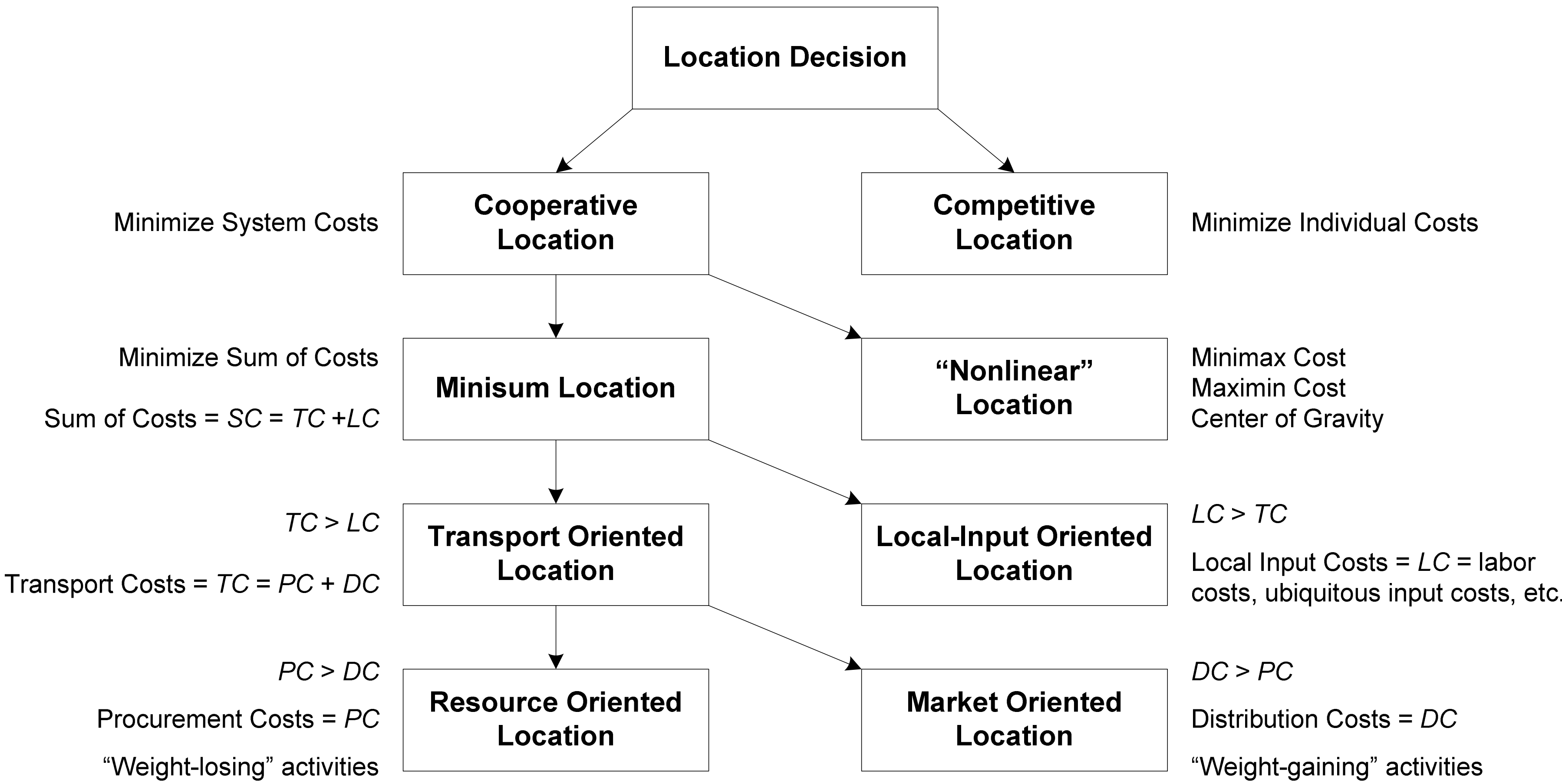
Location 1:
Types of Location Problems

- For most private-industry-related applications, minimizing the sum of distances is the most appropriate objective for determining the optimal location
  - This is because transport cost (roughly speaking) increases directly proportional to distance (it’s *linear*)
    - Truck drivers are paid by the mile
  - In many public or personal applications, costs increase faster than distance (they’re *nonlinear*)
    - Most people would prefer 20 thirty-minute driving trips to one 10-hour trip

Why Are Cities Located Where They Are?

