# Waiting Time Approximation in Public Logistics Network

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### Abstract

A public logistics network (PLN) is proposed as an alternate network to private logistics networks for the ground transport of parcels. A PLN can be modeled as a queuing network composed of packages queuing at each distribution center (DC). Since packages try to bid high to get priority in transit and trucks transport multiple packages during each trip, the PLN represents a priority queuing network with batch arrivals and batch services. Since closed form solutions for average waiting time are difficult to obtain in such a queuing model, an approximate analytical model has been developed to estimate average waiting time, and a simulation model is used for its validation.

### Keywords

Logistics Networks, Queuing Networks with bulk arrivals and bulk services, Waiting Time Estimation

### **1. Introduction**

A public logistics network (PLN) is proposed as an alternative to private logistics networks for the ground transport of parcels. Using the analogy between the packages transported in the network and the packets transmitted through the Internet, a package would be sent from a store and then routed through a sequence of public distribution centers (DCs) located throughput a metropolitan area and then delivered to a home in a matter of hours. The DCs, functioning like routers in the Internet, could also be located at major highway interchanges for longer distance transport. Unlike conventional logistics firms like UPS and FedEx using centralized control, in a PLN, decentralized control is adopted and different functions of the network would be separated so that a single firm is not required for coordination. This would enable scale economies to be realized in performing each logistics function since each element of the network has access to potentially all of the network's demand. However, one of major potential problems of a PLN is huge initial building cost for new DCs and the associated loading and unloading of packages at each DC [1,2,3].

In a PLN, packages bid for transport. Higher bids get higher priority and lead that package closer to the head of a queue. Within each DC, packages arriving on a truck from another DC arrive in batches and are transported in batches to subsequent DCs. Individual packages initiate and end their routes at origin and destination DCs, respectively. A PLN could be viewed as a priority open queuing network with bulk arrivals and bulk services. Due to complexity of this problem, a closed form solution is hard to obtain. An approximate analytical model has been developed to estimate average waiting time, and a simulation model is used for its validation.

Figure 1 shows hypothetical public logistics network with 36 public DCs covering the southeastern portion of the U.S. and connected via interstate highways. Each of the interstate DCs would serve as a hub in a sub-network of local DCs (not shown) covering the region surrounding the interstate DC [1].

### **2** Analytical Model

Simulation is a good tool to simulate real world networks, but it is hard to discover what is going on behind the scenes solely by analyzing its output. Therefore, an approximate analytical model has been developed with Matlab. The analytical model calculates the number of trucks needed to meet a specific package demand, which could be used as input parameter in simulation to target a desired truck load factor instead of as a "hit and miss" procedure to get to desired load factor level.

Analytical model has seven input parameters:

timeLU:	loading and unloading time within DC; default is 5 minutes.
ProxFac:	Proximity factor; default is 0.
nopkg:	total number of packages; default is 3000.
TrCap:	truck capacity; default is 10.
TrSpeed:	truck traveling speed; default is 60.
LdFacA:	load factor for trucks; default is 80%.
HWRatio:	headway ratio, which is used to determine average waiting time from headway; default is 0.5.

Outputs from analytical model are as follows:

Headway: 7.55, AvgTransTime: 11.11, NoTrucks: 115

It shows, under the current parameter set, the headway is 7.55 hours, average transit time for packages is 11.11 hours. In order to reach 80% load factor, 115 trucks are needed. This number of trucks will serve as one input parameter into simulation to obtain the same load factor for simulation model. Figure 2 shows outputs from analytical model with one parameter set.

As mentioned above, since closed form solution is hard to obtain for the PLN, a headway idea is proposed to estimate average waiting time. The headway between vehicles in transit systems is the amount of time that elapses between two vehicles passing the same point traveling in the same direction on a given route. On average, average waiting time is a function of headway. So the problem boils down to finding out a good way to estimate headway.

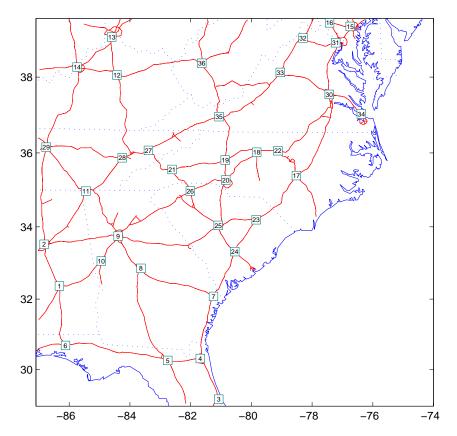


Figure 1 Hypothetical Public Logistics Network Coving Southeastern Portion of the USA

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				HWRatio:	0.50	
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				AvgTransTime:	11.11	
				NoTrucks:	115.00	

Figure 2 Output Snapshot from Analytical Model

### 3. Simulation model

#### 3.1 Simulation model setup

Simulation model is built using the Microsoft .NET environment with C# [1]. It is composed of DCs, trucks, packages, loading areas, etc. Everything is treated as objects. It simulates PLN operation on a daily basis.

Simulation model has six input parameters:

timeLU:	loading and unloading time within DC, default is 5 minutes.
ProxFac:	Proximity factor, default is 0.
nopkg:	total number of packages per day (simulation length is 100 days), default is 3000.
TrCap:	truck capacity, default is 10.
TrSpeed:	truck traveling speed, default is 60.
NoTrucks:	number of trucks. Used number of trucks suggested by analytical model, 115.

