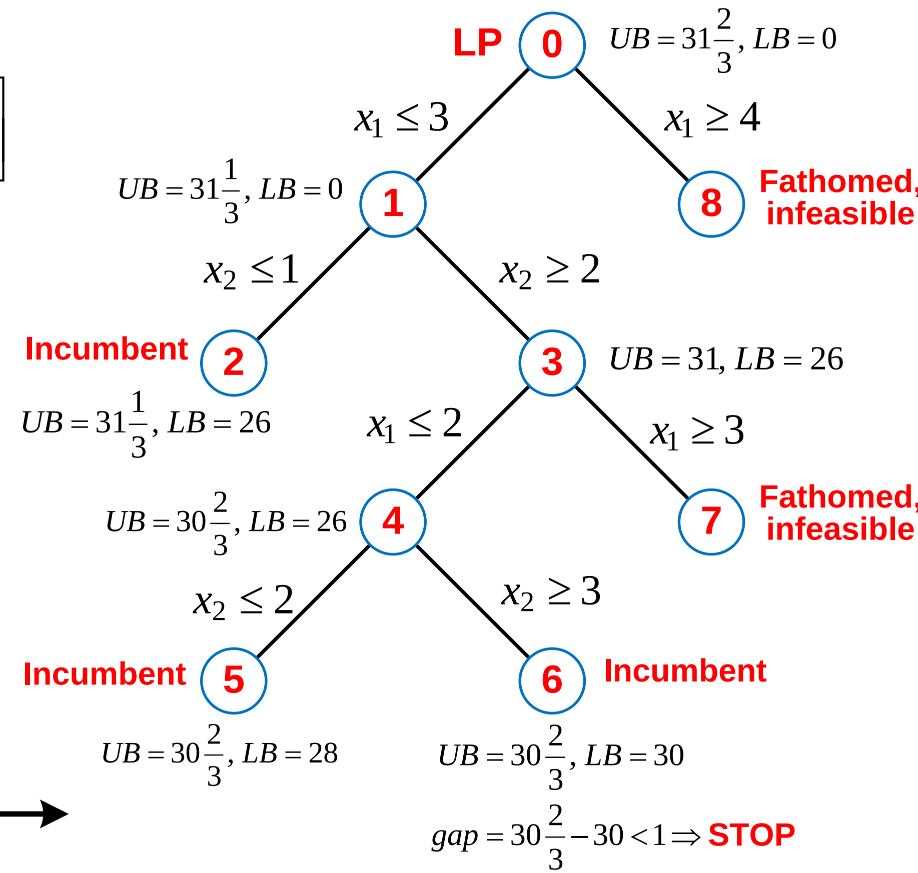
Branch and Bound

$$\begin{aligned} \max \quad & 6x_1 + 8x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\quad \le 7 \\ & x_1, x_2 \ge 0 \\ & x_1, x_2 \text{ integer} \end{aligned}$$

$$\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$





MILP Solvers

LP: $\max\ \mathbf{c}'\mathbf{x}$
s.t. $\mathbf{A}\mathbf{x} \le \mathbf{b}$
$\mathbf{x} \ge 0$

MILP: some x_i integer

ILP: $\mathbf{x}$ integer

BLP: $\mathbf{x} \in \{0,1\}$

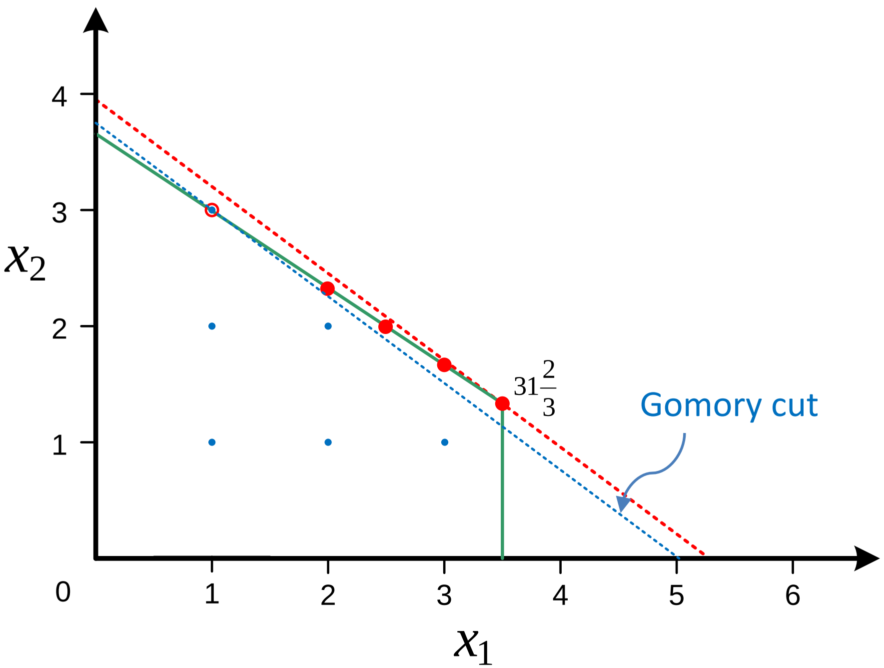
intlinprog: $\min\ \mathbf{c} \quad (\max\ -\mathbf{c})$
s.t. $\mathbf{A}_{\text{lt}} \le \mathbf{b}_{\text{lt}}$
$\mathbf{A}_{\text{eq}} = \mathbf{b}_{\text{eq}}$
$LB \le \mathbf{x} \le UB$
integer variable indices

cplex: $\mathbf{c}$ (sense *min* or *max*)
s.t. $lhs \le \mathbf{A} \le rhs$
$LB \le \mathbf{x} \le UB$
$$\text{variable:}\begin{cases} C & \text{continuous} \\ B & \text{binary} \\ I & \text{general integer} \end{cases}$$

lhs	rhs		
$-\infty$	$\mathbf{b}$	$\Rightarrow$	$\le$
$\mathbf{b}$	∞	$\Rightarrow$	$\ge$
$\mathbf{b}$	$\mathbf{b}$	$\Rightarrow$	$=$

gurobi: $\mathbf{c}$ (modelsense *min* or *max*)
$$\text{s.t.}\quad \mathbf{A}\begin{Bmatrix} < \\ = \\ > \end{Bmatrix}\mathbf{b}$$
$LB \le \mathbf{x} \le UB$
$$\text{variable:}\begin{cases} C & \text{continuous} \\ B & \text{binary} \\ I & \text{general integer} \end{cases}$$

MILP Solvers



- **Presolve**: eliminate variables

$$2x_1 + 2x_2 \leq 1,\ x_1, x_2 \geq 0 \text{ and integer}$$

$$\Rightarrow x_1 = x_2 = 0$$

- **Cutting planes**: keeps all integer solutions and cuts off LP solutions (Gomory cut)
- **Heuristics**: find good initial incumbent solution (Hybrid UFL)
- **Parallel**: use separate cores to solve nodes in B&B tree
- **Speedup** from 1990-2014:
  - 320,000 $\times$ computer speed
  - 580,000 $\times$ algorithm improvements

  $\Rightarrow$ 10 days of 24/7 processing $\rightarrow$ 1 sec

- Cplex (IBM, comm first solver)
- Gurobi (dev Robert Bixby)
- Xpress (used by LLamasoft)
- SAS/OR (part of SAS system)
- Symphony (open source)
- Matlab's `intlinprog`

MILP Formulation of UFL

$$\begin{aligned}
\min \quad & \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} x_{ij} = 1, \qquad j \in M \\
& \cancel{m y_i \ge \sum_{j \in M} x_{ij}, \qquad i \in N} \\
& 0 \le x_{ij} \le 1, \qquad i \in N, j \in M \\
& y_i \in \{0,1\}, \qquad i \in N
\end{aligned}$$

```
%% UFL MILP Matlab code, given k and C
mp.addobj('min',k,C)
for j = M
   mp.addcstr(0,{':',j},'=',1)
end
for i = N
   mp.addcstr({m,{i}},'>=',{i,':'})
end
mp.addub(1,1)
mp.addctype('B','C')
```

$$\boxed{y_i \ge x_{ij}, \qquad i \in N, j \in M}$$

where

k_i = fixed cost of NF at site $i \in N = \{1,\ldots,n\}$

c_{ij} = variable cost from i to serve EF $j \in M = \{1,\ldots,m\}$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

x_{ij} = fraction of EF j demand served from NF at site i.

Capacitated Facility Location (CFL)

$$
\begin{aligned}
\min \quad & \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} x_{ij} = 1, \qquad j \in M \\
& K_i y_i \geq \sum_{j \in M} f_j x_{ij}, \ i \in N \\
& 0 \leq x_{ij} \leq 1, \qquad i \in N,\ j \in M \\
& y_i \in \{0,1\}, \qquad i \in N
\end{aligned}
$$

where

$$k_i = \text{fixed cost of NF at site } i \in N = \{1, \ldots, n\}$$

$$c_{ij} = \text{variable cost from } i \text{ to serve EF } j \in M = \{1, \ldots, m\}$$

$$K_i = \text{capacity of NF at site } i \in N = \{1, \ldots, n\}$$

$$f_j = \text{demand EF } j \in M = \{1, \ldots, m\}$$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij} = \text{fraction of EF } j \text{ demand served from NF at site } i.$$

- CFL does not have simple and effective heuristics, unlike UFL
- Other types of constraints:
  - Fix NF *i* at site *j*: set *LB* and *UB* of x_{ij} to 1
  - Convert UFL to p-Median: set all *k* to 0 and add constraint sum{y_i} = *p*

Matlog's Milp

- Executing `mp = Milp` creates a *Milp* object that can be used to define a MILP model that is then passed to a Solver
  - Similar syntax to math notation for MILP
  - *AMPL* and *OPL* algebraic modeling languages provide similar capabilities, but *Milp* integrated into MATLAB

Milp

```
Milp Mixed-integer linear programming model.
 This class stores Milp models and provides methods to create the models
 and format solutions for output.
 Milp Properties:
    Model        Milp model (same structure as Cplex model).
 Milp Methods:
    Milp         Constructor for Milp objects.
    addobj       Add variable cost arrays to objective function.
    addcstr      Add constraint to model.
    addlb        Add lower bounds for each variable array.
    addub        Add upper bounds for each variable array.
    addctype     Specify type of each variable array.
    namesolution Convert solution to named field arrays.
    dispmodel    Display matrix view of model.
    lp2milp      Convert LP model to Milp model.
    milp2lp      Convert Milp model to LINPROG inputs.
    milp2ilp     Convert Milp model to INTLINPROG inputs.
    milp2gb      Convert Milp model to Gurobi input structure.
```

Illustrating Milp syntax

```
c = [1:4], C = reshape(5:10,2,3)
mp = Milp('Example');
mp.addobj('min',c,C)

mp.addcstr(0,1,'=',100)
mp.addcstr(c,-C,'>=',0)
mp.addcstr(c,'>=',C)
mp.addcstr([c; 2*c],repmat(C(:)',2,1),'<=',[400 500])
mp.addcstr({3},{2,2},'<=',600)
mp.addcstr({2,{3}},{3*3,{2,2}},'<=',700)
mp.addcstr({[2 3],{[3 4]}},{4,{2,':'}},'=',800)
mp.addcstr(0,{C(:,[2 3]),{':',[2 3]}},'>=',900)

mp.addlb(-10,0)
mp.addub(10,Inf)
mp.addctype('B','C')
mp.dispmodel
```

$$\mathrm{addobj}\left(\text{'min'},\mathbf{k},\mathbf{C}\right)$$

$$\mathrm{addobj}\left(\text{'min'},\mathbf{y},\mathbf{X}\right)$$

$$\mathrm{addcstr}\left(my_3,nx_{2,4},\text{'='},7\right)$$

$$\mathrm{addcstr}\left(\{m,\{3\}\},\{n,\{2,4\}\},\text{'='},7\right)$$

$$\mathrm{addcstr}\left(0\mathbf{y},1x_{2,4},\text{'='},7\right)$$

$$\mathrm{addcstr}\left(0,\{2,4\},\text{'='},7\right)$$

```
% c =  1     2     3     4
% C =
%      5     7     9
%      6     8    10
%
% Example:  lhs  B    B    B    B    C    C    C    C    C    C   rhs
% -------:-----------------------------------------------------------
%     Min:         1    2    3    4    5    6    7    8    9   10
%       1:  100    0    0    0    0    1    1    1    1    1    1  100
%       2:    0    1    2    3    4   -5   -6   -7   -8   -9  -10  Inf
%       3:    0    1    2    3    4   -5   -6   -7   -8   -9  -10  Inf
%       4: -Inf    1    2    3    4    5    6    7    8    9   10  400
%       5: -Inf    2    4    6    8    5    6    7    8    9   10  500
%       6: -Inf    0    0    1    0    0    0    0    1    0    0  600
%       7: -Inf    0    0    2    0    0    0    0    9    0    0  700
%       8:  800    0    0    2    3    0    4    0    4    0    4  800
%       9:  900    0    0    0    0    0    0    7    8    9   10  Inf
%      lb:       -10  -10  -10  -10    0    0    0    0    0    0
%      ub:        10   10   10   10  Inf  Inf  Inf  Inf  Inf  Inf
```

Ex 10: UFL MILP

```
k = [8      8     10      8      9      8];
C = [0      3      7     10      6      4
     3      0      4      7      6      7
     7      4      0      3      6      8
    10      7      3      0      7      8
     6      6      6      7      0      2
     4      7      8      8      2      0];
mp = Milp('UFL');
mp.addobj('min',k,C)
[n m] = size(C);
for j = 1:m
   mp.addcstr(0,{':',j},'=',1)
end
for i = 1:n
   mp.addcstr({m,{i}},'>=',{i,':'})  % Weak formulation
end
mp.addub(Inf,1)
mp.addctype('B','C')
[x,TC,nevals,XFlg] = milplog(mp); TC,nevals,XFlg
x = mp.namesolution(x), xC = x.C
TC = k*x.k' + sum(sum(C.*xC))
```

$$
\begin{aligned}
\min \quad & \sum_{i\in N} k_i y_i + \sum_{i\in N}\sum_{j\in M} c_{ij}x_{ij} \\
\text{s.t.} \quad & \sum_{i\in N} x_{ij} = 1, && j\in M \\
& m y_i \ge \sum_{j\in M} x_{ij}, && i\in N \\
& 0 \le x_{ij} \le 1, && i\in N,\, j\in M \\
& y_i \in \{0,1\}, && i\in N
\end{aligned}
$$

```
TC =
   31.0000
nevals =
    67
XFlg =
     1
x =
  struct with fields:

    k: [0 0 1 0 0 1]
    C: [6×6 double]
xC =
     0     0     0     0     0     0
     0     0     0     0     0     0
     0     1     1     1     0     0
     0     0     0     0     0     0
     0     0     0     0     0     0
     1     0     0     0     1     1
TC =
    31
```

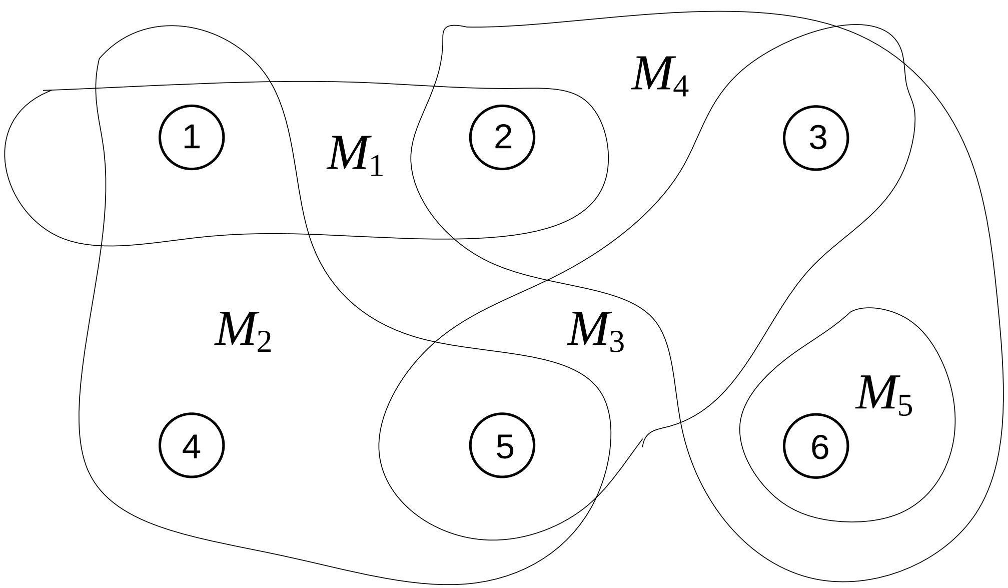
(Weighted) Set Covering

$$M = \{1,\ldots,m\}, \quad \text{objects to be covered}$$

$$M_i \subseteq M,\, i \in N = \{1,\ldots,n\}, \quad \text{subsets of } M$$

$$c_i = \text{cost of using } M_i \text{ in cover}$$

$$I^* = \arg\min_I \left\{ \sum_{i \in I} c_i : \bigcup_{i \in I} M_i = M \right\}, \quad \text{min cost covering of } M$$



$$M = \{1,\ldots,6\}$$

$$i \in N = \{1,\ldots,5\}$$

$$M_1 = \{1,2\}, M_2 = \{1,4,5\}, M_3 = \{3,5\}$$

$$M_4 = \{2,3,6\}, M_5 = \{6\}$$

$$c_i = 1, \quad \text{for all } i \in N$$

$$I^* = \arg\min_I \left\{ \sum_{i \in I} c_i : \bigcup_{i \in I} M_i = M \right\}$$

$$= \{2,4\}$$

$$\sum_{i \in I^*} c_i = 2$$

(Weighted) Set Covering

$$M = \{1, \ldots, m\}, \quad \text{objects to be covered}$$

$$M_i \subseteq M, i \in N = \{1, \ldots, n\}, \quad \text{subsets of } M$$

$$c_i = \text{cost of using } M_i \text{ in cover}$$

$$I^* = \arg\min_I \left\{ \sum_{i \in I} c_i : \bigcup_{i \in I} M_i = M \right\}, \quad \text{min cost covering of } M$$

$$\begin{aligned} \min \quad & \sum_{i \in N} c_i x_i \\ \text{s.t.} \quad & \sum_{i \in N} a_{ji} x_i \ge 1, \qquad j \in M \\ & x_i \in \{0,1\}, \qquad i \in N \end{aligned}$$

where

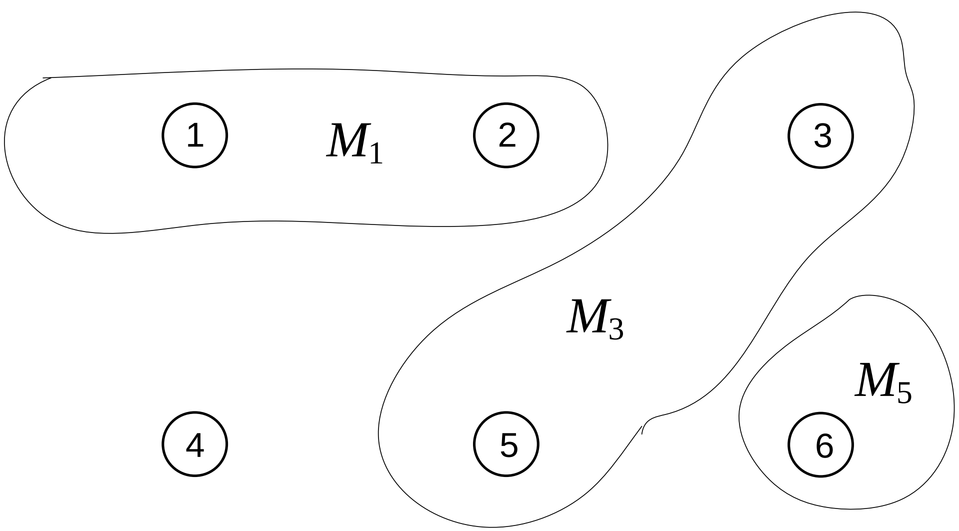
$$x_i = \begin{cases} 1, & \text{if } M_i \text{ is in cover} \\ 0, & \text{otherwise} \end{cases}$$

$$a_{ji} = \begin{cases} 1, & \text{if } j \in M_i \\ 0, & \text{otherwise.} \end{cases}$$

```
%% Set Covering BLP Matlab code,
%  given c and A
mp = Milp('Set Cover')
mp.addobj('min',c)
mp.addcstr(A,'>=',1)
mp.addctype('B')
```

Set Packing

- Maximize the number of mutually disjoint sets
  - Dual of Set Covering problem
  - Not all objects required in a packing
  - Limited logistics engineering application (c.f. bin packing)



$$\begin{aligned} \max \quad & \sum_{i \in N} x_i \\ \text{s.t.} \quad & \sum_{i \in N} a_{ji} x_i \le 1, \qquad j \in M \\ & x_i \in \{0,1\}, \qquad i \in N \end{aligned}$$

Bin Packing

$$M = \{1,\ldots,m\}, \quad \text{objects to be packed}$$
$$v_j = \text{volume of object } j$$
$$V = \text{volume of each bin } B_i \left(\max v_j \le V\right)$$
$$B^* = \arg\min_B \left\{ |B| : \sum_{j\in B_i} v_j \le V,\ \bigcup_{B_i \in B} B_i = M \right\}, \quad \text{min packing of } M$$

$$\begin{aligned}
\min \quad & \sum_{i\in M} y_i \\
\text{s.t.} \quad & V y_i \ge \sum_{j\in M} v_j x_{ij}, && i \in M \\
& \sum_{i\in M} x_{ij} = 1, && j \in M \\
& y_i \in \{0,1\}, && i \in M \\
& x_{ij} \in \{0,1\}, && i \in M,\ j \in M
\end{aligned}$$

where

$$y_i = \begin{cases} 1, & \text{if bin } B_i \text{ is used in packing} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if object } j \text{ packed into bin } B_i \\ 0, & \text{otherwise.} \end{cases}$$

```
%% Bin Packing BLP Matlab code,
%  given v and V
mp = Milp('Bin Packing')
mp.addobj('min',ones(1,m),zeros(m))
for i = M
   mp.addcstr({V,{i}},'>=',{v,{i,':'}})
end
for j = M
   mp.addcstr(0,{':',j},'=',1)
end
mp.addctype('B','B')
```

Topics

1. Introduction
2. Facility location
3. **Freight transport**
   – Exam 1 (take home)
4. Network models
5. Routing
   – Exam 2 (take home)
6. Warehousing
   – Final exam (in class)

Logistics Engineering Design Constants

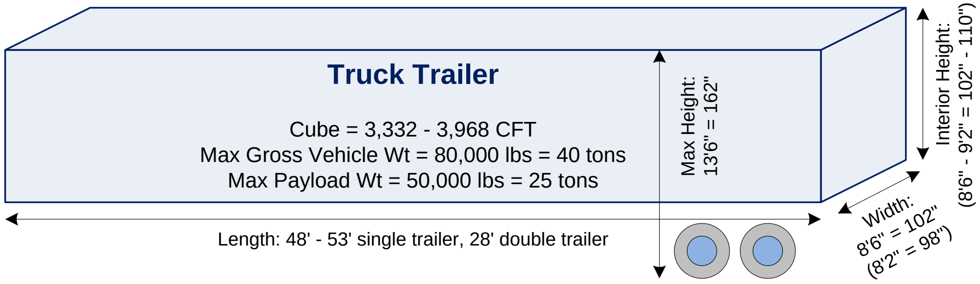
1. Circuity Factor: 1.2 (g)
   – 1.2 × GC distance ≈ actual road distance
2. Local vs. Intercity Transport:
   – Local: < 50 mi ⇒ use actual road distances
   – Intercity: > 50 mi ⇒ can estimate road distances
      • 50-250 mi ⇒ return possible (11 HOS)
      • > 250 mi ⇒ always one-way transport
      • > 500-750 mi ⇒ intermodal rail possible
3. Inventory Carrying Cost (h) = funds + storage + obsolescence
   – 16% average (no product information, per U.S. Total Logistics Costs)
      • (16% ≈ 5% funds + 6% storage + 5% obsolescence)
   – 5-10% low-value product (construction)
   – 25-30% general durable manufactured goods
   – 50+% computer/electronic equipment
   – >> 100% perishable goods (produce)

Logistics Engineering Design Constants

4. $\dfrac{\text{Value}}{\text{Transport Cost}} \gg 1$: $1 ft³ $\approx \dfrac{\text{\$2,620 Shanghai-LA/LB shipping cost}}{\text{2,400 ft}^3 \text{ 40' ISO container capacity}}$

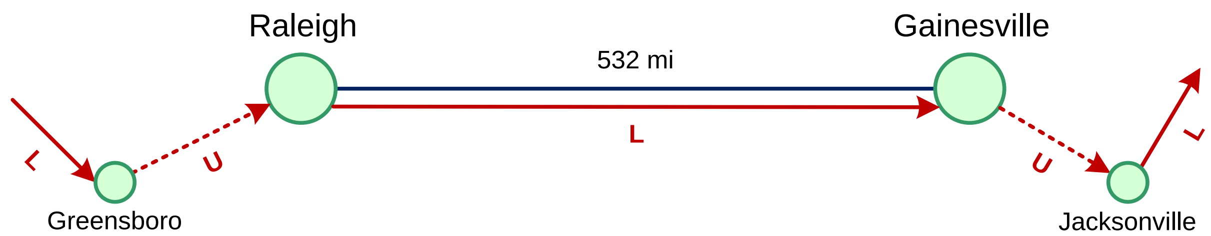
5. TL Weight Capacity: 25 tons (K_{wt})
   - (40 ton max per regulation) – (15 ton tare for tractor-trailer) = 25 ton max payload
   - Weight capacity = 100% of physical capacity

6. TL Cube Capacity: 2,750 ft³ (K_{cu})
   - Trailer physical capacity = 3,332 ft³
   - Effective capacity = 3,332 × 0.80 ≈ 2,750 ft³
   - Cube capacity = 80% of physical capacity



Logistics Engineering Design Constants

7. TL Revenue per Loaded Truck-Mile: $2/mi in 2004 (r)
   – TL revenue for the carrier is your TL cost as a shipper



$$15\%, \quad \text{average deadhead travel}$$

$$\$1.60, \quad \text{cost per mile in 2004}$$

$$\frac{\$1.60}{1-0.15} = \$1.88, \quad \text{cost per loaded-mile}$$

$$6.35\%, \quad \text{average operating margin for trucking}$$

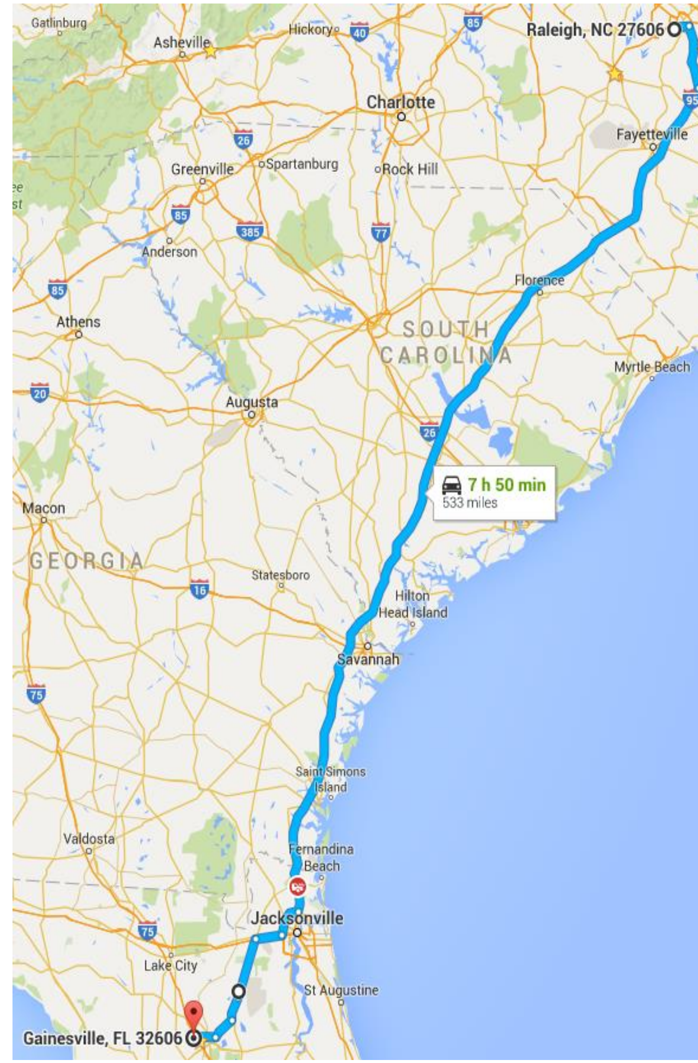
$$\frac{\$1.88}{1-0.0635} \approx \$2.00, \quad \text{revenue per loaded-mile}$$

One-Time vs Periodic Shipments

- **One-Time Shipments** (*operational* decision): know shipment size q
  - Know when and how much to ship, need to determine if TL and/or LTL to be used
  - Must contact carrier or have agreement to know charge
    - Can/should estimate charge before contacting carrier
- **Periodic Shipments** (*tactical* decision): know demand rate f, must determine size q
  - Need to determine how often and how much to ship
  - Analytical transport charge formula allow “optimal” size (and shipment frequency) to be estimated
    - U.S. Bureau of Labor Statistic's *Producer Price Index* (PPI) for TL and LTL used to estimate transport charges

Truck Shipment Example

- Product shipped in cartons from Raleigh, NC (27606) to Gainesville, FL (32606)
- Each identical unit weighs 40 lb and occupies 9 ft^3 (its *cube*)
  - Don't know linear dimensions of each unit for TL and LTL
- Units can be stacked on top of each other in a trailer
- Additional info/data is presented only when it is needed to determine answer



Truck Shipment Example: One-Time

1. Assuming that the product is to be shipped P2P TL, what is the maximum payload for each trailer used for the shipment?

$$q_{\max}^{wt} = K_{wt} = 25 \text{ ton}$$

$$K_{cu} = 2750 \text{ ft}^3$$

$$s = \frac{40 \text{ lb/unit}}{9 \text{ ft}^3\text{/unit}} = 4.4444 \text{ lb/ft}^3$$

$$K_{cu} = \frac{q_{\max}^{cu}}{\left(\dfrac{s}{2000}\right)} \Rightarrow q_{\max}^{cu} = \frac{sK_{cu}}{2000}$$

$$q_{\max} = \min\left\{q_{\max}^{wt}, q_{\max}^{cu}\right\} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\}$$

$$= \min\left\{25, \frac{4.4444(2750)}{2000}\right\} = 6.1111 \text{ ton}$$

Truck Shipment Example: One-Time

2. On Jan 10, 2018, 320 units of the product were shipped. How many truckloads were required for this shipment?

$$q = 320\frac{40}{2000} = 6.4 \text{ ton}, \quad \left\lceil \frac{q}{q_{\max}} \right\rceil = \left\lceil \frac{6.4}{6.1111} \right\rceil = 2 \text{ truckloads}$$

3. Before contacting the carrier (and using Jan 2018 PPI), what is the estimated TL transport charge for this shipment?

$$d = 532 \text{ mi}$$

$$r_{TL} = \frac{PPI_{TL}^{\text{Jan 2018}}}{PPI_{TL}^{2004}} \times r_{2004} = \frac{PPI_{TL}}{102.7} \times \$2.00/\text{mi}$$

$$= \frac{131.0}{102.7} \times \$2.00/\text{mi} = \$2.5511/\text{mi}$$

$$c_{TL} = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d = \left\lceil \frac{6.4}{6.1111} \right\rceil (2.5511)(532) = \$2,714.39$$

Truck Shipment Example: One-Time

UNITED STATES DEPARTMENT OF LABOR

BUREAU OF LABOR STATISTICS

Home ▾ | Subjects ▾ | Data Tools ▾ | Publications ▾ | Economic Releases ▾ | Students ▾ | Beta ▾

Databases, Tables & Calculators by Subject

Change Output Options: From: 2008 ▼ To: 2018 ▼

☐ include graphs ☐ include annual averages

Data extracted on: September 5, 2018 (4:22:19 PM)

PPI Industry Data

```
Series Id:      PCU484121484121
Series Title:   PPI industry data for General freight trucking, long-distance TL, not seasonally adjusted
Industry:       General freight trucking, long-distance TL
Product:        General freight trucking, long-distance TL
Base Date:      200312
```

Download: xlsx

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	116.0	115.9	116.5	117.8	120.5	123.0	124.0	124.0	121.8	121.3	117.8	115.1
2009	113.2	112.1	110.4	109.7	109.8	110.1	111.4	111.0	111.7	110.8	111.5	110.9
2010	110.8	111.0	111.9	112.2	113.2	113.5	113.4	113.7	113.8	114.4	115.8	116.1
2011	116.5	117.4	119.3	121.0	121.7	121.4	121.3	121.2	122.0	122.0	123.2	123.3
2012	124.0	124.6	126.2	126.7	127.0	125.8	125.6	126.8	127.4	127.2	126.9	127.0
2013	126.7	127.2	128.0	127.5	127.8	127.6	127.6	127.6	127.1	127.2	127.6	127.4
2014	127.9	128.2	128.7	129.5	130.6	130.8	130.3	130.4	130.4	129.7	129.8	128.9
2015	126.7	126.0	126.0	126.2	126.3	127.1	126.9	126.2	125.9	125.5	125.8	124.8
2016	124.6	123.4	123.2	123.6	122.8	122.7	123.0	123.0	123.3	124.1	124.1	124.2
2017	124.4	124.7	124.2	124.3	124.0	124.2	124.2	125.9	126.6	126.6	128.5	130.3
2018	131.0	132.0	132.0	132.6(P)	133.6(P)	135.9(P)	138.6(P)					

P : Preliminary. All indexes are subject to revision four months after original publication.

Truck Shipment Example: One-Time

4. Using the Jan 2018 PPI LTL rate estimate, what was the transport charge to ship the fractional portion of the shipment LTL (i.e., the last partially full truckload portion)?

$$q_{\text{frac}} = q - q_{\text{max}} = 6.4 - 6.1111 = 0.2889 \text{ ton}$$

$$r_{LTL} = PPI_{LTL}\left[\frac{\dfrac{s^2}{8}+14}{\left(q_{\text{frac}}^{\frac{1}{7}} d^{\frac{15}{29}} - \dfrac{7}{2}\right)\left(s^2+2s+14\right)}\right]$$

$$= 177.4\left[\frac{\dfrac{4.44^2}{8}+14}{\left(0.2889^{\frac{1}{7}} 532^{\frac{15}{29}} - \dfrac{7}{2}\right)\left(4.44^2+2(4.44)+14\right)}\right] = \$3.8014 / \text{ton-mi}$$

$$c_{LTL} = r_{LTL}\, q_{\text{frac}}\, d = 3.8014(0.2889)(532) = \$584.23$$

Truck Shipment Example: One-Time

5. What is the change in total charge associated with the combining TL and LTL as compared to just using TL?

$$
\begin{aligned}
\Delta c &= c_{TL} - \left(c_{TL-1} + c_{LTL} \right) \\
&= \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d - \left(\left\lfloor \frac{q}{q_{\max}} \right\rfloor r_{TL} d + r_{LTL}\, q_{\text{frac}}\, d \right) \\
&= \$772.96
\end{aligned}
$$

Truck Shipment Example: One-Time

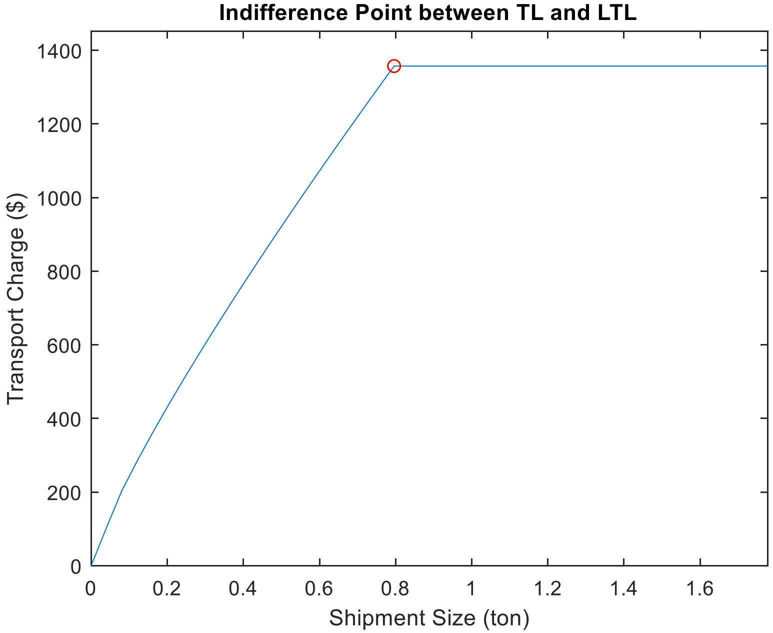
6. What would the fractional portion have to be so that the TL and LTL charges are equal?

$$c_{TL}(q) = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d$$

$$r_{LTL}(q) = PPI_{LTL} \left[\frac{\dfrac{s^2}{8} + 14}{\left(q^{\frac{1}{7}} d^{\frac{15}{29}} - \dfrac{7}{2} \right)\left(s^2 + 2s + 14 \right)} \right]$$

$$c_{LTL}(q) = r_{LTL}(q)\, q d$$

$$\begin{aligned} q_I &= \arg\min_q \left(\left\| c_{TL}(q) - c_{LTL}(q) \right\| \right) \\ &= 0.7960 \text{ ton} \end{aligned}$$



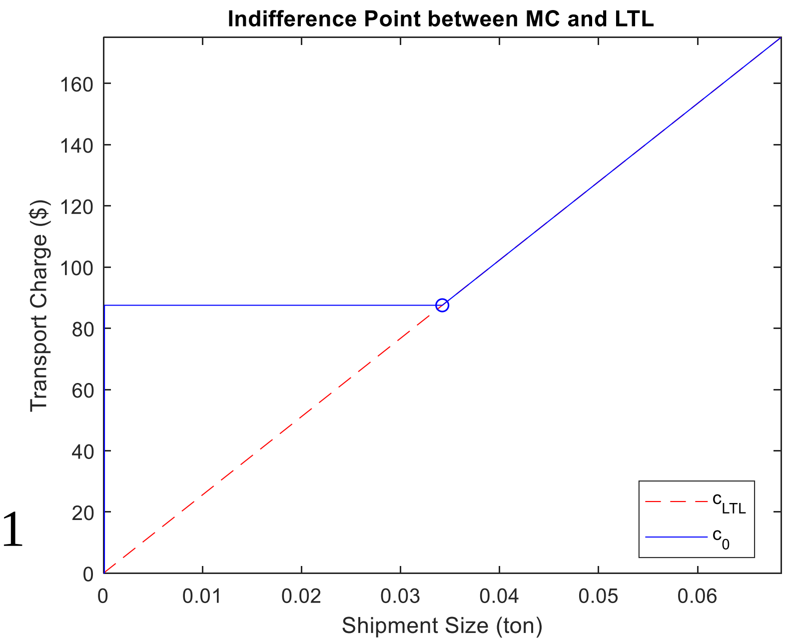
Truck Shipment Example: One-Time

7. What are the TL and LTL minimum charges?

$$MC_{TL} = \left(\frac{r_{TL}}{2}\right) 45 = \$57.40$$

$$MC_{LTL} = \left(\frac{PPI_{LTL}}{104.2}\right)\left(45 + \frac{d^{\frac{28}{19}}}{1625}\right)$$

$$= \left(\frac{177.4}{104.2}\right)\left(45 + \frac{532^{\frac{28}{19}}}{1625}\right) = \$87.51$$



- Why do these charges not depend on the size of the shipment?
- Why does only the LTL minimum charge depend of the distance of the shipment?

Truck Shipment Example: One-Time

- Independent Transport Charge ($):

$$c_0(q) = \min\left\{\max\left\{c_{TL}(q), MC_{TL}\right\}, \max\left\{c_{LTL}(q), MC_{LTL}\right\}\right\}$$

Truck Shipment Example: One-Time

8. Using the same LTL shipment, find online one-time (spot) LTL rate quotes using the FedEx LTL website

$$q_{\text{frac}} = 0.2889 \text{ ton}$$

$$= 0.2889(2000) = 578 \text{ lb}$$

$$\text{no. units} = \left\lceil \frac{0.2889(2000)}{40} \right\rceil = 15 \text{ cartons}$$

- Most likely freight class:

$$s = \frac{40 \text{ lb/unit}}{9 \text{ ft}^3\text{/unit}} = 4.4444 \text{ lb/ft}^3$$

$$\Rightarrow \text{Class 200}$$

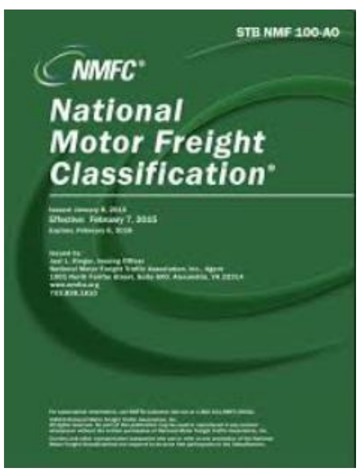
- What is the rate quote for the reverse trip from Gainesville (32606) to Raleigh (27606)?

Class-Density Relationship

Class	Load Density (lb/ft^3) Minimum	Load Density (lb/ft^3) Average	Max Physical Weight (tons)	Max Effective Cube (ft^3)
500	–	0.52	0.72	*2,750*
400	1	1.49	2.06	*2,750*
300	2	2.49	3.43	*2,750*
250	3	3.49	4.80	*2,750*
200	4	4.49	6.17	*2,750*
175	5	5.49	7.55	*2,750*
150	6	6.49	8.92	*2,750*
125	7	7.49	10.30	*2,750*
110	8	8.49	11.67	*2,750*
100	9	9.72	13.37	*2,750*
92.5	10.5	11.22	15.43	*2,750*
85	12	12.72	17.49	*2,750*
77.5	13.5	14.22	19.55	*2,750*
70	15	18.01	24.76	*2,750*
65	22.5	25.50	*25*	1,961
60	30	32.16	*25*	1,555
55	35	39.68	*25*	1,260
50	50	56.18	*25*	890

Truck Shipment Example: One-Time

- The *National Motor Freight Classification* (NMFC) can be used to determine the product class
- Based on:
  1. Load density
  2. Special handling
  3. Stowability
  4. Liability



Item	Description	Class	NMFC	Sub
Abietic Acid	Abietic Acid, in drums	55	42605	-
Accordions	Accordions, in boxes	125	138820	-
Acetonitrile	Acetonitrile, in boxes or drums. See item 60000 for class dependent upon released value	85	42645	-
Acetylene	in steel cylinders	70	85520	-
Acid Fish Scrap	Fish Scrap, NOI, dry, not ground, pulverized nor screened, or Acid Fish Scrap, in bags	77.5	69980	-
Aircraft Parts	metal, struts, skins, panels	200	11790	01
Aluminum Channel	U channel	60	13340	-
Aluminum Table Set	aluminum table SU	200	82105	01
Ambulance Stretcher	stretcher	200	56920	06
Arches Support	Iron Steel	60	52460	-
Architectural Details	6 - 8 lbs per cubic foot	125	56290	05
Architectural Details	2 - 4 lbs per cubic ft	250	56290	03
Assembled Furniture	Bathroom cabinet set up	300	39220	01
Assembled Furniture	Highboys, dressers, wooden set up	125	80120	01
Assembled Furniture	Wood furniture 4-6 Lbs per cu ft	150	82270	04
Assembled Furniture	Chairs wooden setup w/out upholstery	300	80770	01
Assembled Furniture	Chairs wooden setup w/out upholstery KD	125	80770	03
Assembled Furniture	Couch w/ back & arms put together	175	80865	03
Assembled Furniture	Chairs put together w/ upholstery	200	79255	01
Assembled Furniture	Metal cabinets in boxes	110	39270	06
Assembled Furniture	18 gauge steel cabinet	70	39340	-
Assembled Furniture	Benches, cabinets, tables for workstations	125	23410	-
Assembled Furniture	Buffets, china cabinets put together	125	80080	-
Assembled Furniture	Cabinets of metal or plastic for storage	92.5	39235	-
Assembled Furniture	Tanning bed	150	109050	-
Assembled Furniture	Mattresses, in packages or boxes	200	79550	-
Athletic / Sporting Goods	Gym equipment, playground, sports items. Density Item			
Attachments: Backhoe	NOI: Attachments, backhoe (Backhoes), tractor or truck, on lift truck skids or pallets:	175	114217	01
Attachments: Backhoe	Attachments, backhoe (Backhoes), tractor or truck, on lift truck skids or pallets: Each shipped with all components secured to a single pallet, platform or skid, weighing 1100 pounds or more and having a density of 8 pounds or greater per cubic foot	100	114217	02

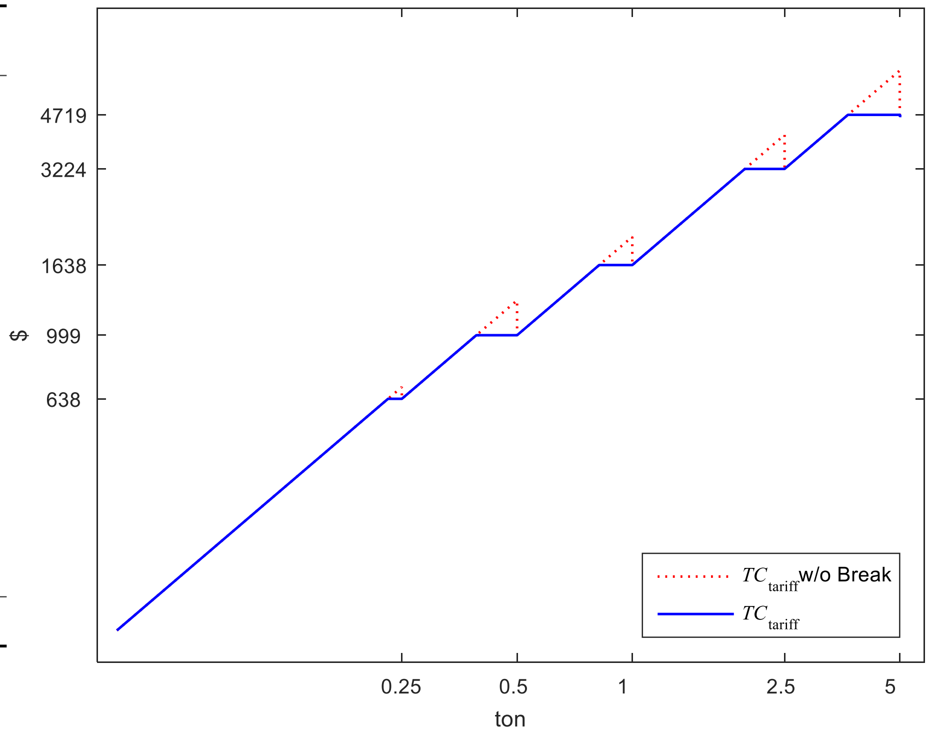
Truck Shipment Example: One-Time

- CzarLite tariff table for O-D pair 27606-32606

$$cwt = \text{hundredweight} = 100 \text{ lb} = \frac{100}{2000} = \frac{1}{20} \text{ ton}$$

Tariff (in \$/cwt) from Raleigh, NC (27606) to Gainesville, FL (32606)
(532 mi, CzarLite DEMOCZ02 04-01-2000, minimum charge = \$95.23)

Freight Class	Rate Breaks (i)								
	1	2	3	4	5	6	7	8	9&10
500	341.42	314.14	245.80	201.48	158.60	112.37	55.66	55.66	55.66
400	273.88	251.99	197.19	161.61	127.22	91.12	45.10	45.10	45.10
300	206.34	189.85	148.56	121.76	95.85	69.47	34.43	34.43	34.43
250	172.56	158.77	124.23	101.83	80.15	58.03	28.79	28.79	28.79
200	138.78	127.69	99.92	81.89	64.47	47.19	23.40	23.40	23.40
175	121.37	111.68	87.39	71.62	56.38	41.27	20.39	20.39	20.39
150	104.49	96.13	75.22	61.66	48.53	35.96	17.75	17.75	17.75
125	87.59	80.60	63.07	51.69	40.69	30.24	15.00	15.00	15.00
110	77.57	71.37	55.85	45.77	36.04	28.61	14.40	14.40	14.40
100	71.23	65.55	51.29	42.04	33.09	27.58	14.03	10.80	9.90
92	66.48	61.18	47.88	39.24	30.89	25.75	13.68	10.52	9.66
85	61.74	56.80	44.45	36.43	28.68	23.91	13.20	10.15	9.32
77	56.99	52.44	41.04	33.63	26.48	22.07	12.60	9.68	8.89
70	52.77	48.55	37.99	31.14	24.51	20.43	12.00	9.23	8.47
65	50.07	46.08	36.05	29.56	23.04	19.39	11.87	9.14	8.39
60	47.44	43.64	34.15	28.00	21.82	18.37	11.76	9.04	8.30
55	44.75	41.17	32.22	26.40	20.59	17.32	11.64	8.96	8.22
50	41.57	38.26	29.94	24.54	19.12	16.10	11.52	8.85	8.14
Tons (q_i^B)	0.25	0.5	1	2.5	5	10	15	20	∞