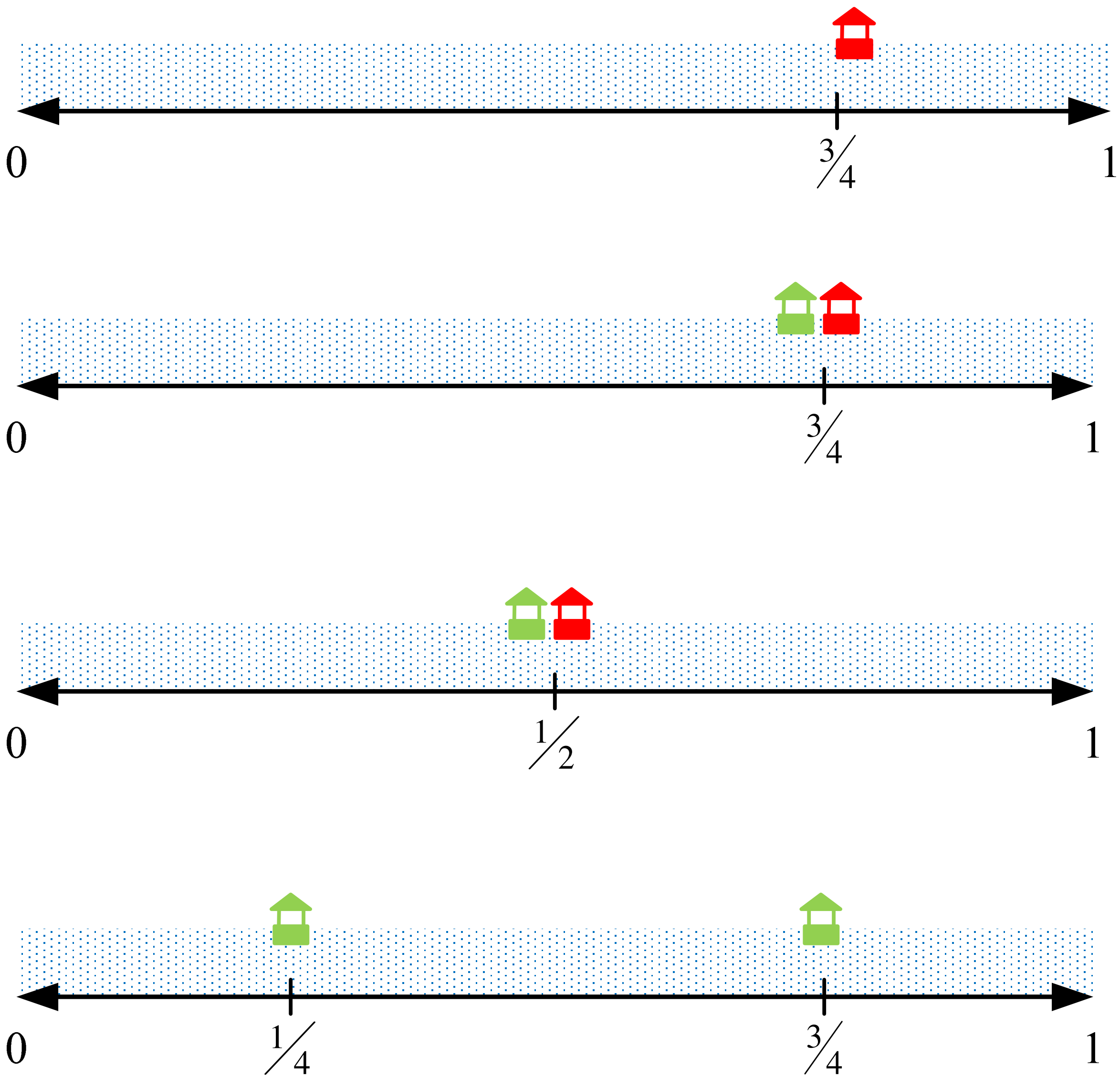
- Minimizing total logistics costs is often principle factor