Outputs from simulation are as follows:

LdFacS: 0.82, AvgTransTime: 17.96, AvgWaitTime: 10.34

As shown in figure 3, with current parameter set, load factor is 82%, which is close to 80% from analytical model. Average transit time for packages is 17.96 hours; average wait time for packages is 10.34 hours.

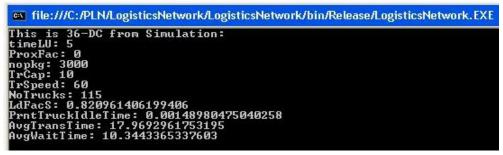


Figure 3 Output Snapshot from Simulation Model

#### 3.2 Simulation model verification

Before further research is carried out with simulation model, it is necessary to be testified it is working correctly. Little's law and factory physics laws were used to verify simulation.

Little's Law: The average number of customers in a stable system (over some time interval), *WIP*, is equal to their average arrival rate,  $\lambda$ , multiplied by their average cycle time in the system, *CT* [4],

 $WIP = \lambda CT$ 

Simulation keeps track of *WIP* for PLN, and compares it with theoretic value given  $\lambda$  and *CT*. The difference is small and acceptable, around -2.67%.

Figure 4 shows output screen for verification against Little's Law.

Average # Packages	ime (hours): 2.7873686428864 in System: 2159.82185448602 from Little's Law: -2.67% : 23.2155641225313	-
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Figure 4 Verification of Little's Law

If truck capacity is constrained to be one, and every package bids the same fixed amount for trips, PLN network reduces to first-in-first-out(FIFO) network. Classic factory physics laws (feasibility analysis) should apply to it. Simulation model is compared against theoretical values from feasibility analysis. As show in table 1, for each level of load factor, simulation tracks factory physics metrics closely.

	Factory	Physics	Simulatio	on (2DC)
Load	Pkg Transit		Pkg Transit Time	# of trucks
Factor	Time(hr)	# of trucks	(hr)	(capacity=1)
95%	7.18	13	5.22	13
88%	3.98	14	3.23	14
82%	3.42	15	2.88	15
77%	3.21	16	2.72	16
73%	3.11	17	2.64	17
68%	3.05	18	2.58	18
65%	3.03	19	2.68	19
62%	3.01	20	2.54	20
59%	3.00	21	2.66	21
56%	2.99	22	2.52	22
53%	2.99	23	2.62	23
51%	2.98	24	2.5	24

Table 1 Comparison between Simulation and Feasibility Analysis Values

Based on the comparisons above, it is safe to claim simulation is working logically and correctly.

#### 4. Headway Estimation from Analytical and Simulation models

So far, there are two tools on the table: analytical model and simulation model. Since both of them have their own limitations, we combined those two tools to obtain some insights into average waiting time estimation with headways.

In the analytical model, five input parameters are used, which are: timeLU, nopkg, TrCap, TrSpeed, LdFac.

In the simulation, five input parameters are used, which are: timeLU, nopkg, TrCap, TrSpeed, notrucks. By varying each parameter in high, median, and low levels, a full set of parameters is obtained. Run this full parameter set with analytical and simulation model separately, a dataset containing levels of each parameter and headway is obtained and shown as figure 5.

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Figure 5 Dataset from Analytical and Simulation Model Run

By applying SAS 9.1 Proc Corr procedure, as shown in figure 6, correlation between Packages average waiting time and headway is 0.37, and it is very statistically significant.

By using SAS 9.1 Proc Reg procedure, it shows that with headway present, adding TimeLu, nopkgm, TrCap, and TrSpeed does not help explains data much (adjusted R-square increases 1% from 0.1421 to 0.1436 by adding those four variables, as shown in figures 7 and 8). So headway could be used as a first order estimation for waiting in stead of using those four variables.

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#### Figure 6 Correlation between Headway and Package Average Waiting Time

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > [t]
Intercept	Intercept	1	0.86731	4.79650	0.18	0.8565
TIMELU		1	-0.22754	0.10645	-2.14	0.0326
NOPKGY		1	0.00137	0.00024422	5.59	<.0001
TRCAP		1	0.04142	0.17481	0.24	0.8127
TRSPEED		1	0.09651	0.06969	1.38	0.1661
Headway	Headway	1	2.87556	0.05681	50.62	<.0001
arcdemand		1	-0.00034870	0.00003703	-9.42	<.0001
arcdis		1	-0.03199	0.00919	-3.48	0.0005

#### Parameter Estimates

Figure 7 Regression with TimeLu, Nopkgm TrCap, TrSpeed Variables

Parameter Estimates

Variable	Labe 1	DF	Parameter Estimate	Standard Error	t Value	Pr > [t]
Intercept	Intercept	1	10.88282	1.37111	7.94	<.0001
Headway	Headway	1	2.88583	0.05652	51.06	<.0001
arcdemand	•	1	-0.00024729	0.00003231	-7.65	<.0001
arcdis		1	-0.02703	0.00916	-2.95	0.0032

Figure 8 Regression without TimeLu, nopkgm TrCap, TrSpeed Variables

### 4. Conclusion and Future Work

Packages average waiting time approximation is difficult in PLN. As shown above, headway is heavily related to average waiting time and headway encapsulates information in other variables, such as TimeLu, nopkgm, TrCap, and TrSpeed, so headway could be used as a first order approximation for average waiting time. It could greatly reduce factors needed to estimate average waiting time and narrow down the problem space. Although the relationship between average waiting time and headway needs more research, it is one step further down the road to consolidate those four factors into one factor—headway.

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