Truck Shipment Example: One-Time

9. Using the same LTL shipment, what is the transport cost found using the undiscounted CzarLite tariff?

$$q = 0.2889, \quad class = 200$$

$$disc = 0, \quad MC = 95.23$$

Freight Class	Rate Breaks (i) 1	2	3	4	5	6	7	8	9&10
500	341.42	314.14	245.80	201.48	158.60	112.37	55.66	55.66	55.66
400	273.88	251.99	197.19	161.61	127.22	91.12	45.10	45.10	45.10
300	206.34	189.85	148.56	121.76	95.85	69.47	34.43	34.43	34.43
250	172.56	158.77	124.23	101.83	80.15	58.03	28.79	28.79	28.79
200	138.78	127.69	99.92	81.89	64.47	47.19	23.40	23.40	23.40
175	121.37	111.68	87.39	71.62	56.38	41.27	20.39	20.39	20.39
50	41.57	38.26	29.94	24.54	19.12	16.10	11.52	8.85	8.14
Tons (q_i^B)	0.25	0.5	1	2.5	5	10	15	20	∞

$$i = \arg\left\{ q_i^B \,\middle|\, q_{i-1}^B \le q < q_i^B \right\}$$

$$= \arg\left\{ q_2^B \,\middle|\, q_1^B \le q < q_2^B \right\}$$

$$= \arg\left\{ q_2^B \,\middle|\, 0.25 \le 0.2889 < 0.5 \right\} = 2$$

$$c_{\text{tariff}} = (1 - disc)\max\left\{ MC, \min\left\{ OD(class,i)\,20q,\, OD(class,i+1)\,20q_i^B \right\}\right\}$$

$$= (1-0)\max\left\{ 95.23, \min\left\{ OD(200,2)\,20(0.2889),\, OD(200,3)\,20(0.5) \right\}\right\}$$

$$= \max\left\{ 95.23, \min\left\{ (127.69)\,20(0.2889),\, (99.92)20(0.5) \right\}\right\}$$

$$= \max\left\{ 95.23, \min\left\{ 737.76,\, 999.20 \right\}\right\} = \$737.76$$

Truck Shipment Example: One-Time

10. What is the implied discount of the estimated charge from the CzarLite tariff cost?

$$disc = \frac{c_{\text{tariff}} - c_{LTL}}{c_{\text{tariff}}}$$

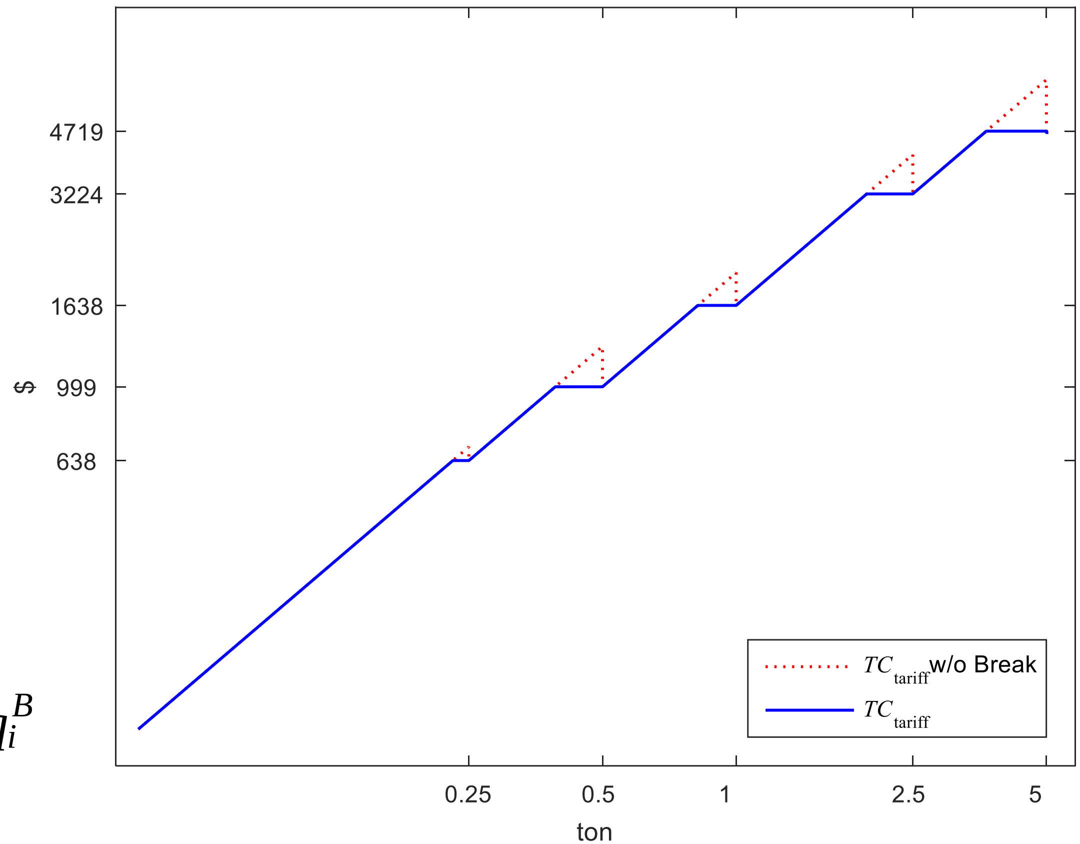
$$= \frac{737.76 - 584.23}{737,76}$$

$$= 20.81\%$$

- What is the weight break between the rate breaks?

$$q_i^W = \frac{OD(class, i+1)}{OD(class, i)} q_i^B$$

$$= \frac{99.92}{127.69}(1) = 0.3913 \text{ ton}$$



Truck Shipment Example: One-Time

- **PX: Package Express**
  - (Undiscounted) charge c_{PX} based rate tables, R, for each service (2-day ground, overnight, etc.)
  - Rate determined by on *chargeable weight*, wt_{chrg}, and *zone*
  - All PX carriers (FedEX, UPS, USPS, DHL) use *dimensional weight*, wt_{dim}
  - wt_{dim} > 150 lb is prorated per-lb rate
  - Actual weight 1−70 lb (UPS, FedEx home), 1−150 lb (FedEx commercial)
  - Carrier sets a *shipping factor*, which is min cubic volume per pound
  - Zone usually determined by O-D distance of shipment
  - Supplemental charges for home delivery, excess declared value, etc.

$$c_{PX} = R\left(wt_{\text{chrg}}, zone\right)$$

$$wt_{\text{chrg}} = \left\lceil \max\left\{wt_{\text{act}}, wt_{\text{dim}}\right\} \right\rceil \text{ (lb)}$$

$$wt_{\text{act}} = \text{actual weight (1 to 150 lb)}$$

$$wt_{\text{dim}} = \frac{l \times w \times d \ (\text{in}^3)}{sf \ (\text{in}^3/\text{lb})} \text{ (lb)}$$

$$l, w, d = \text{length, width, depth (in)}$$

$$l \ge w, \quad l \times w \times d \ge \text{actual cube}$$

$$sf = \text{shipping factor (in}^3/\text{lb)}$$

$$= 12^3 \big/ s, \text{ inverse of density}$$

$$= 139 \text{ FedEx (2019)}$$

$$\Rightarrow s = 12.43 \text{ lb/ft}^3 \text{ (Class 85)}$$

$$= 194 \text{ USPS } \Rightarrow s = 8.9 \text{ lb/ft}^3$$

Truck Shipment Example: One-Time

- (Undisc.) charge to ship a single carton via FedEx?

$$wt_{\text{act}} = 40 \text{ lb}, cu = 9 \text{ ft}^3$$

$$d = 532 \text{ mi} \Rightarrow zone = 4$$

$$\text{carton} \Rightarrow l \times w \times d = \text{actual cube} \Rightarrow$$

$$l \times w \times d = 9 \times 12^3 = 15{,}552 \text{ in}^3 = 32 \times 27 \times 18$$

$$wt_{\text{dim}} = \frac{l \times w \times d}{sf} = \frac{15{,}552}{139} = 111.9 \text{ lb}$$

$$wt_{\text{chrg}} = \left\lceil \max\left\{ wt_{\text{act}}, wt_{\text{dim}} \right\} \right\rceil$$

$$= \left\lceil \max\left\{ 40, 111.9 \right\} \right\rceil = 112 \text{ lb}$$

$$c_{PX} = R\left(wt_{\text{chrg}}, zone \right)$$

$$= R\left(112, 4\right) = \$64.27$$

FedEx Standard List Rates (eff. Jan. 7, 2019)

Service	FedEx Ground® and FedEx Home Delivery® (up to 70 lbs.)						
Delivery Commitment	1–5 days based on distance to destination						
Zones[1]	2	3	4	5	6	7	8
Maximum Weight in Lbs.	0–150 miles	151–300 miles	301–600 miles	601–1,000 miles	1,001–1,400 miles	1,401–1,800 miles	1,801-plus miles
1 lb.	$ 7.85	$ 8.23	$ 8.96	$ 9.36	$ 9.68	$ 9.80	$ 9.96
2 lbs.	8.52	9.48	10.15	10.37	10.82	11.24	11.43
3	8.87	9.89	10.70	11.14	11.59	11.98	12.57
4	9.13	[illegible]	11.04	11.75	12.08	12.87	13.47
5	9.37	[illegible]	[illegible]	[illegible]	[illegible]	13.46	14.22
6	9.68	[illegible]	[illegible]	[illegible]	[illegible]	13.81	14.48
7	10.23	[illegible]	[illegible]	[illegible]	[illegible]	14.18	15.18
8	10.43	11.24	12.32	13.28	13.74	14.61	15.69
9	10.59	11.40	12.48	13.39	14.04	15.21	16.52
10	10.84	11.51	12.60	13.76	14.33	16.10	17.62
111	59.41	59.89	64.26	67.20	75.20	82.60	92.25
112	60.62	61.13	64.27	67.21	75.84	83.31	92.36
113	60.68	61.18	64.98	67.83	76.52	84.00	94.04
114	61.32	62.45	66.33	69.15	77.81	85.41	94.65
115	61.99	63.16	66.34	69.33	77.82	85.42	94.66
146	82.51	84.98	88.95	89.15	98.04	105.96	118.85
147	83.66	85.00	89.66	89.86	98.74	106.69	119.66
148	84.68	85.63	90.61	90.62	100.20	107.40	120.46
149	84.84	86.38	91.26	91.28	100.42	108.08	121.81
150[2]	84.85	87.16	92.76	94.33	100.95	108.83	122.60

Note: No Zone 1 (usually < 50 mi local)

Truck Shipment Example: Periodic

11. Continuing with the example: assuming a constant annual demand for the product of 20 tons, what is the number of full truckloads per year?

$$f = 20 \text{ ton/yr}$$

$$q = q_{\max} = 6.1111 \text{ ton/ TL} \quad (\text{full truckload} \Rightarrow q \equiv q_{\max})$$

$$n = \frac{f}{q} = \frac{20}{6.1111} = 3.2727 \text{ TL/yr}, \quad \text{average shipment frequency}$$

- Why should this number not be rounded to an integer value?

Truck Shipment Example: Periodic

12. What is the shipment interval?

$$t = \frac{1}{n} = \frac{q}{f} = \frac{6.1111}{20} = 0.3056 \text{ yr/TL}, \quad \text{average shipment interval}$$

- How many days are there between shipments?

$$365.25 \text{ day/yr}$$

$$t \times 365.25 = \frac{365.25}{n} = 111.6042 \text{ day/TL}$$

Truck Shipment Example: Periodic

13. What is the annual full-truckload transport cost?

$$d = 532 \text{ mi}, \quad r_{TL} = \$2.5511 / \text{mi}$$

$$r_{FTL} = \frac{r_{TL}}{q_{\max}} = \frac{2.5511}{6.1111} = \$0.4175 / \text{ton-mi}$$

$$TC_{FTL} = f\, r_{FTL}\, d = n\, r_{TL} d \quad \left(= w\, d,\ w = \text{monetary weight in \$/mi}\right)$$

$$= 3.2727\,(2.5511)\,532 = \$4,441.73/\text{yr}$$

- What would be the cost if the shipments were to be made at least every three months?

$$t_{\max} = \frac{3}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 4 \text{ TL/yr} \quad \Rightarrow \quad q = \frac{f}{\max\left\{n, n_{\min}\right\}}$$

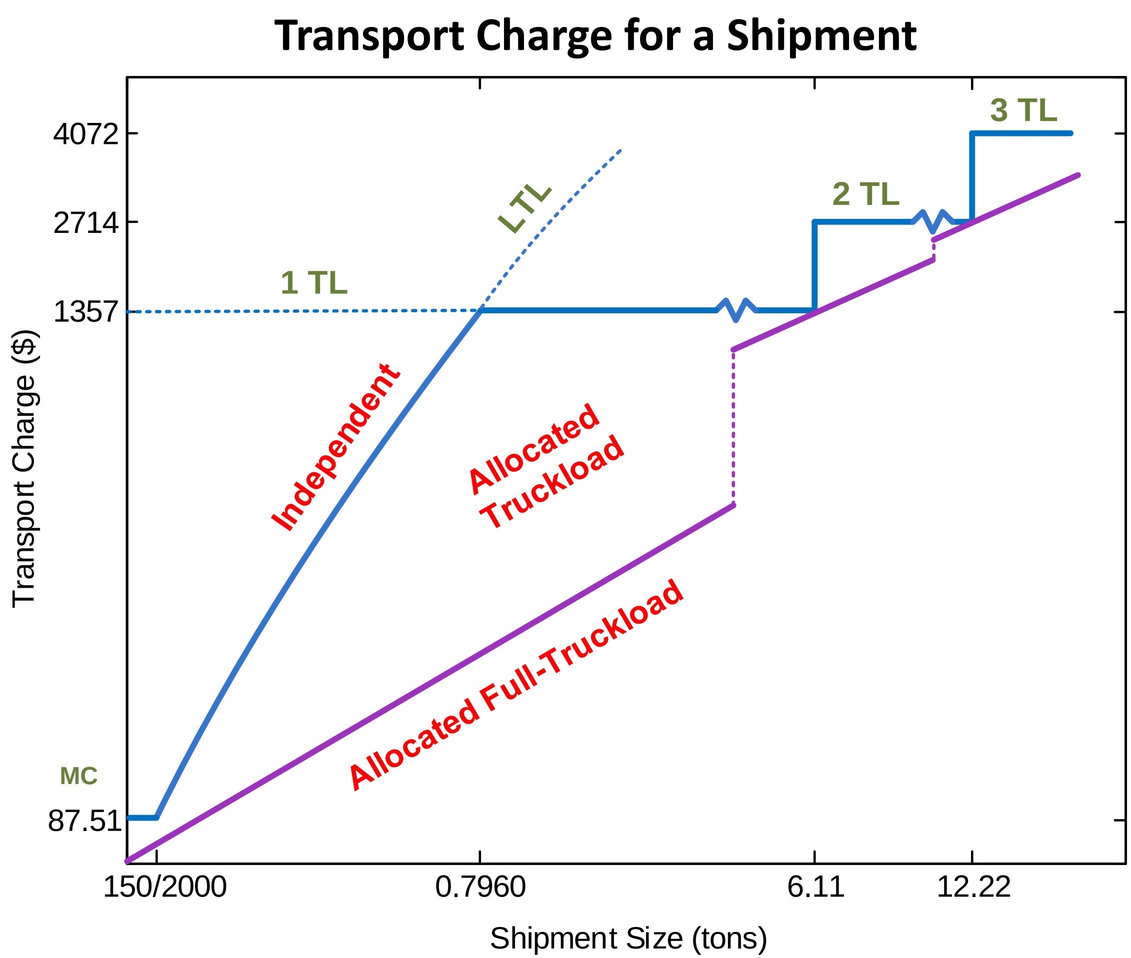
$$TC'_{FTL} = \max\left\{n, n_{\min}\right\} r_{TL} d$$

$$= \max\left\{3.2727, 4\right\} 2.5511(532) = \$5,428.78/\text{yr}$$

Truck Shipment Example: Periodic

- Independent and allocated full-truckload charges:

$$q \le q_{\max} \Rightarrow [UB, LB] = [c_0(q), q r_{FTL} d]$$



Truck Shipment Example: Periodic

- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$
\begin{aligned}
TLC &= TC + IC + PC \\
TC &= \text{transport cost} \\
IC &= \text{inventory cost} \\
&= IC_{\text{cycle}} + IC_{\text{pipeline}} + IC_{\text{safety}} \\
PC &= \text{purchase cost}
\end{aligned}
$$

- *Cycle inventory*: held to allow cheaper large shipments
- *Pipeline inventory*: goods in transit or awaiting transshipment
- *Safety stock*: held due to transport uncertainty
- *Purchase cost*: can be different for different suppliers

Truck Shipment Example: Periodic

- Same units of inventory can serve multiple roles at each position in a production process

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)
- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)
- *Safety stock:* held to buffer effects of uncertainty
  - (decoupling, MRO (maintenance, repair, and operations))

Truck Shipment Example: Periodic

14. Since demand is constant throughout the year, one half of a shipment is stored at the destination, on average. Assuming that the production rate is also constant, one half of a shipment will also be stored at the origin, on average. Assuming each ton of the product is valued at $25,000, what is a "reasonable estimate" for the total annual cost for this cycle inventory?

$$IC_{\text{cycle}} = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$

$$= (vh)(\alpha q)$$

$$v = \text{unit value of shipment (\$/ton)}$$

$$h = \text{inventory carrying rate, the cost per dollar of inventory per year (1/yr)}$$

$$\alpha = \text{average inter-shipment inventory fraction at Origin and Destination}$$

$$q = \text{shipment size (ton)}$$

Truck Shipment Example: Periodic

- **Inv. Carrying Rate** (*h*) = interest + warehousing + obsolescence
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate (*yr*) h = 0.3 $\Rightarrow h_{\text{obs}} \approx$ 0.2 (mfg product)
  - *Low* FGI cost (*yr*): $h = h_{\text{int}} + h_{\text{wh}} + h_{\text{obs}}$
  - *High* FGI cost (*hr*): $h \approx h_{\text{obs}}$, can ignore interest & warehousing
    - $(h_{\text{int}}+h_{\text{wh}})/H$ = (0.05+0.06)/2000 = 0.000055 (H = oper. hr/yr)
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h_{obs}

$$h_{\text{obs}} t_h v = x_h v \Rightarrow h_{\text{obs}} t_h = x_h \Rightarrow \boxed{h_{\text{obs}} = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h_{\text{obs}}}$$

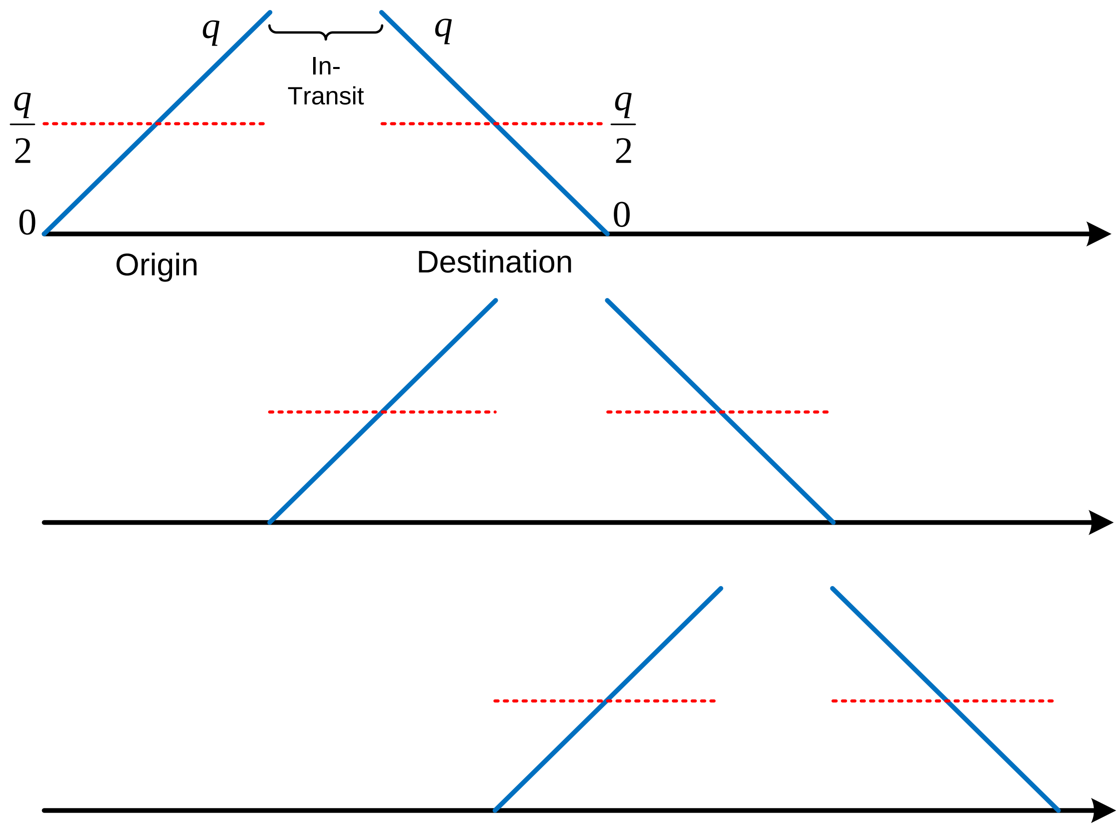
  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as t_{CT}

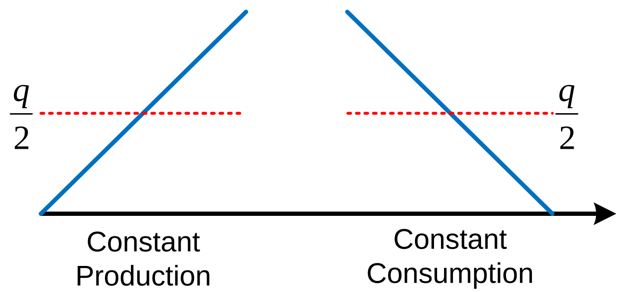
Truck Shipment Example: Periodic

- Average annual inventory level $= \frac{q}{2} + \frac{q}{2} = \left(\frac{1}{2} + \frac{1}{2}\right)q = (1)q \Rightarrow \alpha = 1$

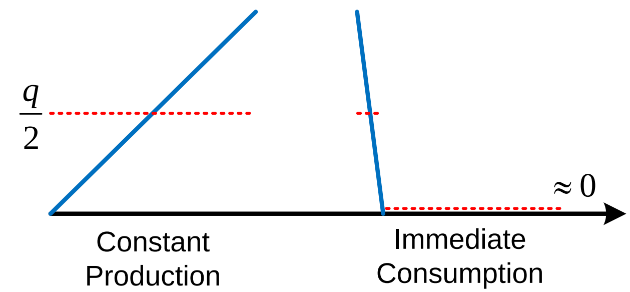


Truck Shipment Example: Periodic

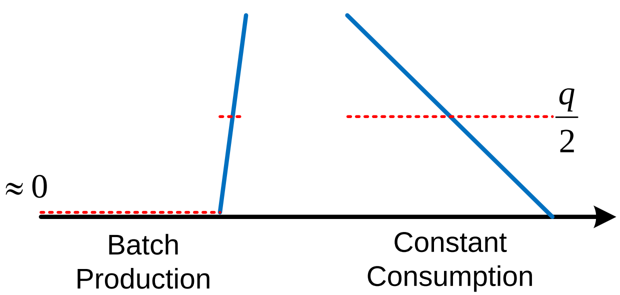
- Inter-shipment inventory fraction alternatives: $\alpha = \alpha_O + \alpha_D$



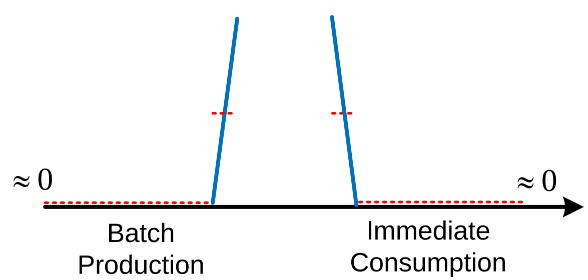
$$\alpha = \frac{1}{2} + \frac{1}{2} = 1$$



$$\alpha = \frac{1}{2} + 0 = \frac{1}{2}$$



$$\alpha = 0 + \frac{1}{2} = \frac{1}{2}$$



$$\alpha = 0 + 0 = 0$$

Truck Shipment Example: Periodic

- "Reasonable estimate" for the total annual cost for the cycle inventory:

$$
\begin{aligned}
IC_{\text{cycle}} &= \alpha v h q \\
&= (1)(25{,}000)(0.3)6.1111 \\
&= \$45{,}833.33 \,/\, \text{yr}
\end{aligned}
$$

where

$$\alpha = \frac{1}{2} \text{ at Origin } + \frac{1}{2} \text{ at Destination} = 1$$

$$v = \$25{,}000 = \text{unit value of shipment (\$/ton)}$$

$$h = 0.3 = \text{estimated carrying rate for manufactured products (1/yr)}$$

$$q = q_{\max} = 6.111 = \text{FTL shipment size (ton)}$$

Truck Shipment Example: Periodic

15. What is the annual total logistics cost (TLC) for these full-truckload TL shipments?

$$
\begin{aligned}
TLC_{FTL} &= TC_{FTL} + IC_{\text{cycle}} \\
&= n\, r_{TL} d + \alpha v h q \\
&= 3.2727\,(2.5511)\,532 + (1)(25,000)(0.3)6.1111 \\
&= 4,441.73 + 45,833.33 \\
&= \$50,275.06 / \, yr
\end{aligned}
$$

Truck Shipment Example: Periodic

16. What is minimum possible annual total logistics cost for TL shipments, where the shipment size can now be less than a full truckload?

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha vhq = \frac{f}{q} rd + \alpha vhq$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q_{TL}^* = \sqrt{\frac{f\, r_{TL} d}{\alpha vh}} = \sqrt{\frac{20(2.5511)532}{(1)25000(0.3)}} = 1.9024 \text{ ton}$$

$$TLC_{TL}(q_{TL}^*) = \frac{f}{q_{TL}^*} r_{TL} d + \alpha vhq_{TL}^*$$

$$= \frac{20}{1.8553}(2.5511)532 + (1)25000(0.3)1.8553$$

$$= 14,268.12 + 14,268.12$$

$$= \$28,536.25 / \text{yr}$$

Truck Shipment Example: Periodic

- Including the minimum charge and maximum payload restrictions:

$$q_{TL}^{*} = \min\left\{\sqrt{\frac{f \max\left\{r_{TL}d, MC_{TL}\right\}}{\alpha v h}}, q_{\max}\right\} \approx \sqrt{\frac{f r_{TL} d}{\alpha v h}}$$

- What is the TLC if this size shipment could be made as an allocated full-truckload?

$$TLC_{AllocFTL}(q_{TL}^{*}) = \frac{f}{q_{TL}^{*}}\left(q_{TL}^{*} r_{FTL} d\right) + \alpha v h q_{TL}^{*} = f\,\frac{r_{TL}}{q_{\max}}d + \alpha v h q_{TL}^{*}$$

$$= 20\,\frac{2.5511}{6.1111}532 + (1)25000(0.3)1.9024$$

$$= 4,441.73 + 14,268.12$$

$$= \$18,709.85\,/\,\text{yr} \quad \left(\text{vs. } \$28,536.25 \text{ as independent P2P TL}\right)$$

Truck Shipment Example: Periodic

17. What is the optimal LTL shipment size?

$$TLC_{LTL}(q) = TC_{LTL}(q) + IC(q) = \frac{f}{q} c_{LTL}(q) + \alpha v h q$$

$$q_{LTL}^{*} = \arg\min_{q} TLC_{LTL}(q) = 0.7622 \text{ ton}$$

- Must be careful in picking starting point for optimization since LTL formula only valid for limited range of values:

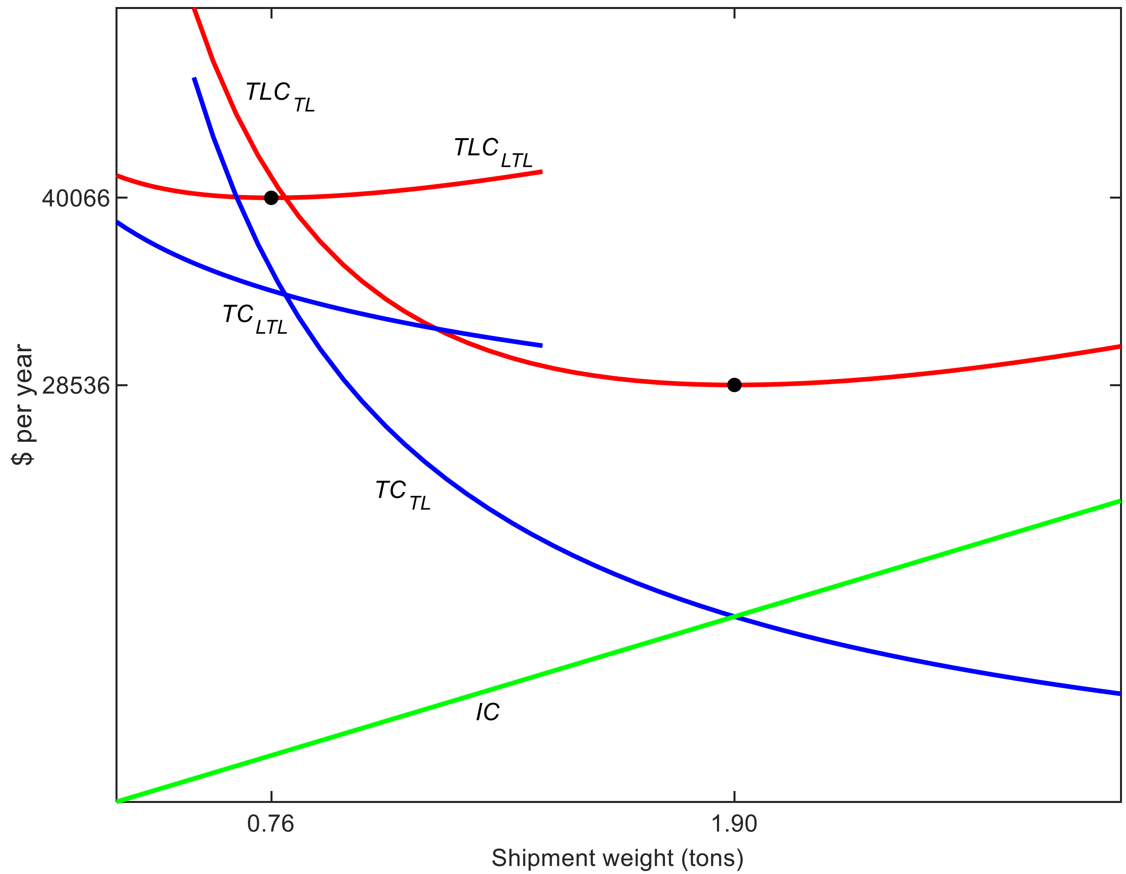
$$r_{LTL} = PPI_{LTL}\left[\frac{\dfrac{s^2}{8}+14}{\left(q^{\frac{1}{7}} d^{\frac{15}{29}} - \dfrac{7}{2}\right)\left(s^2+2s+14\right)}\right], \quad \begin{cases} 37 \le d \le 3354 \text{ (dist)} \\ \dfrac{150}{2,000} \le q \le \dfrac{10,000}{2,000} \text{ (wt)} \\ 2000\dfrac{q}{s} \le 650 \text{ ft}^3 \text{ (cube)} \end{cases}$$

$$\frac{150}{2000} \le q \le \min\left\{\frac{10,000}{2,000}, \frac{650s}{2000}\right\} \Rightarrow 0.075 \le q \le 1.44$$

Truck Shipment Example: Periodic

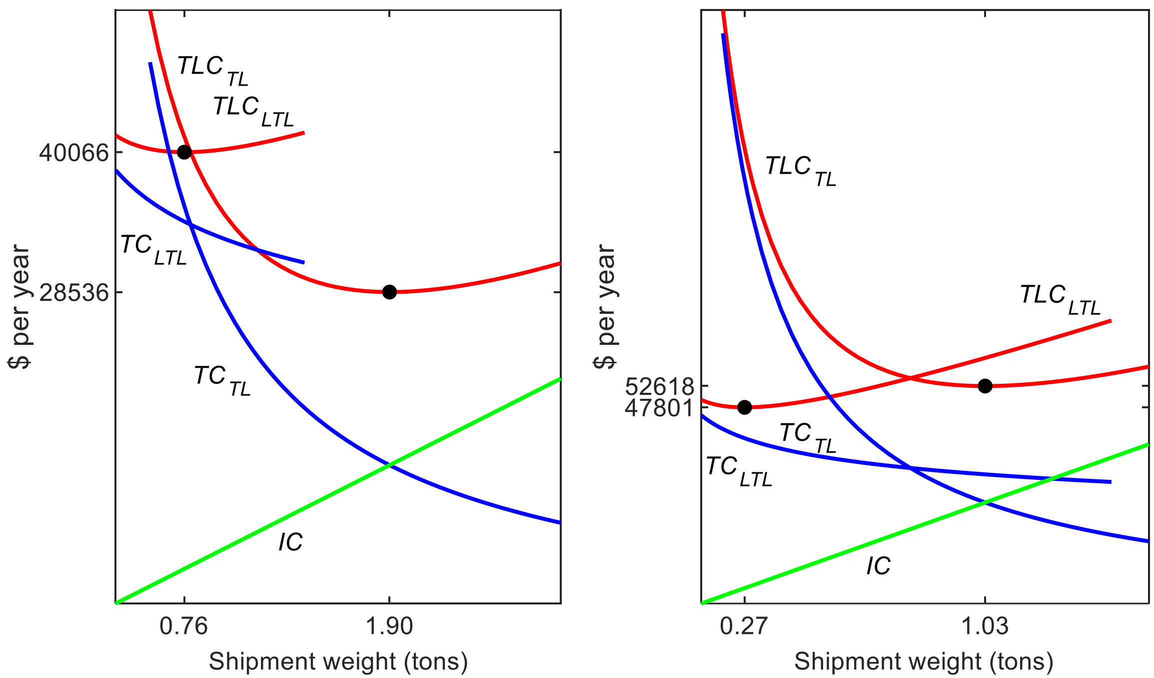
18. Should the product be shipped TL or LTL?

$$TLC_{LTL}(q^*_{LTL}) = TC_{LTL}(q^*_{LTL}) + IC(q^*_{LTL}) = 34{,}349.19 + 5{,}716.40 = \$40{,}065.59\,/\,\text{yr}$$



Truck Shipment Example: Periodic

19. If the value of the product increased to \$85,000 per ton, should the product be shipped TL or LTL?

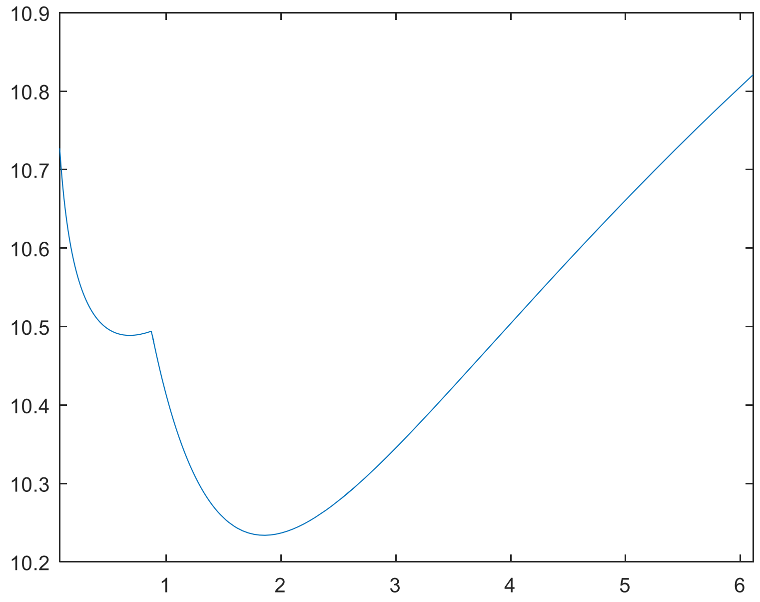


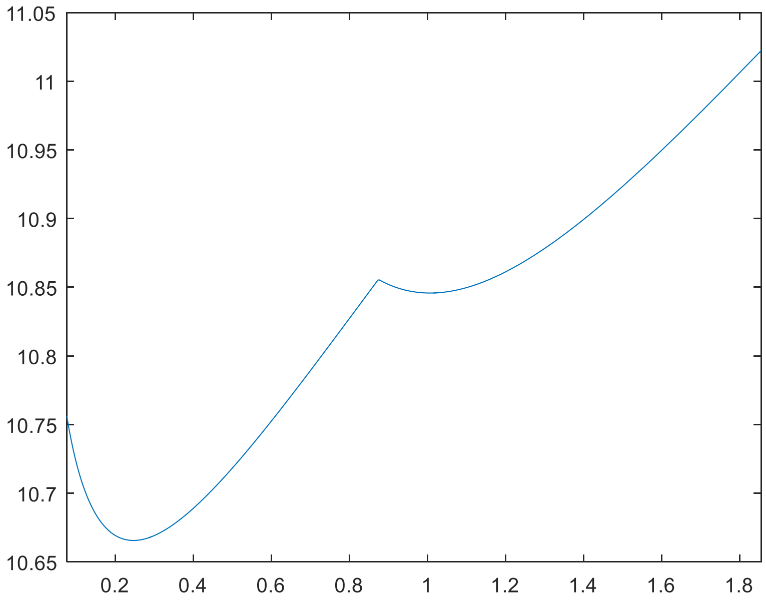
(a) \$25000 value per ton (b) \$85000 value per ton

Truck Shipment Example: Periodic

- Better to pick from separate optimal TL and LTL because independent charge has two local minima:

$$q_0^* = \arg\min_q \left\{ TLC_{TL}(q), TLC_{LTL}(q) \right\} \qquad q_0^* \overset{!}{=} \arg\min_q \left\{ \frac{f}{q} c_0(q) + \alpha v h q \right\}$$





Truck Shipment Example: Periodic

20. What is optimal independent shipment size to ship 80 tons per year of a Class 60 product valued at $5000 per ton between Raleigh and Gainesville?

$$s = 32.16 \text{ lb/ft}^3$$

$$q_0^* = \arg\min_q \left\{ TLC_{TL}(q), TLC_{LTL}(q) \right\} = 8.5079 \text{ ton}$$

$$TLC_{TL}(q_0^*) = \$25,523.60 \,/\, \text{yr} < TLC_{LTL}(q_0^*)$$

Truck Shipment Example: Periodic

21. What is the optimal shipment size if both shipments will always be shipped together on the same truck (with same shipment interval)?

$$d_1 = d_2, \quad h_1 = h_2, \quad \alpha_1 = \alpha_2$$

$$f_{\text{agg}} = f_1 + f_2 = 20 + 80 = 100 \text{ ton}$$

$$s_{\text{agg}} = \frac{(\text{aggregate weight, in lb})}{(\text{aggregate cube, in ft}^3)} = \frac{f_{\text{agg}}}{\frac{f_1}{s_1} + \frac{f_2}{s_2}} = \frac{100}{\frac{20}{4.44} + \frac{80}{32.16}} = 14.31 \text{ lb/ft}^3$$

$$v_{\text{agg}} = \frac{f_1}{f_{\text{agg}}} v_1 + \frac{f_2}{f_{\text{agg}}} v_2 = \frac{20}{100} 85,000 + \frac{80}{100} 5000 = \$21,000 \text{ / ton}$$

$$q_{TL}^* = \sqrt{\frac{f_{\text{agg}} r d}{\alpha v_{\text{agg}} h}} = \sqrt{\frac{100(2.5511)532}{(1)21000(0.3)}} = 4.6414 \text{ ton}$$

Truck Shipment Example: Periodic

- Summary of results:

	f	s	v	qmax	TLC	q	t
1:	20	4.44	85,000	6.11	47,801.01	0.27	5.00
2:	80	32.16	5,000	25.00	25,523.60	8.51	38.84
1+2:					73,324.60		
Aggregate:	100	14.31	21,000	19.68	58,481.90	4.64	16.95

Ex 11: FTL vs Interval Constraint

- On average, 200 tons of components are shipped 750 miles from your fabrication plant to your assembly plant each year. The components are produced and consumed at a constant rate throughout the year. Currently, full truckloads of the material are shipped. What would be the impact on total annual logistics costs if TL shipments were made every two weeks? The revenue per loaded truck-mile is \$2.00; a truck's cubic and weight capacities are 3,000 ft^3 and 24 tons, respectively; each ton of the material is valued at \$5,000 and has a density of 10 lb per ft^3; the material loses 30% of its value after 18 months; and in-transit inventory costs can be ignored.

$$f = 200, \quad d = 750, \quad \alpha = \frac{1}{2} + \frac{1}{2} = 1, \quad r_{TL} = 2, \quad K_{cu} = 3000, \quad K_{wt} = 24, \quad v = 5000, \quad s = 10$$

$$h_{\text{obs}} = \frac{x_h}{t_h} = \frac{0.3}{1.5} = 0.2 \Rightarrow h = 0.05 + 0.06 + 0.2 = 0.31, \quad q_{FTL} = q_{\max} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\} = 15$$

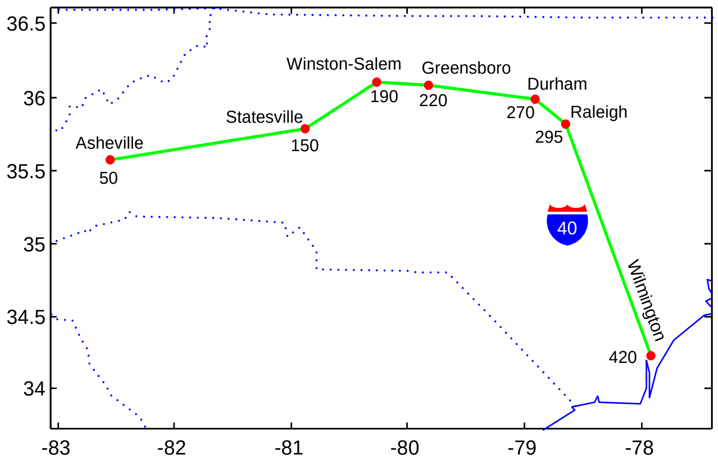
$$n_{FTL} = \frac{f}{q_{FTL}} = 13.33, TLC_{FTL} = n_{FTL} r_{TL} d + \alpha v h q_{FTL} = 43{,}250, \quad \text{2-wk TL} \Rightarrow \text{LTL not considered}$$

$$t_{\max} = \frac{2 \cdot 7}{365.25} \Rightarrow n_{\min} = 26.09, \quad q_{2\text{wk}} = \frac{f}{n_{\min}} = 7.67, \quad TLC_{2\text{wk}} = n_{\min} r_{TL} d + \alpha v h q_{2\text{wk}} = 51{,}016$$

$$\Delta TLC = TLC_{2\text{wk}} - TLC_{FTL} = \$7{,}766 \text{ per year increase with two-week interval constraint}$$

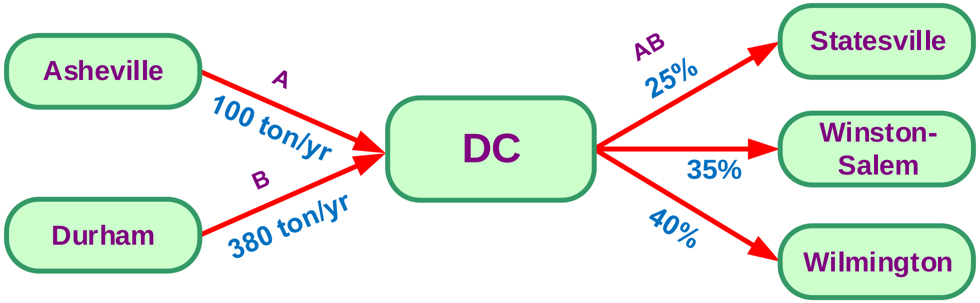
Ex 12: FTL Location

- Where should a DC be located in order to minimize transportation costs, given:



1. FTLs containing mix of products A and B shipped P2P from DC to customers in Winston-Salem, Durham, and Wilmington
2. Each customer receives 20, 30, and 50% of total demand
3. 100 tons/yr of A shipped FTL P2P to DC from supplier in Asheville
4. 380 tons/yr of B shipped FTL P2P to DC from Statesville
5. Each carton of A weighs 30 lb, and occupies 10 ft^3
6. Each carton of B weighs 120 lb, and occupies 4 ft^3
7. Revenue per loaded truck-mile is $2
8. Each truck's cubic and weight capacity is 2,750 ft^3 and 25 tons, respectively

Ex 12: FTL Location



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

$$\underset{(\$/\text{mi-yr})}{w_i} = \underset{(\text{ton/yr})}{f_i} \times \underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \underset{(\text{TL/yr})}{n_i} \times \underset{(\$/\text{TL-mi})}{r_i}$$

$$\underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \frac{r}{q_{\max}}, \quad n = \frac{f}{q_{\max}}$$

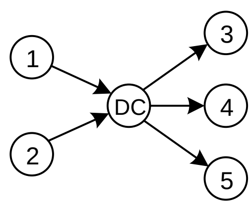
$$r = \$2/\text{TL-mi}, \quad f_{agg} = f_A + f_B = 100 + 380 = 480 \text{ ton/yr}, \quad s_{agg} = \frac{f_{agg}}{\frac{f_A}{s_A} + \frac{f_B}{s_B}} = \frac{480}{\frac{100}{3} + \frac{380}{30}} = 10.4348 \text{ lb/ft}^3, \quad q_{\max} = \left\{25, \frac{10.4348(2750)}{2000}\right\} = 14.3478$$

$$s_1 = \frac{30}{10} = 3 \text{ lb/ft}^3, \ q_{\max} = \min\left\{25, \frac{3(2750)}{2000}\right\} = 4.125 \text{ ton}$$

$$f_1 = 100, \ n_1 = \frac{100}{4.125} = 24.24, \ w_1 = 24.24(2) = 48.48$$

$$s_2 = \frac{120}{4} = 30 \text{ lb/ft}^3, \ q_{\max} = \min\left\{25, \frac{30(2750)}{2000}\right\} = 25 \text{ ton}$$

$$f_2 = 380, \ n_2 = \frac{380}{25} = 15.2, \ w_2 = 15.2(2) = 30.4$$

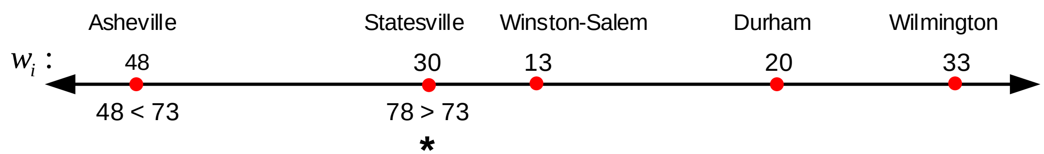


$$f_3 = 0.20 f_{agg} = 96, \ n_3 = \frac{96}{14.3478} = 6.69, \ w_3 = 6.69(2) = 13.38$$

$$f_4 = 0.30 f_{agg} = 144, \ n_4 = \frac{144}{14.3478} = 10.04, \ w_4 = 10.04(2) = 20.07$$

$$f_5 = 0.50 f_{agg} = 240, \ n_5 = \frac{240}{14.3478} = 16.73, \ w_5 = 16.73(2) = 33.45$$

$$W = 146, \quad \frac{W}{2} = 73$$



$$\text{(Montetary) Weight Losing:} \quad \Sigma w_{in} = 79 > \Sigma w_{out} = 67 \quad \left(\Sigma n_{in} = 39 > \Sigma n_{out} = 33\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{in} = 480 = \Sigma f_{out} = 480$$

Ex 12: FTL Location

- Include monthly outbound frequency constraint:
  - Outbound shipments must occur at least once each month
  - Implicit means of including inventory costs in location decision

$$t_{\max} = \frac{1}{12}\ \text{yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 12\ \text{TL/yr}$$

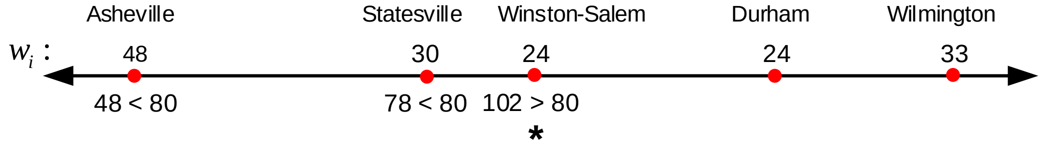
$$TC'_{FTL} = \max\left\{n, n_{\min}\right\} r d$$

$$n_3 = \max\left\{6.69, 12\right\} = 12, w_3 = 12(2) = 24$$

$$n_4 = \max\left\{10.04, 12\right\} = 12, w_4 = 12(2) = 24$$

$$n_5 = \max\left\{16.73, 12\right\} = 16.73, w_5 = 16.73(2) = 33.45$$

$$W = 160, \quad \frac{W}{2} = 80$$



$$\text{(Montetary) Weight Gaining:} \quad \Sigma w_{\text{in}} = 79 < \Sigma w_{\text{out}} = 81 \quad \left(\Sigma n_{\text{in}} = 39 < \Sigma n_{\text{out}} = 41\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$$

Location and Transport Costs

- Monetary weights *w* used for location are, in general, a function of the location of a NF
  - Distance *d* appears in optimal TL size formula
  - TC & IC functions of location $\Rightarrow$ Need to minimize TLC instead of TC
  - FTL (since size is fixed at max payload) results in only constant weights for location $\Rightarrow$ Need to only minimize TC since IC is constant in TLC