Taxonomy of Location Problems

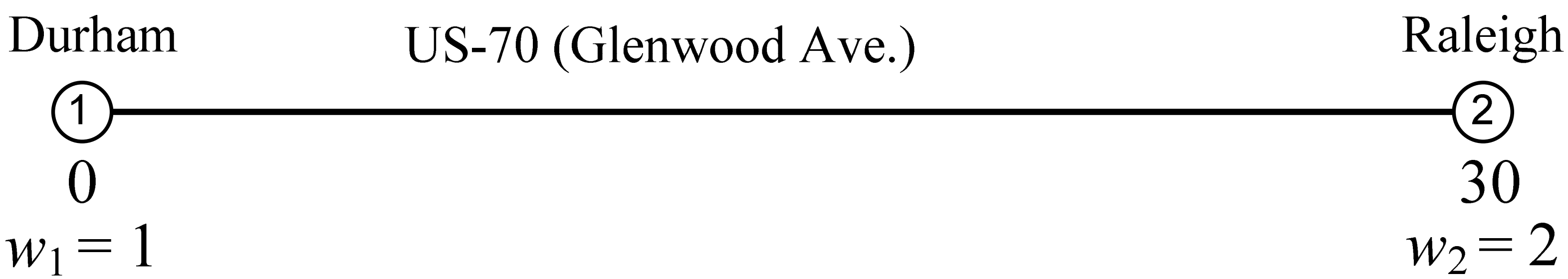


Location where cost of producing good is more than cost of distributing it, then have resource oriented location (think of production of metal ore...needs to be close to source of raw materials). Market orientation location has greater concern with distributing goods since production costs are not as high or transportation costs are high (think of distribution center for Amazon where its more beneficial to have your facility located closer to the market). [S. March]

Hotelling's Law



1-D Cooperative Location



$$\text{Min } TC = \sum w_i \, d_i$$

$$\text{Min } TC = \sum w_i \, d_i^2$$

$$a_1 = 0, \quad a_2 = 30$$

$$TC = \sum w_i \, d_i^2 = \sum w_i \left(x - a_i \right)^2$$

$$\frac{dTC}{dx} = 2 \sum w_i \left(x - a_i \right) = 0 \Rightarrow$$

$$x \sum w_i = \sum w_i \, a_i \Rightarrow$$

Squared−Euclidean Distance ⇒ Center of Gravity: $x^* = \dfrac{\sum w_i \, a_i}{\sum w_i} = \dfrac{1(0) + 2(30)}{1+2} = 20$

Linear vs Nonlinear Location

$$\text{Linear:} \quad \min \sum w_i\, d_i \qquad \text{Nonlinear:} \begin{cases} \min \sum w_i\, d_i^2 & \text{center of gravity} \\ \min\{\max d_i\} & \text{minimax} \\ \max\{\min d_i\} & \text{maximin} \\ \min \sum k_i + w_i\, d_i & \text{fixed cost (affine)} \\ \min \sum w_i \sqrt{d_i} & \text{economy of scale} \end{cases}$$