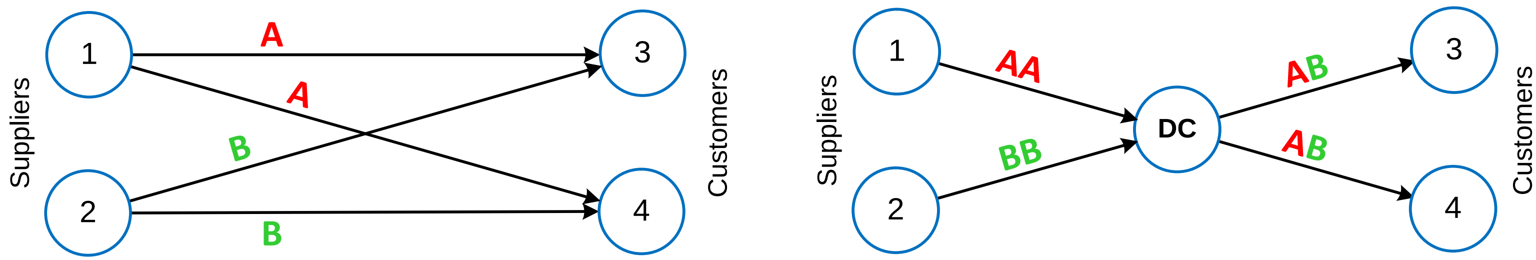
$$TLC_{TL}(\mathbf{x}) = \sum_{i=1}^{m} w_i(\mathbf{x}) d_i(\mathbf{x}) + \alpha v h q_i(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_i(\mathbf{x})} r d_i(\mathbf{x}) + \alpha v h\, q_i(\mathbf{x})$$

$$= \sum_{i=1}^{m} \frac{f_i}{\sqrt{\dfrac{f_i r d_i(\mathbf{x})}{\alpha v h}}} r d_i(\mathbf{x}) + \alpha v h \sqrt{\frac{f_i r d_i(\mathbf{x})}{\alpha v h}} = \sum_{i=1}^{m} \sqrt{f_i r d_i(\mathbf{x})} \left(\frac{1}{\sqrt{\alpha v h}} + \sqrt{\alpha v h} \right)$$

$$TLC_{FTL}(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_{\max}} r d_i(\mathbf{x}) + \alpha v h\, q_{\max} = \sum_{i=1}^{m} w_i d_i(\mathbf{x}) + \alpha v h\, q_{\max} = TC_{FTL}(\mathbf{x}) + \text{constant}$$

Transshipment

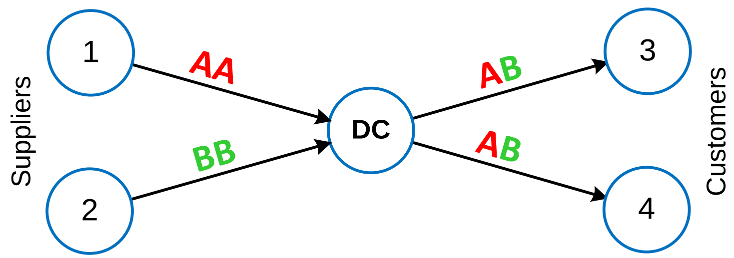
- *Direct*: P2P shipments from Suppliers to Customers



- *Transshipment*: use DC to consolidate outbound shipments
  - *Uncoordinated*: determine separately each optimal inbound and outbound shipment $\Rightarrow$ hold inventory at DC
  - *(Perfect) Cross-dock*: use single shipment interval for all inbound and outbound shipments $\Rightarrow$ no inventory at DC (usually only cross-dock a selected subset of shipments)

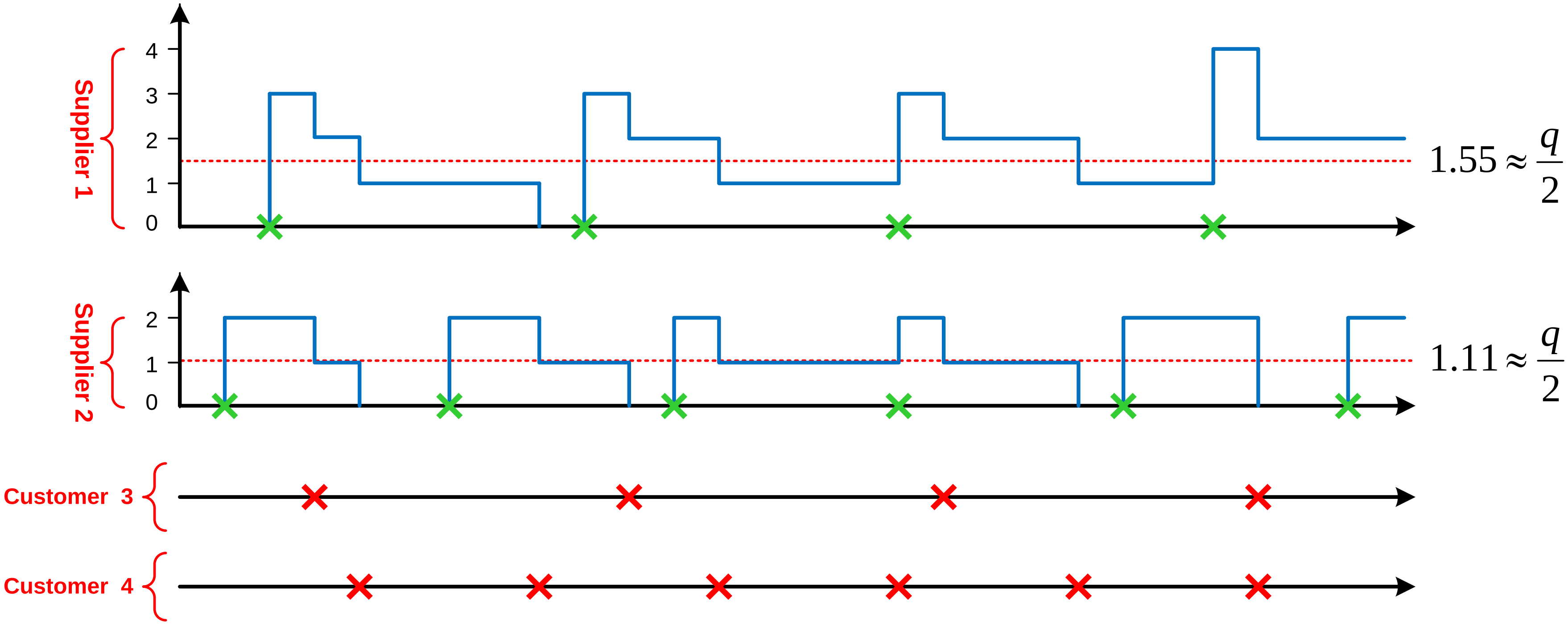
Uncoordinated Inventory

- Average pipeline inventory level at DC:



$$\alpha = \alpha_O + \alpha_D$$

$$= \begin{cases} \alpha_O + \frac{1}{2}, & \text{inbound} \\ 0 + \alpha_D, & \text{outbound} \end{cases}$$



TLC with Transshipment

- Uncoordinated: $TLC_i = TLC$ of supplier/customer i

$$q_i^* = \arg\min_q TLC_i(q)$$

$$TLC^* = \sum TLC_i\left(q_i^*\right)$$

- Cross-docking: $t = \dfrac{q}{f}$, shipment interval

$$TLC_i(t) = \frac{c_0(t)}{t} + \alpha v h f t \quad \left(\text{cf. } TLC_i(q) = \frac{f}{q} c_0(q) + \alpha v h q\right)$$

$$c_0(t) = \text{independent transport charge as function of } t$$

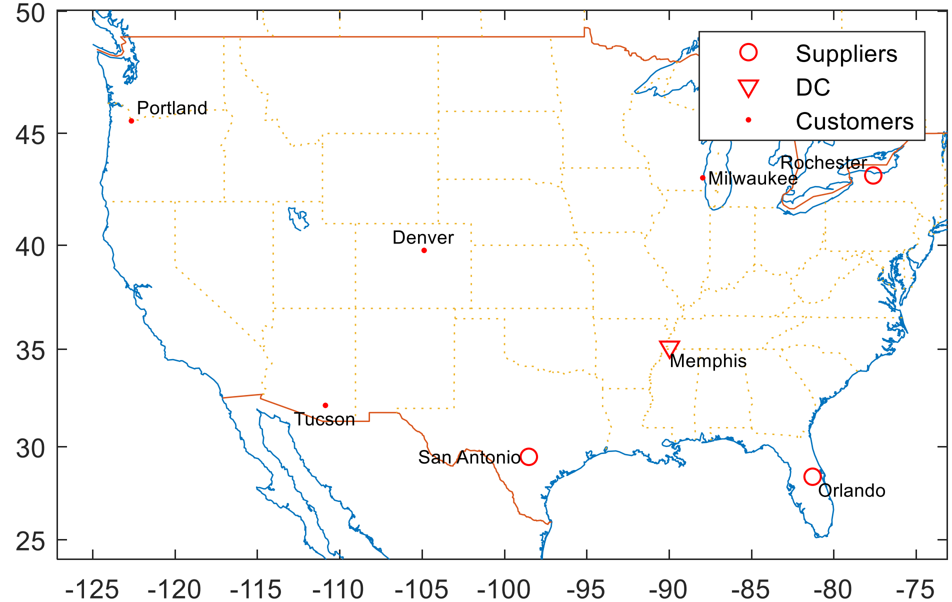
$$\alpha = \begin{cases} \alpha_O + 0, & \text{inbound} \\ 0 + \alpha_D, & \text{outbound} \end{cases}$$

$$t^* = \arg\min_t \sum TLC_i(t)$$

$$TLC^* = \sum TLC_i\left(t^*\right)$$

Ex 13: Direct vs Transhipment

- 3 different products supplied to 4 customers, compare:
  1. Direct shipments
  2. Uncoordinated at existing DC in Memphis
  3. Cross-docking at Memphis
  4. Uncoordinated at optimal DC location
  5. Cross-docking at optimal location



Transshipment Example:	TLC	t	LTL
Direct:	783,139.06	50.71	4
Uncoordinated at Memphis:	797,281.81	18.67	0
Cross-Docking at Memphis:	634,470.88	18.26	0
Uncoordinated at Optimal Location:	754,102.09	15.65	0
Cross-Docking at Optimal Location:	598,052.01	18.26	0

TLC and Location

- TLC should include all logistics-related costs

  $\Rightarrow$ TLC can be used as sole objective for network design (incl. location)

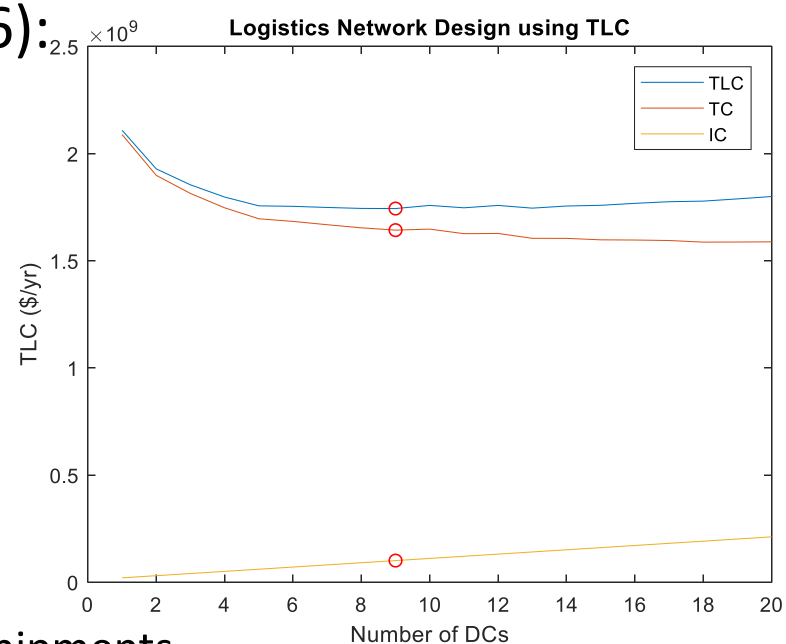
- Facility fixed costs, two options:
  1. Use non-transport-related facility costs (mix of top-down and bottom-up) to estimate fixed costs via linear regression
  2. For DCs, might assume public warehouses to be used for all DCs

     $\Rightarrow$ Pay only for time each unit spends in WH $\Rightarrow$ No fixed cost at DC

- Transport fixed costs:
  - Costs that are independent of shipment size (e.g., \$/mi vs. \$/ton-mi)
    - Costs that make it worthwhile to incur the inventory cost associated with larger shipment sizes in order to spread out the fixed cost
  - Main transport fixed cost is the indivisible labor cost for a human driver
    - Why many logistics networks (e.g., Walmart, Lowes) designed for all FTL transport

Ex 14: Optimal Number DCs for Lowe's

- Example of logistics network design using TLC
- Lowe's logistics network (2016):
  - Regional DCs (15)
  - Costal holding facilities
  - Appliance DCs and Flatbed DCs
  - Transloading facilities



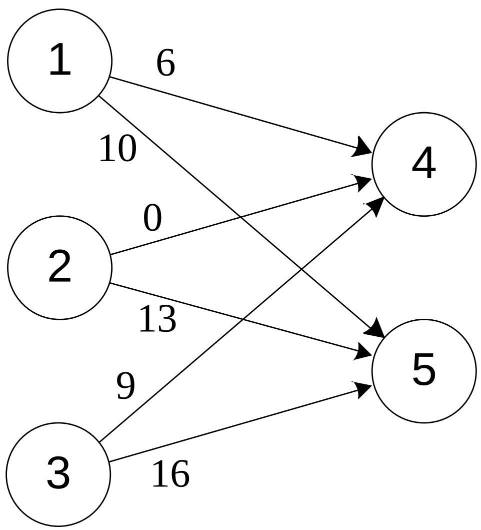
- Modeling approach:
  - Focus only on Regional DCs
  - Mix of top-down (COGS) and bottom-up (typical load/TL parameters)
  - FTL for all inbound and outbound shipments
  - ALA used to determine TC for given number of DCs
  - $IC = \alpha v h q_{\max} \times$ (number of suppliers $\times$ number of DCs + number stores)
  - Assume uncoordinated DC inventory, no cross-docking
  - Ignoring max DC-to-store distance constraints, consolidation, etc.
- Determined 9 DCs min TLC (15 DCs $\Rightarrow$ 0.87% increase in TLC)

Topics

1. Introduction
2. Facility location
3. Freight transport
   – Exam 1 (take home)
4. **Network models**
5. Routing
   – Exam 2 (take home)
6. Warehousing
   – Final exam (in class)

Graph Representations

- Complete bipartite directed (or digraph):
  - Suppliers to multiple DCs, single mode of transport




```
C:   1    2
-:--------
1:   6   10
2:   0   13
3:   9   16
```

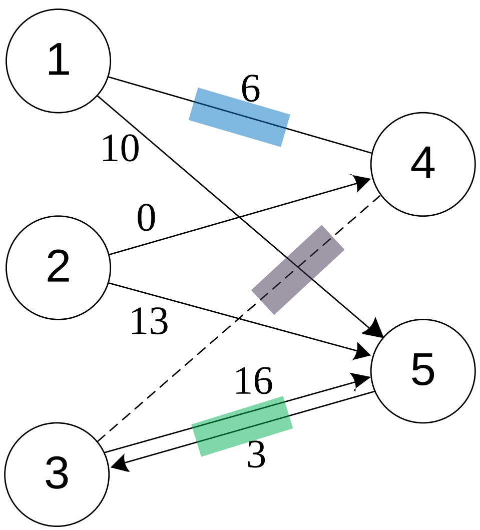
Interlevel matrix

$$W = \begin{matrix} 0 & 0 & 0 & 6 & 10 \\ 0 & 0 & 0 & \text{NaN} & 13 \\ 0 & 0 & 0 & 9 & 16 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Weighted adjacency matrix

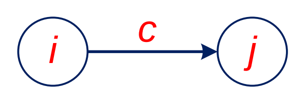
Graph Representations

- Bipartite:
  - One- or two-way connections between nodes in two groups



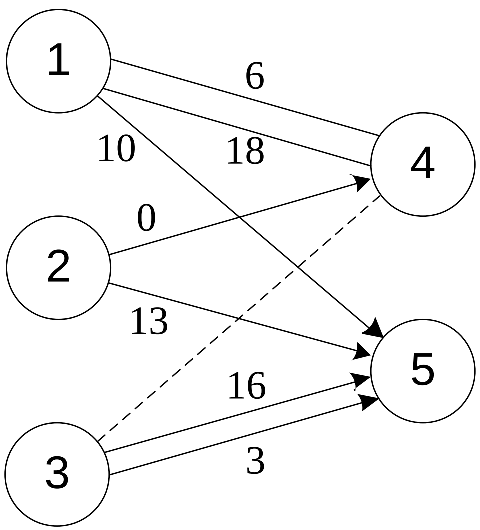

```
W =     0     0     0     6    10
        0     0     0   NaN    13
        0     0     0     0    16
        6     0     0     0     0
        0     0     3     0     0

IJC =
        4     1     6
        5     3     3
        1     4     6
        2     4     0
        1     5    10
        2     5    13
        3     5    16
```

Arc list matrix



Graph Representations

- Multigraph:
  - Multiple connections, multiple modes of transport

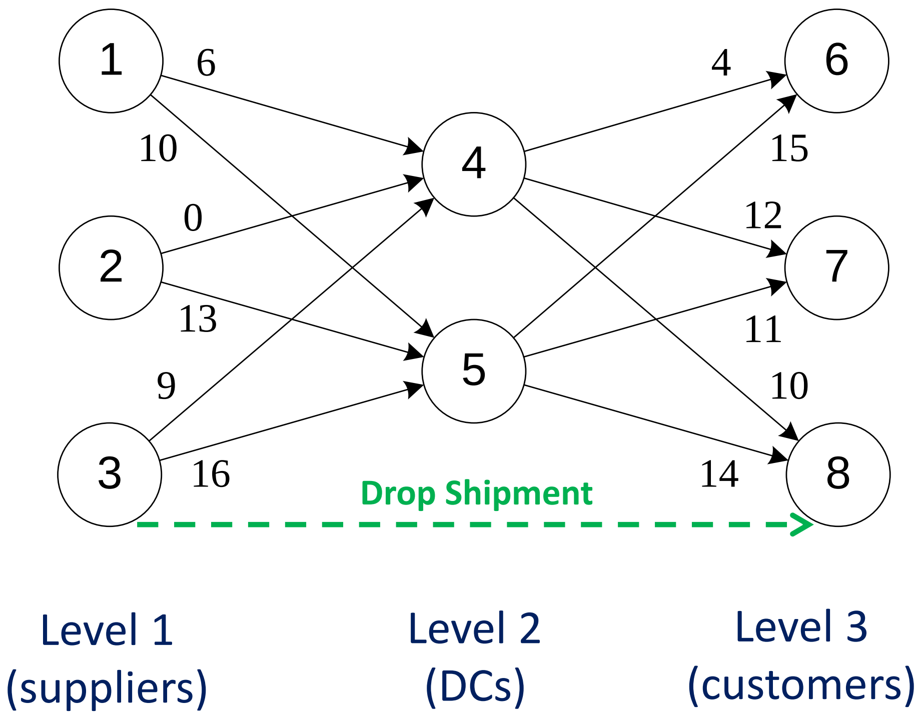



```
IJC = 1    -4    6
      1    -4   18
      1     5   10
      2     4    0
      2     5   13
      3     5   16
      3     5    3
no_W =
      0     0     0    24    10
      0     0     0   NaN    13
      0     0     0     0    19
     24     0     0     0     0
      0     0     0     0     0
```

Can’t represent using adjacency matrix

Graph Representations

- Complete multipartite directed:
  - Typical supply chain (no drop shipments)



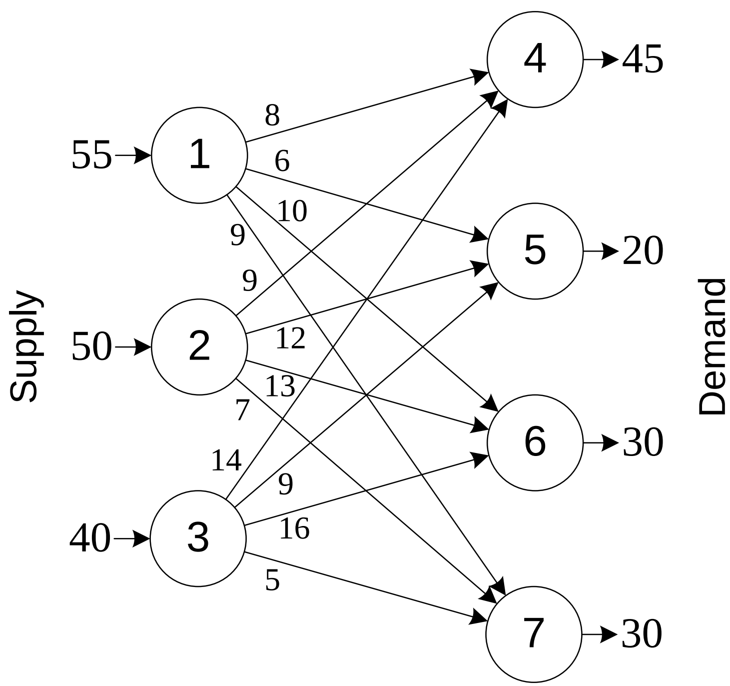
C12:	1	2
1:	6	10
2:	0	13
3:	9	16

C23:	1	2	3
1:	4	12	10
2:	15	11	14

W:	1	2	3	4	5	6	7	8
1:	0	0	0	6	10	0	0	0
2:	0	0	0		13	0	0	0
3:	0	0	0	9	16	0	0	0
4:	0	0	0	0	0	4	12	10
5:	0	0	0	0	0	15	11	14
6:	0	0	0	0	0	0	0	0
7:	0	0	0	0	0	0	0	0
8:	0	0	0	0	0	0	0	0

Transportation Problem

- Satisfy node demand from supply nodes
  - Can be used for allocation in ALA when NFs have capacity constraints
  - Min cost/distance allocation ⇔ infinite supply at each node



Trans	4	5	6	7	Supply
1	8	6	10	9	55
2	9	12	13	7	50
3	14	9	16	5	40
Demand	45	20	30	30	

Greedy Solution Procedure

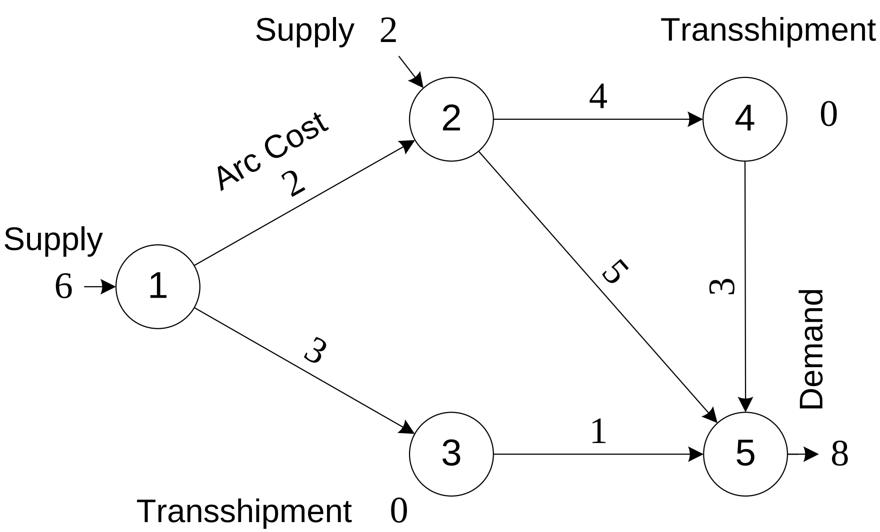
- Procedure for transportation problem: *Continue to select lowest cost supply until all demand is satisfied*
  - Fast, but not always optimal for transportation problem
  - Dijkstra's shortest path and simplex method for LP are optimal greedy procedures

Trans	4	5	6	7	Supply	
1	8	6	10	9	55	-20 = 35 -35 = 0
2	9	12	13	7	50	-10 = 40 -30 = 10
3	14	9	16	5	40	-30 = 10
Demand	~~45~~	~~20~~	~~30~~	~~30~~		
	~~10~~ 0	0	0	0		

$$TC = 5(30) + 6(20) + 8(35) + 9(10) + 13(30) = 1,030 \quad \left(\text{vs } 970 \text{ optimal}\right)$$

Min Cost Network Flow (MCNF) Problem

- Most general network problem, can solve using any type of graph representation



$$s_i = \text{net supply of node } i$$

$$= \begin{cases} > 0, & \text{supply node} \\ < 0, & \text{demand node} \\ = 0, & \text{transshipment node} \end{cases}$$

$$\text{Arc cost: } \mathbf{c} = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 & 3 \end{bmatrix}'$$

$$\text{Net node supply: } \mathbf{s} = \begin{bmatrix} 6 & 2 & 0 & 0 & -8 \end{bmatrix}'$$

$$\text{Incidence Matrix: } \mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

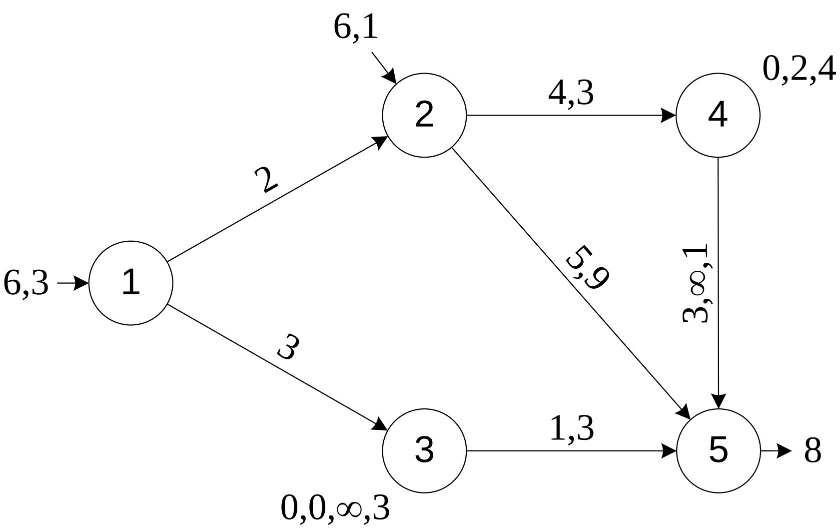
MCNF:	lhs	C	C	C	C	C	C	rhs
Min:		2	3	4	5	1	3	
1:	6	1	1	0	0	0	0	6
2:	2	-1	0	1	1	0	0	2
3:	0	0	-1	0	0	1	0	0
4:	0	0	0	-1	0	0	1	0
lb:		0	0	0	0	0	0	
ub:		Inf	Inf	Inf	Inf	Inf	Inf	

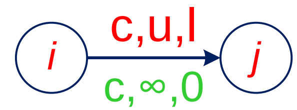
Row for node 5 is redundant

$$\begin{aligned} \text{MCNF:} \quad & \max \ \mathbf{c}'\mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} = \mathbf{s} \\ & \mathbf{x} \geq 0 \end{aligned}$$

MCNF with Arc/Node Bounds and Node Costs

- Bounds on arcs/nodes can represent capacity constraints in a logistic network
- Node cost can represent production cost or intersection delay

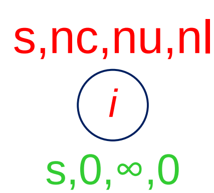




IJCUL:	1	2	3	4	5
1:	1	2	2	Inf	0
2:	1	3	3	Inf	0
3:	2	4	4	3	0
4:	2	5	5	9	0
5:	3	5	1	3	0
6:	4	5	3	Inf	1

$$s_i = \text{net supply of node } i$$

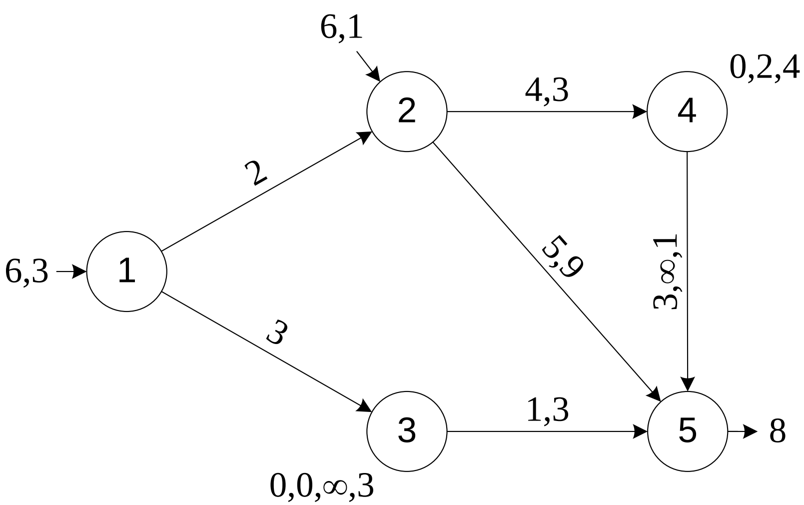
$$= \begin{cases} > 0, & \text{supply node} \\ < 0, & \text{demand node} \\ = 0, & \text{transshipment node} \end{cases}$$

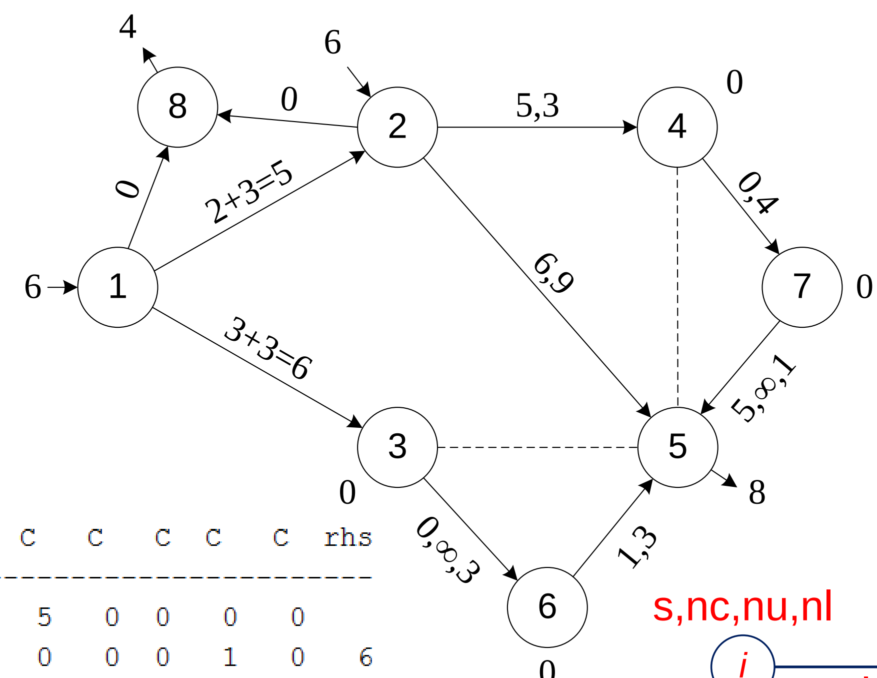


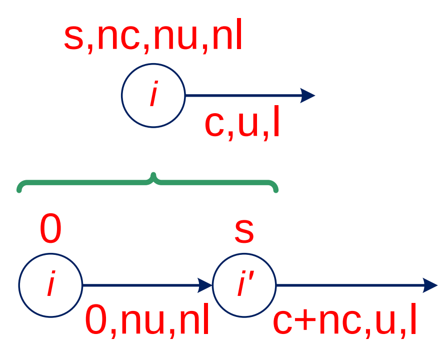
SCUL:	1	2	3	4
1:	6	3	Inf	0
2:	6	1	Inf	0
3:	0	0	Inf	3
4:	0	2	4	0
5:	-8	0	Inf	0

Expanded-Node Formulation of MCNF

- Node cost/constraints converted to arc cost/constraints
  - Dummy node (8) added so that supply = demand



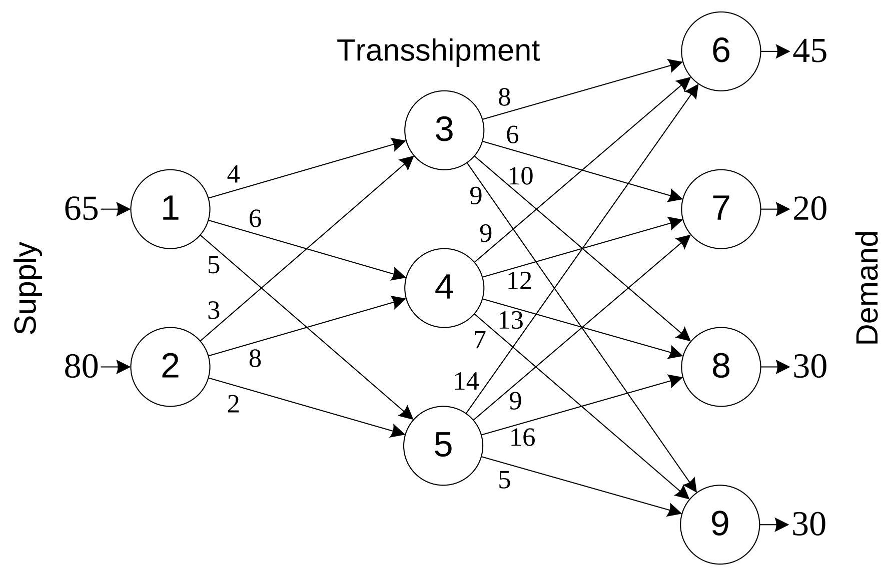




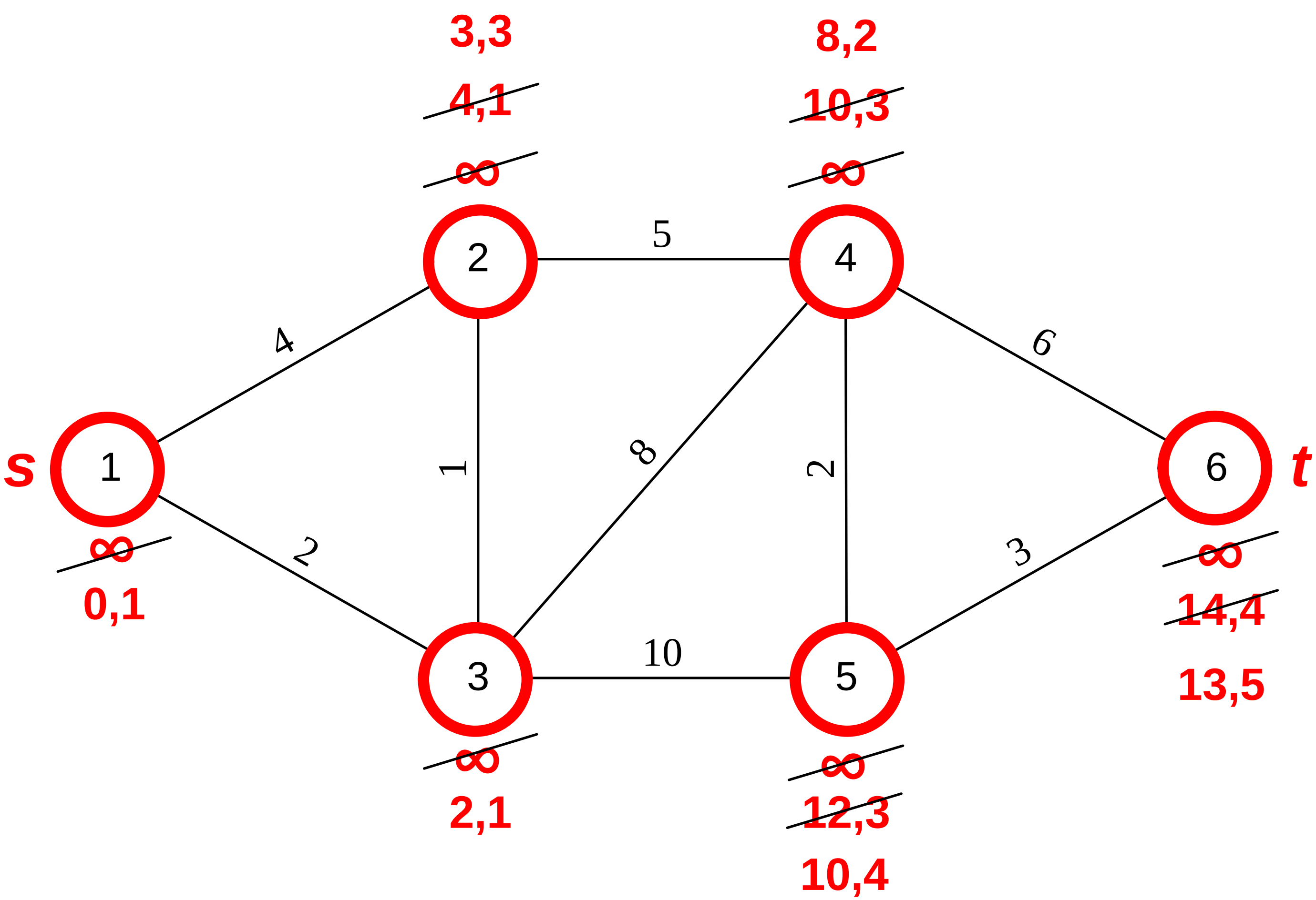
MCNF:	lhs	C	C	C	C	C	C	C	C	C	C	rhs
Min:		5	6	5	6	1	5	0	0	0	0	
1:	6	1	1	0	0	0	0	0	0	1	0	6
2:	6	-1	0	1	1	0	0	0	0	0	1	6
3:	0	0	-1	0	0	0	0	1	0	0	0	0
4:	0	0	0	-1	0	0	0	0	1	0	0	0
5:	-8	0	0	0	-1	-1	-1	0	0	0	0	-8
6:	0	0	0	0	0	1	0	-1	0	0	0	0
7:	0	0	0	0	0	0	1	0	-1	0	0	0
lb:		0	0	0	0	0	1	3	0	0	0	
ub:		Inf	Inf	3	9	3	Inf	Inf	4	Inf	Inf	

Solving an MCNF as an LP

- Special procedures more efficient than LP were developed to solve MCNF and Transportation problems
  - e.g., Network simplex algorithm (MCNF)
  - e.g., Hungarian method (Transportation and *Transshipment*)
- Now usually easier to transform into LP since solvers are so good, with MCNF just aiding in formulation of problem:
  - Trans $\Rightarrow$ MCNF $\Rightarrow$ LP
  - Special, very efficient procedures only used for shortest path problem (Dijkstra)



Dijkstra Shortest Path Procedure



Path: $1 \leftarrow 3 \leftarrow 2 \leftarrow 4 \leftarrow 5 \leftarrow 6:\quad 13$

Dijkstra Shortest Path Procedure

procedure $dijkstra(\mathbf{W}, n, s)$

$S \leftarrow \{\ \}, \quad \bar{S} \leftarrow \{1, \ldots, n\}$

for $i \in \bar{S}$, $d(i) \leftarrow \infty$, **endfor**

$d(s) \leftarrow 0, \quad pred(s) \leftarrow 0$

while $|S| < n$

$\quad i \leftarrow \arg\min_j \{d(j) : j \in \bar{S}\}$

$\quad S \leftarrow S \cup i, \quad \bar{S} \leftarrow \bar{S} \setminus i$

$\quad$ **for** $j \in \arg\{W_{i(j)} : W_{ij} \neq 0\}$

$\quad\quad$ **if** $d(j) > d(i) + W_{ij}$

$\quad\quad\quad d(j) \leftarrow d(i) + W_{ij}$

$\quad\quad\quad pred(j) \leftarrow i$

$\quad\quad$ **endif**

$\quad$ **endfor**

endwhile

return d, pred

```
%% DIJKSTRA Matlab code,
%  given W, n, and s
S = []; nS = 1:n;
d = inf(1,n);
d(s) = 0; pred(s) = 0;
while length(S) < n
    [di,idx] = min(d(nS));
    i = nS(idx);
    S = [S i];
    nS(idx) = [];
    pred(d > di + W(i,:)) = i;
    d = min(d,di + W(i,:));
end
d, pred
```

Index to index vector nS

Order important

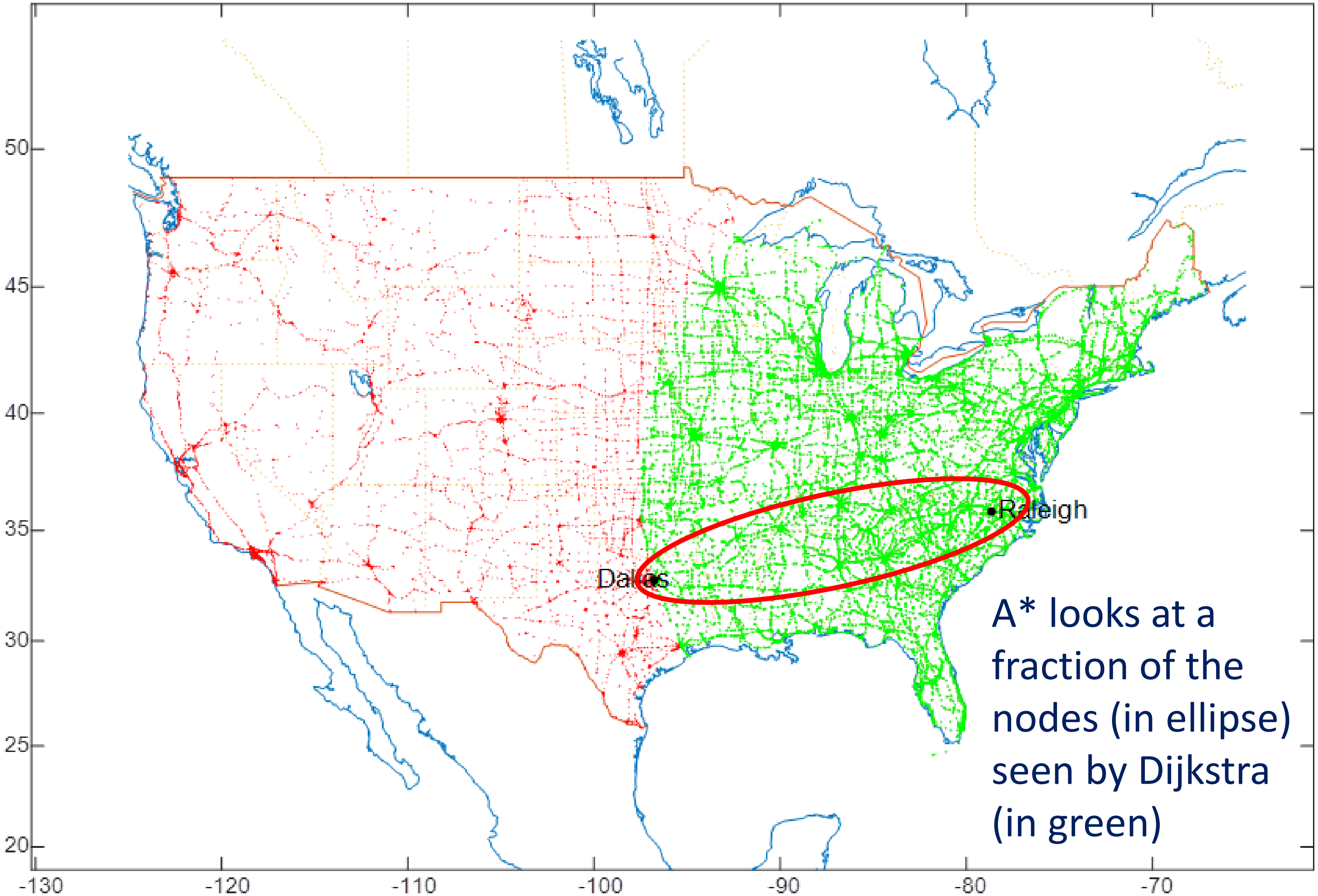
$O(2^n)$	Simplex (LP)
$O(n^4)$	Ellipsoid (LP)
$O(n^3)$	Hungarian (transportation)
$O(n^2)$	Dijkstra (linear min)
$O(m \log n)$	Dijkstra (Fibonocci heap)
m	no. arcs

Other Shortest Path Procedures

- Dijkstra requires that all arcs have nonnegative lengths
  - It is a "label setting" algorithm since step to final solution made as each node labeled
  - Can find longest path (used, e.g., in CPM) by negating *all* arc lengths
- Networks with only *some* negative arcs require slower "label correcting" procedures that repeatedly check for optimality at all nodes or detect a negative cycle
  - Requires $O(n^3)$ via Floyd-Warshall algorithm (cf., $O(n^2)$ Dijkstra)
  - Negative arcs used in project scheduling to represent maximum lags between activities
- A* algorithm adds to Dijkstra an heuristic LB estimate of each node's remaining distance to destination
  - Used in AI search for all types of applications (tic-tac-toe, chess)
  - In path planning applications, great circle distance from each node to destination could be used as LB estimate of remaining distance

A* Path Planning Example 1

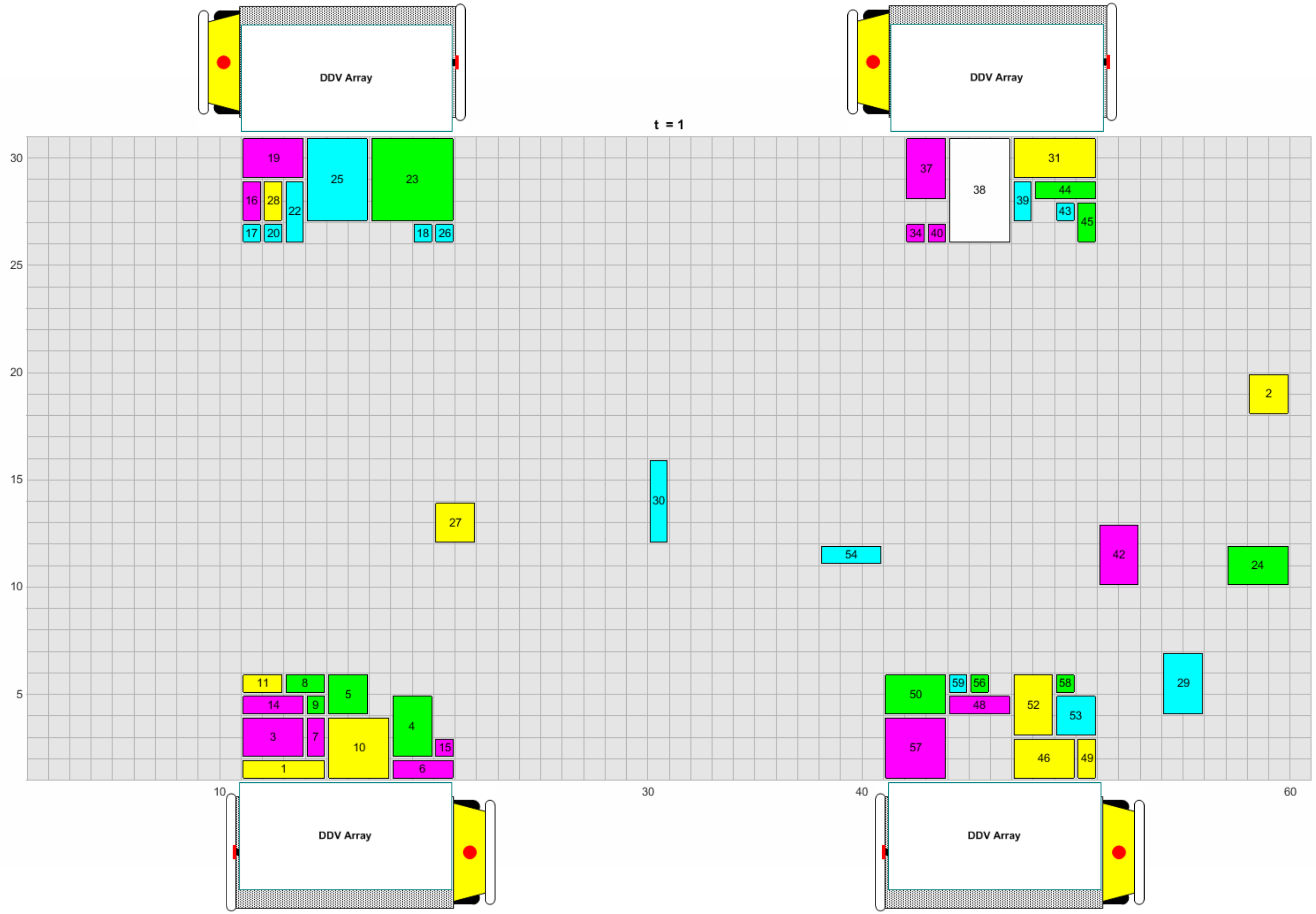
$$d_{A^*}(\text{Raleigh, Dallas}) = d_{dijk}(\text{Raleigh}, i) + d_{GC}(i, \text{Dallas}), \quad \text{for each node } i$$



A* Path Planning Example 2

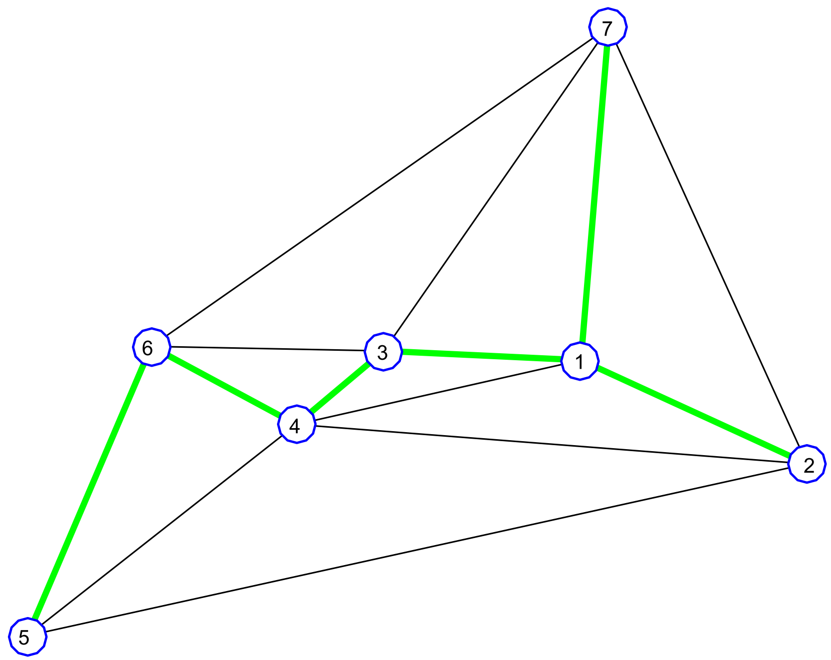
- 3-D (x,y,t) A* used for planning path of each container in a DC
- Each container assigned unique priority that determines planning sequence
  - Paths of higher-priority containers become obstacles for subsequent containers

A* Path Planning Example 2



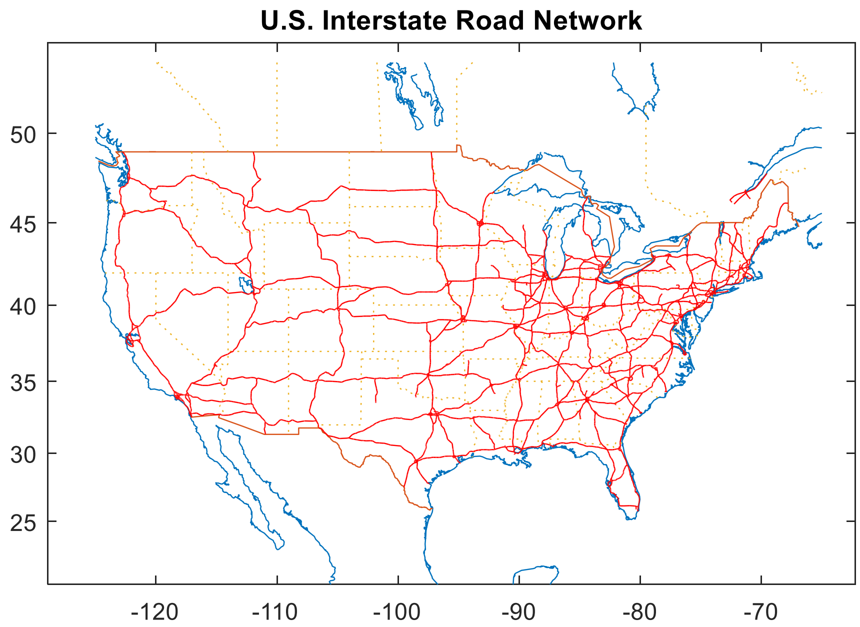
Minimum Spanning Tree

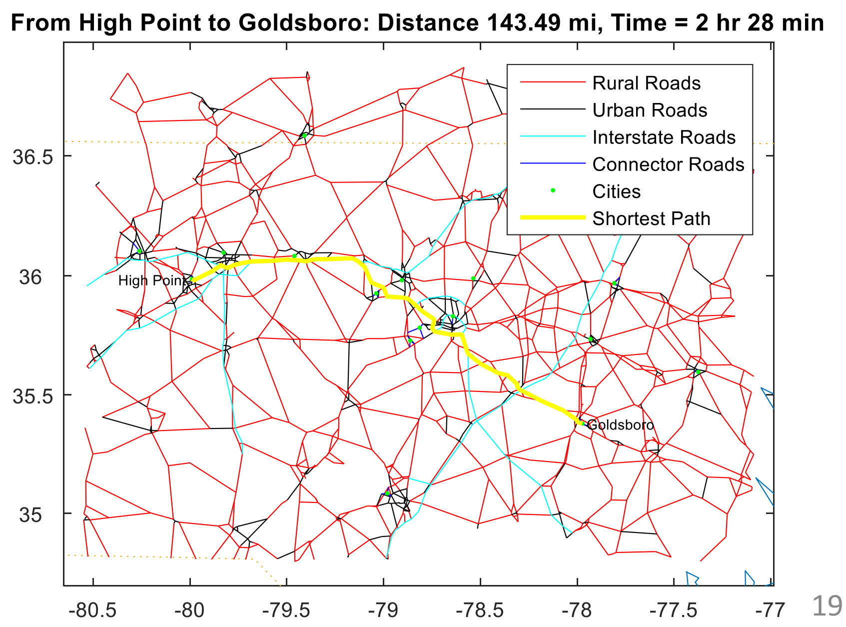
- Find the minimum cost set of arcs that connect all nodes
  - Undirected arcs: Kruskal's algorithm (easy to code)
  - Directed arcs: Edmond's branching algorithm (hard to code)



U.S. Highway Network

- Oak Ridge National Highway Network
  - Approximately 500,000 miles of roadway in US, Canada, and Mexico
  - Created for truck routing, does not include residential
  - Nodes attributes: XY, FIPS code
  - Arc attributes: IJD, Type (Interstate, US route), Urban





FIPS Codes

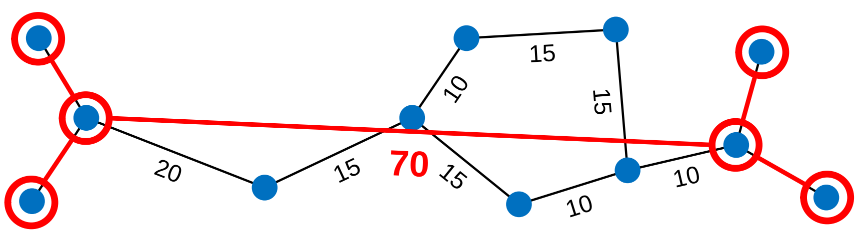
- Federal Information Processing Standard (FIPS) codes used to uniquely identify states (2-digit) and counties (3-digit)
  - 5-digit Wake county code = 2-digit state + 3-digit county = 37183 = 37 NC FIPS + 183 Wake FIPS

1	AL	Alabama	22	LA	Louisiana	40	OK	Oklahoma
2	AK	Alaska	23	ME	Maine	41	OR	Oregon
4	AZ	Arizona	24	MD	Maryland	42	PA	Pennsylvania
5	AR	Arkansas	25	MA	Massachusetts	44	RI	Rhode Island
6	CA	California	26	MI	Michigan	45	SC	South Carolina
8	CO	Colorado	27	MN	Minnesota	46	SD	South Dakota
9	CT	Connecticut	28	MS	Mississippi	47	TN	Tennessee
10	DE	Delaware	29	MO	Missouri	48	TX	Texas
11	DC	Dist Columbia	30	MT	Montana	49	UT	Utah
12	FL	Florida	31	NE	Nebraska	50	VT	Vermont
13	GA	Georgia	32	NV	Nevada	51	VA	Virginia
15	HI	Hawaii	33	NH	New Hampshire	53	WA	Washington
16	ID	Idaho	34	NJ	New Jersey	54	WV	West Virginia
17	IL	Illinois	35	NM	New Mexico	55	WI	Wisconsin
18	IN	Indiana	36	NY	New York	56	WY	Wyoming
19	IA	Iowa	37	NC	North Carolina	72	PR	Puerto Rico
20	KS	Kansas	38	ND	North Dakota	88		Canada
21	KY	Kentucky	39	OH	Ohio	91		Mexico

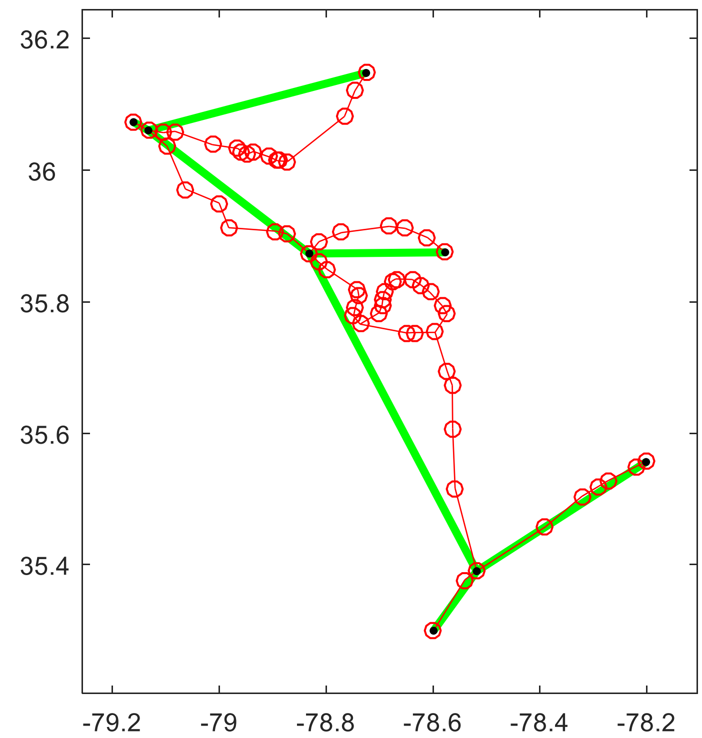
Road Network Modifications

1. Thin

- Remove all degree-2 nodes from network
- Add cost of both arcs incident to each degree-2 node
- If results in multiple arcs between pair of nodes, keep minimum cost



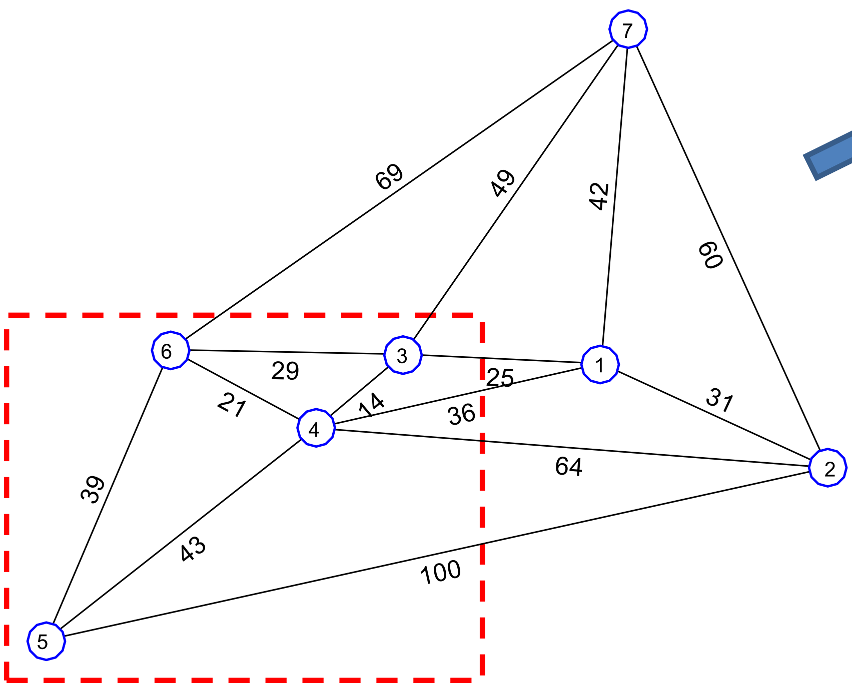
Thinned I-40 Around Raleigh



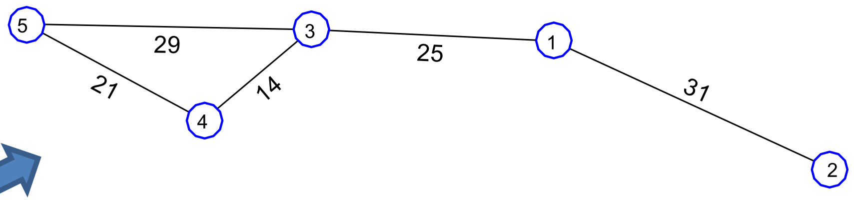
Road Network Modifications

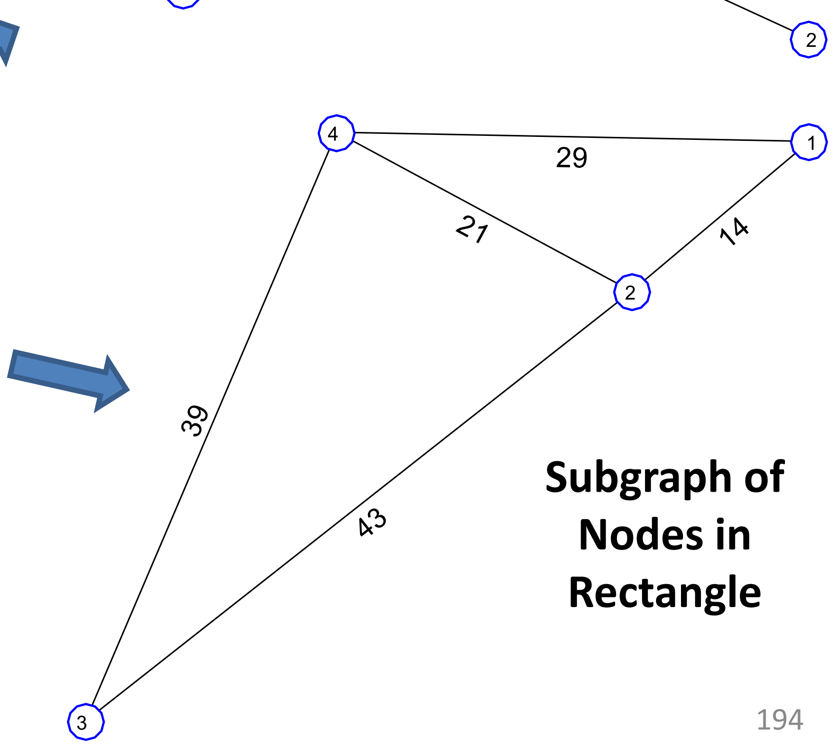
2. Subgraph

– Extract portion of graph with only those nodes and/or arcs that satisfy some condition



Subgraph of Arcs < 35

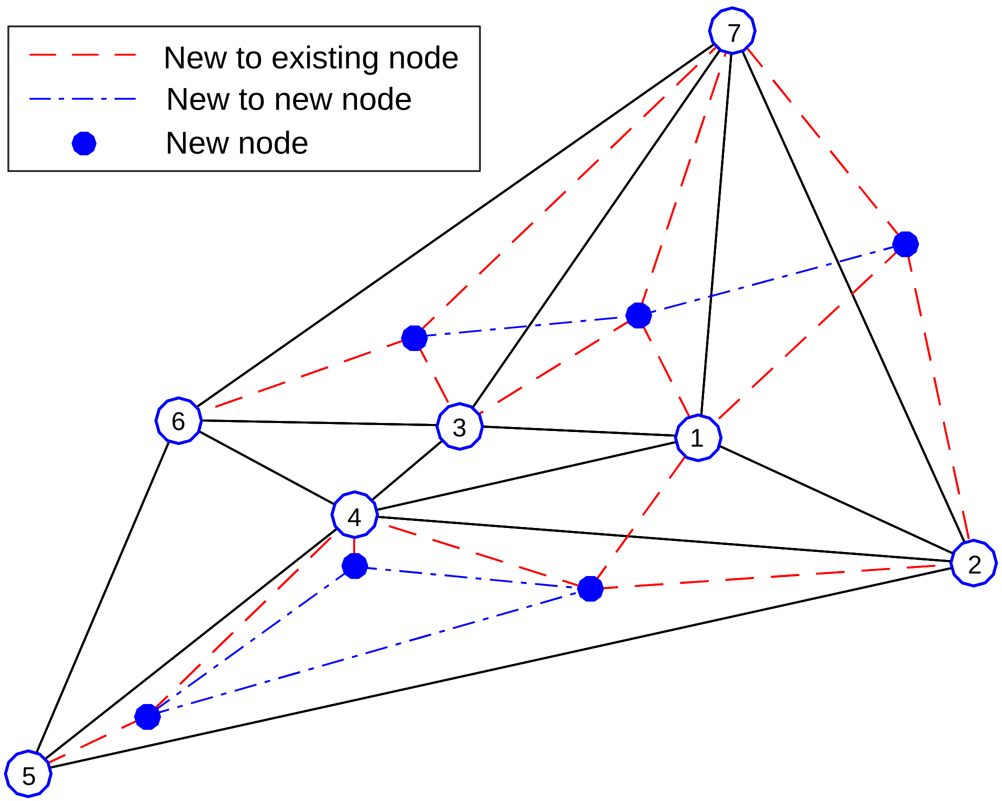




Subgraph of Nodes in Rectangle

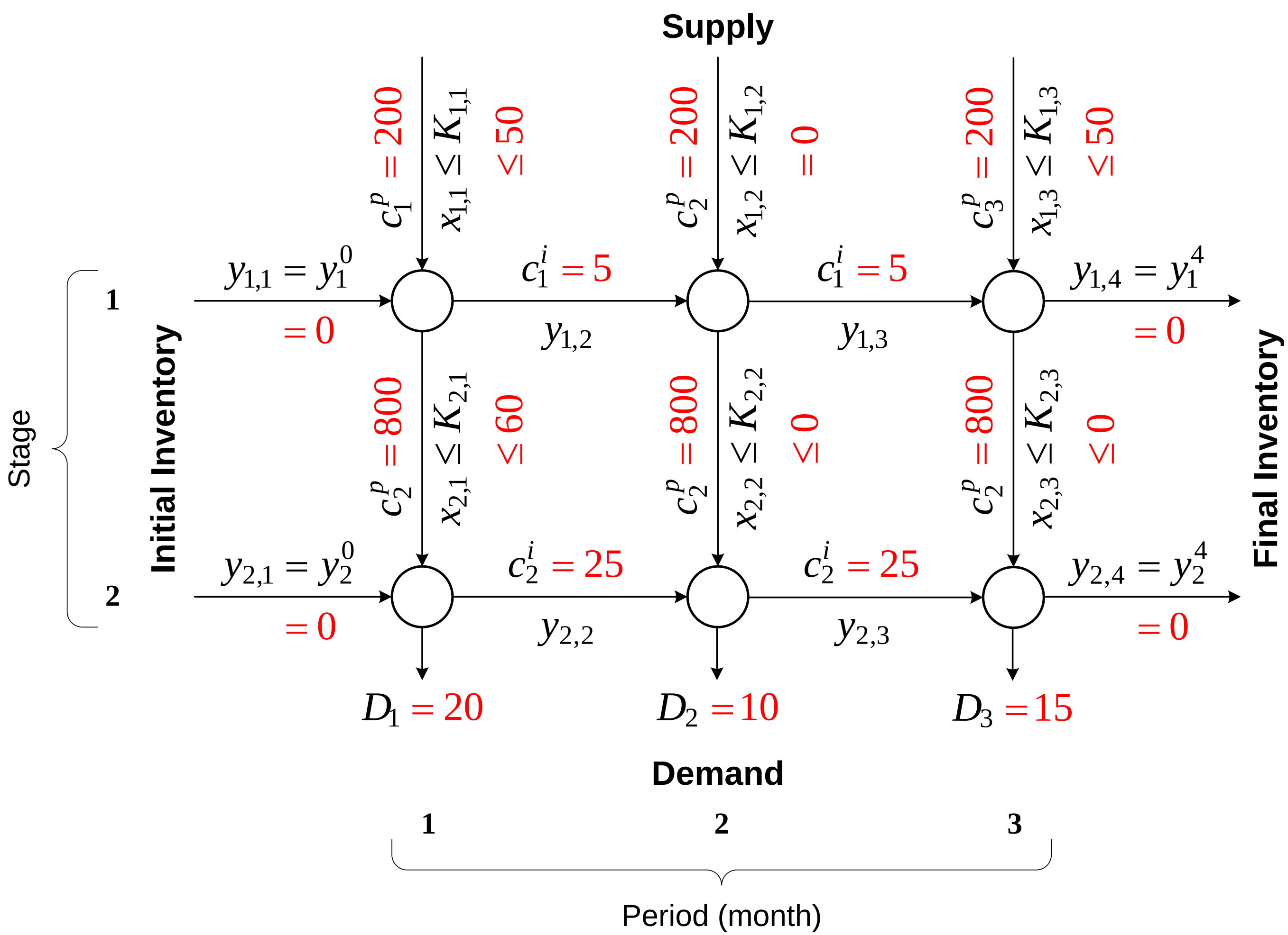
Road Network Modifications

3. **Add connector**
   - Given new nodes, add arcs that connect the new nodes to the existing nodes in a graph and to each other



- Distance of connector arcs = GC distance x circuity factor (1.5)
- New node connected to 3 closest existing nodes, except if
  - Ratio of closest to 2nd and 3rd closest < threshold (0.1)
  - Distance shorter using other connector and graph

Production and Inventory: One Product



$$h = 0.3\frac{\$}{\$\text{-yr}} = 0.3$$

$$\frac{h}{T} = \frac{0.3}{12}\,\frac{\$}{\$\text{-month}} = 0.025$$

$$c_m^i = \frac{h}{T}\sum_{j=1}^{m} c_j^p$$

$$c_1^i = \frac{0.3}{12}200 = 5$$

$$c_2^i = \frac{0.3}{12}\left(200 + 800\right)$$

$$= 25$$

Production and Inventory: One Product

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T} c_m^p x_{mt} + \sum_{m=1}^{M}\sum_{t=1}^{T+1} c_m^i y_{mt}$$

subject to

Flow balance

$$x_{mt} - x_{(m+1)t} + y_{mt} - y_{m(t+1)} = 0, \qquad m = 1,\ldots,M-1;\ t = 1,\ldots,T$$

$$x_{Mt} + y_{Mt} - y_{M(t+1)} = D_t, \qquad t = 1,\ldots,T$$

Capacity

$$x_{mt} \le K_{mt}, \quad m = 1,\ldots,M;\ t = 1,\ldots,T$$

Initial/Final inventory

$$y_{m1} = y_m^0, \qquad m = 1,\ldots,M$$

$$y_{m(T+1)} = y_m^{T+1}, \quad m = 1,\ldots,M$$

$$x, y \ge 0 \text{ and continuous}$$

Use var. LB & UB instead of constraints

where

M = number of production stages

c_m^p = production cost in stage m (\$/ton)

x_{mt} = production at stage m in period t (ton)

c_m^i = inventory cost for stage m (\$/ton)

y_{mt} = stage-m inventory period $t-1$ to t (ton)

T = number of periods of production

D_t = demand in period t (ton)

K_{mt} = capacity of stage m in period t (ton)

y_m^0 = initial inventory of stage m (ton)

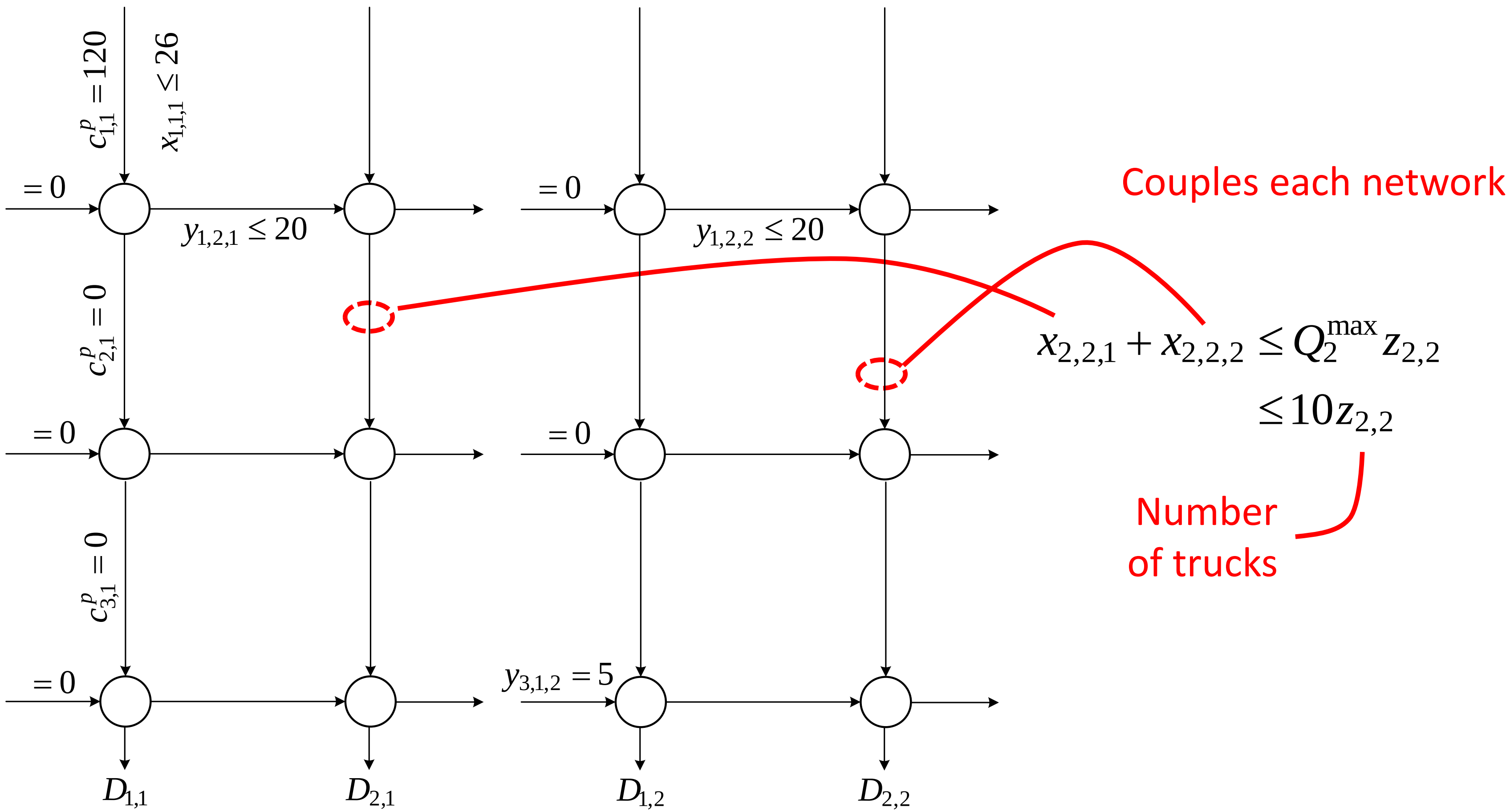
y_m^{T+1} = final inventory of stage m (ton)

Ex 15: Coupled Networks via Truck Capacity

- Facility that extracts two different raw materials for pharmaceuticals
  1. Extracted material to be sent over rough terrain in a truck to a staging station where it is then loaded onto a tractor trailer for transport to its final destination
  2. Facility can extract up to 26 and 15 tons per week of each material, respectively, at a cost of \$120 and \$200 per ton
  3. Annual inventory carrying rate is 0.15
  4. Facility can store up to 20 tons of each material on site, and unlimited amounts of material can be stored at the staging station and the final destination
  5. Currently, five tons of the second material is in inventory at the final destination and this same amount should be in inventory at the end of the planning period
  6. Costs \$200 for a truck to make the roundtrip from the facility to the staging station, and it costs \$800 for each truckload transported from the station to the final destination
  7. Each truck and tractor trailer can carry up to 10 and 25 tons of material, respectively, and each load can contain both types of material
- Determine the amount of each material that should be extracted and when it should be transported in order to minimize total costs over the planning horizon
- Separate networks for two products are coupled via sharing truck capacity

Ex 15: Coupled Networks via Truck Capacity

- Separate networks for each raw material are coupled via sharing the same trucks (added as constraint to model)



Ex 15: Coupled Networks via Truck Capacity

- Math programming model:

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{g=1}^{G} c_{mg}^{p}\, x_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T+1}\sum_{g=1}^{G} c_{mg}^{i}\, y_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T} c_{m}^{t} z_{mt}$$

subject to

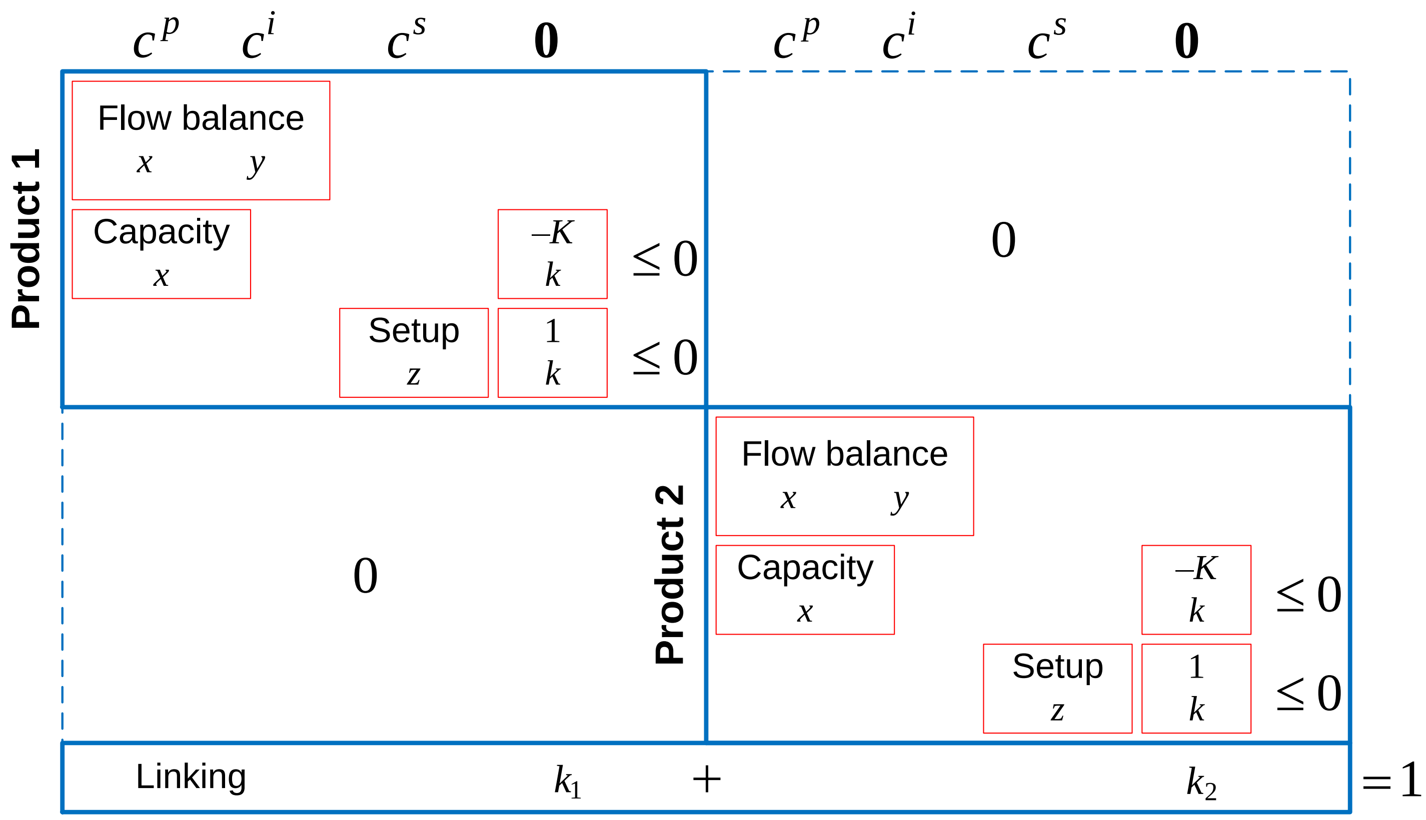
$$x_{mtg} - x_{(m+1)tg} + y_{mtg} - y_{m(t+1)g} = 0, \qquad m = 1,\ldots,M-1; \quad t = 1,\ldots,T; \; g = 1,\ldots,G$$

$$x_{Mtg} + y_{Mtg} - y_{M(t+1)g} = D_{tg}, \qquad t = 1,\ldots,T; \; g = 1,\ldots,G$$

$$\sum_{g=1}^{G} x_{mtg} \le Q_{m}^{\max} z_{mt}, \quad m = 1,\ldots,M; t = 1,\ldots,T$$

Use LB,UB for capacity constraints

Production and Inventory: Multiple Products



Production and Inventory: Multiple Products

$k_{mtg} \in \{0,1\}$, production indicator

k_{mtg}	1	2	3	4	5	6	7
1	0	1	1	0	1	1	1
2	0	0	0	1	0	0	0

$z_{mtg} \in \{0,1\}$, setup indicator

z_{mtg}	1	2	3	4	5	6	7
1	0	1	0	0	1	0	0
2	0	0	0	1	0	0	0

	$-z_t$	$+\ k_t$	$-\ k_{t-1}$	$\leq\ 0$	
	0	0	0	0	
	0	0	1	–1	
Don't want (not feasible)	0	1	0	1	
	0	1	1	0	
	1	0	0	–1	Feasible, but not min cost
	1	0	1	–2	Feasible, but not min cost
Want (feasible)	1	1	0	0	
	1	1	1	–1	Feasible, but not min cost

Production and Inventory: Multiple Products

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{g=1}^{G} c^{p}_{mg}\, x_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T+1}\sum_{g=1}^{G} c^{i}_{mg}\, y_{mtg} + \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{g=1}^{G} c^{s}_{mg}\, z_{mtg} \left(\underbrace{+\mathbf{0}_{MTG}}_{\text{dummy}}\, k_{mtg}\right)$$

subject to

$$\text{Flow balance}\left\{\begin{array}{rll} x_{mtg} - x_{(m+1)tg} + y_{mtg} - y_{m(t+1)g} &= 0, & m = 1,\dots,M-1;\quad t = 1,\dots,T;\ g = 1,\dots,G \\ x_{Mtg} + y_{Mtg} - y_{M(t+1)g} &= D_{tg}, & t = 1,\dots,T;\ g = 1,\dots,G \end{array}\right.$$

$$\text{Capacity}\left\{\begin{array}{rll} x_{mtg} &\le K_{mg} k_{mtg}, & m = 1,\dots,M;\ t = 1,\dots,T;\ g = 1,\dots,G \end{array}\right.$$

$$\text{Setup}\left\{\begin{array}{rll} -z_{m1g} + k_{m1g} &\le k^{0}_{mg}, & m = 1,\dots,M;\ g = 1,\dots,G \\ -z_{mtg} + k_{mtg} - k_{m(t-1)g} &\le 0, & m = 1,\dots,M;\ t = 2,\dots,T;\ g = 1,\dots,G \end{array}\right.$$

$$\text{Linking}\left\{\begin{array}{rll} \sum_{g=1}^{G} k_{mtg} &= 1, & m = 1,\dots,M;\ t = 1,\dots,T \end{array}\right.$$

$$y_{m1g} = y^{0}_{mg}, \qquad m = 1,\dots,M;\ g = 1,\dots,G$$

$$y_{m(T+1)g} = y^{T+1}_{mg}, \qquad m = 1,\dots,M;\ g = 1,\dots,G$$

$$x, y \ge 0 \text{ and continuous; } k, z \text{ binary}$$

MILP

Production and Inventory: Multiple Products

where $\mathbf{0}_{MTG}$ is a matrix of zeroes and

M = number of production stages

T = number of periods of production

G = number of products produced

c^p_{mg} = production cost of product g at stage m (\$/ton)

x_{mtg} = production at stage m in period t of product g (ton)

c^i_{mg} = inventory cost of product g for stage m (\$/ton)

y_{mtg} = inventory at stage m between periods $t-1$ and t of product g (ton)

c^s_{mg} = stage-m product-g setup cost (\$)

z_{mtg} = setup indicator at stage m in period t for product g

k_{mtg} = production indicator at stage m in period t for product g

D_{tg} = demand for product g in period t (ton)

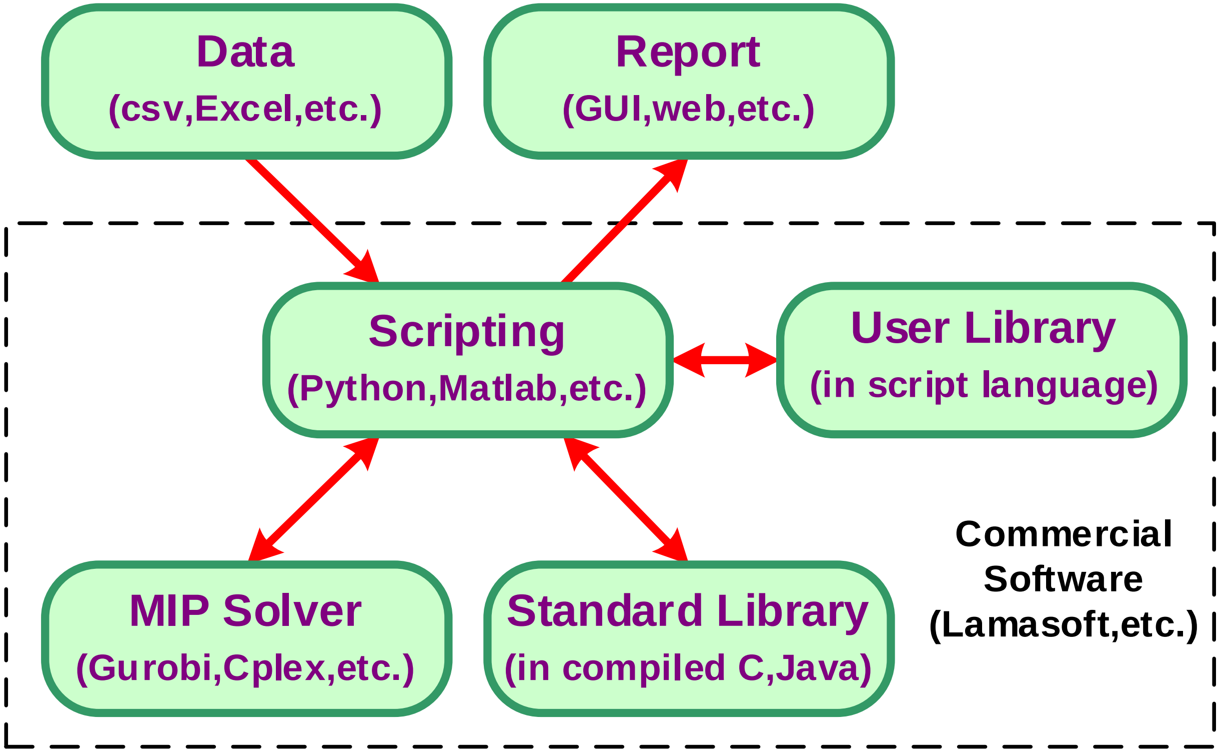
K_{mg} = capacity for product g in stage m (ton)

k^0_{mg} = initial setup at stage m for product g

y^0_{mg} = initial product g inventory at stage m (ton)

y^{T+1}_{mg} = final product g inventory at stage m (ton)

Example of Logistics Software Stack

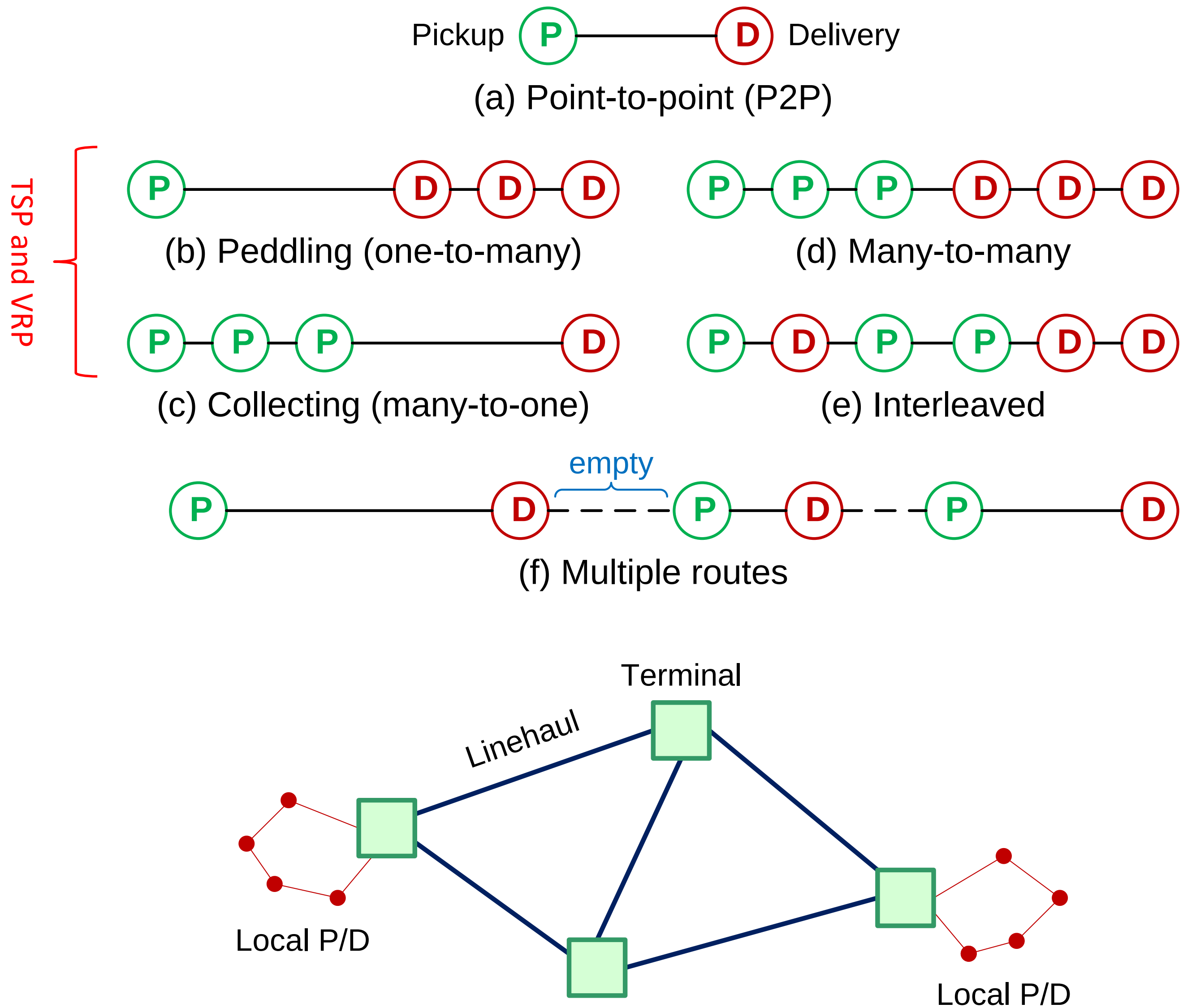


- **Flow:** *Data → Model → Solver → Output → Report*
  - reports are run on a regular period-to-period, *rolling-horizon* basis as part of normal operations management
  - model only changed when logistics network changes

Topics

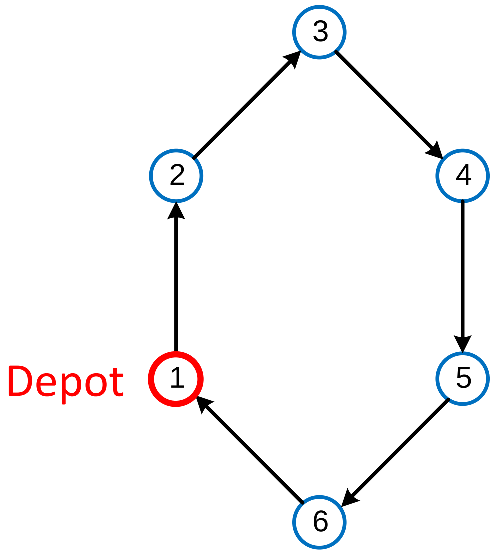
1. Introduction
2. Facility location
3. Freight transport
   – Exam 1 (take home)
4. Network models
5. **Routing**
   – Exam 2 (take home)
6. Warehousing
   – Final exam (in class)

Routing Alternatives



TSP

- Problem: find connected sequence through all nodes of a graph that minimizes total arc cost
  - Subroutine in most vehicle routing problems
  - Node sequence can represent a route only if all pickups and/or deliveries occur at a single node (depot)



Node sequence = permutation + start node

1	2	3	4	5	6	1

$n = 6 \Rightarrow (n-1)! = 120$ possible solutions

TSP

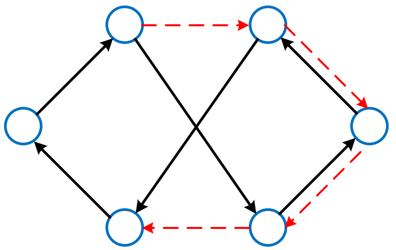
- TSP can be solved by a mix of *construction* and *improvement* procedures
  - BIP formulation has an exponential number of constraints to eliminate subtours ($\Rightarrow$ column generation techniques)
- Asymmetric: only best-known solutions for large *n*

$$(n-1)! \quad n=13 \Rightarrow \approx \frac{1}{2}\text{billion solutions}$$

- Symmetric: solved to optimal using BIP

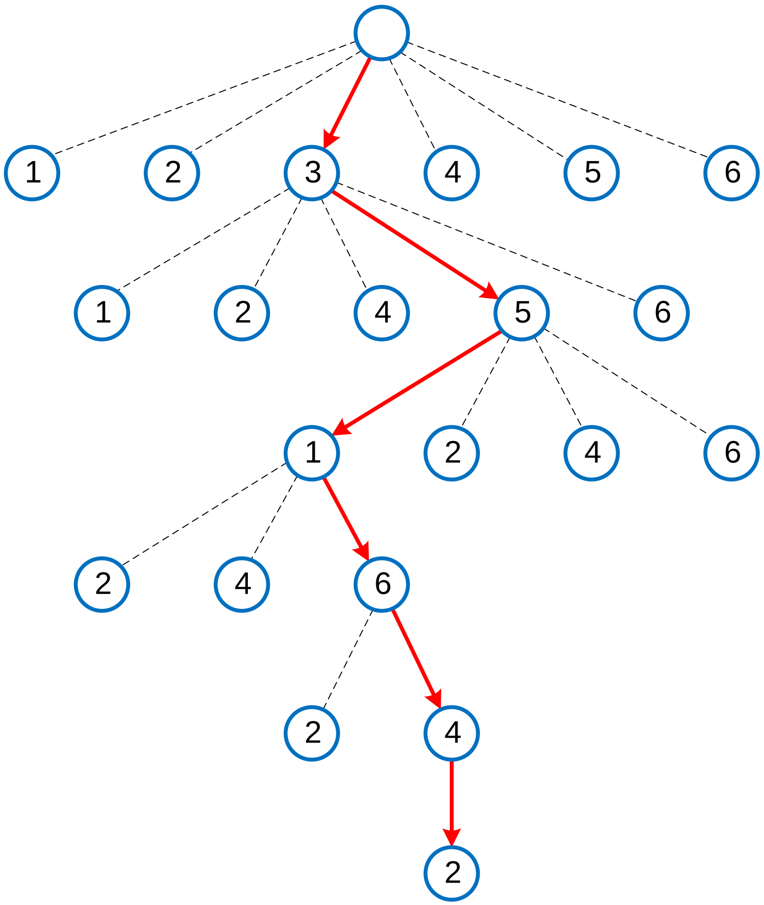
$$c_{ij}=c_{ji} \Rightarrow \frac{(n-1)!}{2} \quad \text{solutions}$$

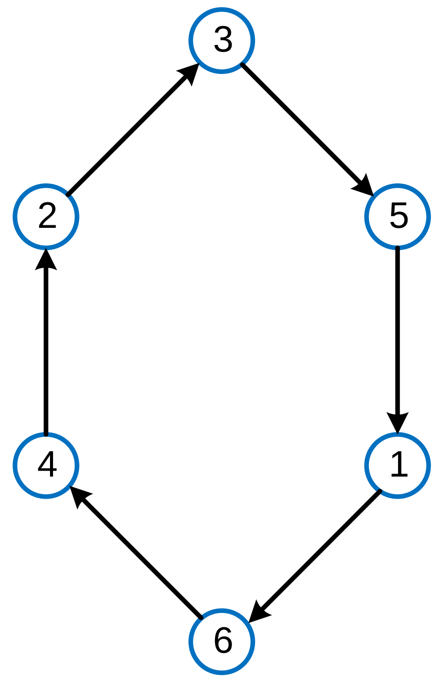
- Euclidean: arcs costs = distance between nodes



TSP Construction

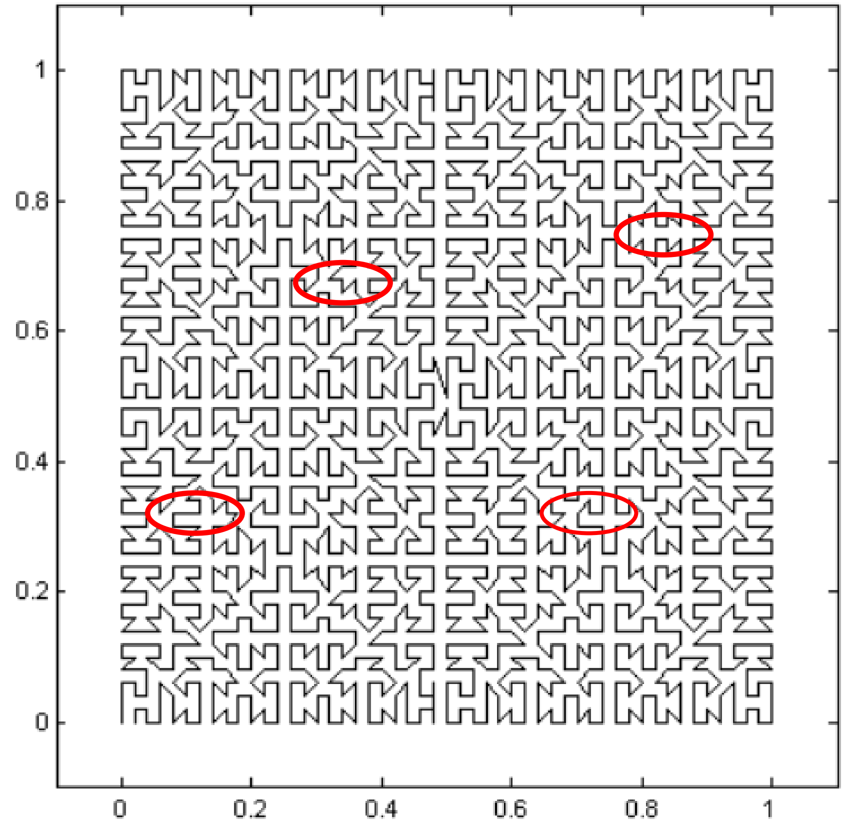
- Construction easy since any permutation is feasible and can then be improved