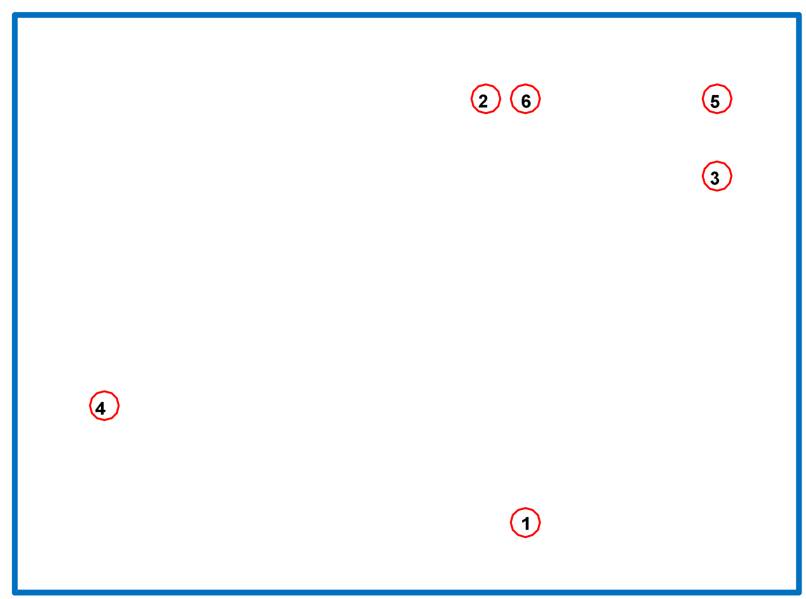
- Cost $\propto$ distance
- Private firm pays driver
- Easy to solve:
  - convex $\Rightarrow$ easy continuous location
  - LP $\Rightarrow$ easy discrete location

- Cost $\not\propto$ distance
- "Psychological cost"
  - Marchetti's constant: avg. commute 1 hr/day
- Public/personal
- More difficult to solve

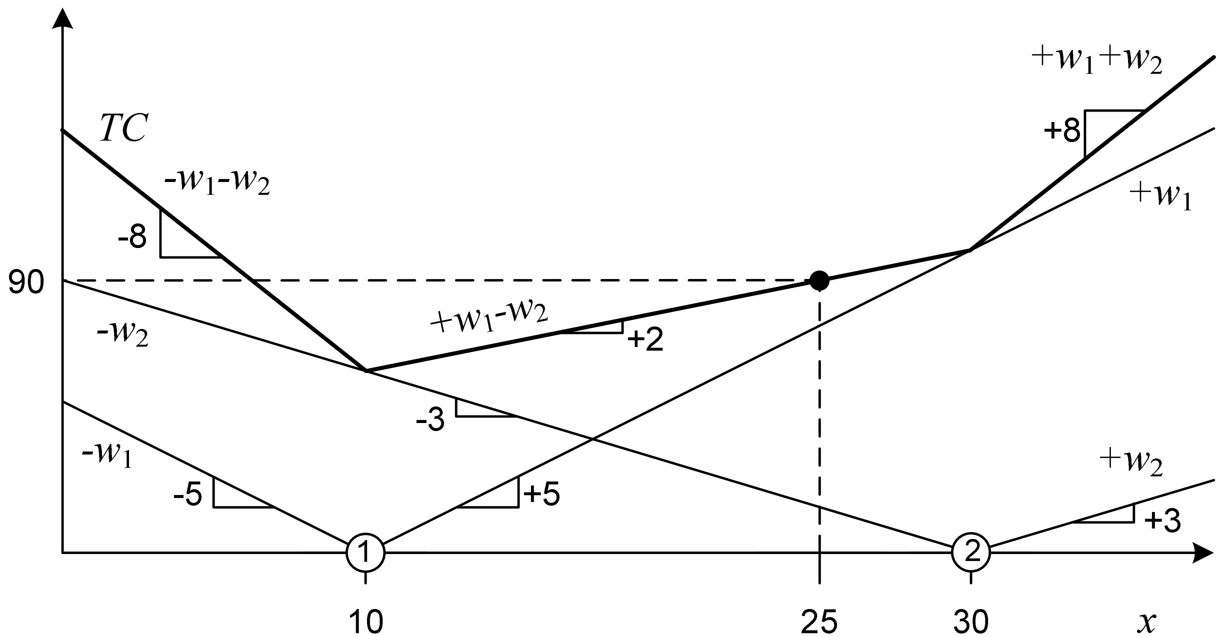
Minimax and Maximin Location

- Minimax
  - Min max distance
  - Set covering problem
  - optimal point is usually halfway between two points that are furthest apart (points 4 and 5)