Spacefilling Curve

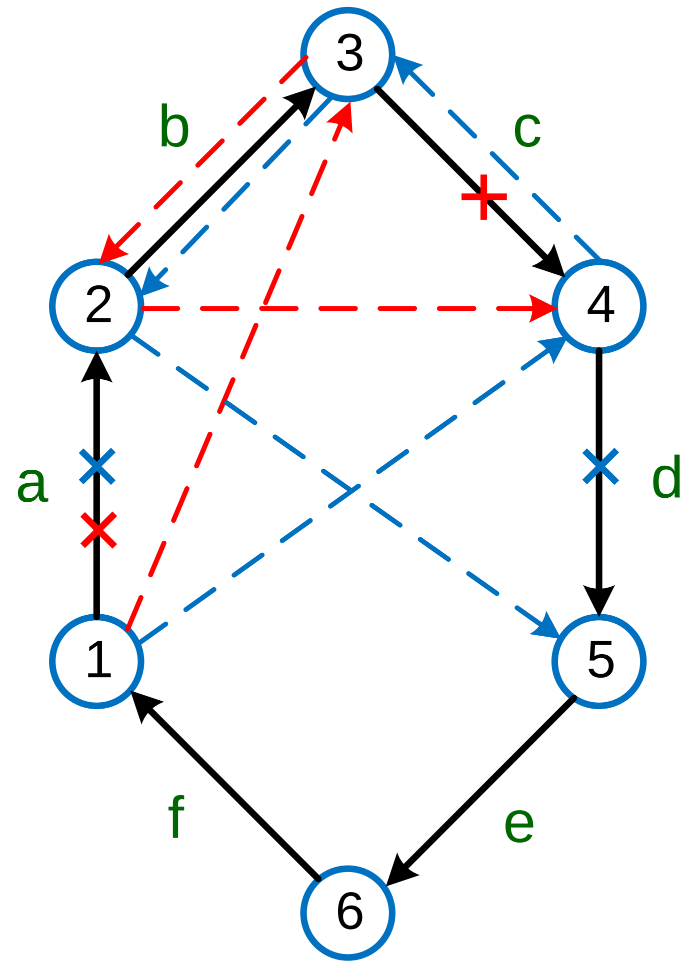
1.0	0.250	0.254	0.265	0.298	0.309	0.438	0.441	0.452	0.485	0.496	0.500
0.9	0.246	0.257	0.271	0.292	0.305	0.434	0.445	0.458	0.479	0.493	0.504
0.8	0.235	0.229	0.279	0.283	0.333	0.423	0.417	0.467	0.471 1	0.521	0.515
0.7	0.202	0.208	0.158	0.154 3	0.354	0.390	0.396	0.596	0.592	0.542	0.548
0.6	0.191	0.180	0.167	0.146	0.132	0.379	0.618	0.604	0.583	0.570	0.559
0.5	0.188	0.184	0.173	0.140	0.129	0.375	0.621	0.610	0.577	0.566	0.563
0.4	0.059	0.070	0.083	0.104	0.118	0.871	0.632	0.646	0.667	0.680	0.691
0.3	0.048	0.042	0.092	0.096	0.896	0.860	0.854	0.654	0.658	0.708	0.702
0.2	0.015	0.021 2	0.971	0.967	0.917	0.827	0.833	0.783 4	0.779	0.729	0.735
0.1	0.004	0.993	0.979	0.958	0.945	0.816	0.805	0.792	0.771	0.757	0.746
0.0	0.000	0.996	0.985	0.952	0.941	0.813	0.809	0.798	0.765	0.754	0.750
θ	**0.0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**



Sequence determined by sorting position along 1-D line covering 2-D space

2: 0.021
3: 0.154
1: 0.471
4: 0.783

Two-Opt Improvement



	a	b	c	d	e	f	
	1	2	3	4	5	6	1
a-c	1	3	2	4	5	6	1
a-d	1	4	3	2	5	6	1
a-e	1	5	4	3	2	6	1
b-d							
b-e							
b-f							
c-e							
c-f							
d-f							

Sequences considered at end to verify local optimum: n nodes $\Rightarrow \overset{\text{first arc } a}{\sum_{j=3}^{n-1}(1)} + \overbrace{\sum_{i=2}^{n-2}\sum_{j=i+2}^{n}}^{\text{arcs } b \text{ to } n-2}(1) = \frac{n(n-3)}{2} = 9 \text{ for } n = 6$

Ex 16: Two-Opt Improvement

- Order in which *twoopt* considers each sequence:

1:	1	2	3	4	5	6	1	38
2:	1	3	2	4	5	6	1	39
3:	1	4	3	2	5	6	1	32
4:	1	3	4	2	5	6	1	31
5:	1	4	3	2	5	6	1	32
6:	1	2	4	3	5	6	1	31
7:	1	5	2	4	3	6	1	21
8:	1	2	5	4	3	6	1	21
9:	1	4	2	5	3	6	1	32
10:	1	3	4	2	5	6	1	31
11:	1	5	4	2	3	6	1	12
12:	1	4	5	2	3	6	1	34
13:	1	2	4	5	3	6	1	40
14:	1	3	2	4	5	6	1	39
15:	1	5	2	4	3	6	1	21
16:	1	5	3	2	4	6	1	30
17:	1	5	6	3	2	4	1	31
18:	1	5	4	3	2	6	1	13
19:	1	5	4	6	3	2	1	18
20:	1	5	4	2	6	3	1	20

Local optimal sequence

D:	1	2	3	4	5	6
1:	0	8	6	9	1	5
2:	3	0	1	5	4	2
3:	9	2	0	3	1	1
4:	8	2	1	0	10	6
5:	6	7	10	1	0	10
6:	6	2	5	2	1	0

Note: Not symmetric

Sequences considered at end to verify local optimum: n nodes $\Rightarrow$

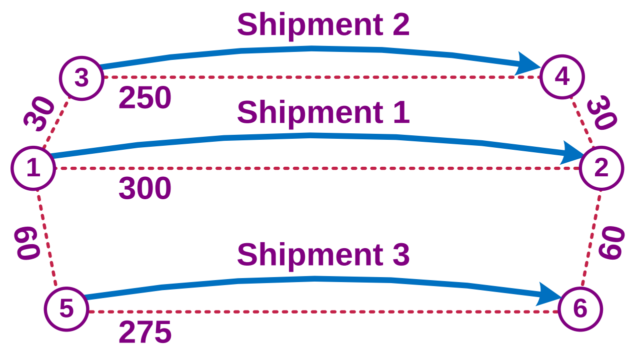
$$\sum_{j=3}^{n-1}(1)+\sum_{i=2}^{n-2}\sum_{j=i+2}^{n}(1)=\frac{n(n-3)}{2}=9 \text{ for } n=6$$

TSP Comparison

	TSP Procedure	Total Cost
1	Spacefilling curve	482.7110
2	1 + 2-opt	456
3	Convex hull insert + 2-opt	452
4	Nearest neighbor + 2-opt	439.6
5	Random construction + 2-opt	450, 456
6	Eil51 in TSPLIB	426* optimal

Multi-Stop Routing

- Each shipment might have a different origin and/or destination $\Rightarrow$ node/location sequence not adequate



$$L = (y_1, \ldots, y_n) = (1, 2, 3) \qquad n\text{-element shipment sequence}$$

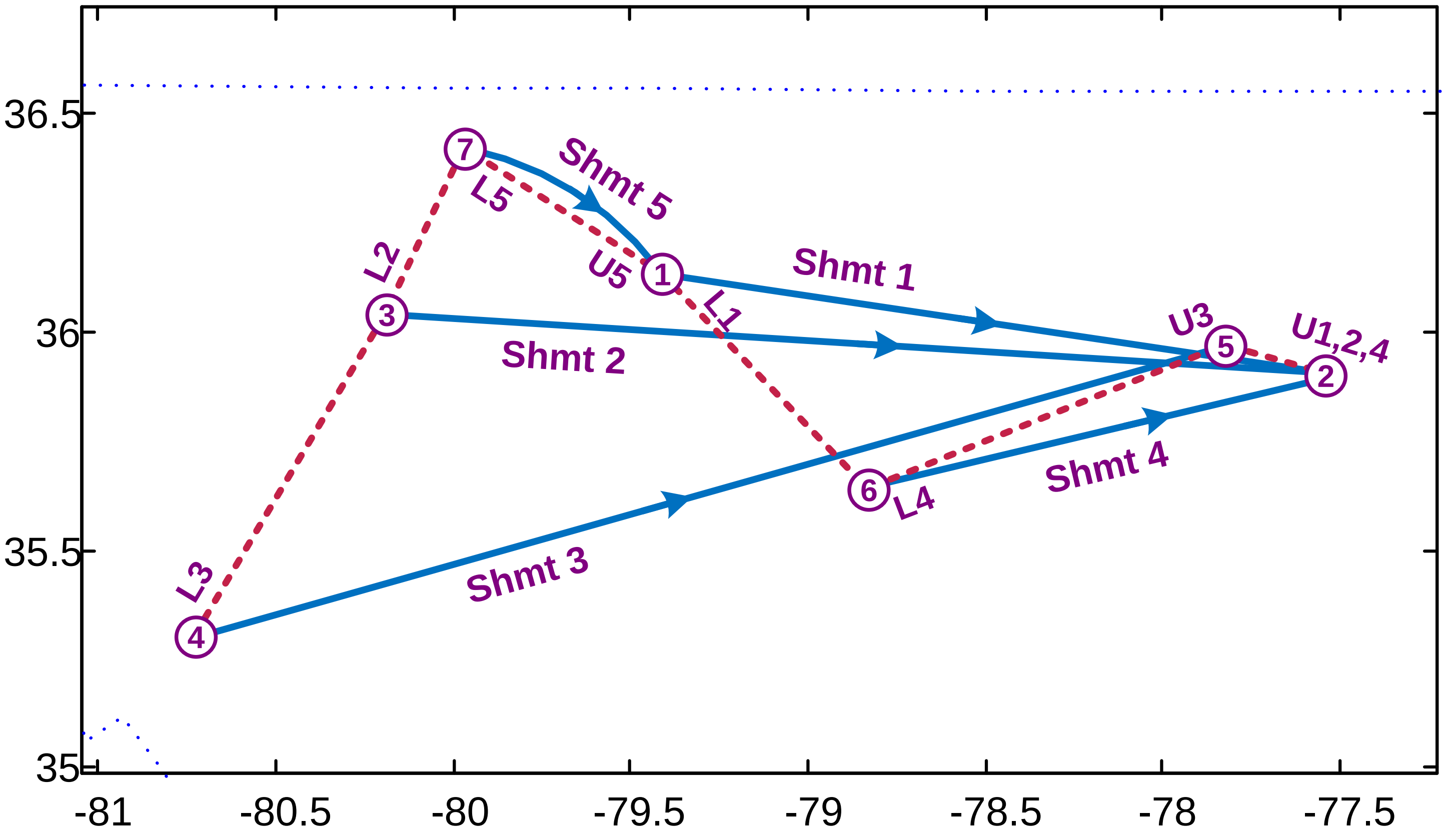
$$R = (z_1, \ldots, z_{2n}) = (3, 1, 2, 2, 1, 3) \qquad 2n\text{-element route sequence}$$

$$X = (x_1, \ldots, x_{2n}) = (5, 1, 3, 4, 2, 6) \qquad 2n\text{-element location (node) sequence}$$

$$c_{ij} = \text{cost between locations } i \text{ and } j$$

$$c(R) = \sum_{i=1}^{2n-1} c_{x_i, x_{i+1}} = 60 + 30 + 250 + 30 + 60 = 430, \quad \text{total cost of route } R$$

5-Shipment Example



Route sequence: $R = (3, 2, 5, 5, 1, 4, 3, 1, 2, 4)$

Location sequence: $X = (4, 3, 7, 1, 1, 6, 5, 2, 2, 2)$

Route Sequencing Procedures

- Online procedure: add a shipment to an existing route as it becomes available
  - Insert and Improve: for each shipment,
    insert where it has the least increase in cost for route and then improve (`mincostinsert` → `twoopt`)
- Offline procedure: consider all shipments to decide order in which each added to route
  - Savings and Improve: using all shipments,
    determine insert ordering based on "savings," then improve final route (`savings` → `twoopt`)

Min Cost Insert

		1			1			
1		●			●	2	2	×
2	2	●			●	2		c_2
3		●	2	2	●			c_3^*
4		●		2	●	2		c_4
5	2	●	2		●			c_5

		1			2		2		1	
1	3	●	3		●		●		●	
2	3	●			●	3	●		●	
3	3	●			●		●	3	●	
4	3	●			●		●		●	3
5		●	3	3	●		●		●	
6		●		3	●	3	●			
7		●		3	●		●	3		
⋮		⋮			⋮		⋮		⋮	

Insert and Improve Online Procedure

- To route each shipment added to load:
  - Minimum Cost Insertion
  - Two-opt improvement
- Different shipment sequences L can result in different routes
  - Order shipment joins load important

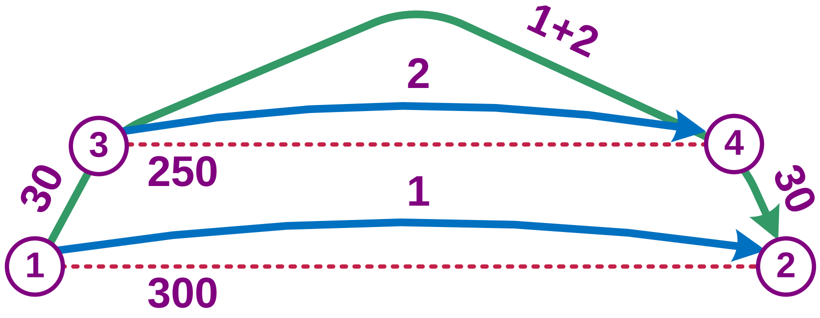
procedure *insertImprove*$(y_i \in L)$
$R = (y_1, y_1)$
for $i = 2, \ldots, |L|$
  $R = \text{minCostInsert}(y_i, R)$
  $R = \text{twoOpt}(R)$
endfor
return R

subprocedure *minCostInsert*$(y, z_i \in R)$
$c_R = c(R)$
for $i = 1, \ldots, |R|+1$, **for** $j = 1, \ldots, |R|+1$
  $R' = (z_1, \ldots, z_{i-1}, y, z_i, \ldots, z_{j-1}, y, z_j, \ldots, z_{|R|})$
  if $c(R') < c_R$, $c_R = c(R')$, $R = R'$, **endif**
endfor, **endfor**
return R

subprocedure *twoOpt*$(z_i \in R)$
$c_R = c(R)$
repeat
  done = true, $i = 1$, $j = 2$
  while *done* and $i < |R|$
    while *done* and $j < |R|+1$
      $R' = (z_1, \ldots, z_{i-1}, \text{reverseSequence}(z_i, \ldots, z_j), z_{j+1}, \ldots, z_{|R|})$
      if $c(R') < c_R$
        $c_R = c(R')$, $R = R'$, *done* = false
      endif
      $j = j + 1$
    endwhile
    $i = i + 1$, $j = i + 1$
  endwhile
until *done* = true
return R

First improvement (cf. steepest descent)

Pairwise Savings



$$s_{ij} = \text{pairwise savings between shipments } i \text{ and } j$$

$$= c_i + c_j - c_{ij} > 0$$

$$s_{1,2} = 300 + 250 - 310$$

$$= 240$$

Clark-Wright (Offline) Savings Procedure

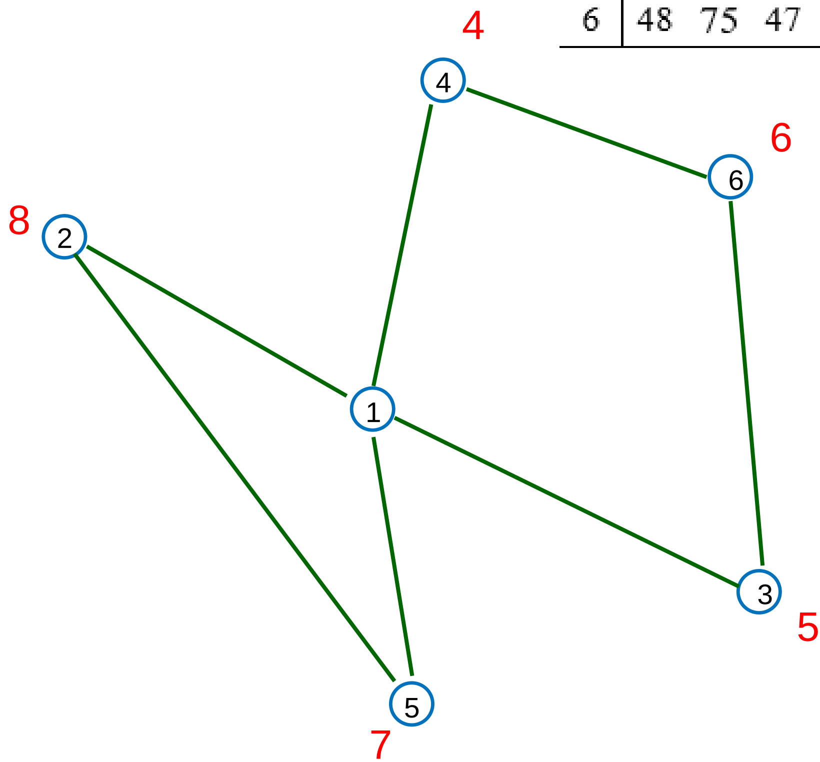
- First (1964), and still best, offline routing procedure if only have vehicle capacity constraints (`vrpsavings`)
- Pairs of shipments ordered in terms of their decreasing (positive) pairwise savings
- Given savings pair i-j, without exceeding capacity constraint, either:
    1. Create new route if i and j not in any existing route
    2. Add i to route only if j at beginning or end of route
    3. Combine routes only if i and j are endpoints of each route

Ex 17: Clark-Wright Savings Procedure

- Node 1 is depot, nodes 2-6 customers
- Customer demands 8, 3, 4, 7, 6, resp.
- Vehicle capacity is 15
- Symmetric costs

	1	2	3	4	5	6
1	0	40	48	38	33	48
2	40	0	87	46	65	75
3	48	87	0	67	41	47
4	38	46	67	0	70	34
5	33	65	41	70	0	69
6	48	75	47	34	69	0

i	j	s_{ij}
2	3	$40+48-87=1$
~~2~~	~~4~~	$40+38-46=32$
$\boxed{2}$	$\boxed{5}$	8
~~2~~	~~6~~	13
~~3~~	~~4~~	19
~~3~~	~~5~~	40
$\boxed{3}$	$\boxed{6}$	49
4	5	1
$\boxed{4}$	$\boxed{6}$	52
~~5~~	~~6~~	12



Multi-Stop (Offline) Savings Procedure

- Pairs of shipments ordered by their decreasing pairwise savings to create **i** and **j** (`pairwisesavings`)
- Creates set of multi-shipment routes (`savings`)

$$R = \{R_1, \ldots, R_m\}$$

  - Shipments with no pairwise savings are not included (use `sh2rte` to add)
- Clark-Wright only adds to beginning or end of a route
  - Multi-stop savings considers adding anywhere in route via min cost insert
  - More computation required, but can include sequence-dependent constraints like time windows (capacity not sequence dependent)

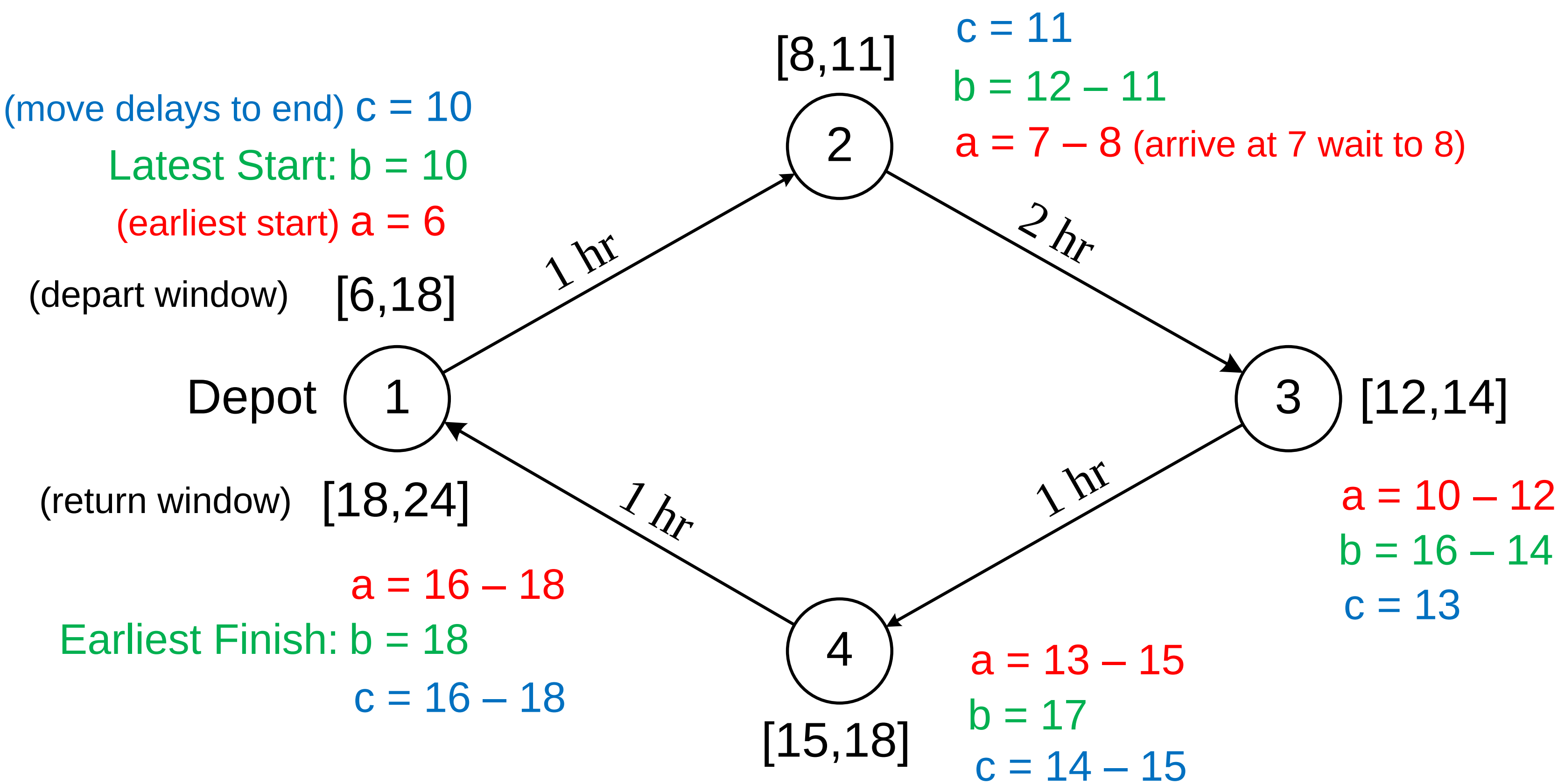
procedure $savings(\mathbf{i}, \mathbf{j})$
$R \leftarrow \{\ \}$
for $k = \{1, \ldots, |\mathbf{i}|\}$
  if $i_k \notin R$ and $j_k \notin R$ **1. Form new route**
    $R \leftarrow R \cup minCostInsert(i_k, j_k)$
  elseif $(i_k \notin R$ and $j_k \in R)$ or $(i_k \in R$ and $j_k \notin R)$
    if $j_k \notin R$ **2. Add shipment to route**
      $temp \leftarrow i_k, \quad i_k \leftarrow j_k, \quad j_k \leftarrow temp$
    endif
    $h \leftarrow \arg\{R_l : j_k \in R_l\}$
    $R' \leftarrow minCostInsert(i_k, R_h)$
    if $c(R') < c(i_k) + c(R_h)$
      $R_h \leftarrow R'$
    endif
  else   **3. Combine two routes**
    $g \leftarrow \arg\{R_l : i_k \in R_l\}, \quad h \leftarrow \arg\{R_l : j_k \in R_l\}$
    if $g \neq h$
      $R' \leftarrow minCostInsert(R_g, R_h)$
      if $c(R') < c(R_g) + c(R_h)$
        $R_g \leftarrow \{\ \}, \quad R_h \leftarrow R'$
      endif
    endif
  endif
endfor
return R

Vehicle Routing Problem

- VRP = TSP + vehicle constraints
- Constraints:
  - Capacity (weight, cube, etc.)
  - Maximum TC (HOS: 11 hr max)
  - Time windows (with/without delay at customer)
    - VRP uses absolute windows that can be checked by simple scanning
    - Project scheduling uses relative windows solved by shortest path with negative arcs
  - Maximum number of routes/vehicles (hard)
- Criteria:
  1. Number of routes/vehicles
  2. TC (time or distance)
- VRP solution can be one time or periodic
  - One time (operational) VRP minimizes TC
  - Periodic (tactical) VRP minimizes TLC (sometimes called a “milk run”)

Ex 18: VRP with Time Windows

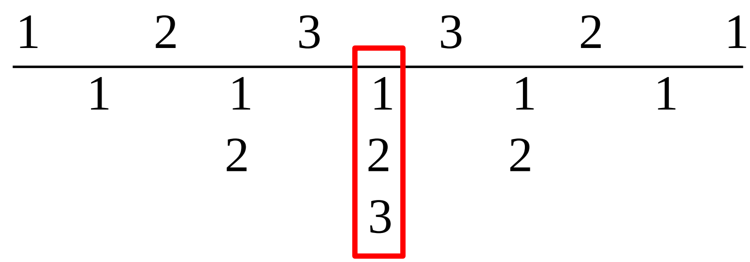
[0,24] hr; Loading/unloading time = 0; Capacity = ∞; LB = 5 hr



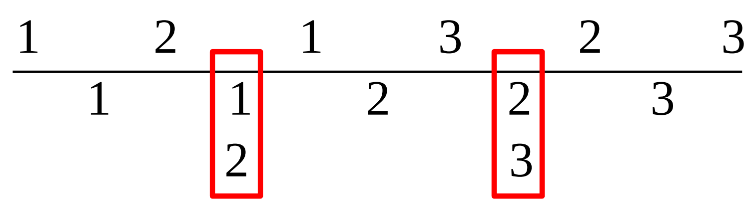
Earliest Finish – Latest Start = 18 – 10 = 8 hr = 5 travel + 3 delay

Periodic Multi-Stop Routing

- Periodic consolidated shipments that have the same frequency/interval
- Min TLC of aggregate shipment may not be feasible
  - Different combinations of shipments (*load mix*) may be on board during each segment of route
  - Minimum TLC of unconstrained aggregate of all shipments first determined
  - If needed, all shipment sizes reduced in proportion to load mix with the minimum max payload (to keep common frequency)



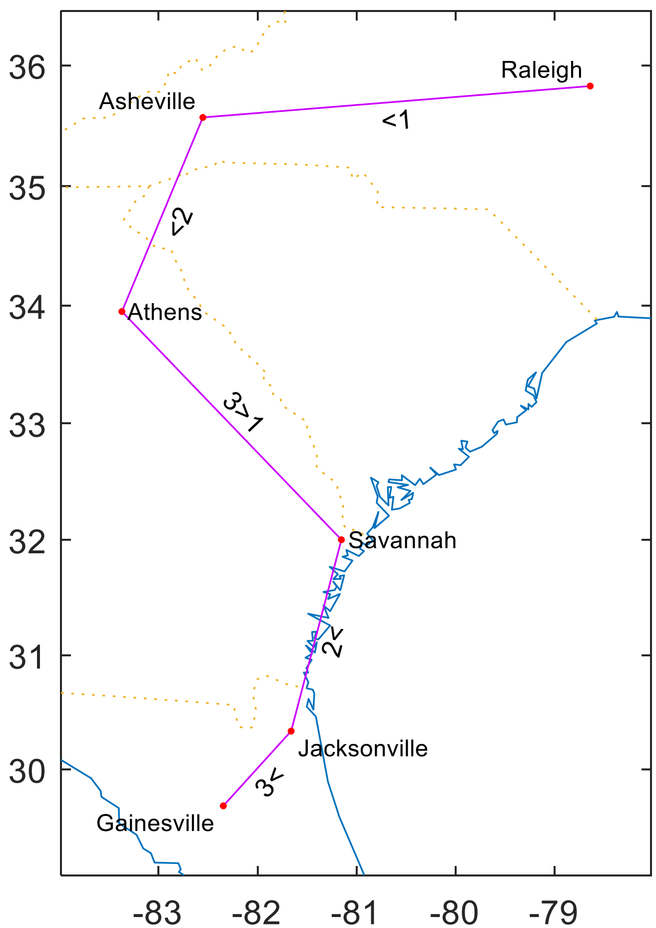
Single load mix



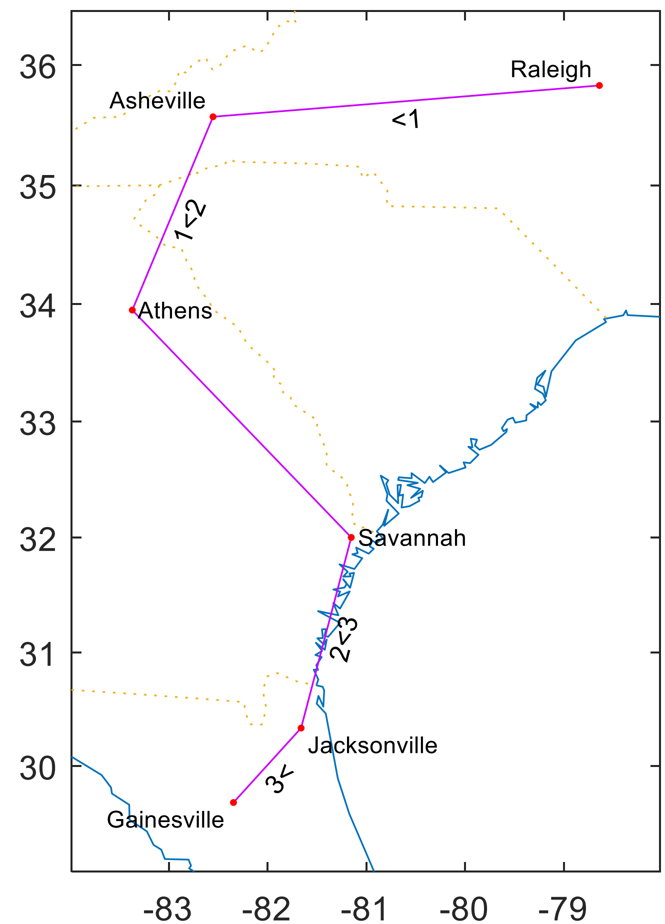
First load mix Second load mix

Load Mix Example

Single Load-Mix Instance



Two Load-Mix Instance



TLC Calculation for Multi-Stop Route

- How *minTLC* determines TLC for a route:

$$q_{agg} = \sqrt{\frac{f_{agg} \max\left\{r_{TL}\, d_{agg}, MC\right\}}{\alpha v_{agg} h}}$$ (no truck capacity constraints, only min charge)

$$q_i = q_{agg} \frac{f_i}{f_{agg}}$$ (allocate based on demand)

$$s_{L_j} = \sum_{i \in L_j} f_i \Big/ \sum_{i \in L_j} \frac{f_i}{s_i}$$ (aggregate density of shipments in load-mix L_j)

$$k = \min_{L_j}\left\{1,\quad \frac{\min\left\{K_{wt}, \dfrac{s_{L_j} K_{cu}}{2000}\right\}}{\sum_{i \in L_j} q_i}\right\}$$ (min ratio of max payload to size of shipments in load-mix)

$$q_i^* = k\, q_i$$ (apply truck capacity deduction factor)

$$TLC^* = \frac{f_{agg}}{\sum q_i^*} r_{TL}\, d_{agg} + \alpha v_{agg} h \sum q_i^*$$ (d_{agg} = distance of entire route)

Ex 19: Periodic Two Load-Mix Instance

D:	Raleigh	Athens	Asheville	Jax	Savannah	Gville
Raleigh:	0	357	264	501	361	571
Athens:	357	0	145	322	223	361
Asheville:	264	145	0	438	311	488
Jacksonville:	501	322	438	0	143	73
Savannah:	361	223	311	143	0	210
Gainesville:	571	361	488	73	210	0

sh:	f	s	b	e	v	a	h
1:	100	10	1	2	500	1	0.3
2:	200	15	3	4	200	1	0.3
3:	50	7	5	6	800	1	0.3

$$q_{agg} = \sqrt{\frac{f_{agg} \max\left\{r_{TL}\, d_{agg}, MC\right\}}{\alpha v_{agg} h}} = \sqrt{\frac{350 \max\left\{2(848), 135\right\}}{1\,(371.4286)\,0.3}} = 72.9875, \quad q_i = q_{agg} \frac{f_i}{f_{agg}} = 20.85, 41.71, 10.43$$

$$L_1 = \left\{1,2\right\}: \quad s_{L_1} = \sum_{i \in L_1} f_i \Big/ \sum_{i \in L_j} \frac{f_i}{s_i} = 12.8571, \quad k = 1$$

$$k = \min\left\{k, \frac{\min\left\{K_{wt}, \frac{s_{L_1} K_{cu}}{2000}\right\}}{\sum_{i \in L_1} q_i}\right\} = \min\left\{1, \frac{\min\left\{25, \frac{12.8571(2750)}{2000}\right\}}{62.5607}\right\} = \min\left\{1, 0.2826\right\} = 0.2826$$

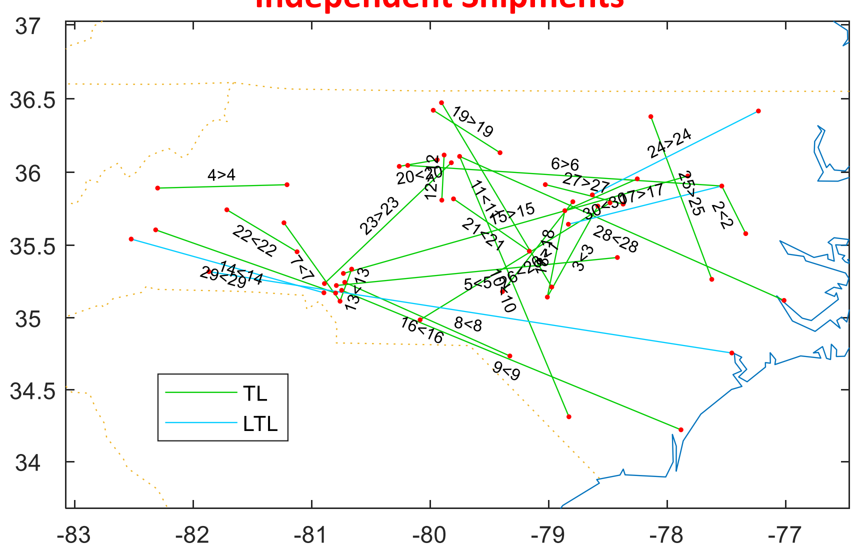
$$L_2 = \left\{2,3\right\}: \quad s_{L_2} = 12.2093, \quad k = \min\left\{0.2826, \frac{\min\left\{25, \frac{12.2093(2750)}{2000}\right\}}{52.1339}\right\} = \min\left\{0.2826,\ 0.3220\right\} = 0.2826$$

$$q_i^* = k\, q_i = 5.8929, 11.7857, 2.9464, \quad TLC^* = \frac{f_{agg}}{\sum q_i^*} r_{TL}\, d_{agg} + \alpha v_{agg} h \sum q_i^* = \$31,079$$

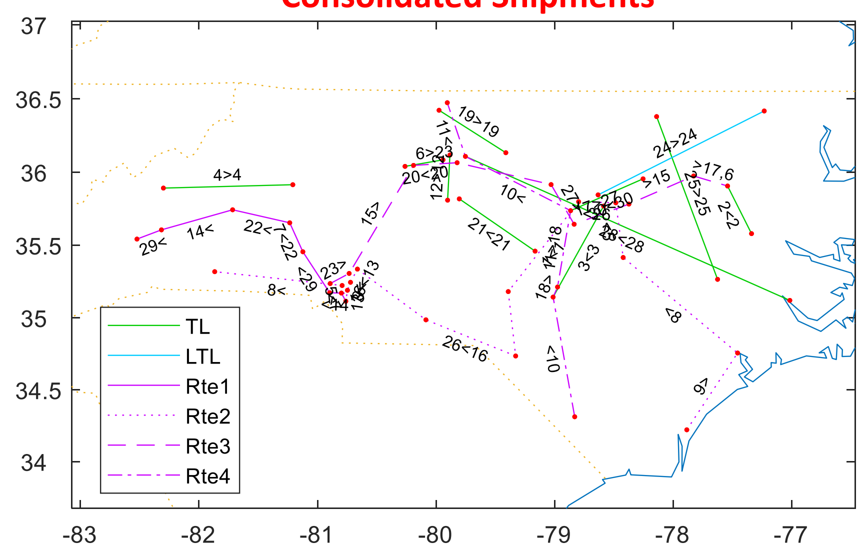
Ex 20: 30 Periodic NC Shipments

sh:	b	e	f	s	v	a	h	d	TLC1	q1	t1	isLTL	qmax
1:	15	42	2.13	1.17	683.19	0.5	0.3	64.97	337.20	1.61	0.76	0	1.61
2:	26	27	4.22	5.46	691.30	0.5	0.3	30.24	325.51	1.57	0.37	0	7.51
3:	23	40	6.27	15.23	5,843.73	0.5	0.3	59.27	1,614.46	0.92	0.15	0	20.94
4:	53	51	6.41	5.71	383.71	0.5	0.3	73.29	464.93	4.04	0.63	0	7.85
5:	17	32	6.32	13.82	2,776.55	0.5	0.3	161.07	1,842.23	2.21	0.35	0	19.00
28:	48	10	1.36	5.22	152.43	0.5	0.3	201.70	224.11	4.90	3.60	0	7.17
29:	37	54	4.92	26.52	8,327.51	0.5	0.3	114.03	1,599.38	0.24	0.05	1	25.00
30:	14	13	2.46	13.09	2,164.49	0.5	0.3	44.83	535.60	0.82	0.33	0	18.00

Independent Shipments

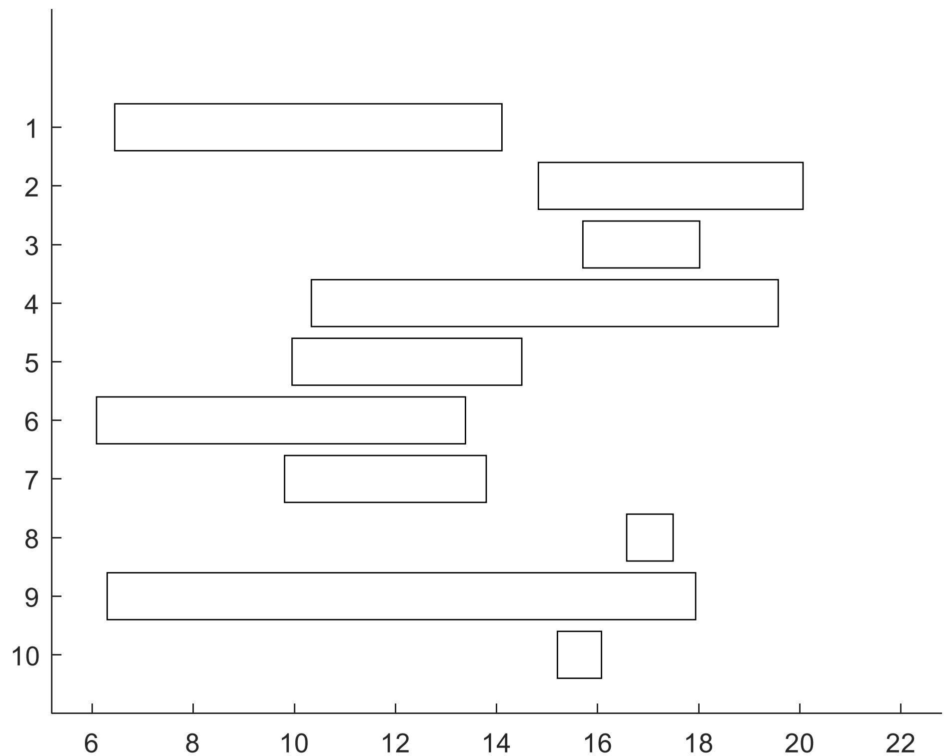


Consolidated Shipments



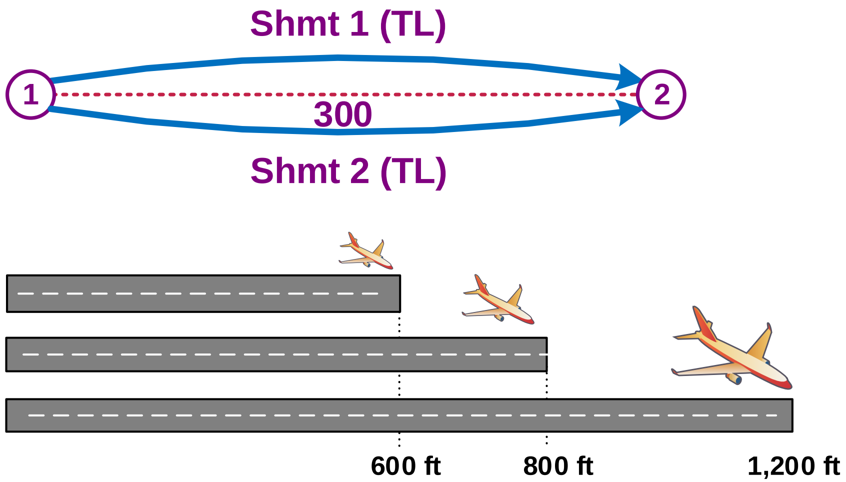
Ex 21: Minimize Number of Trucks

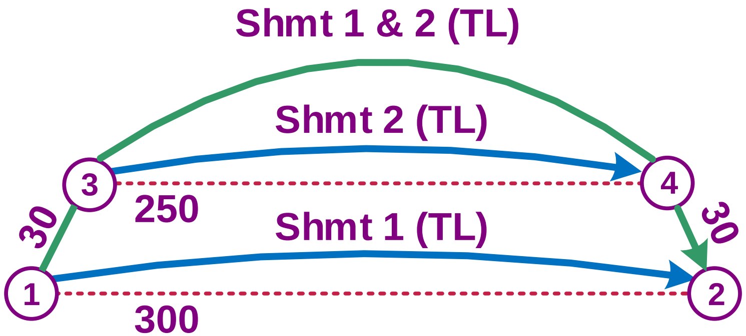
- Given begin-end times for 10 routes, determine minimum number of trucks needed
  - Trucks begin and end at the depot
  - Optimal solution via directed-arc minimum spanning tree
  - Greedy procedure usually works fine for small instances



Cost Allocation for Routing

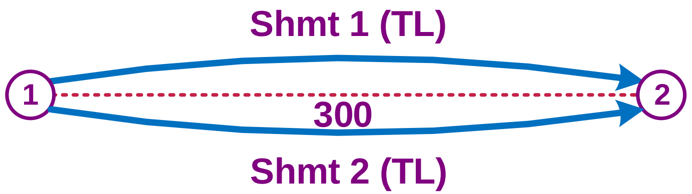
- **Allocation Problem:** If shipments from different firms are sharing the same vehicle, how much should each shipment contribute to the total cost paid to carrier?
  - What is a “fair” allocation?
  - Allocated cost should not exceed cost as an independent shipment (its *reservation price*)
  - Examples:



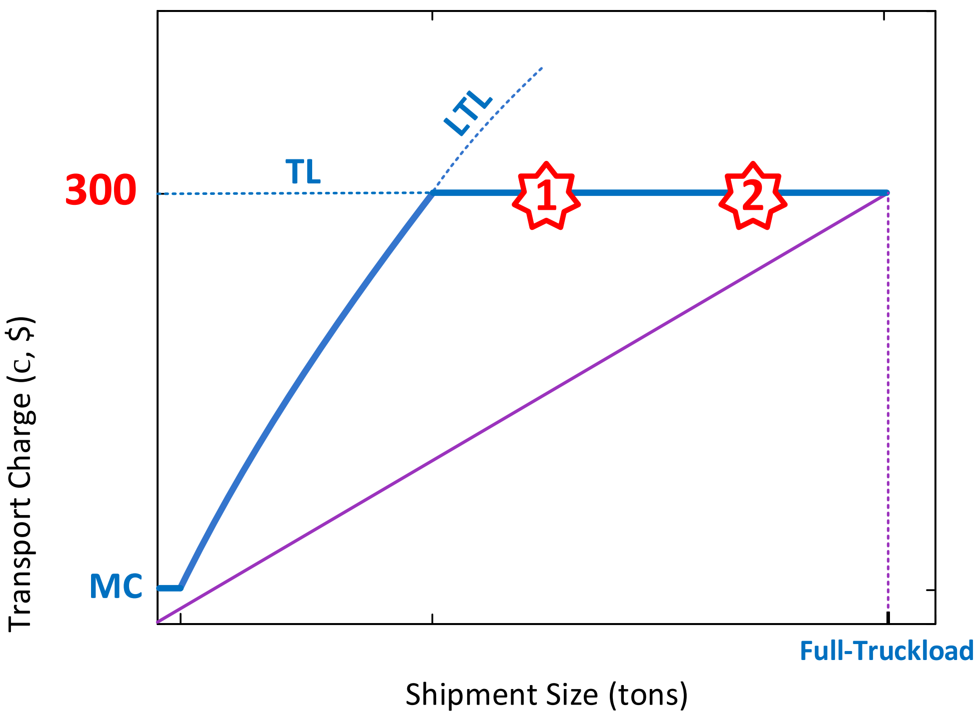


Ex 22: TL + TL Same O/D

- Shipment 1
  - sets r = 1, d = 300, TL, max c = 300
- Shipment 2
  - same O/D, TL, max c = 300



$$c = \max\left\{\min\left\{c_{LTL}, c_{TL}\right\}, MC\right\}$$

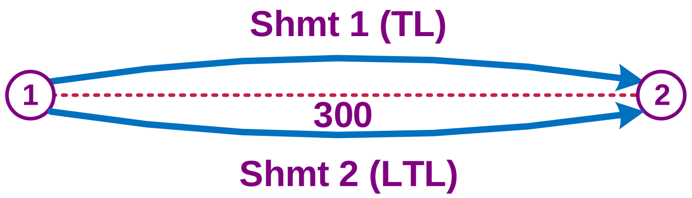


$$c = c_1 = 300$$

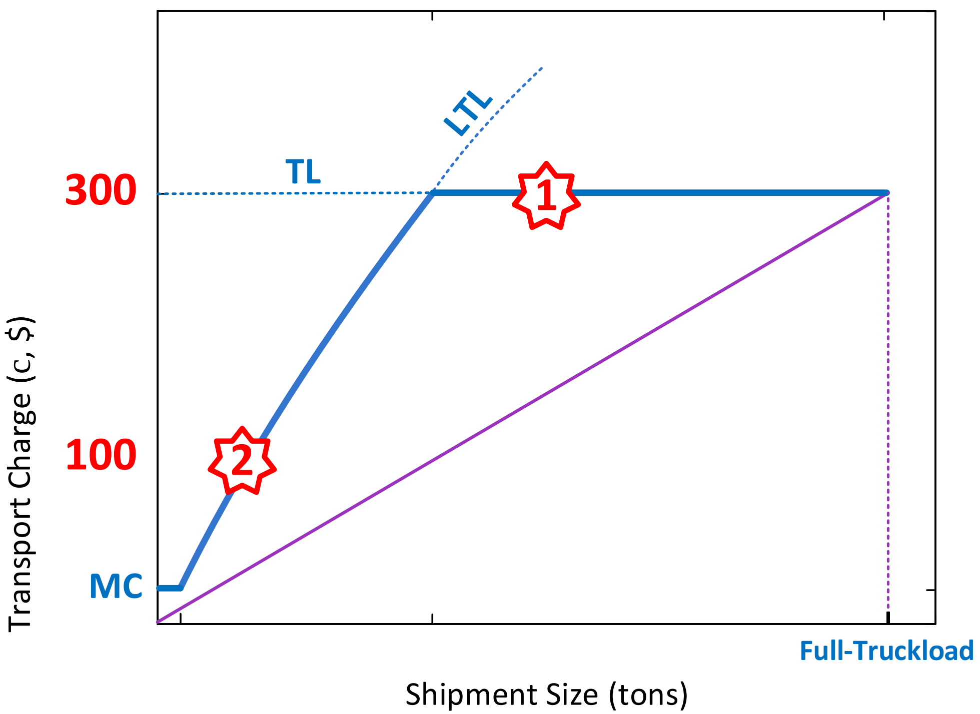
$$c = 300 = c_1 + c_2, \quad c_1 = c_2 = \frac{c}{2} = 150$$

Ex 23: TL + LTL Same O/D

- Shipment 1
  - sets r = 1, d = 300, TL, max c = 300
- Shipment 2
  - same O/D, LTL, max c = 100



$$c = \max\left\{\min\left\{c_{LTL}, c_{TL}\right\}, MC\right\}$$



$$c = 300 = c_1 + c_2$$

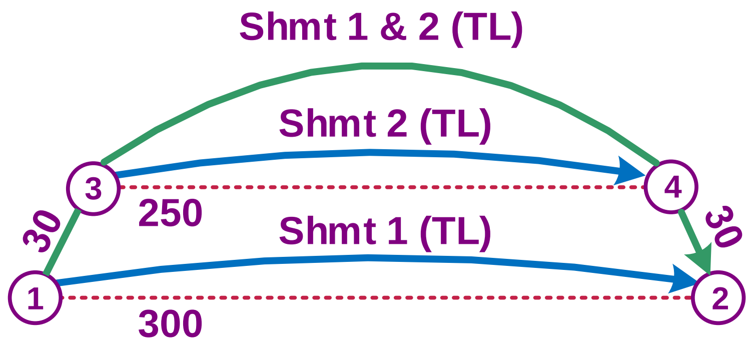
	1	2
1 2	300	0
2 1	200	100
	250	50

$$c_1 = 250, \quad c_2 = 50$$

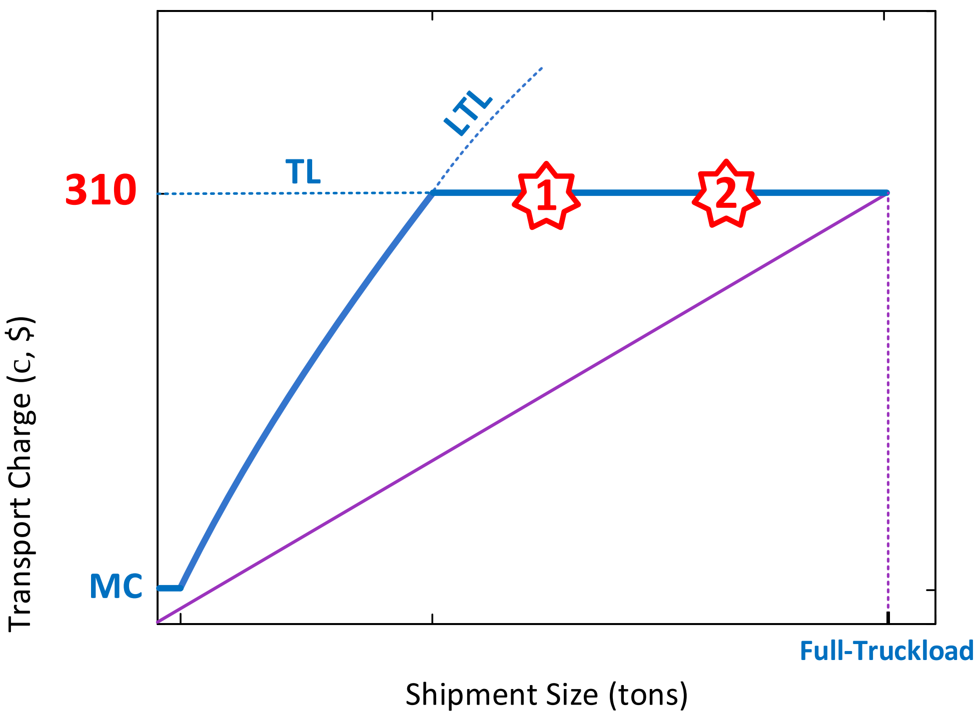
(Shapley value allocation)

Ex 24: TL + TL Different O/D

- Shipment 1
  - sets r = 1, d = 300, TL, max c = 300
- Shipment 2
  - different O/D, TL, max c = 250



$$c = \max\left\{\min\left\{c_{LTL}, c_{TL}\right\}, MC\right\}$$



$$c = 310 = c_1 + c_2$$

	1	2
1 2	300	10
2 1	60	250
	180	130

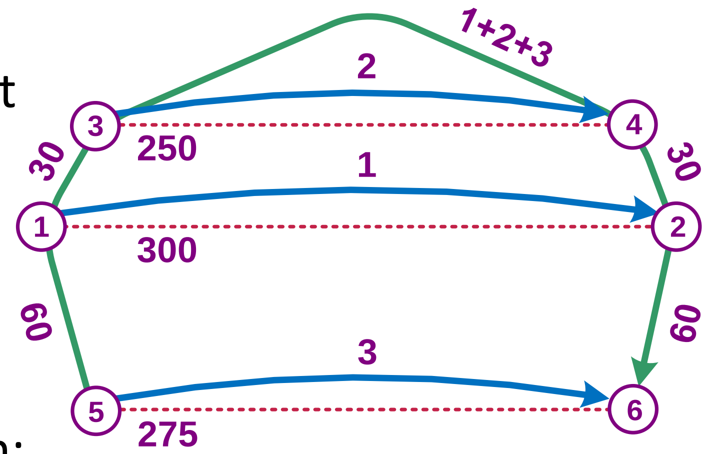
$$c_1 = 180, \quad c_2 = 130$$

Shapley Value Approximation

- Shapley value
  - Average additional cost each shipment imposes by joining route
  - Exact value requires $n!$

$$\alpha_i = \sum_{0 \le m \le n-1} \frac{m!(n-m-1)!}{n!} \sum_{\substack{M \subset N \setminus i \\ |M|=m}} \left(\sigma_{M \cup \{i\}} - \sigma_M \right)$$

  - Use n^2 pairwise savings approximation:



$$c_L^{\text{sav}} = \sum_{i=1}^{n} c_i^0 - c_L$$

$$= 300 + 250 + 275 - 430$$

$$= 825 - 430 = \$395 \text{ savings for load}$$

$$c_i^{\text{sav}} = \frac{c_L^{\text{sav}}}{n} + \frac{1}{n-1} \sum_{j=1}^{n} \frac{c_{ij}^{\text{sav}} + c_{ji}^{\text{sav}}}{2} - \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk}^{\text{sav}}$$

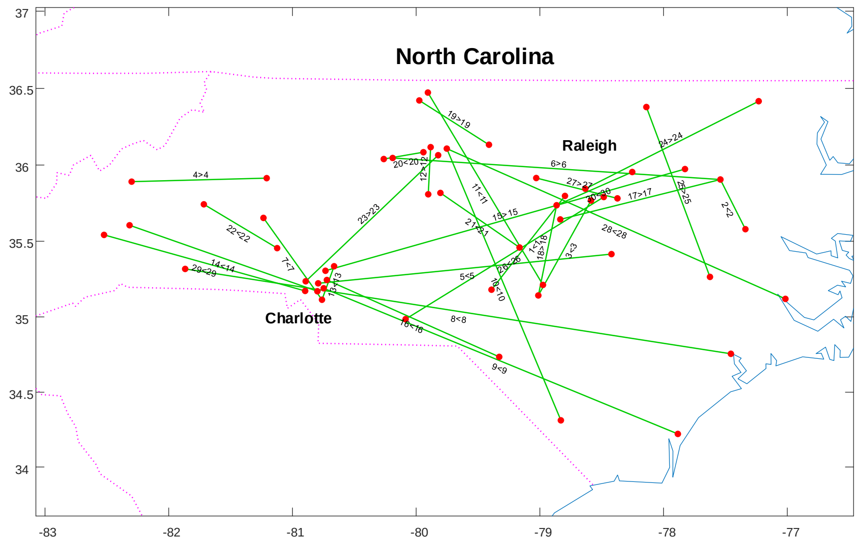
$$c_{ij}^{\text{sav}} = c_i^0 + c_j^0 - c_{(i,j)}, \quad c_L^{\text{sav}} = \sum_{i=1}^{n} c_i^0 - c_L$$

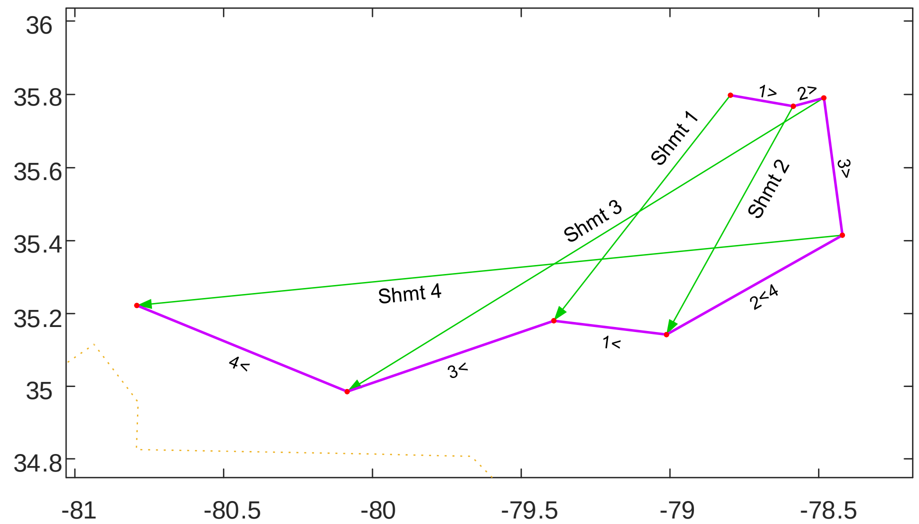
:	c0	c_equal	c_eq_sav	c_Shap_exact	c_Shap_approx
1:	300	143.33	168.33	130.00	130.00
2:	250	143.33	118.33	122.50	122.50
3:	275	143.33	143.33	177.50	177.50
Total:	825	430.00	430.00	430.00	430.00
Avg:	275	143.33	143.33	143.33	143.33

C:	1	2	3
123:	300	10	120
132:	300	35	95
213:	60	250	120
231:	0	250	180
312:	120	35	275
321:	0	155	275

Ex 25: Intercity Trucking

- 4 out 30 available shipments form consolidated load
  - Savings of 824.81 – 452.47 = 372.34 from consolidation
  - Pairwise approximation differs from exact Shapley value





Shmt :	c0	c_equal	(%)	c_eq_sav	(%)	c_Shap_exact	(%)	c_Shap_approx
1:	130	113	13	37	72	62	52	52
2:	119	113	5	25	79	53	55	50
3:	254	113	56	161	37	117	54	123
4:	322	113	65	229	29	220	32	227
Total:	825	452		452		452		452

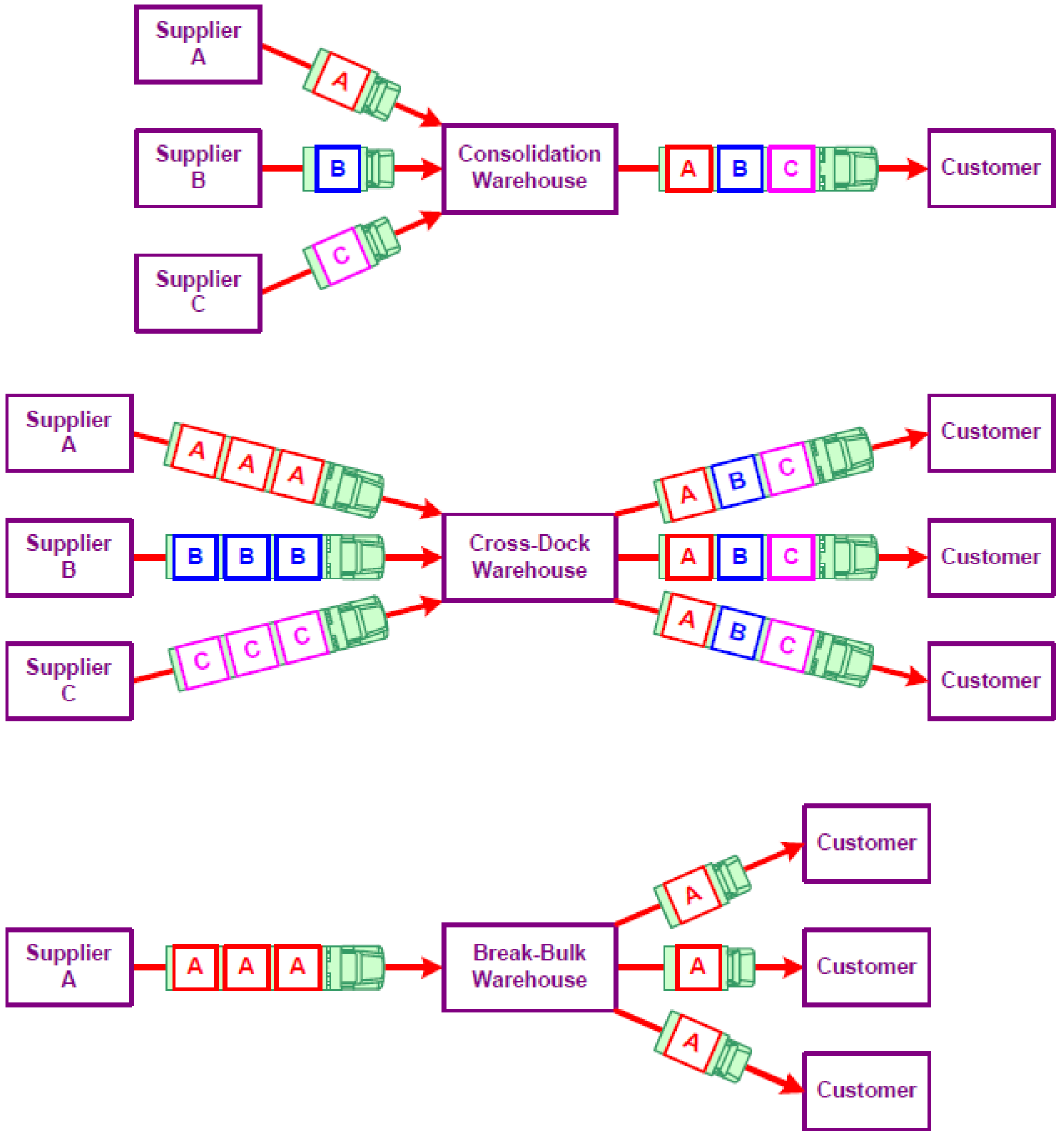
Topics

1. Introduction
2. Facility location
3. Freight transport
   – Exam 1 (take home)
4. Network models
5. Routing
   – Exam 2 (take home)
6. **Warehousing**
   – Final exam (in class)

Warehousing

- *Warehousing* are the activities involved in the design and operation of warehouses
- A *warehouse* is the point in the supply chain where raw materials, work-in-process (WIP), or finished goods are stored for varying lengths of time.
- Warehouses can be used to add value to a supply chain in two basic ways:
  1. Storage. Allows product to be available where and when its needed.
  2. Transport Economies. Allows product to be collected, sorted, and distributed efficiently.
- A *public warehouse* is a business that rents storage space to other firms on a month-to-month basis. They are often used by firms to supplement their own *private warehouses*.

Types of Warehouses



Warehouse Design Process

- The objectives for warehouse design can include:
  - maximizing cube utilization
  - minimizing total storage costs (including building, equipment, and labor costs)
  - achieving the required storage throughput
  - enabling efficient order picking
- In planning a storage layout: either a storage layout is required to fit into an existing facility, or the facility will be designed to accommodate the storage layout.

Warehouse Design Elements

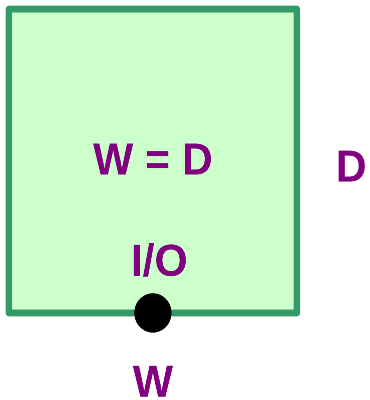
- The design of a new warehouse includes the following elements:
  1. Determining the layout of the storage locations (i.e., the warehouse layout).
  2. Determining the number and location of the input/output (I/O) ports (e.g., the shipping/receiving docks).
  3. Assigning items (stock-keeping units or *SKUs*) to storage locations (*slots*).
- A typical objective in warehouse design is to minimize the overall storage cost while providing the required levels of service.

Design Trade-Off

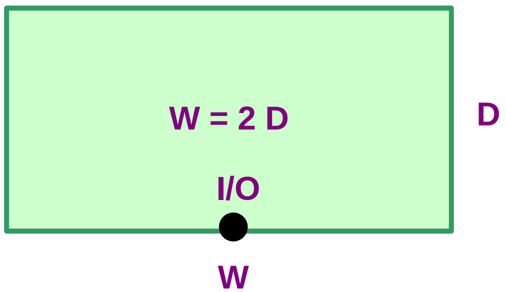
- Warehouse design involves the trade-off between building and handling costs:

min Building Costs	vs.	min Handling Costs
⇕		⇕
max Cube Utilization	vs.	max Material Accessibility

Shape Trade-Off



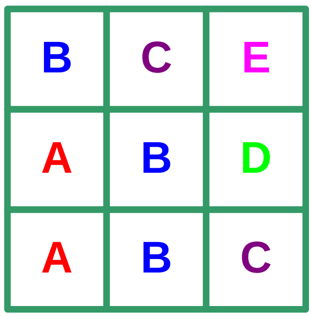
vs.



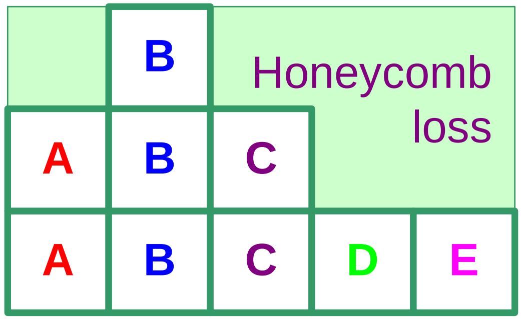
Square shape minimizes perimeter length for a given area, thus minimizing building costs

Aspect ratio of 2 (W = 2D) min. expected distance from I/O port to slots, thus minimizing handling costs

Storage Trade-Off



vs.



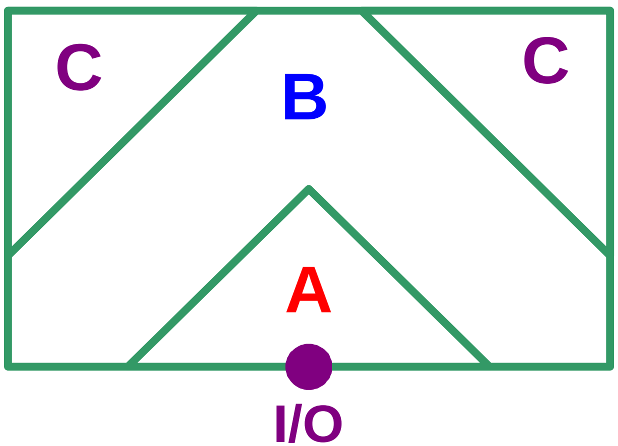
Maximizes cube utilization, but minimizes material accessibility

Making at least one unit of each item accessible decreases cube utilization

Storage Policies

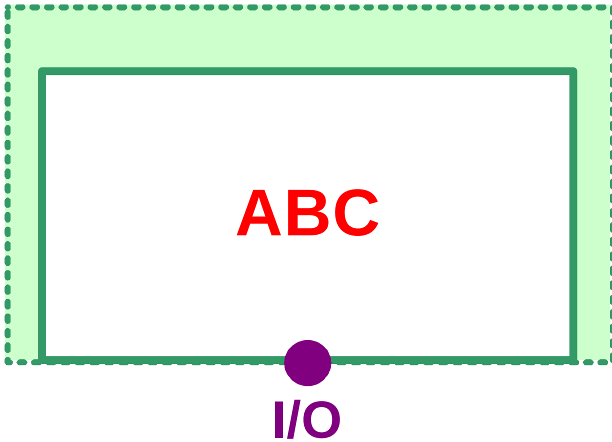
- A storage policy determines how the slots in a storage region are assigned to the different SKUs to the stored in the region.
- The differences between storage polices illustrate the trade-off between minimizing building cost and minimizing handling cost.
- Type of policies:
  - Dedicated
  - Randomized
  - Class-based

Dedicated Storage



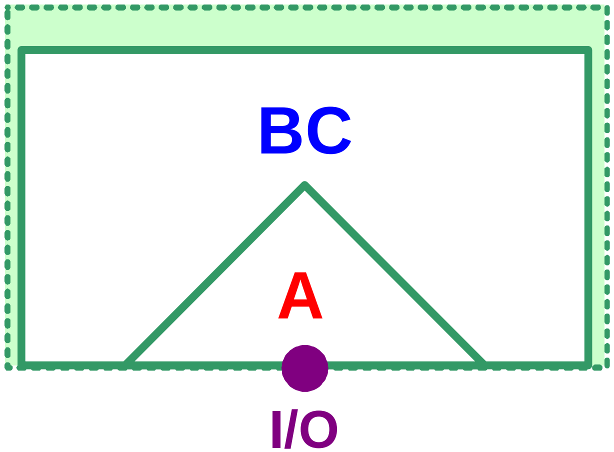
- Each SKU has a predetermined number of slots assigned to it.
- Total capacity of the slots assigned to each SKU must equal the storage space corresponding to the maximum inventory level of each *individual* SKU.
- Minimizes handling cost.
- Maximizes building cost.

Randomized Storage



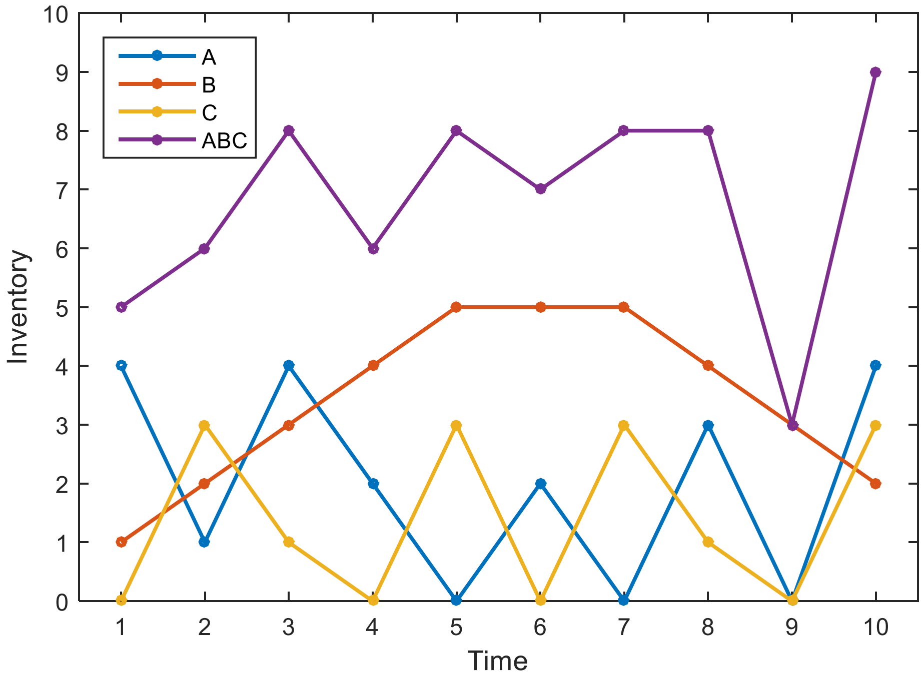
- Each SKU can be stored in any available slot.
- Total capacity of all the slots must equal the storage space corresponding to the maximum *aggregate* inventory level of all of the SKUs.
- Maximizes handling cost.
- Minimizes building cost.

Class-based Storage



- Combination of dedicated and randomized storage, where each SKU is assigned to one of several different storage classes.
- Randomized storage is used for each SKU within a class, and dedicated storage is used between classes.
- Building and handling costs between dedicated and randomized.

Individual vs Aggregate SKUs



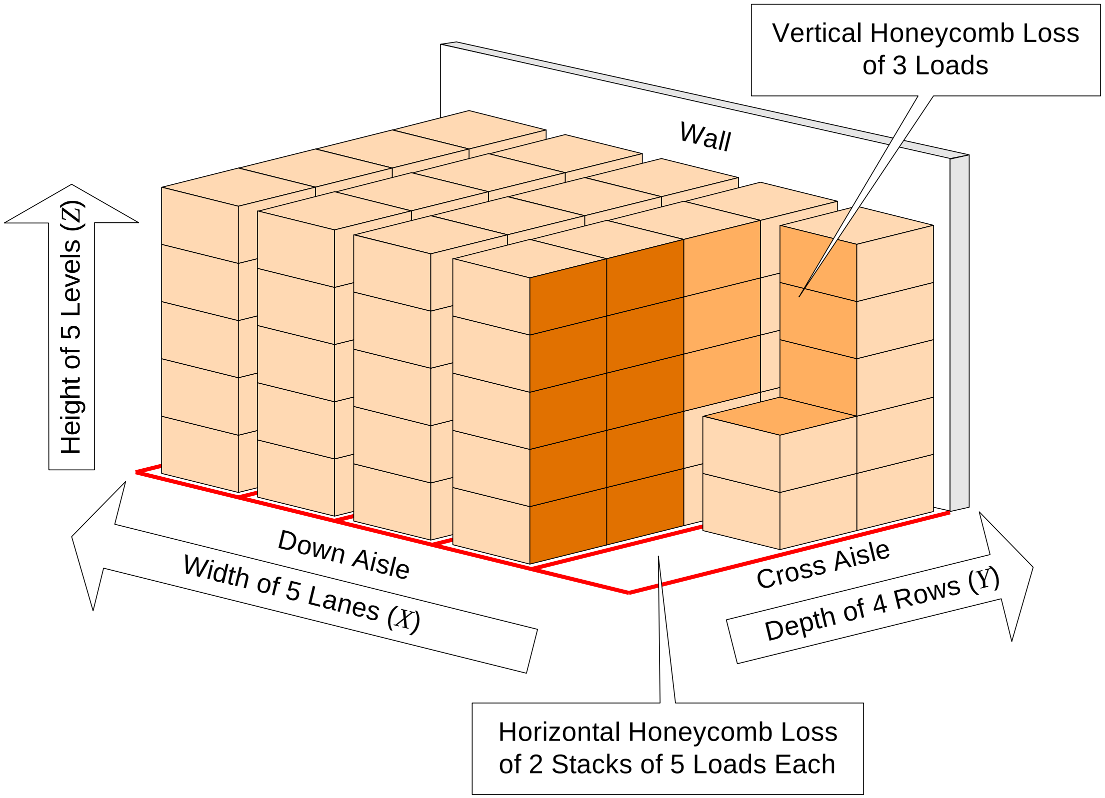
	Dedicated			Random	Class-Based		
Time	A	B	C	ABC	AB	AC	BC
1	4	1	0	5	5	4	1
2	1	2	3	6	3	4	5
3	4	3	1	8	7	5	4
4	2	4	0	6	6	2	4
5	0	5	3	8	5	3	8
6	2	5	0	7	7	2	5
7	0	5	3	8	5	3	8
8	3	4	1	8	7	4	5
9	0	3	0	3	3	0	3
10	4	2	3	9	6	7	5
M_i	4	5	3	9	7	7	8

Cube Utilization

- *Cube utilization* is percentage of the total space (or “cube”) required for storage actually occupied by items being stored.
- There is usually a trade-off between cube utilization and material accessibility.
- Bulk storage using block stacking can result in the minimum cost of storage, but material accessibility is low since only the top of the front stack is accessible.
- Storage racks are used when support and/or material accessibility is required.

Honeycomb Loss

- *Honeycomb loss*, the price paid for accessibility, is the unusable empty storage space in a lane or stack due to the storage of only a single SKU in each lane or stack



Estimating Cube Utilization

- The (3-D) cube utilization for dedicated and randomized storage can estimated as follows:

$$\text{Cube utilization} = \frac{\text{item space}}{\text{total space}} = \frac{\text{item space}}{\text{item space} + \begin{pmatrix}\text{honeycomb} \\ \text{loss}\end{pmatrix} + \begin{pmatrix}\text{down aisle} \\ \text{space}\end{pmatrix}}$$

$$CU(\text{3-D}) = \begin{cases} \dfrac{x \cdot y \cdot z \cdot \sum_{i=1}^{N} M_i}{TS(D)}, & \text{dedicated} \\[2ex] \dfrac{x \cdot y \cdot z \cdot M}{TS(D)}, & \text{randomized} \end{cases}$$

$$CU(\text{2-D}) = \begin{cases} \dfrac{x \cdot y \cdot \sum_{i=1}^{N} \left\lceil \dfrac{M_i}{H} \right\rceil}{TA(D)}, & \text{dedicated} \\[2ex] \dfrac{x \cdot y \cdot \left\lceil \dfrac{M}{H} \right\rceil}{TA(D)}, & \text{randomized} \end{cases}$$

where

x = lane/unit-load width

y = unit-load depth

z = unit-load height

M_i = maximum number of units of SKU i

M = maximum number of units of all SKUs

N = number of different SKUs

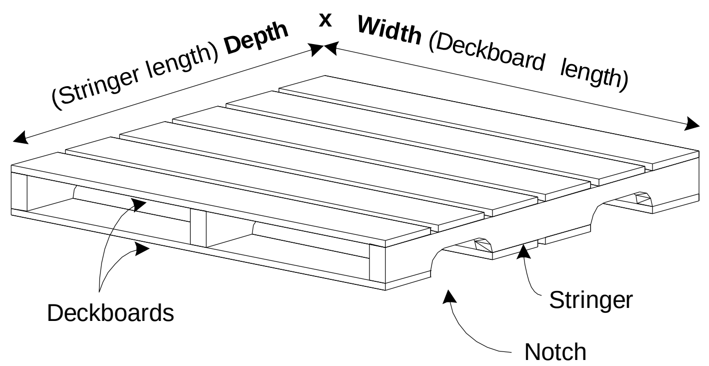
D = number of rows

$TS(D)$ = total 3-D *space* (given D rows of storage).

$TA(D)$ = total 2-D *area* (given D rows of storage).

Unit Load

- *Unit load*: single unit of an item, or multiple units restricted to maintain their integrity
- Linear dimensions of a unit load:

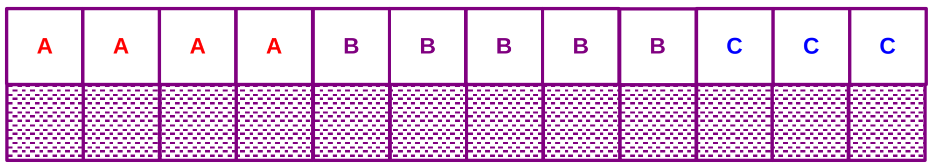
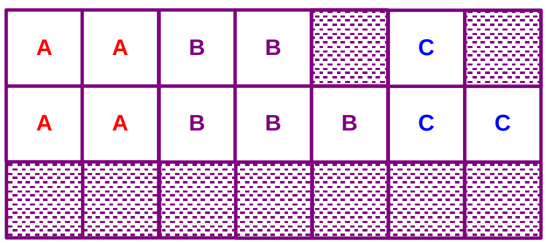
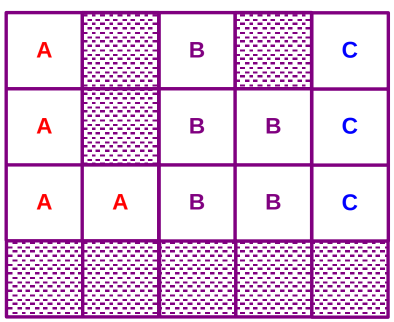


Depth (stringer length) × *Width* (deckboard length)

$$y \times x$$

- Pallet height (5 in.) + load height gives z: $y \times x \times z$

Cube Utilization for Dedicated Storage

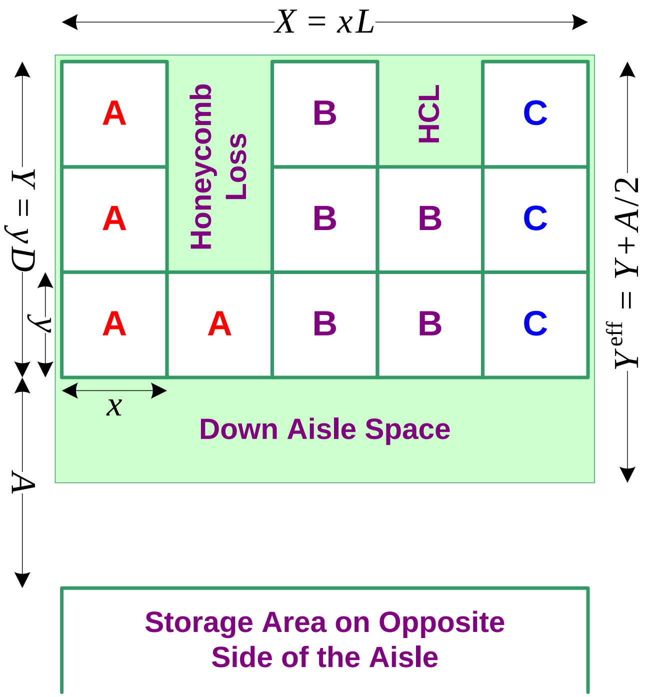
	Storage Area at Different Lane Depths	Item Space	Lanes	Total Space	Cube Util.
$D = 1$ $A/2 = 1$		12	12	24	50%
$D = 2$ $A/2 = 1$		12	7	21	57%
$D = 3$ $A/2 = 1$		12	5	20	60%

Total Space/Area

- The total space required, as a function of lane depth *D*:

$$\text{Total space (3-D):}\quad TS(D) = X \cdot \underbrace{\left(Y + \frac{A}{2}\right)}_{\text{Eff. lane depth}} \cdot Z = xL(D) \cdot \left(yD + \frac{A}{2}\right) \cdot zH$$

$$\text{Total area (2-D):}\quad TA(D) = \frac{TS(D)}{Z} = X \cdot Y^{\text{eff}} = xL(D) \cdot \left(yD + \frac{A}{2}\right)$$



where

$X =$ width of storage region (row length)

$Y =$ depth of storage region (lane depth)

$Z =$ height of storage region (stack height)

$A =$ down aisle width

$L(D) =$ number of lanes (given D rows of storage)

$H =$ number of levels.

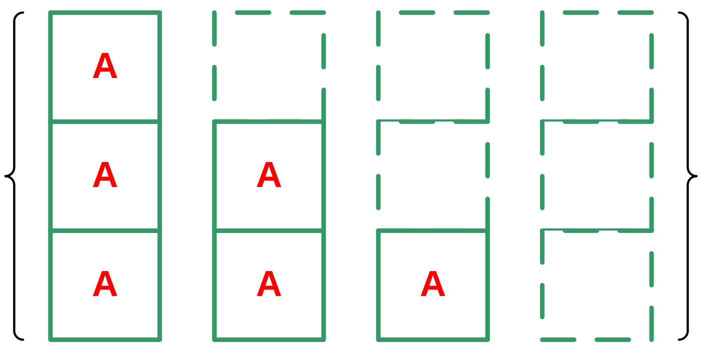
Number of Lanes

- Given *D*, estimated total number of lanes in region:

$$\text{Number of lanes:}\quad L(D)=\begin{cases}\displaystyle\sum_{i=1}^{N}\left\lceil\frac{M_i}{DH}\right\rceil, & \text{dedicated}\\[2ex] \left\lceil\dfrac{M+NH\left(\dfrac{D-1}{2}\right)+N\left(\dfrac{H-1}{2}\right)}{DH}\right\rceil, & \text{randomized } (N>1)\end{cases}$$

- Estimated HCL:

$D=3$



doesn't occur because slots are used by another SKU

Probability: $\frac{1}{D}$ $\times$ + $\frac{1}{D}$ $\times$ + $\frac{1}{D}$ $\times$

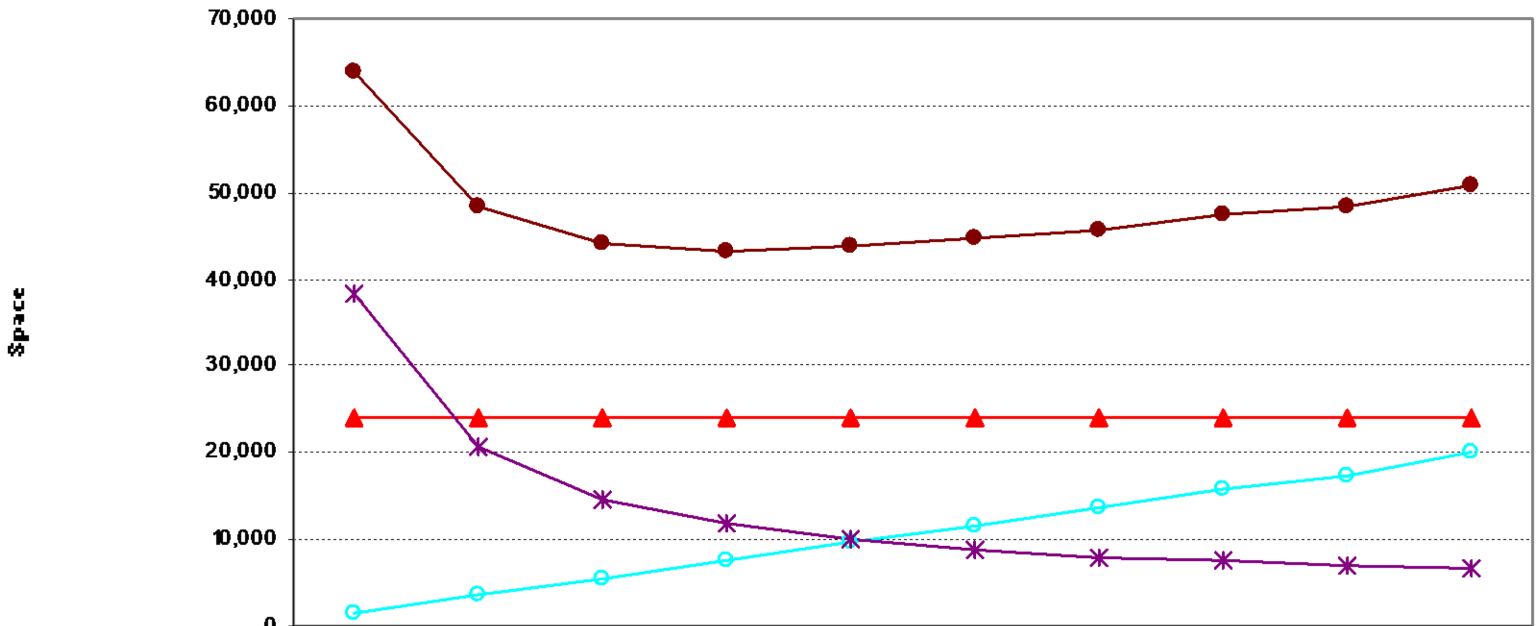
Unit Honeycomb Loss: 0 $\quad(D-2)\quad(D-1)$

$$\text{Expected Loss: }\frac{1}{D}\big[(D-2)+(D-1)\big]=\frac{1}{D}(1+2)=\frac{1}{D}\sum_{i=1}^{D-1}i=\frac{1}{D}\left(\frac{(D-1)D}{2}\right)=\frac{D-1}{2}=1$$

Optimal Lane Depth

- Solving for D in $dTS(D)/dD = 0$ results in:

Optimal lane depth for randomized storage (in rows): $D^* = \left\lfloor \sqrt{\dfrac{A(2M-N)}{2NyH}} + \dfrac{1}{2} \right\rfloor$



	1	2	3	4	5	6	7	8	9	10
Item Space	24,000	24,000	24,000	24,000	24,000	24,000	24,000	24,000	24,000	24,000
Honeycomb Loss	1,536	3,648	5,376	7,488	9,600	11,712	13,632	15,936	17,472	20,160
Aisle Space	38,304	20,736	14,688	11,808	10,080	8,928	8,064	7,488	6,912	6,624
Total Space	63,840	48,384	44,064	43,296	43,680	44,640	45,696	47,424	48,384	50,784

Lane Depth (in Rows)

Max Aggregate Inventory Level

- Usually can determine max inventory level for each SKU:
  - M_i = maximum number of units of SKU *i*
- Since usually don't know M directly, but can estimate it **if**
  - SKUs' inventory levels are uncorrelated
  - Units of each item are either stored or retrieved at a constant rate

$$M = \left\lfloor \sum_{i=1}^{N} \frac{M_i}{2} + \frac{1}{2} \right\rfloor$$

- Can add include safety stock for each item, SS_i
  - For example, if the order size of three SKUs is 50 units and 5 units of each item are held as safety stock

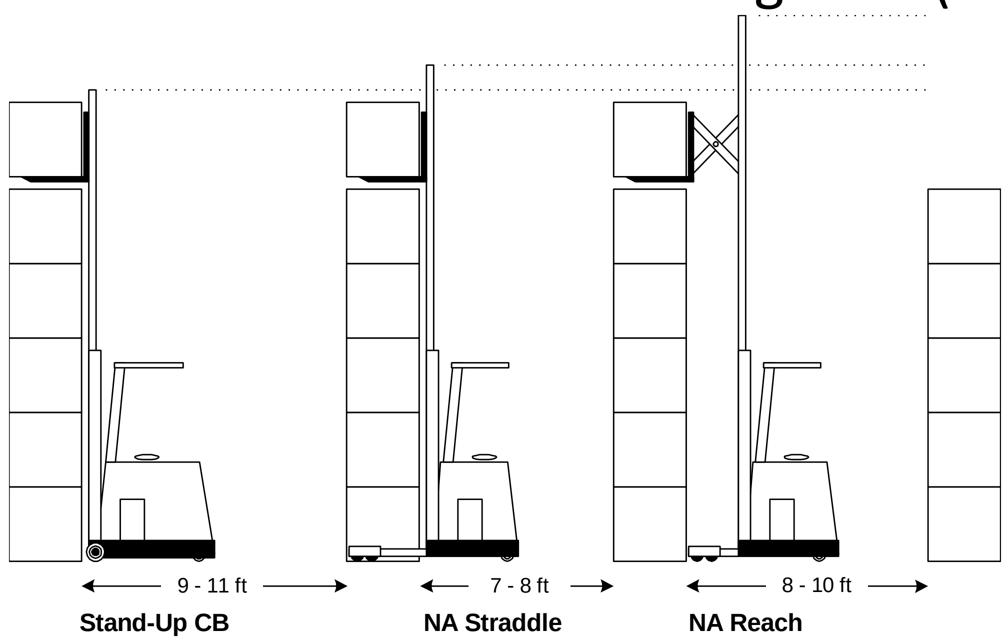
$$M = \left\lfloor \sum_{i=1}^{N} \left(\frac{M_i - SS_i}{2} + SS_i \right) + \frac{1}{2} \right\rfloor = \left\lfloor 3\left(\frac{50}{2} + 5 \right) + \frac{1}{2} \right\rfloor = 90$$

Steps to Determine Area Requirements

1. For randomized storage, assumed to know N, H, x, y, z, A, and all M_i
   – Number of levels, H, depends on building clear height (for block stacking) or shelf spacing
   – Aisle width, A, depends on type of lift trucks used
2. Estimate maximum aggregate inventory level, M
3. If D not fixed, estimate optimal land depth, D^*
4. Estimate number of lanes required, $L(D^*)$
5. Determine total 2-D area, $TA(D^*)$

Aisle Width Design Parameter

- Typically, A (and sometimes H) is a parameter used to evaluate different overall design alternatives
- Width depends on type of lift trucks used, a narrower aisle truck
  - reduces area requirements (building costs)
  - costs more and slows travel and loading time (handling costs)



Ex 26: Area Requirements

Units of items A, B, and C are all received and stored as 42 × 36 × 36 in. (*y* × *x* × *z*) pallet loads in a storage region that is along one side of a 10-foot-wide down aisle in the warehouse of a factory. The shipment size received for each item is 31, 62, and 42 pallets, respectively. Pallets can be stored up to three deep and four high in the region.

$$x = \frac{36}{12} = 3' \qquad M_A = 31 \qquad A = 10'$$

$$y = 3.5' \qquad M_B = 62 \qquad D = 3$$

$$z = 3' \qquad M_C = 42 \qquad H = 4$$

$$N = 3$$

Ex 26: Area Requirements

1. If a dedicated policy is used to store the items, what is the 2-D cube utilization of this storage region?

$$L(D) = L(3) = \sum_{i=1}^{N} \left\lceil \frac{M_i}{DH} \right\rceil = \left\lceil \frac{31}{3(4)} \right\rceil + \left\lceil \frac{62}{3(4)} \right\rceil + \left\lceil \frac{42}{3(4)} \right\rceil = 3 + 6 + 4 = 13 \text{ lanes}$$

$$TA(3) = xL(D) \cdot \left(yD + \frac{A}{2} \right) = 3(13) \cdot \left(3.5(3) + \frac{10}{2} \right) = 605 \text{ ft}^2$$

$$CU(3) = \frac{\text{item space}}{TA(3)} = \frac{x \cdot y \cdot \sum_{i=1}^{N} \left\lceil \frac{M_i}{H} \right\rceil}{TA(3)} = \frac{3 \cdot 3.5 \cdot \left(\left\lceil \frac{31}{4} \right\rceil + \left\lceil \frac{62}{4} \right\rceil + \left\lceil \frac{42}{4} \right\rceil \right)}{605} = 61\%$$

Ex 26: Area Requirements

2. If the shipments of each item are uncorrelated with each other, no safety stock is carried for each item, and retrievals to the factory floor will occur at a constant rate, what is an estimate the maximum number of units of all items that would ever occur?

$$M = \left\lfloor \sum_{i=1}^{N} \frac{M_i}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{31+62+42}{2} + \frac{1}{2} \right\rfloor = 68$$

Ex 26: Area Requirements

3. If a randomized policy is used to store the items, what is total 2-D area needed for the storage region?

$$D = 3$$

$$L(3) = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{68 + 3(4)\left(\frac{3-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 8 \text{ lanes}$$

$$TA(3) = xL(D) \cdot \left(yD + \frac{A}{2} \right) = 3(8) \cdot \left(3.5(3) + \frac{10}{2} \right) = 372 \text{ ft}^2$$

Ex 26: Area Requirements

4. What is the optimal lane depth for randomized storage?

$$D^* = \left\lfloor \sqrt{\frac{A(2M - N)}{2NyH}} + \frac{1}{2} \right\rfloor = \left\lfloor \sqrt{\frac{10(2(68) - 3)}{2(3)3.5(4)}} + \frac{1}{2} \right\rfloor = 4$$

5. What is the change in total area associated with using the optimal lane depth as opposed to storing the items three deep?

$$D = 4 \Rightarrow L(4) = \left\lceil \frac{68 + 3(4)\left(\frac{4-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 6 \text{ lanes}$$

$$\Rightarrow TA(4) = 3(6) \cdot \left(3.5(4) + \frac{10}{2}\right) = 342 \text{ ft}^2$$

$$D = 3 \Rightarrow TA(3) = 372 \text{ ft}^2$$

Ex 27: Trailer Loading

How many identical 48 × 42 × 36 in. four-way containers can be shipped in a full truckload? Each container load:

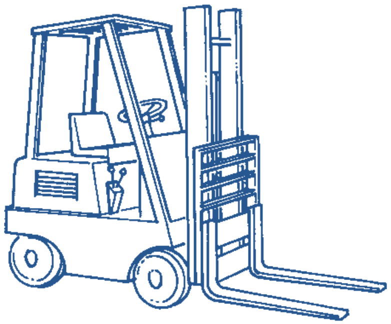
1. Weighs 600 lb
2. Can be stacked up to six high without causing damage from crushing
3. Can be rotated on the trucks with respect to their width and depth.