- Maximin
  - Max min distance
  - AKA obnoxious facility location



2-EF Minisum Location



$$TC(x) = \sum w_i\, d_i = \beta_1(x - x_1) + \beta_2(x - x_2), \quad \text{where } \beta_i = \begin{cases} w_i, & \text{if } x \geq x_i \\ -w_i, & \text{if } x < x_i \end{cases}$$

$$TC(25) = w_1(25-10) + (-w_2)(25-30)$$

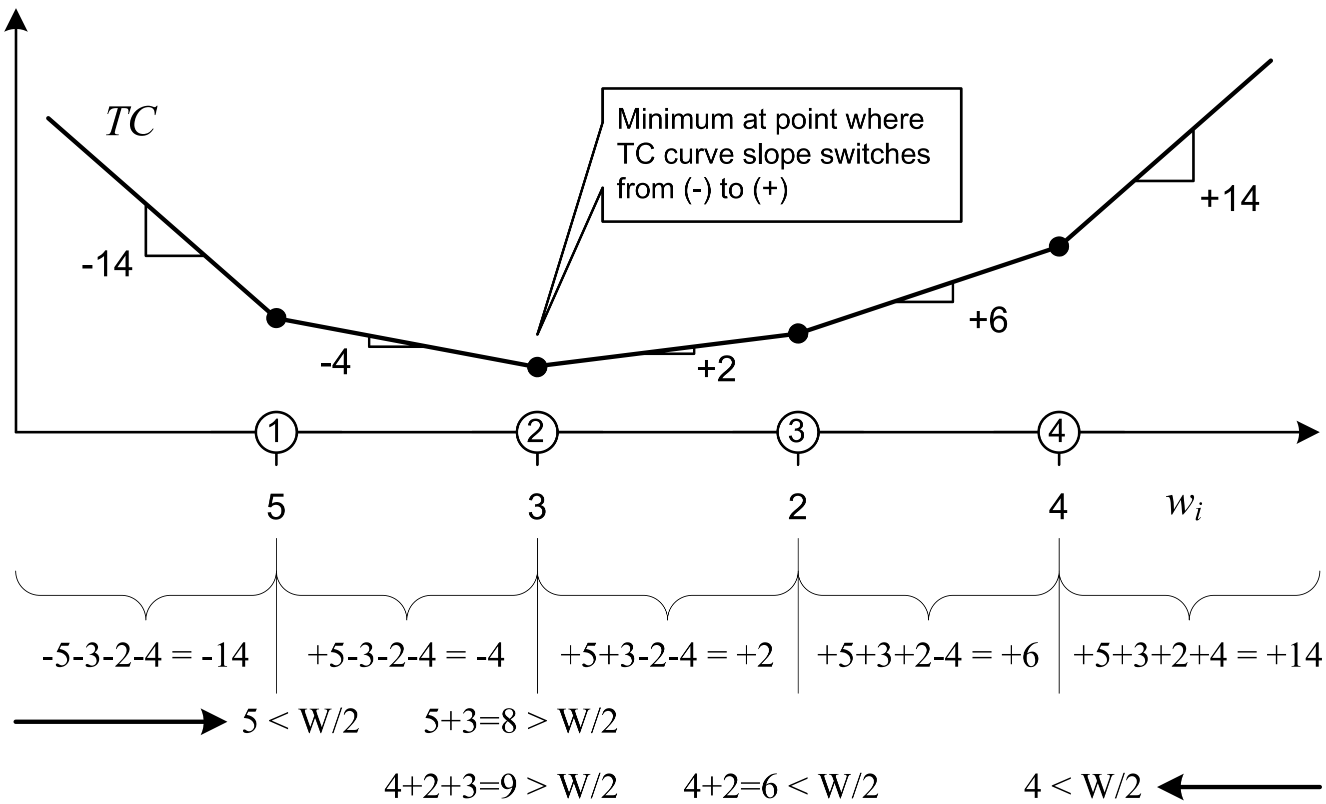
$$= 5(15) + (-3)(-5) = 90$$