X	98/12 =	8.166667	8.166667	ft
Y		53	53	ft
Z	110/12 =	9.166667	9.166667	ft
x	[48,42]/12 =	4	3.5	ft
y	[42,48]/12 =	3.5	4	ft
z	30/12 =	2.5	2.5	ft
L	floor(X/x) =	2	2	
D	floor(Y/y) =	15	13	
H	min(6,floor(Z/z)) =	3	3	
LDH	L*D*H =	90	78	units
wt		600	600	lb
unit/TL	min(LDH, floor(50000/wt)) =	83	78	

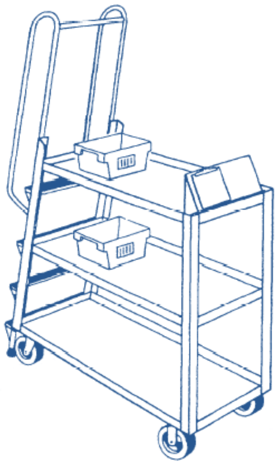
Max of 83 units per TL

Storage and Retrieval Cycle

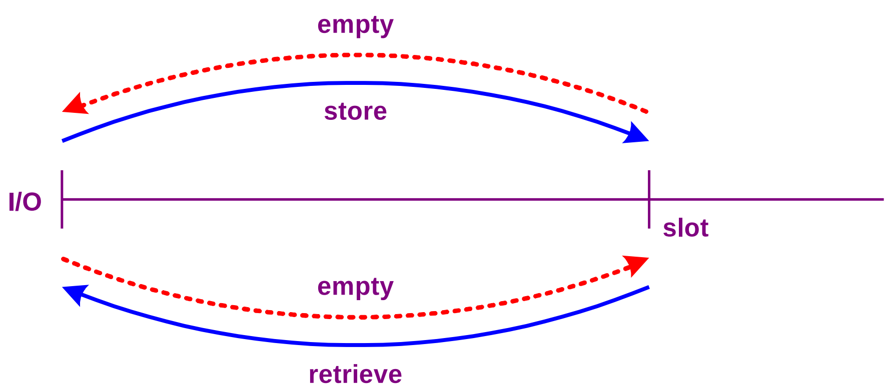
- A storage and retrieval (S/R) cycle is one complete roundtrip from an I/O port to slot(s) and back to the I/O
- Type of cycle depends on load carrying ability:
  - Carrying one load at-a-time (load carried on a pallet):
    - Single command
    - Dual command



  - Carrying multiple loads (order picking of small items):
    - Multiple command



Single-Command S/R Cycle



- Single-command (SC) cycles:
  - Storage: carry one load to slot for storage and return empty back to I/O port, or
  - Retrieval: travel empty to slot to retrieve load and return with it back to I/O port

Expected time for each SC S/R cycle:

$$t_{SC} = \frac{d_{SC}}{v} + t_L + t_U = \frac{d_{SC}}{v} + 2t_{L/U}$$

where

d_{SC} = expected distance per SC cycle

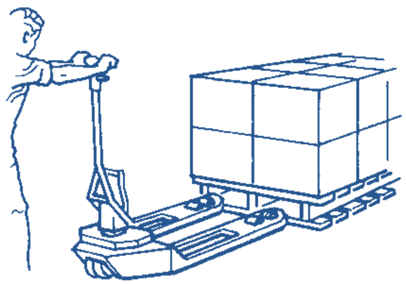
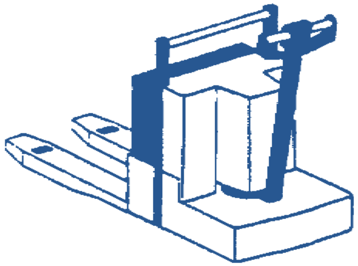
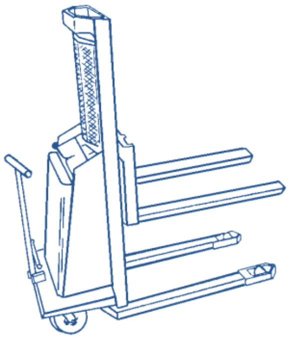
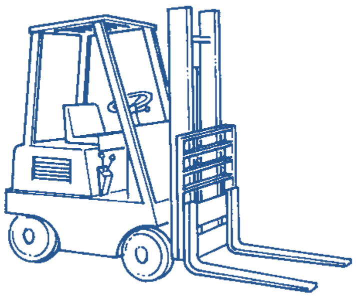
v = average travel speed (e.g.: 2 mph = 176 fpm walking; 7 mph = 616 fpm riding)

t_L = loading time

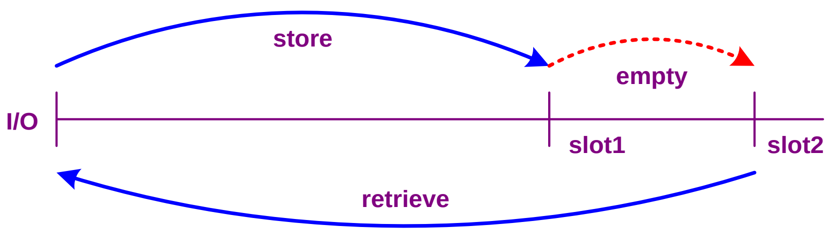
t_U = unloading time

$t_{L/U}$ = loading/unloading time, if same value

Industrial Trucks: Walk vs. Ride

Walk (2 mph = 176 fpm)	Ride (7 mph = 616 fpm)
	
Pallet Jack	Pallet Truck
	
Walkie Stacker	Sit-down Counterbalanced Lift Truck

Dual-Command S/R Cycle

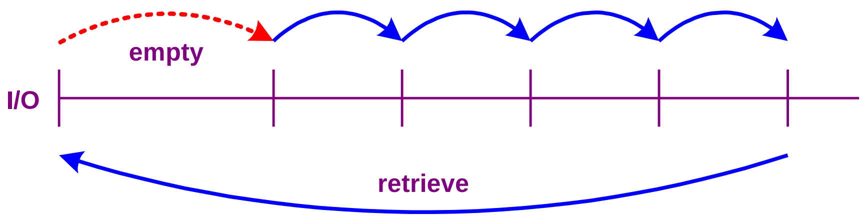


Expected time for each SC S/R cycle:

$$t_{DC} = \frac{d_{DC}}{v} + 2t_L + 2t_U = \frac{d_{DC}}{v} + 4t_{L/U}$$

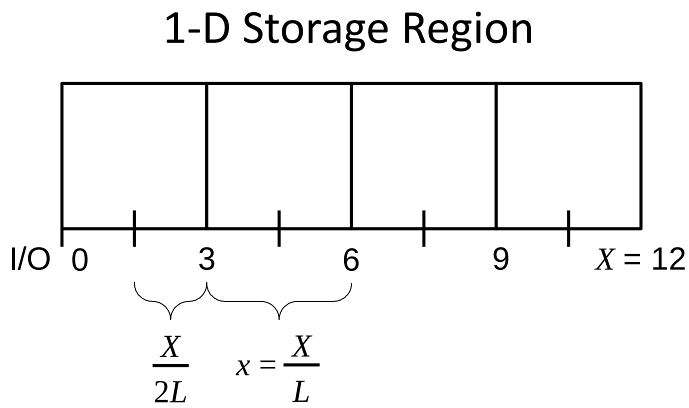
- Dual-command (DC):
- Combine storage with a retrieval:
  - store load in slot 1, travel empty to slot 2 to retrieve load
- Can reduce travel distance by a third, on average
- Also termed task “interleaving”

Multi-Command S/R Cycle



- Multi-command: multiple loads can be carried at the same time
- Used in case and piece order picking
- Picker routed to slots
  - Simple VRP procedures can be used

1-D Expected Distance



$$TD_{1-way} = \sum_{i=1}^{L}\left(i\frac{X}{L} - \frac{X}{2L}\right) = \frac{X}{L}\sum_{i=1}^{L} i - \frac{X}{2L}(1)$$

$$= \frac{X}{L}\left(\frac{L(L+1)}{2}\right) - \frac{X}{2L}(L)$$

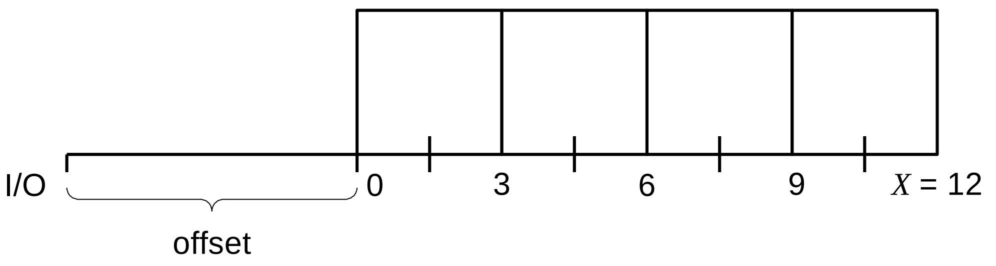
$$= \frac{XL + X - X}{2} = \frac{XL}{2}$$

$$ED_{1-way} = \frac{TD_{1-way}}{L} = \frac{X}{2}$$

$$d_{SC} = 2(ED_{1-way}) = X$$

- Assumptions:
  - All single-command cycles
  - Rectilinear distances
  - Each slot is region used with equal frequency (i.e., randomized storage)
- Expected distance is the average distance from I/O port to midpoint of each slot
  - e.g., [2(1.5) + 2(4.5) + 2(6.5) + 2(10.5)]/4 = 12

Off-set I/O Port



$$d_{SC} = 2(d_{\text{offset}}) + X$$

- If the I/O port is off-set from the storage region, then 2 times the distance of the offset is added the expected distance within the slots

2-D Expected Distances

- Since dimensions *X* and *Y* are independent of each other for rectilinear distances, the expected distance for a 2-D rectangular region with the I/O port in a corner is just the sum of the distance in *X* and in *Y*: $d_{SC}^{rect} = X + Y$

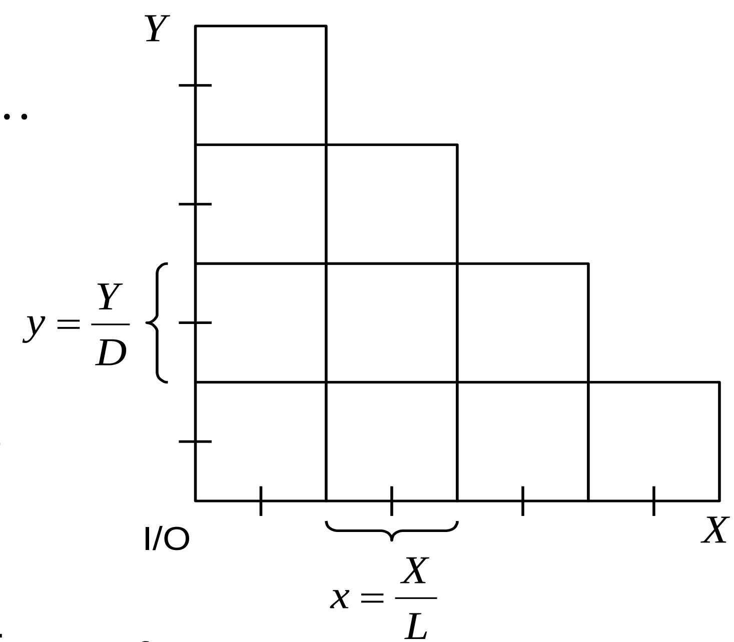
- For a triangular region with the I/O port in the corner:

$$TD_{\text{1-way}} = \sum_{i=1}^{L} \sum_{j=1}^{L-i+1} \left[\left(i\frac{X}{L} - \frac{X}{2L} \right) + \left(j\frac{X}{L} - \frac{X}{2L} \right) \right] = \quad \cdots$$

$$= \frac{X}{6}\left(2L^2 + 3L + 1\right)$$

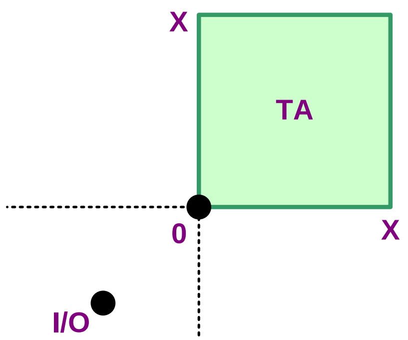
$$ED_{\text{1-way}} = \frac{TD_{\text{1-way}}}{\frac{L(L+1)}{2}} = \frac{2}{3}X + \frac{X}{3L} = \frac{2}{3}X, \quad \text{as } L \to \infty$$

$$d_{SC}^{tri} = 2\left(\frac{2}{3}X\right) = 2\left(\frac{1}{3}X + \frac{1}{3}Y\right) = \frac{2}{3}(X+Y) = \frac{4}{3}X, \quad \text{if } X = Y$$



I/O-to-Side Configurations

Rectangular

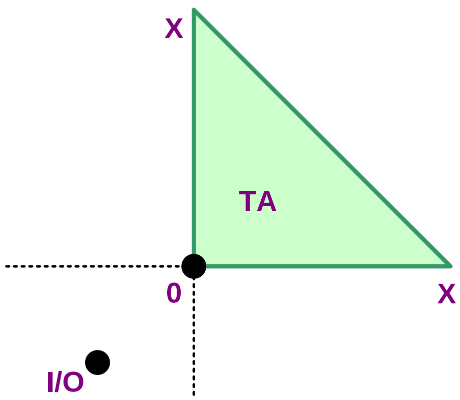


$$TA = X^2$$

$$\Rightarrow X = \sqrt{TA}$$

$$\Rightarrow d_{SC} = 2\sqrt{TA}$$

Triangular



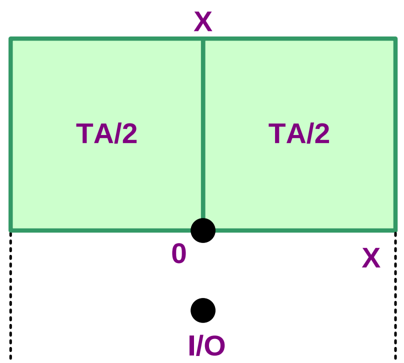
$$TA = \frac{1}{2} X^2$$

$$\Rightarrow X = \sqrt{2TA} = \sqrt{2}\sqrt{TA}$$

$$\Rightarrow d_{SC} = \frac{4}{3}\sqrt{2}\sqrt{TA} = 1.886\sqrt{TA}$$

I/O-at-Middle Configurations

Rectangular

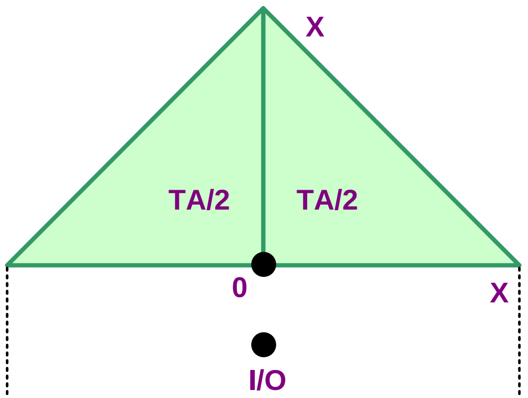


$$\frac{TA}{2} = X^2$$

$$\Rightarrow X = \sqrt{\frac{TA}{2}} = \frac{\sqrt{TA}}{\sqrt{2}}$$

$$\Rightarrow d_{SC} = \sqrt{2}\sqrt{TA} = 1.414\sqrt{TA}$$

Triangular



$$\frac{TA}{2} = \frac{1}{2}X^2$$

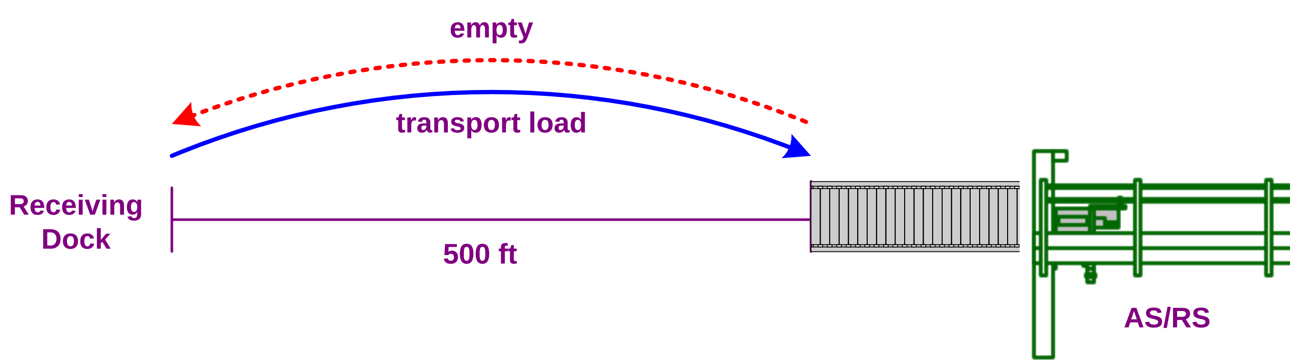
$$\Rightarrow X = \sqrt{TA}$$

$$\Rightarrow d_{SC} = \frac{4}{3}\sqrt{TA} = 1.333\sqrt{TA}$$

Ex 28: Handling Requirements

Pallet loads will be unloaded at the receiving dock of a warehouse and placed on the floor. From there, they will be transported 500 feet using a dedicated pallet truck to the in-floor induction conveyor of an AS/RS. Given

a. It takes 30 sec to load each pallet at the dock
b. 30 sec to unload it at the induction conveyor
c. There will be 80,000 loads per year on average
d. Operator rides on the truck (because a pallet truck)
e. Facility will operate 50 weeks per year, 40 hours per week



Ex 28: Handling Requirements

1. Assuming that it will take 30 seconds to load each pallet at the dock and 30 seconds to unload it at the induction conveyor, what is the expected time required for each single-command S/R cycle?

$$d_{SC} = 2(500) = 1000 \text{ ft/mov}$$

$$t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{1000 \text{ ft/mov}}{616 \text{ ft/min}} + 2\left(\frac{30}{60}\right) \text{min/mov}$$

$$= 2.62 \text{ min/mov} = \frac{2.62}{60} \text{ hr/mov}$$

(616 fpm because operator rides on a pallet truck)

Ex 28: Handling Requirements

2. Assuming that there will be 80,000 loads per year on average and that the facility will operate for 50 weeks per year, 40 hours per week, what is the minimum number of trucks needed?

$$r_{avg} = \frac{80,000 \text{ mov/yr}}{50(40) \text{ hr/yr}} = 40 \text{ mov/hr}$$

$$m = \lfloor r_{avg} t_{SC} + 1 \rfloor$$

$$= \left\lfloor 40\left(\frac{2.62}{60}\right) + 1 \right\rfloor = \lfloor 1.75 + 1 \rfloor$$

$$= 2 \text{ trucks}$$

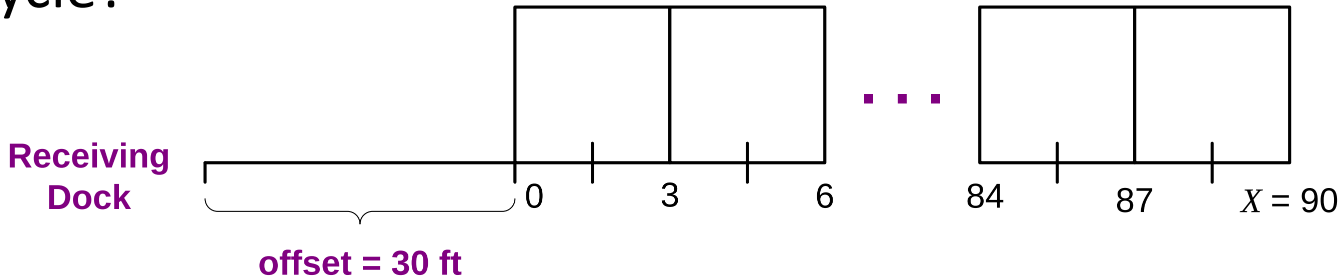
Ex 28: Handling Requirements

3. How many trucks are needed to handle a peak expected demand of 80 moves per hour?

$$
\begin{aligned}
r_{peak} &= 80 \text{ mov/hr} \\
m &= \left\lfloor r_{peak} t_{SC} + 1 \right\rfloor \\
&= \left\lfloor 80\left(\frac{2.62}{60}\right) + 1 \right\rfloor = \left\lfloor 3.50 + 1 \right\rfloor \\
&= 4 \text{ trucks}
\end{aligned}
$$

Ex 28: Handling Requirements

4. If, instead of unloading at the conveyor, the 3-foot-wide loads are placed side-by-side in a staging area along one side of 90-foot aisle that begins 30 feet from the dock, what is the expected time required for each single-command S/R cycle?



$$d_{SC} = 2(d_{\text{offset}}) + X = 2(30) + 90 = 150 \text{ ft}$$

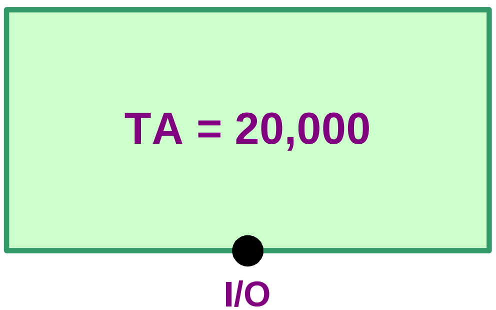
$$t_{SC} = \frac{d_{SC}}{v} + 2t_{L/U} = \frac{150 \text{ ft/mov}}{616 \text{ ft/min}} + 2\left(\frac{30}{60}\right) \text{min/mov}$$

$$= 1.24 \text{ min/mov} = \frac{1.24}{60} \text{ hr/mov}$$

Estimating Handling Costs

- Warehouse design involves the trade-off between building and handling cost.
- Maximizing the cube utilization of a storage region will help minimize building costs.
- Handling costs can be estimated by determining:
  1. Expected time required for each move based on an average of the time required to reach each slot in the region.
  2. Number of vehicles needed to handle a target *peak demand* for moves, e.g., moves per hour.
  3. *Operating costs per hour of vehicle operation*, e.g., labor, fuel (assuming the operators can perform other productive tasks when not operating a truck)
  4. Annual operating costs based on *annual demand* for moves.
  5. Total handling costs as the sum of the annual capital recovery costs for the vehicles and the annual operating costs.

Ex 29: Estimating Handing Cost



$\Uparrow$

Add 20% Cross aisle:

$$TA = TA' \times 1.2$$
$$= 20{,}000\ \text{ft}^2$$

$\Uparrow$

Total Storage Area:

$$D^* \Rightarrow L(D^*) \Rightarrow TA'$$

Expected Distance: $d_{SC} = \sqrt{2}\sqrt{TA} = \sqrt{2}\sqrt{20{,}000} = 200\ \text{ft}$

Expected Time: $t_{SC} = \dfrac{d_{SC}}{v} + 2t_{L/U}$

$$= \frac{200\ \text{ft}}{200\ \text{fpm}} + 2(0.5\ \text{min}) = 2\ \text{min per move}$$

Peak Demand: $r_{\text{peak}} = 75$ moves per hour

Annual Demand: $r_{\text{year}} = 100{,}000$ moves per year

Number of Trucks: $m = \left\lfloor r_{\text{peak}} \dfrac{t_{SC}}{60} + 1 \right\rfloor = \lfloor 3.5 \rfloor = 3$ trucks

Handling Cost: $TC_{\text{hand}} = mK_{\text{truck}} + r_{\text{year}} \dfrac{t_{SC}}{60} C_{\text{labor}}$

$$= 3(\$2{,}500 / \text{tr-yr}) + 100{,}000 \frac{2}{60} (\$10 / \text{hr})$$

$$= \$7{,}500 + \$33{,}333 = \$40{,}833 \text{ per year}$$

Dedicated Storage Assignment (DSAP)

- The assignment of items to slots is termed *slotting*
  - With randomized storage, all items are assigned to all slots
- DSAP (dedicated storage assignment problem):
  - Assign N items to slots to minimize total cost of material flow
- DSAP solution procedure:
  1. *Order Slots:* Compute the expected cost for each slot and then put into nondecreasing order
  2. *Order Items:* Put the flow density (flow per unit of volume) for each item i into nonincreasing order

$$\frac{f_{[1]}}{M_{[1]}s_{[1]}} \geq \frac{f_{[2]}}{M_{[2]}s_{[2]}} \geq \cdots \geq \frac{f_{[N]}}{M_{[N]}s_{[N]}}$$

  3. *Assign Items to Slots:* For $i = 1, \ldots, N$, assign item $[i]$ to the first slots with a total volume of at least $M_{[i]}s_{[i]}$

Ex 30: 1-D Slotting

		A	B	C
Max units	M	4	5	3
Space/unit	s	1	1	1
Flow	f	24	7	21
Flow Density	f/(M x s)	6.00	1.40	7.00

Flow Density	1-D Slot Assignments	Expected Distance	Flow	Total Distance
$\frac{21}{3} = 7.00$	I/O C C C (0, 3)	2(0) + 3 = 3 ×	21 =	63
$\frac{24}{4} = 6.00$	I/O A A A A (-3, 0, 4)	2(3) + 4 = 10 ×	24 =	240
$\frac{7}{5} = 1.40$	I/O B B B B B (-7, 0, 5)	2(7) + 5 = 19 ×	7 =	133
	I/O C C C A A A A B B B B B (0, 3, 7, 12)			436

Ex 30: 1-D Slotting

		Dedicated			Random	Class-Based		
		A	B	C	ABC	AB	AC	BC
Max units	M	4	5	3	9	7	7	8
Space/unit	s	1	1	1	1	1	1	1
Flow	f	24	7	21	52	31	45	28
Flow Density	f/(M x s)	6.00	1.40	7.00	5.78	4.43	6.43	3.50

		1-D Slot Assignments												Total Distance	Total Space
Dedicated (flow density)	I/O	C	C	C	A	A	A	A	B	B	B	B	B	436	12
Dedicated (flow only)	I/O	A	A	A	A	C	C	C	B	B	B	B	B	460	12
Class-based	I/O	C	C	C	AB	AB	AB	AB	AB	AB	AB			466	10
Randomized	I/O	ABC	ABC	ABC	ABC	ABC	ABC	ABC	ABC	ABC				468	9

Ex 31: 2-D Slotting

		A	B	C
Max units	M	4	5	3
Space/unit	s	1	1	1
Flow	f	24	7	21
Flow Density	f/(M x s)	6.00	1.40	7.00

8	7	6	5	4	5	6	7	8
7	6	5	4	3	4	5	6	7
6	5	4	3	2	3	4	5	6
5	4	3	2	1	2	3	4	5
4	3	2	1	0	1	2	3	4

Distance from I/O to Slot

	C	C	B	
C	A	A	B	B
A	A	I/O	B	B

Original Assignment (TD = 215)

	B	B	B	
B	A	C	A	B
A	C	I/O	C	A

Optimal Assignment (TD = 177)

DSAP Assumptions

1. All SC S/R moves
2. For item *i*, probability of move to/from each slot assigned to item is the same
3. The *factoring assumption*:
   a. Handling cost and distances (or times) for each slot are identical for all items
   b. Percent of S/R moves of item stored at slot *j* to/from I/O port *k* is identical for all items

- Depending of which assumptions not valid, can determine assignment using other procedures

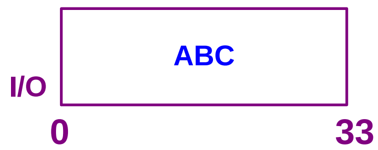
$$\left[\left(\frac{f_i}{M_i}\cdot d_j\right)x_{ij}\right] DSAP \subset \underset{\left(c_{ij}x_{ij}\right)}{LAP} \subset LP \subset \underset{\displaystyle TSP}{\underset{\cup}{QAP}} \left(c_{ijkl}x_{ij}x_{kl}\right)$$

Ex 32: 1-D DSAP

- What is the change in the minimum expected total distance traveled if dedicated, as compared to randomized, block stacking is used, where
  a. Slots located on one side of 10-foot-wide down aisle
  b. All single-command S/R operations
  c. Each lane is three-deep, four-high
  d. 40×36 in. two-way pallet used for all loads
  e. Max inventory levels of SKUs A, B, C are 94, 64, and 50
  f. Inventory levels are uncorrelated and retrievals occur at a constant rate
  g. Throughput requirements of A, B, C are 160, 140, 130
  h. Single I/O port is located at the end of the aisle

Ex 32: 1-D DSAP

- Randomized:



$$M = \left\lfloor \frac{M_A + M_B + M_C}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{94+64+50}{2} + \frac{1}{2} \right\rfloor = 104$$

$$L_{rand} = \left\lceil \frac{M + NH\left(\frac{D-1}{2}\right) + N\left(\frac{H-1}{2}\right)}{DH} \right\rceil$$

$$= \left\lceil \frac{104 + 3(4)\left(\frac{3-1}{2}\right) + N\left(\frac{4-1}{2}\right)}{3(4)} \right\rceil = 11 \text{ lanes}$$

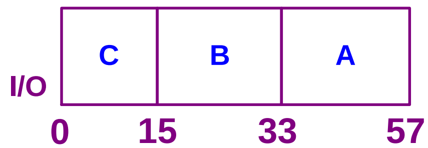
$$X = xL_{rand} = 3(11) = 33 \text{ ft}$$

$$d_{SC} = X = 33 \text{ ft}$$

$$TD_{rand} = \left(f_A + f_B + f_C \right) X = \left(160+140+130\right) 33 = 14{,}190 \text{ ft}$$

Ex 32: 1-D DSAP

- Dedicated:



$$\frac{f_A}{M_A} = \frac{160}{94} = 1.7,\ \frac{f_B}{M_B} = \frac{140}{64} = 2.19,\ \frac{f_C}{M_C} = \frac{130}{50} = 2.6 \quad \Rightarrow \quad C > B > A$$

$$L_A = \left\lceil \frac{M_A}{DH} \right\rceil = \left\lceil \frac{94}{3(4)} \right\rceil = 8,\ L_B = \left\lceil \frac{M_B}{DH} \right\rceil = \left\lceil \frac{64}{3(4)} \right\rceil = 6,\ L_C = \left\lceil \frac{M_C}{DH} \right\rceil = \left\lceil \frac{50}{3(4)} \right\rceil = 5$$

$$X_C = xL_C = 3(5) = 15,\ X_B = xL_B = 3(6) = 18,\ X_A = xL_A = 3(8) = 24$$

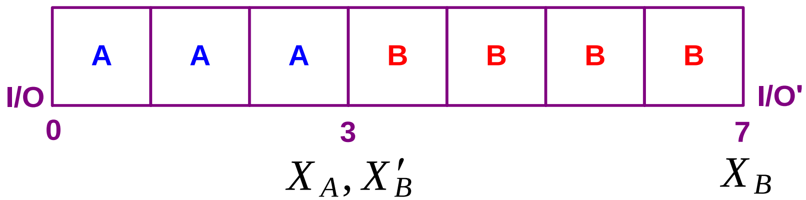
$$d_{SC}^{C} = X_C = 3(5) = 15 \text{ ft}$$

$$d_{SC}^{B} = 2(X_C) + X_B = 2(15) + 18 = 48 \text{ ft}$$

$$d_{SC}^{A} = 2(X_C + X_B) + X_A = 2(15+18) + 24 = 90 \text{ ft}$$

$$TD_{ded} = f_A d_{SC}^{A} + f_B d_{SC}^{B} + f_C d_{SC}^{C} = 160(90) + 140(48) + 130(15) = 23,070 \text{ ft}$$

1-D Multiple Region Expected Distance



- In 1-D, easy to determine the offset
- In 2-D, no single offset value for each region

$$d_{SC}^{A} = d_A = X_A = 3$$

$$d_B = 2d_{offset} + \left(X_B - X_A\right) = 2X_A + \left(X_B - X_A\right) = X_A + X_B = 10$$

$$= 2\left(d_{I/\mathrm{O\ to}\ I/O'}\right) - X'_B = 2\left(7\right) - 4 = 10$$

$$d_{AB} = 7$$

$$TA_A = X_A = 3, \quad TA_B = X_B - X_A = 4, \quad TA_{AB} = TA_A + TA_B = 7$$

$$TM_A = TA_A d_A, \quad TM_B = TA_B d_B$$

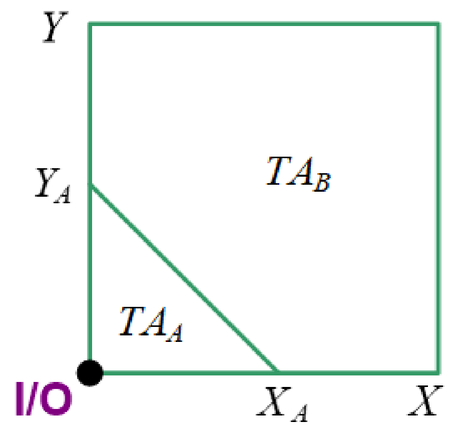
$$TM_{AB} = TA_{AB} d_{AB} = X_B^2 = \left(X_A + X_B - X_A\right)^2 = X_A^2 + \left(X_B - X_A\right)\left(2X_A + X_B - X_A\right)$$

$$= TA_A d_A + TA_B d_B = TM_A + TM_B$$

$$d_B = \frac{TM_B}{TA_B} = \frac{TM_{AB} - TM_A}{TA_B} = \frac{TA_{AB} d_{AB} - TA_A d_A}{TA_B} = \frac{7(7) - 3(3)}{4} = 10$$

2-D Multiple Region Expected Distance

Case: $\boxed{TA_A \le \frac{1}{2} TA_{AB}}$



Let $X = Y, X_A = Y_A, d = d_{sc}$

$$d_A = \frac{2}{3}(X_A + Y_A) = \frac{4}{3} X_A$$

$$d_B = \frac{TM_B}{TA_B} = \frac{TM_{AB} - TM_A}{TA_B} = \frac{TA_{AB}\, d_{AB} - TA_A d_A}{TA_{AB} - TA_A} = \frac{XY(X+Y) - \frac{1}{2} X_A Y_A \frac{2}{3}(X_A + Y_A)}{XY - \frac{1}{2} X_A Y_A}$$

$$= \frac{2}{3} \frac{3X^2Y + 3XY^2 - X_A^2 Y_A - X_A Y_A^2}{2XY - X_A Y_A} = \frac{4}{3} \frac{3X^3 - X_A^3}{2X^2 - X_A^2}$$

If $X = X_A \Rightarrow d_B = \frac{4}{3} \frac{X^2(2X)}{X^2} = \frac{8}{3} X$

2-D Multiple Region Expected Distance

Case: $\boxed{TA_A \geq \frac{1}{2} TA_{AB}}$

Let $X'_B = X - X_B, Y'_B = Y - Y_B$

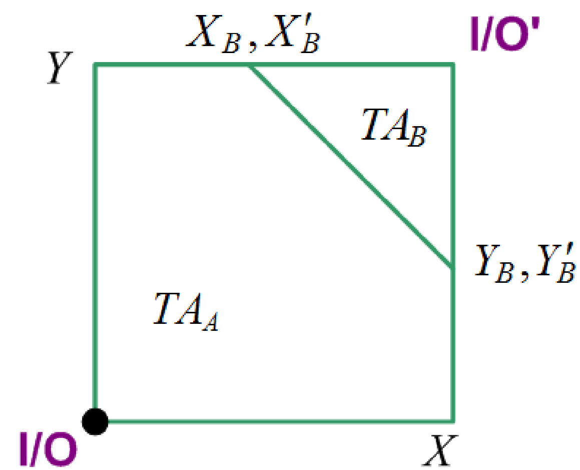
$$d_B = 2\left(d_{I/\text{O to } I/O'}\right) - \frac{2}{3}\left(X'_B + Y'_B\right)$$

$$= 2\left(X + Y\right) - \frac{2}{3}\left[\left(X - X_B\right) + \left(Y - Y_B\right)\right]$$

$$= \frac{4}{3}\left(X + Y\right) + \frac{2}{3}\left(X_B + Y_B\right)$$

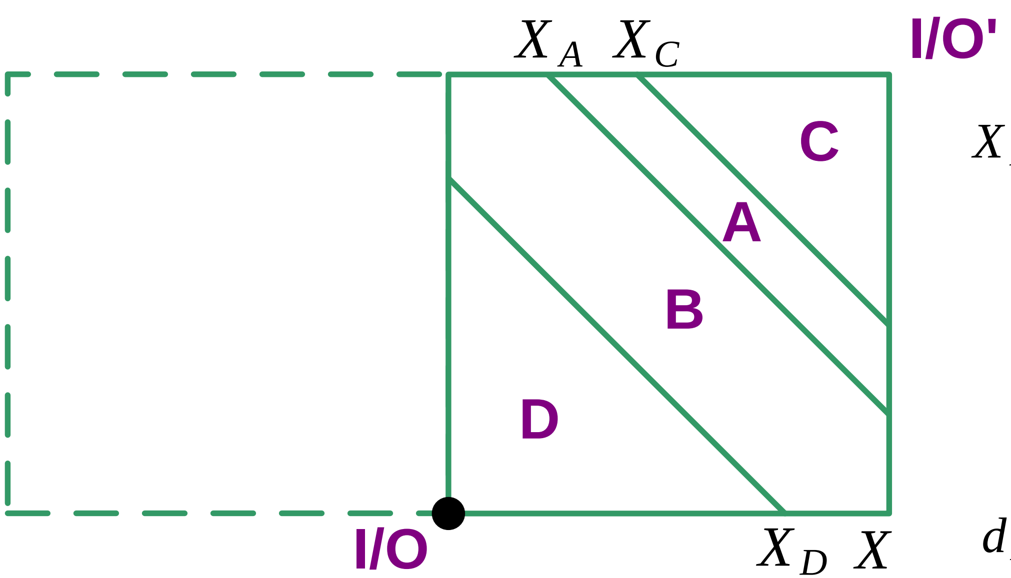
$$= \frac{8}{3}X + \frac{4}{3}X_B, \text{ where } X = Y$$

If $X_B = 0 \Rightarrow d_B = \frac{8}{3}X$



2-D Multiple Region Expected Distance

- If more than two regions:
  - For regions below diagonal (D), start with region closest to I/O
  - For regions above diagonal (A+C), start with regions closest to I/O' (C)
  - For region in the middle (B), solve using whole area less other regions



$$\text{Given } TA_i',\ f_i \Rightarrow \frac{f_i}{TA_i} \Rightarrow D\text{-}B\text{-}A\text{-}C$$

$$TA_i = \frac{TA_i'}{2} \Rightarrow TA = \sum TA_i \Rightarrow X = \sqrt{TA}$$

$$X_D = \sqrt{2}\sqrt{TA_D} \Rightarrow d_D = \frac{4}{3} X_D$$

$$X_C' = \sqrt{2}\sqrt{TA_C} \Rightarrow d_C = 4X - \frac{4}{3} X_C'$$

$$= 4X - \frac{4}{3}\left(X - X_C\right), \quad X_C = X - X_C'$$

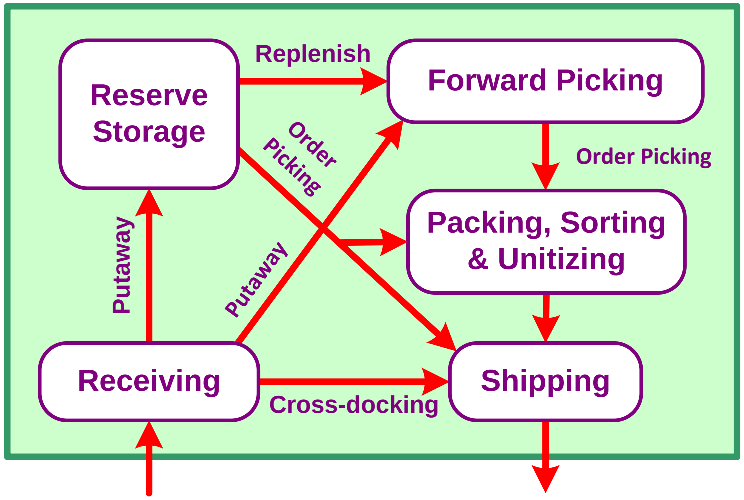
$$X_A' = \sqrt{2}\sqrt{TA_{AC}} \Rightarrow d_{AC} = 4X - \frac{4}{3} X_A'$$

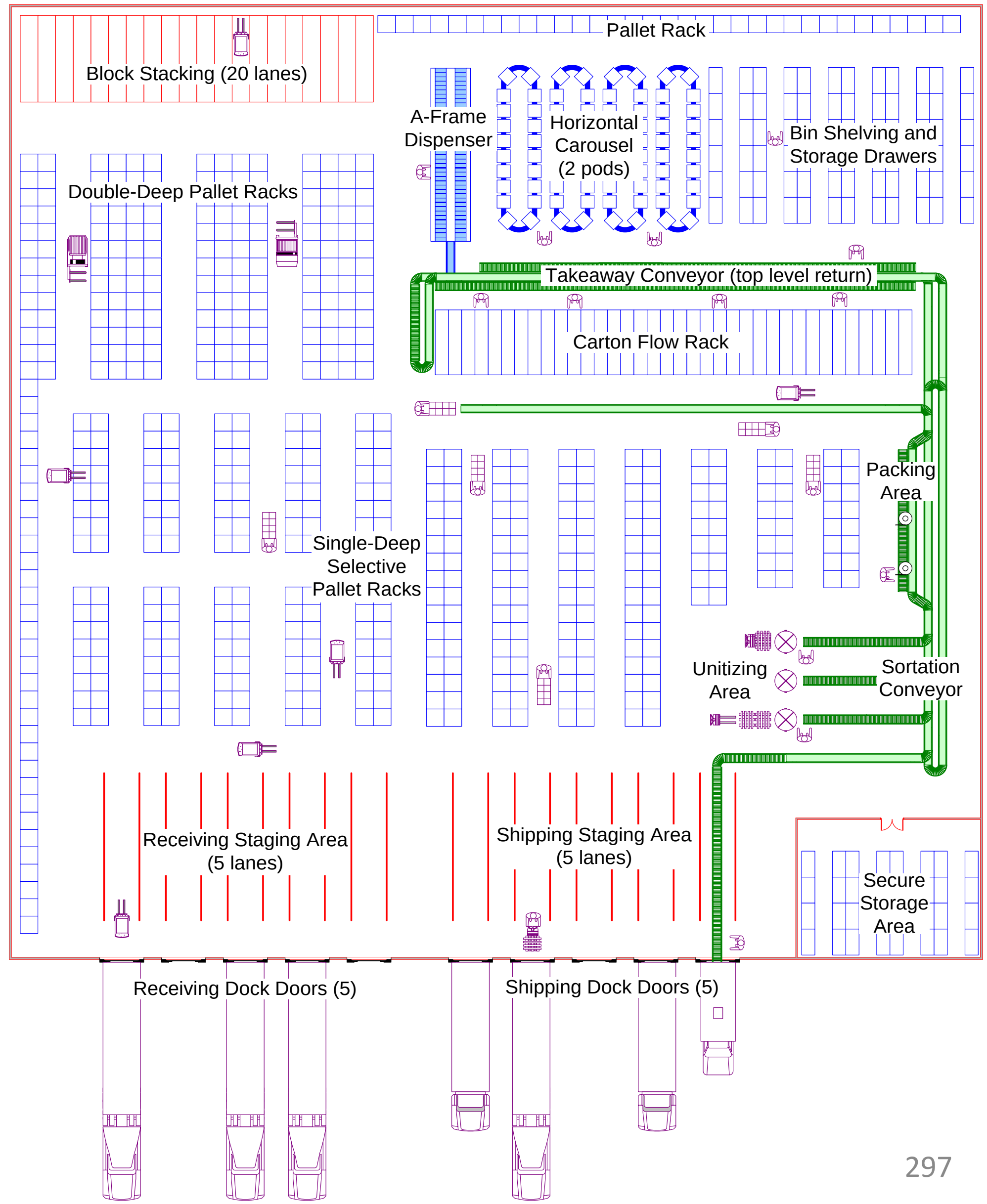
$$\Rightarrow d_A = \frac{TM_A}{TA_A} = \frac{TM_{AC} - TM_C}{TA_A}$$

$$= \frac{TA_{AC} d_{AC} - TA_C d_C}{TA_A}$$

$$d_B = \frac{TM_B}{TA_B} = \frac{TM - TM_{AC} - TM_D}{TA_B}$$

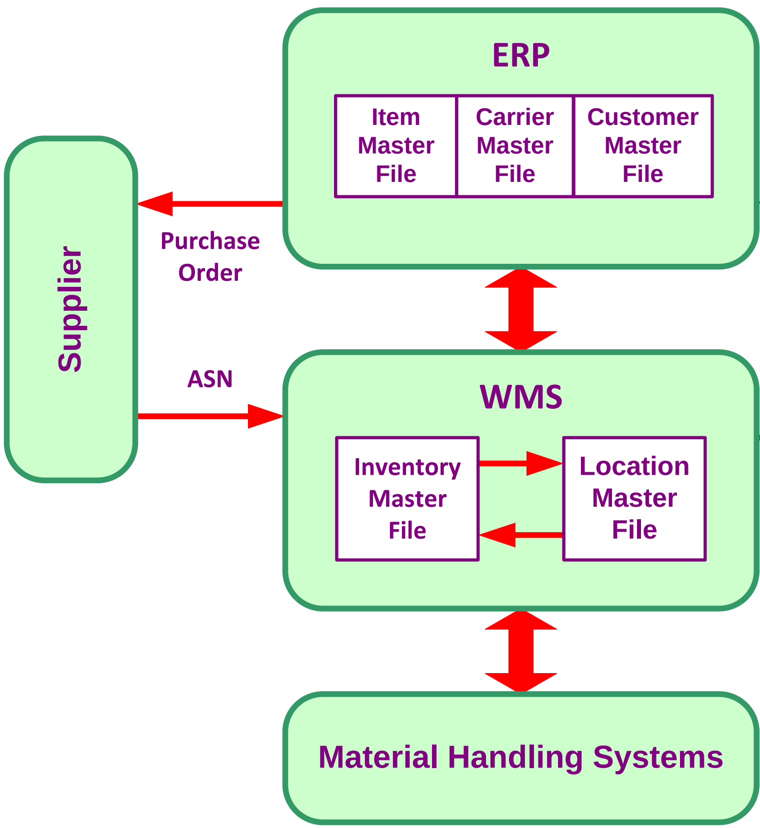
Warehouse Operations





Warehouse Management System

- WMS interfaces with a corporation's enterprise resource planning (ERP) and the control software of each MHS

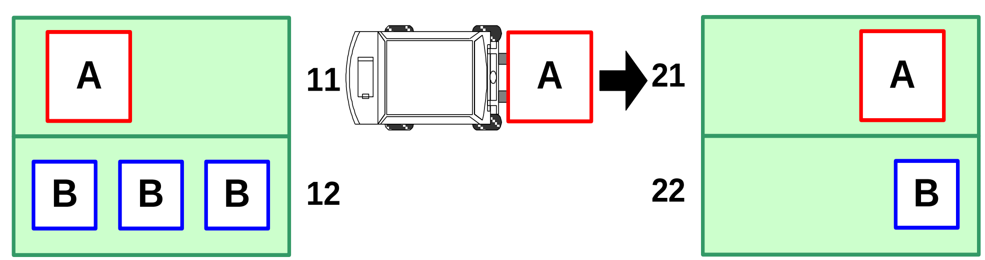


Inventory Master File

Item	On-Hand Balance	In-Transit Qty.	Locations
A	2	1	11,21
B	4	0	12,22

Location Master File

Location	Item	On-Hand Balance	In-Transit Qty.
11	A	1	0
12	B	3	0
21	A	1	1
22	B	1	0



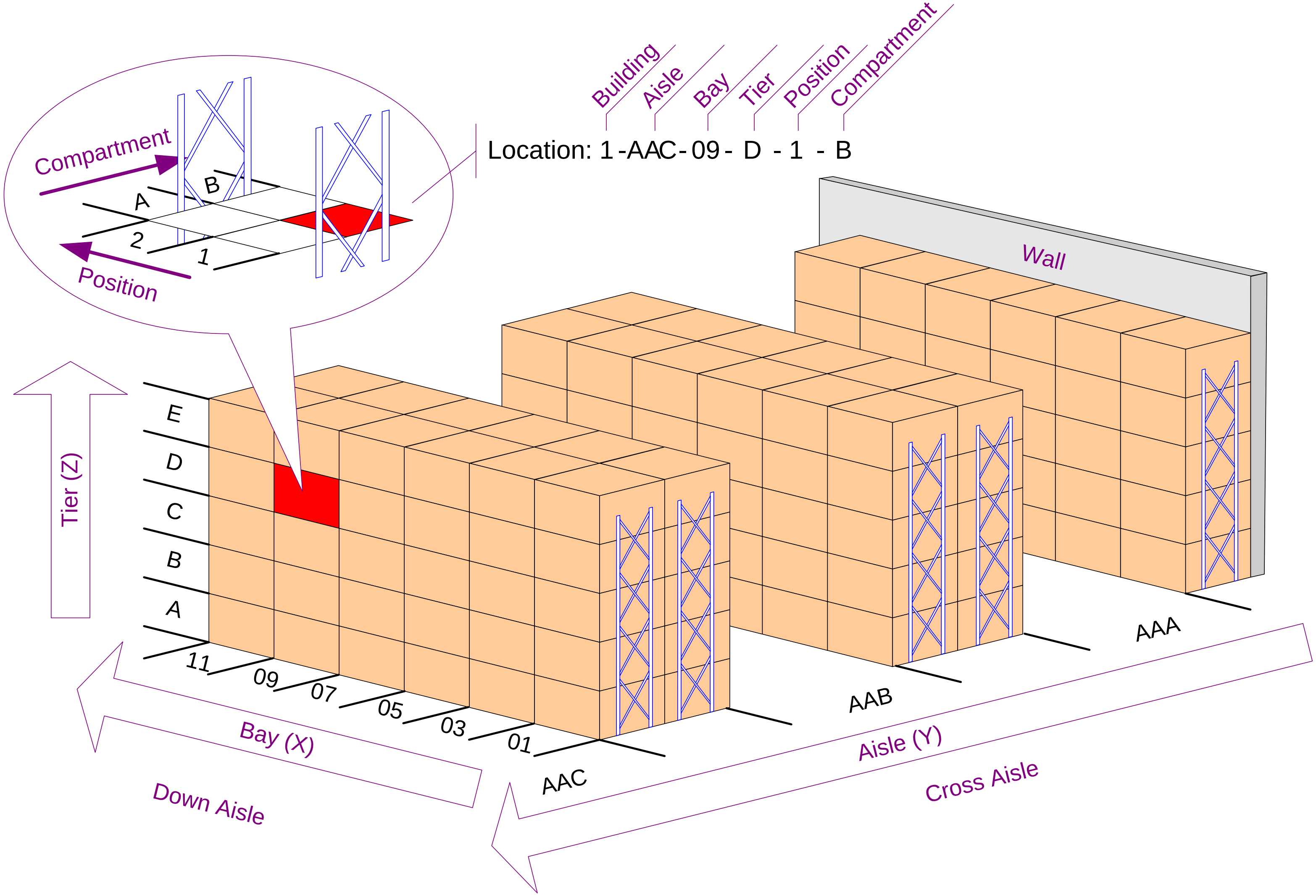
- Advance shipping notice (ASN) is a standard format used for communications

Logistics-related Codes

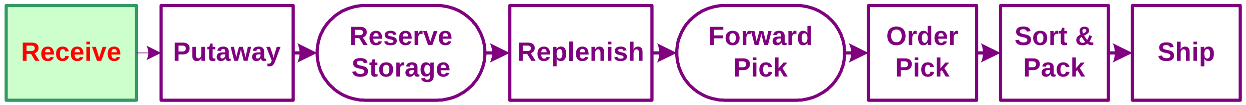
	Commodity Code	Item Code	Unit Code
Level	Category	Class	Instance
Description	Grouping of similar objects	Grouping of identical objects	Unique physical object
Function	Product classification	Inventory control	Object tracking
Names	—	Item number, Part number, SKU, SKU + Lot number	Serial number, License plate
Codes	UNSPSC, GPC	GTIN, UPC, ISBN, NDC	EPC, SSCC

UNSPSC: United Nations Standard Products and Services Code
GPC: Global Product Catalogue
GTIN: Global Trade Item Number (includes UPC, ISBN, and NDC)
UPC: Universal Product Code
ISBN: International Standard Book Numbering
NDC: National Drug Code
EPC: Electronic Product Code (globally unique serial number for physical objects identified using RFID tags)
SSCC: Serial Shipping Container Code (globally unique serial number for identifying *movable units* (carton, pallet, trailer, etc.))

Identifying Storage Locations



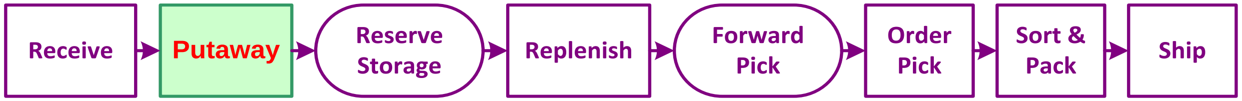
Receiving



- Basic steps:

1. Unload material from trailer.
2. Identify supplier with ASN, and associate material with each moveable unit listed in ASN.
3. Assign inventory attributes to movable unit from item master file, possibly including repackaging and assigning new serial number.
4. Inspect material, possibly including holding some or all of the material for testing, and report any variances.
5. Stage units in preparation for putaway.
6. Update item balance in inventory master and assign units to a receiving area in location master.
7. Create receipt confirmation record.
8. Add units to putaway queue

Putaway



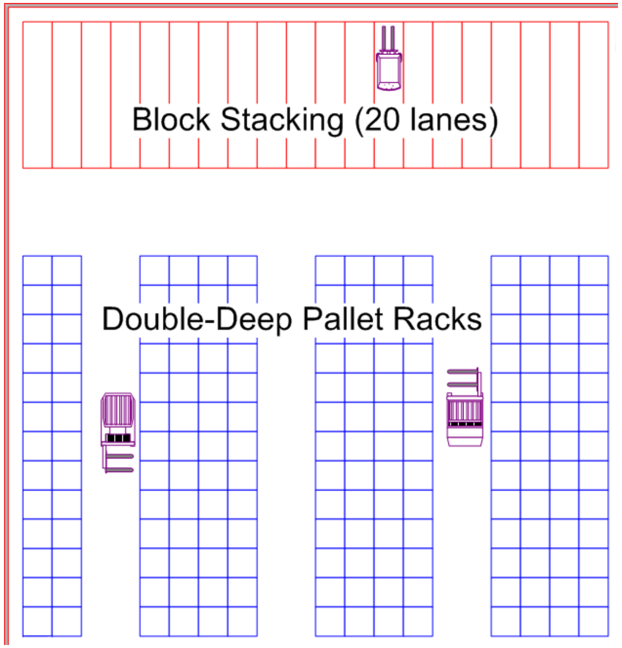
- A putaway algorithm is used in WMS to search for and validate locations where each movable unit in the putaway queue can be stored
- Inventory and location attributes used in the algorithm:
  - *Environment* (refrigerated, caged area, etc.)
  - *Container type* (pallet, case, or piece)
  - *Product processing type* (e.g., floor, conveyable, nonconveyable)
  - *Velocity* (assign to A, B, C based on throughput of item)
  - Preferred putaway zone (item should be stored in same zone as related items in order to improve picking efficiency)

Replenishment

Receive → Putaway → Reserve Storage → **Replenish** → Forward Pick → Order Pick → Sort & Pack → Ship

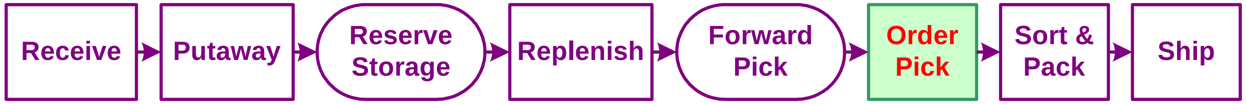
- Replenishment is the process of moving material from reserve storage to a forward picking area so that it is available to fill customer orders efficiently

- Other types of in-plant moves include:
  - Consolidation: combining several partially filled storage locations of an item into a single location
  - Rewarehousing: moving items to different storage locations to improve handling efficiency

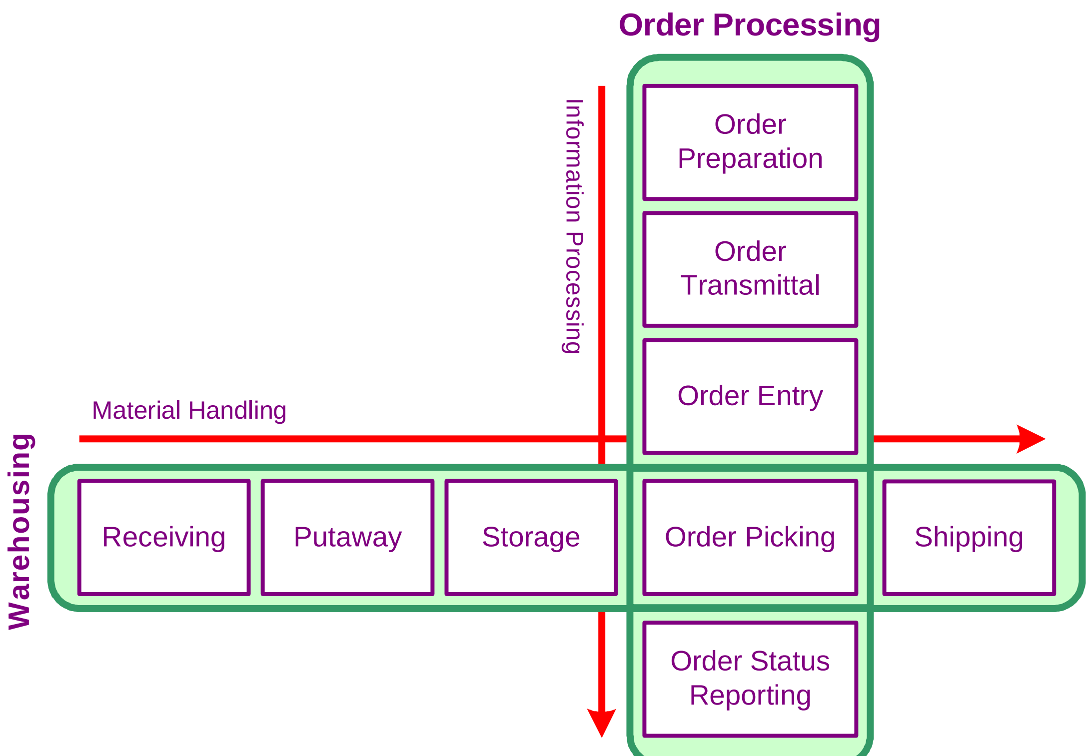


Reserve Storage Area

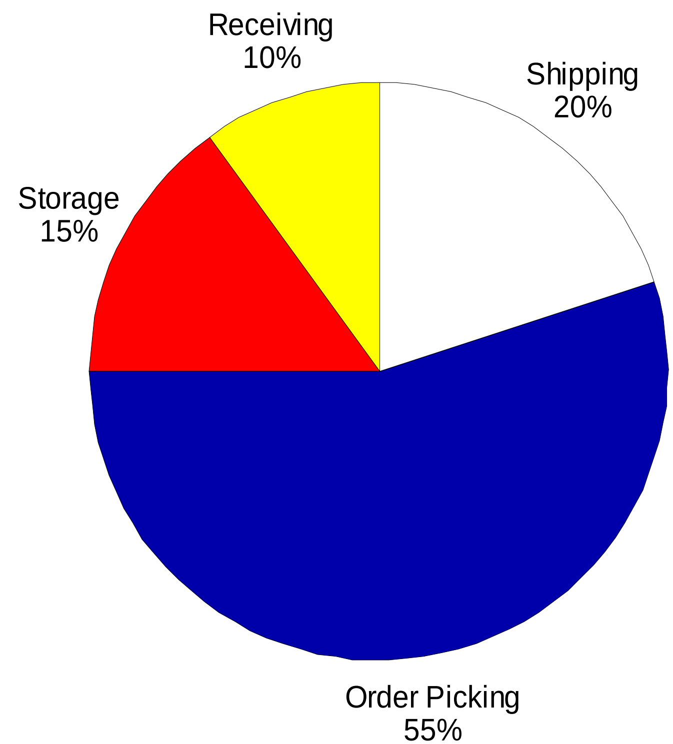
Order Picking



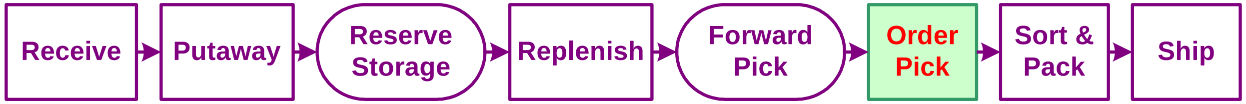
- Order picking is at the intersection of warehousing and order processing



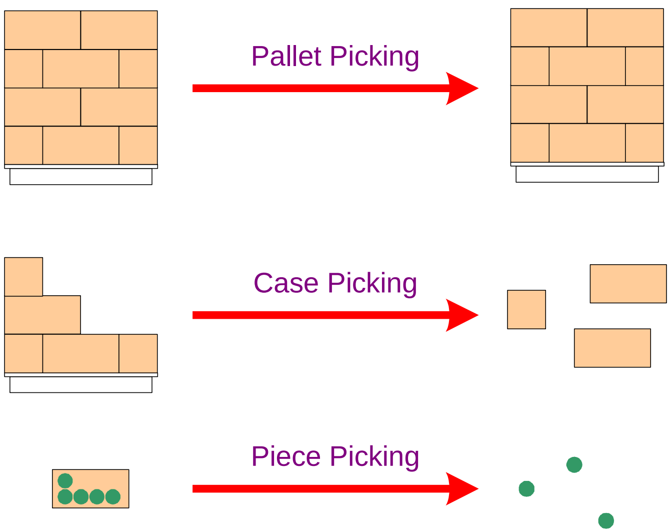
WH Operating Costs

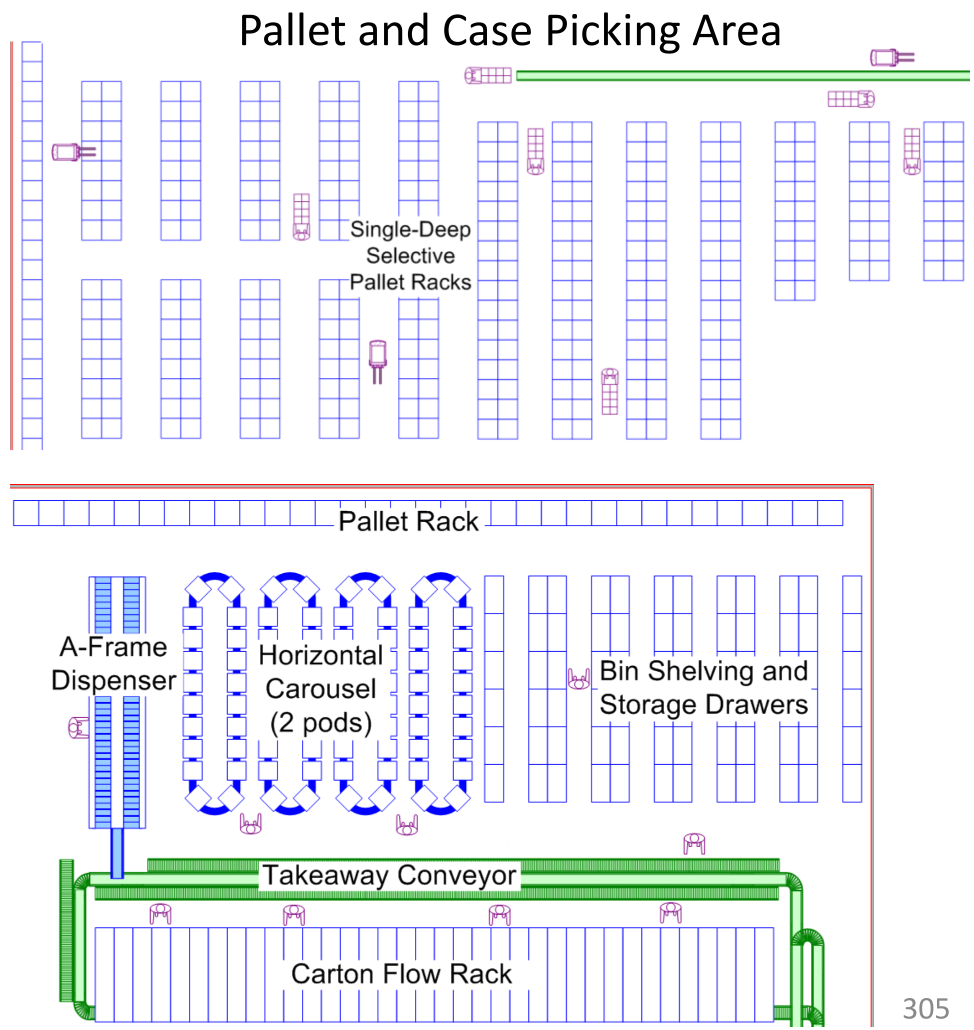


Order Picking



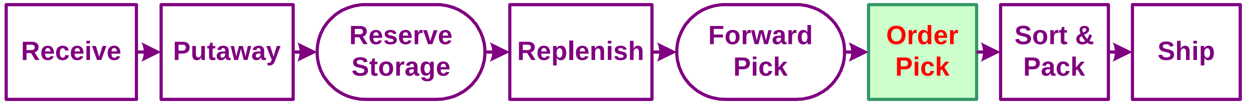
Levels of Order Picking



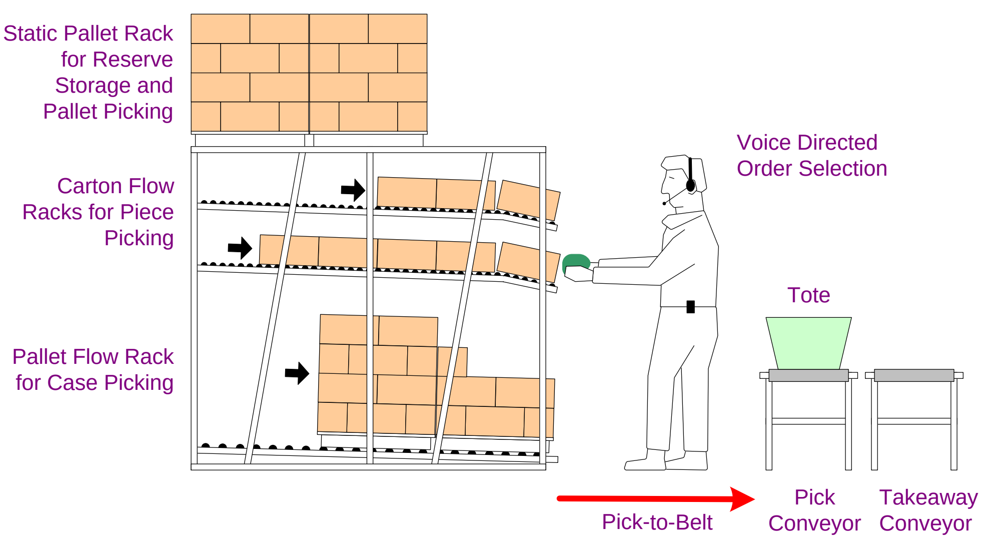


Forward Piece Picking Area

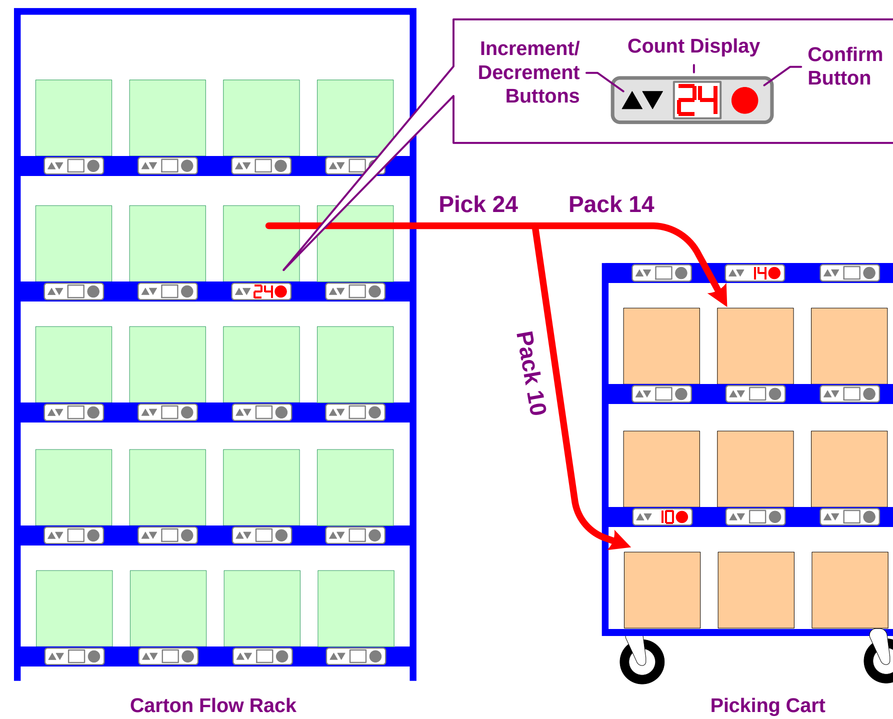
Order Picking



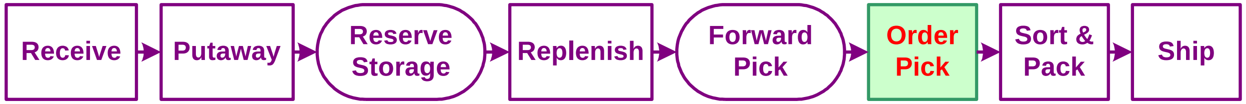
Voice-Directed Piece and Case Picking



Pick-to-Light Piece Picking



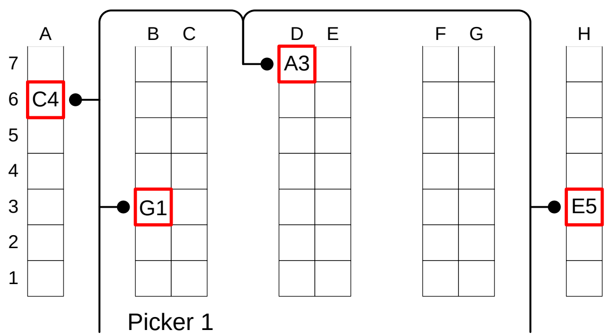
Order Picking



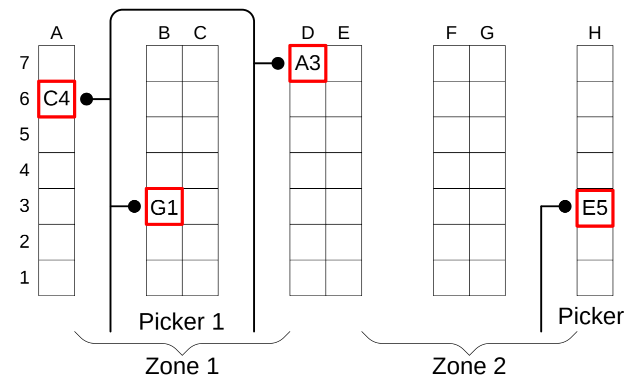
Methods of Order Picking

Method	Pickers per Order	Orders per Picker
Discrete	**Single**	**Single**
Zone	**Multiple**	**Single**
Batch	**Single**	**Multiple**
Zone-Batch	**Multiple**	**Multiple**

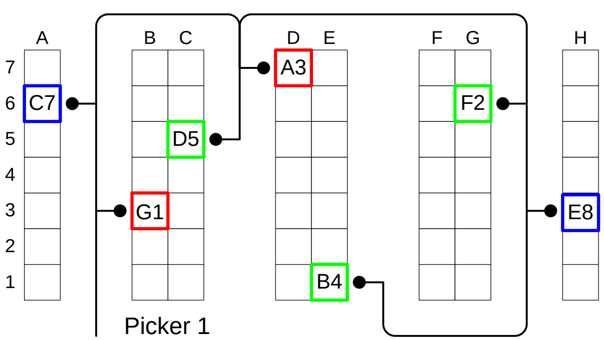
Discrete



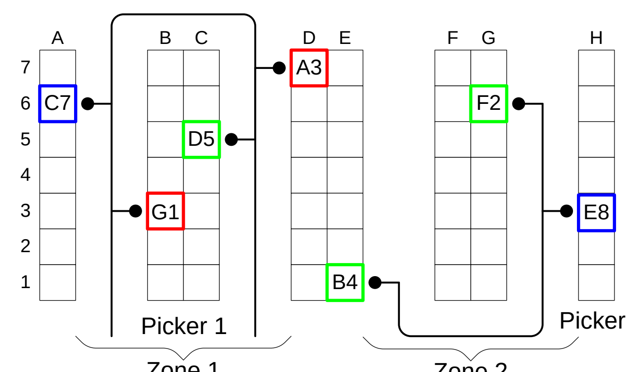
Zone



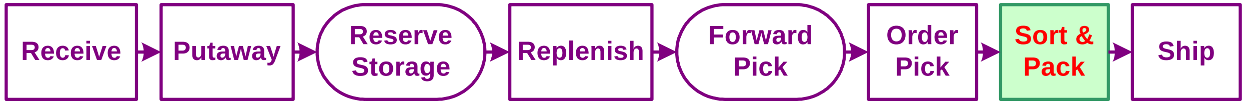
Batch

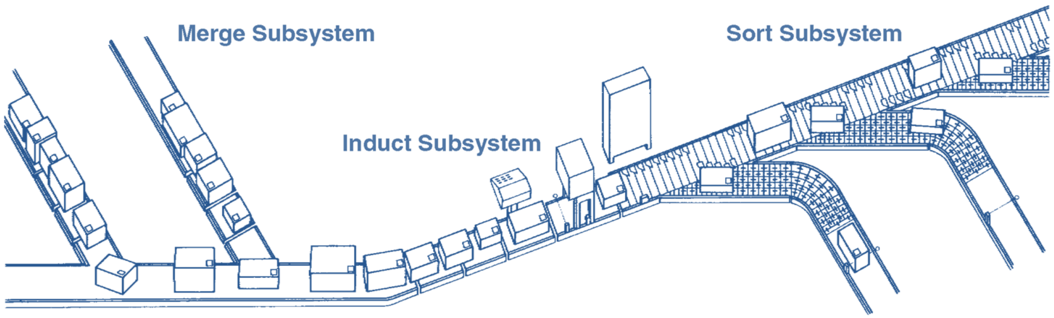


Zone-Batch

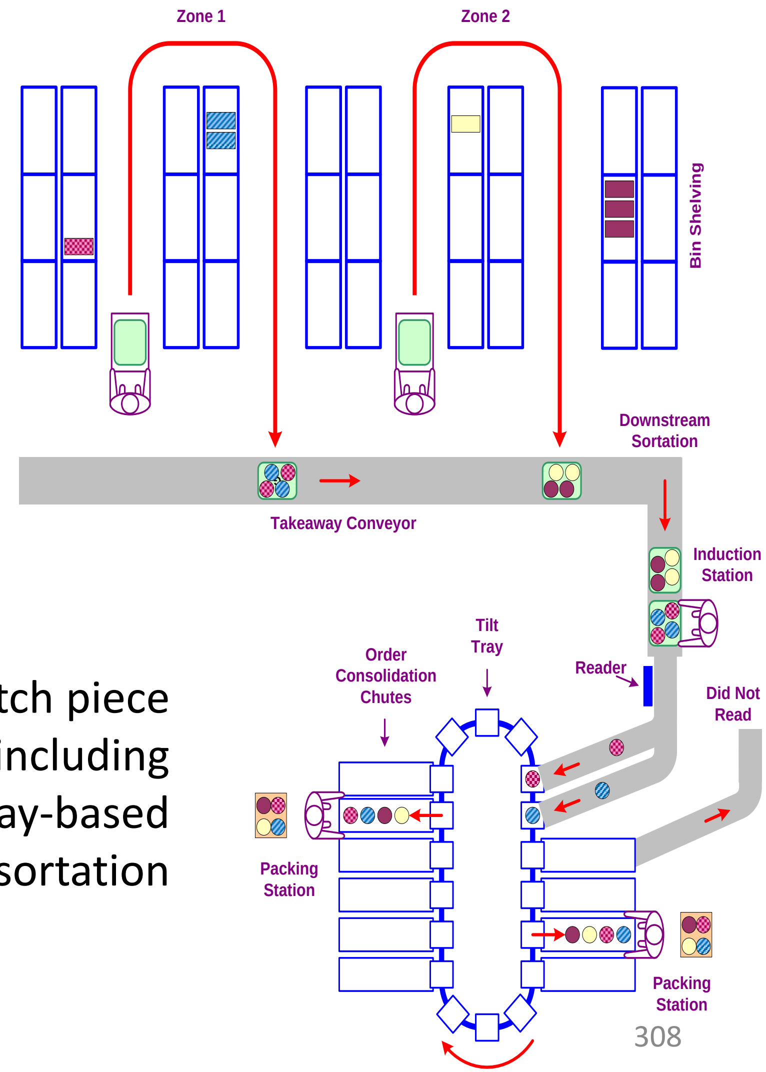


Sortation and Packing



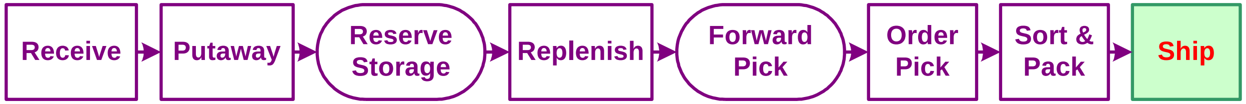


Case Sortation System

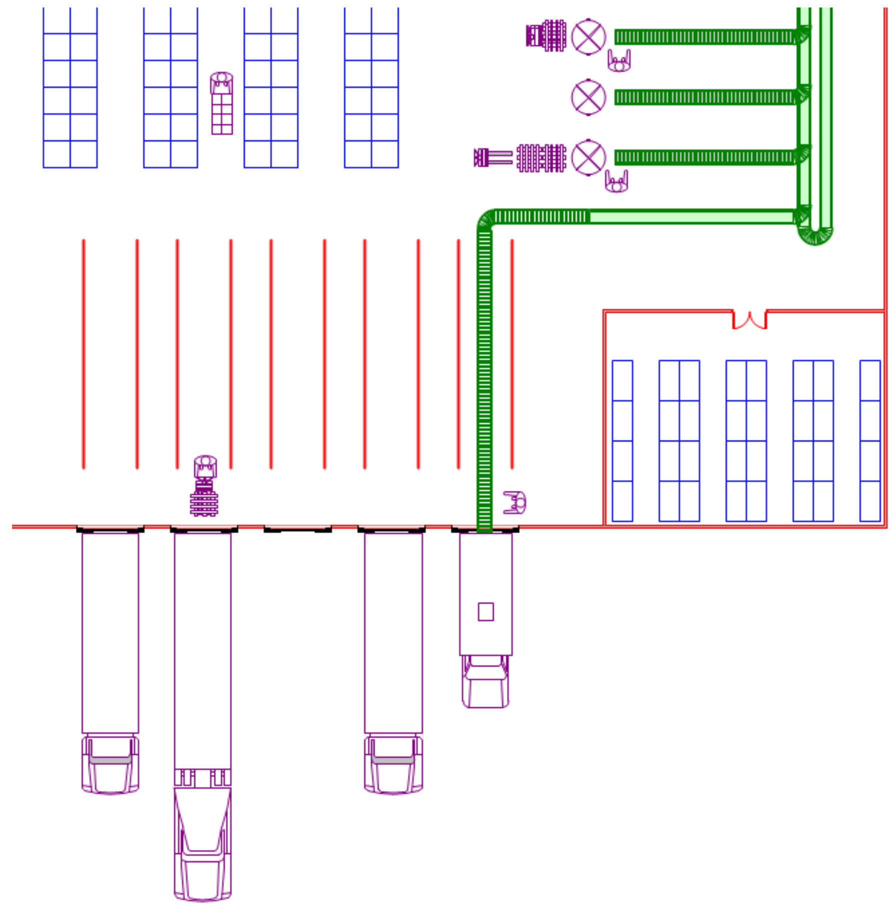


Wave zone-batch piece picking, including downstream tilt-tray-based sortation

Shipping



- Staging, verifying, and loading orders to be transported
  - ASN for each order sent to the customer
  - Customer-specific shipping instructions retrieved from customer master file
  - Carrier selection is made using the rate schedules contained in the carrier master file



Shipping Area

Activity Profiling

- *Total Lines*: total number of lines for all items in all orders
- *Lines per Order*: average number of different items (lines/SKUs) in order
- *Cube per Order*: average total cubic volume of all units (pieces) in order
- *Flow per Item*: total number of S/R operations performed for item
- *Lines per Item* (*popularity*): total number of lines for item in all orders
- *Cube Movement*: total unit demand of item time x cubic volume
- *Demand Correlation*: percent of orders in which both items appear

Item Master

SKU	Length	Width	Depth	Cube	Weight
A	5	3	2	30	1.25
B	3	2	4	24	4.75
C	8	6	5	180	9.65
D	4	4	3	32	6.35
E	6	4	5	120	8.20

Customer Orders

Order: 1	
SKU	Qty
A	5
B	3
C	2
D	6

Order: 2	
SKU	Qty
A	4
C	1

Order: 3	
SKU	Qty
A	2

Order: 4	
SKU	Qty
B	2

Order: 5	
SKU	Qty
C	1
D	12
E	6

Total Lines = 11

Lines per Order = 11/5 = 2.2

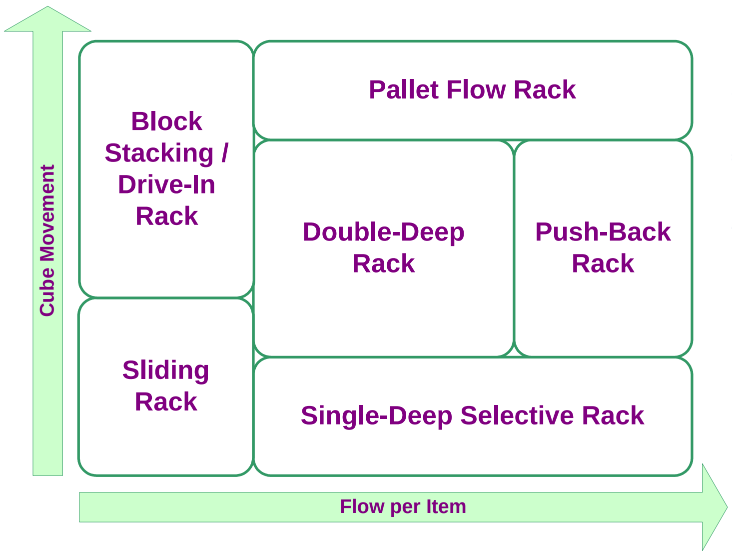
Cube per Order = 493.2

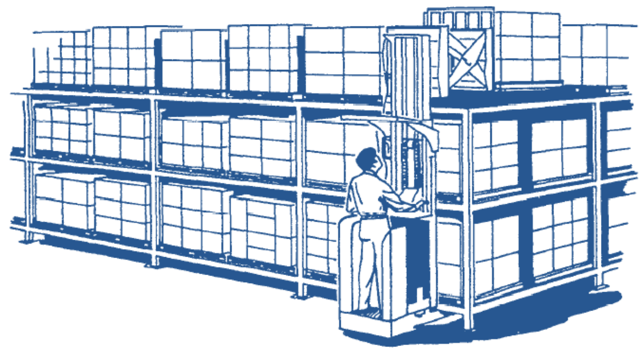
SKU	Flow per Item	Lines per Item	Cube Movement
A	11	3	330
B	5	2	120
C	4	3	720
D	18	2	576
E	6	1	720

Demand Correlation Distibution

SKU	A	B	C	D	E
A		0.2	0.4	0.2	0.0
B			0.2	0.2	0.0
C				0.4	0.2
D					0.2
E					

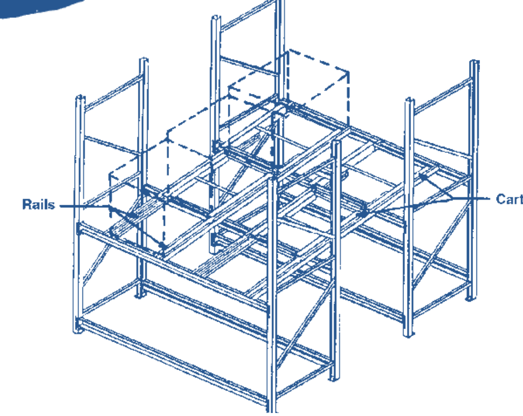
Pallet Picking Equipment

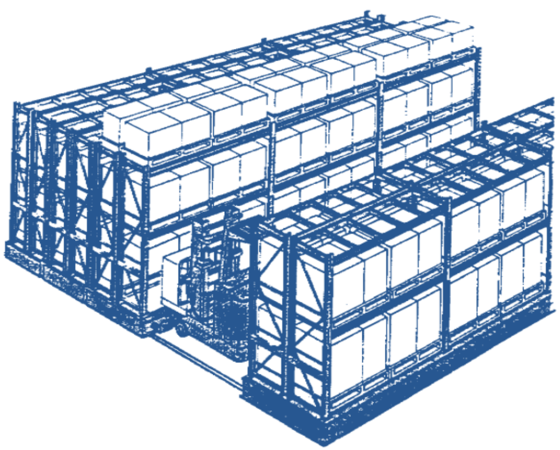




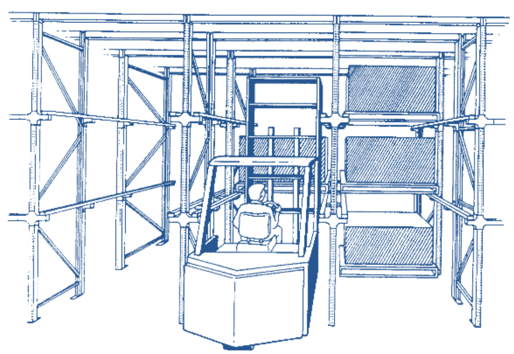
Double-Deep Rack

Push-Back Rack

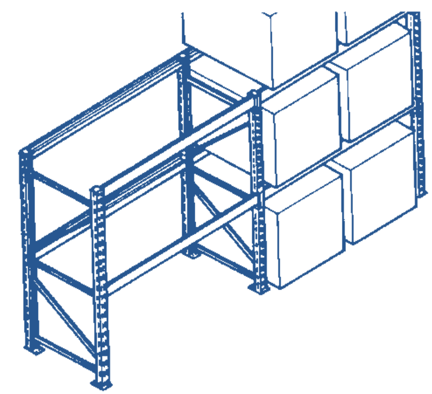




Sliding Rack

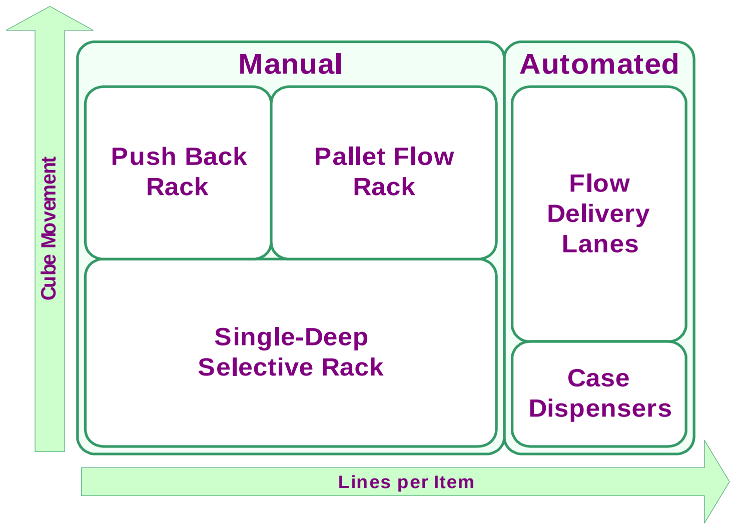


Drive-In Rack

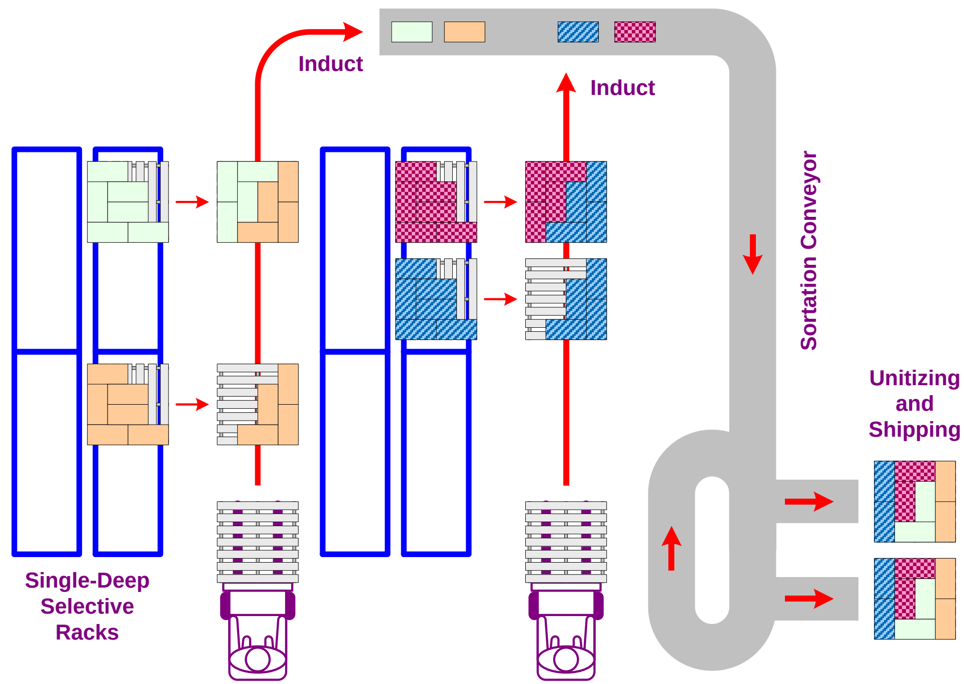


Single-Deep Selective Rack

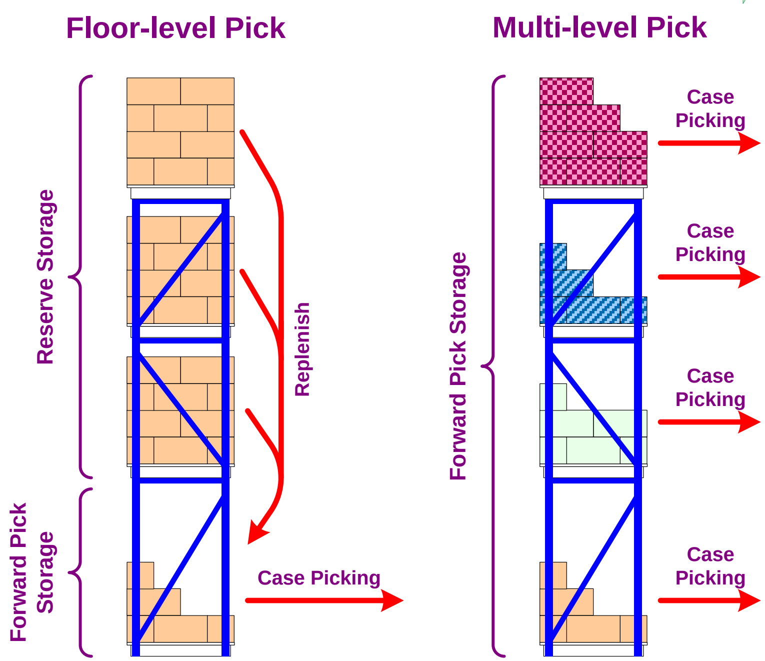
Case Picking



Case Picking Equipment

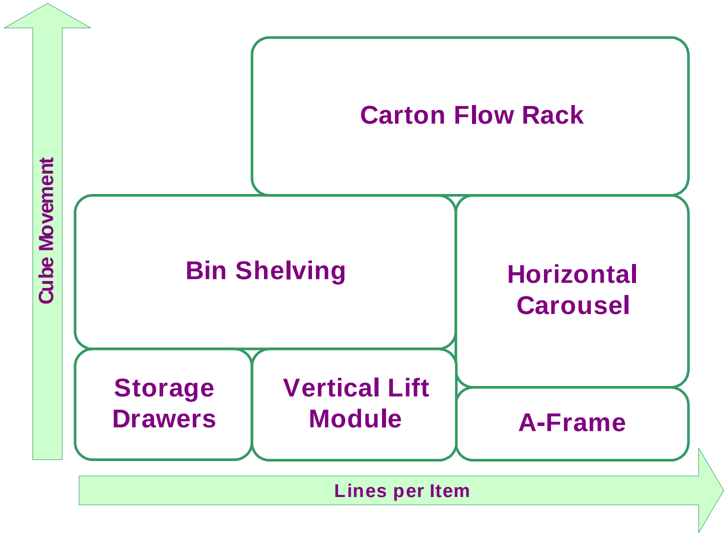


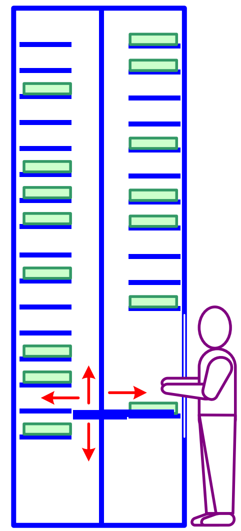
Zone-Batch Pick to Pallet



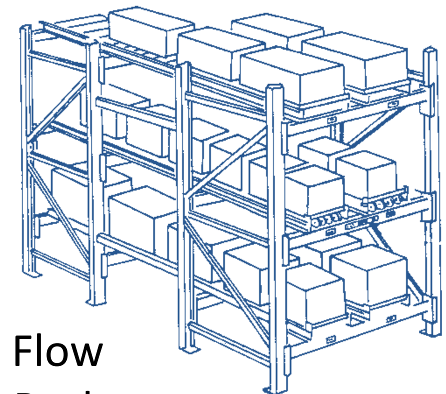
Floor- vs. Multi-level Pick to Pallet

Piece Picking Equipment

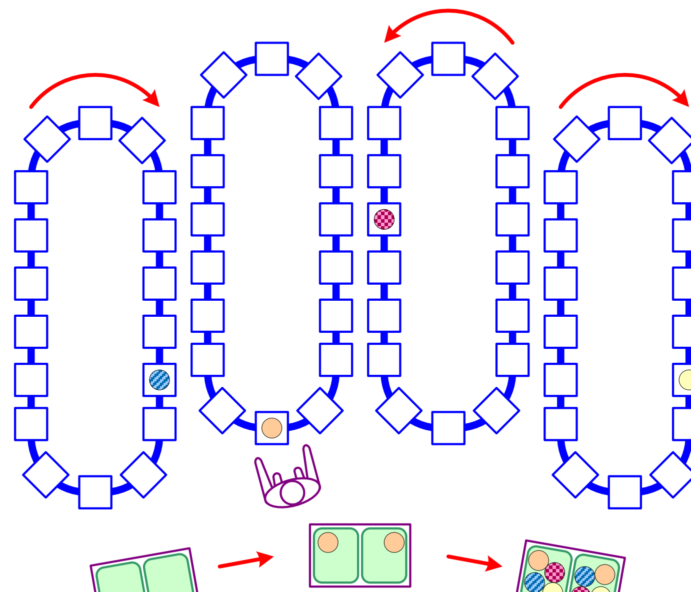




Vertical Lift Module

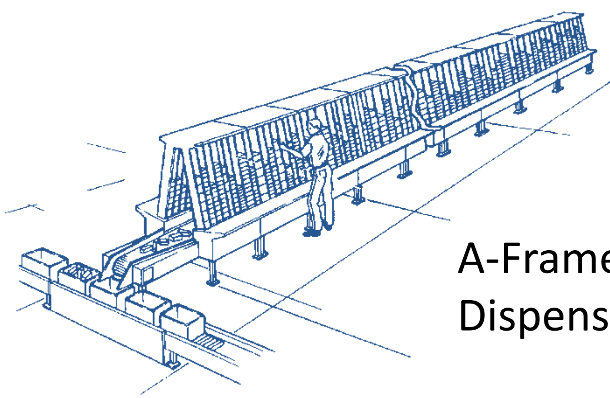


Carton Flow Rack



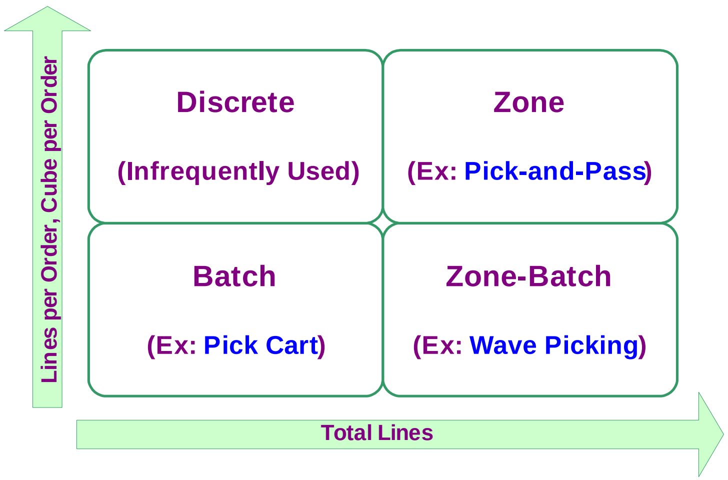
Carousel

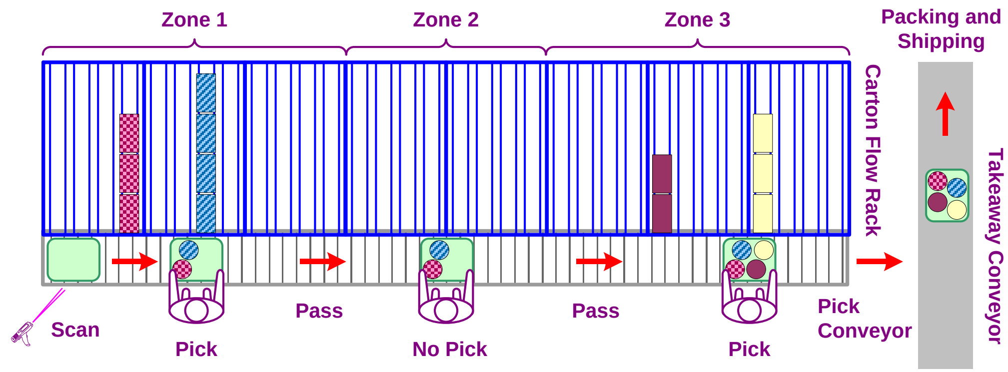
Drawers/Bins



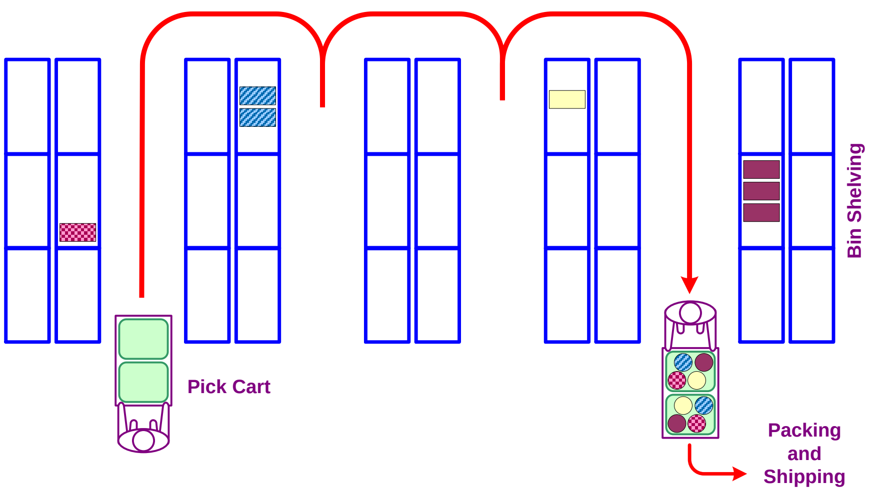
A-Frame Dispenser

Methods of Piece Picking

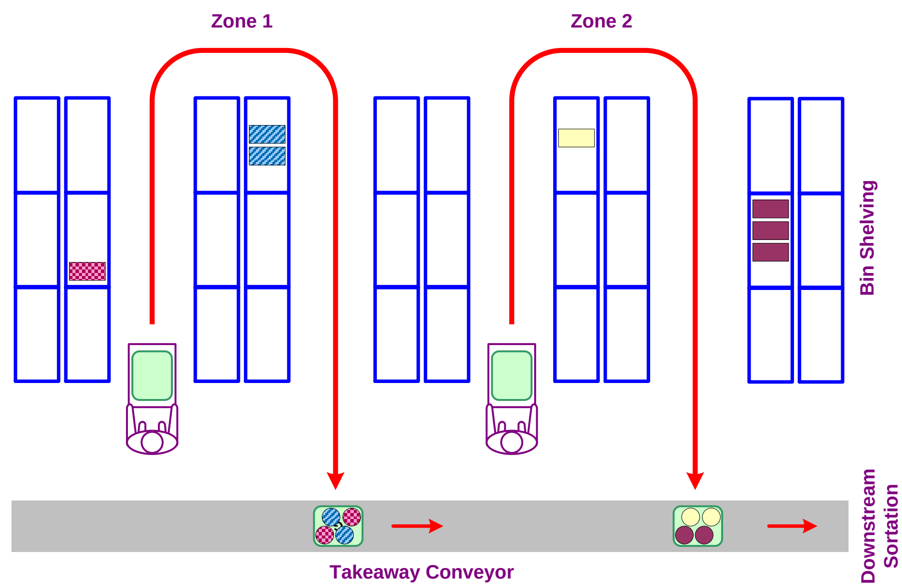




Pick-and-Pass Zone Piece Picking



Pick-cart Batch Piece Picking



Wave Zone-Batch Piece Picking

Warehouse Automation

- Historically, warehouse automation has been a craft industry, resulting highly customized, one-off, high-cost solutions
- To survive, need to
  - adapt mass-market, consumer-oriented technologies in order to realize to economies of scale
  - replace mechanical complexity with software complexity
- How much can be spent for automated equipment to replace one material handler:

$$\$45,432\left(\frac{1-1.017^{-5}}{1-1.017^{-1}}\right)=\$45,432(4.83)=\$219,692$$

  - \$45,432: median moving machine operator annual wage + benefits
  - 1.7% average real interest rate 2005-2009 (real = nominal – inflation)
  - 5-year service life with no salvage (service life for Custom Software)

KIVA Mobile-Robotic Fulfillment System

- Goods-to-man order picking and fulfillment system
- Multi-agent-based control
  - Developed by Peter Wurman, former NCSU CSC professor
- Kiva now called Amazon Robotics
  - purchased by Amazon in 2012 for $775 million