Note: This means of finding the optimal solution only works because *TC* is linear, need to use numerical optimization otherwise

Median Location: 1-D 4 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$



Median Location: 1-D 7 EFs

Median location: For each dimension x of X:

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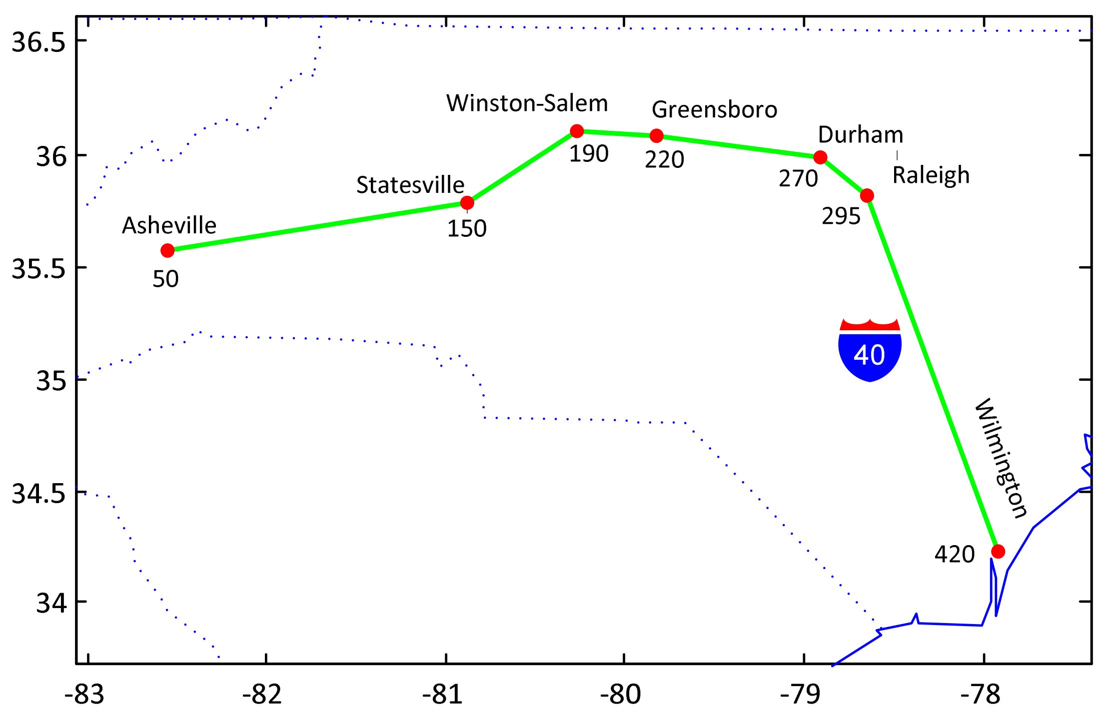
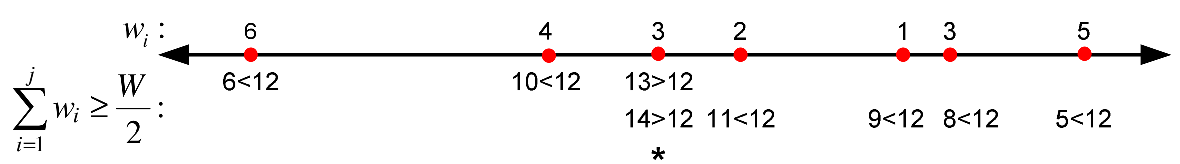
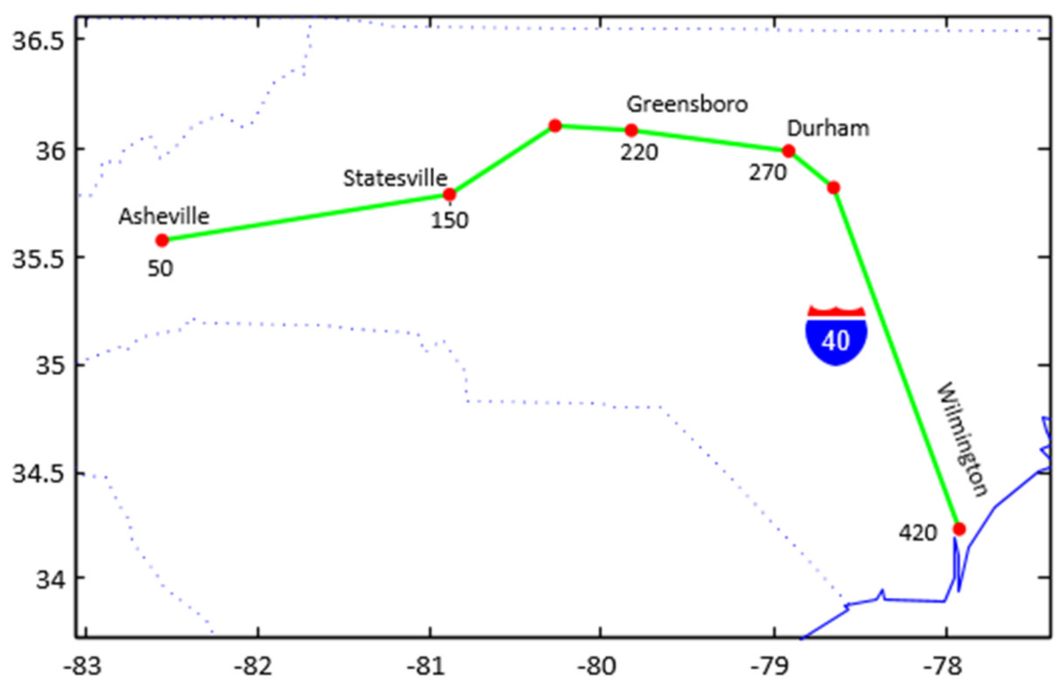




Question 2.1.1

Assuming 5, 15, 10, 10, and 20 trips per month are made to EFs in Asheville, Durham, Greensboro, Statesville, and Wilmington, respectively, determine the minisum location.

a) Asheville
b) Durham
c) Greensboro
d) Statesville
e) Wilmington



Ex: 1-D Median Location

- Traveling north, I-95 passes through or near the following cities: Jacksonville, FL; Savannah, GA; Florence, SC; Lumberton, NC; Fayetteville, NC; Rocky Mount, NC; and Richmond, VA. A company wants to build a facility along I-95 to serve customers in these cities. If the weekly demand in truckloads of customers in each city is 12, 32, 6, 15, 24, 11, and 20, respectively, determine where the facility should be located to minimize the distance traveled to serve the customers assuming that I-95 will be used for all travel.

```
cities = ["Jacksonville, FL", "Savannah, GA", "Florence, SC", "Lumberton, NC",
          "Fayetteville, NC", "Rocky Mount, NC", "Richmond, VA"]
w = [12, 32, 6, 15, 24, 11, 20]
@show sum(w)
@show sum(w)/2
@show cumsum(w)
# Find index of city where cumulative demand first equals or exceeds half of total demand
idx = findfirst(x -> x >= sum(w)/2, cumsum(w))
```

```
sum(w) = 120
sum(w) / 2 = 60.0
cumsum(w) = [12, 44, 50, 65, 89, 100, 120]

4
```

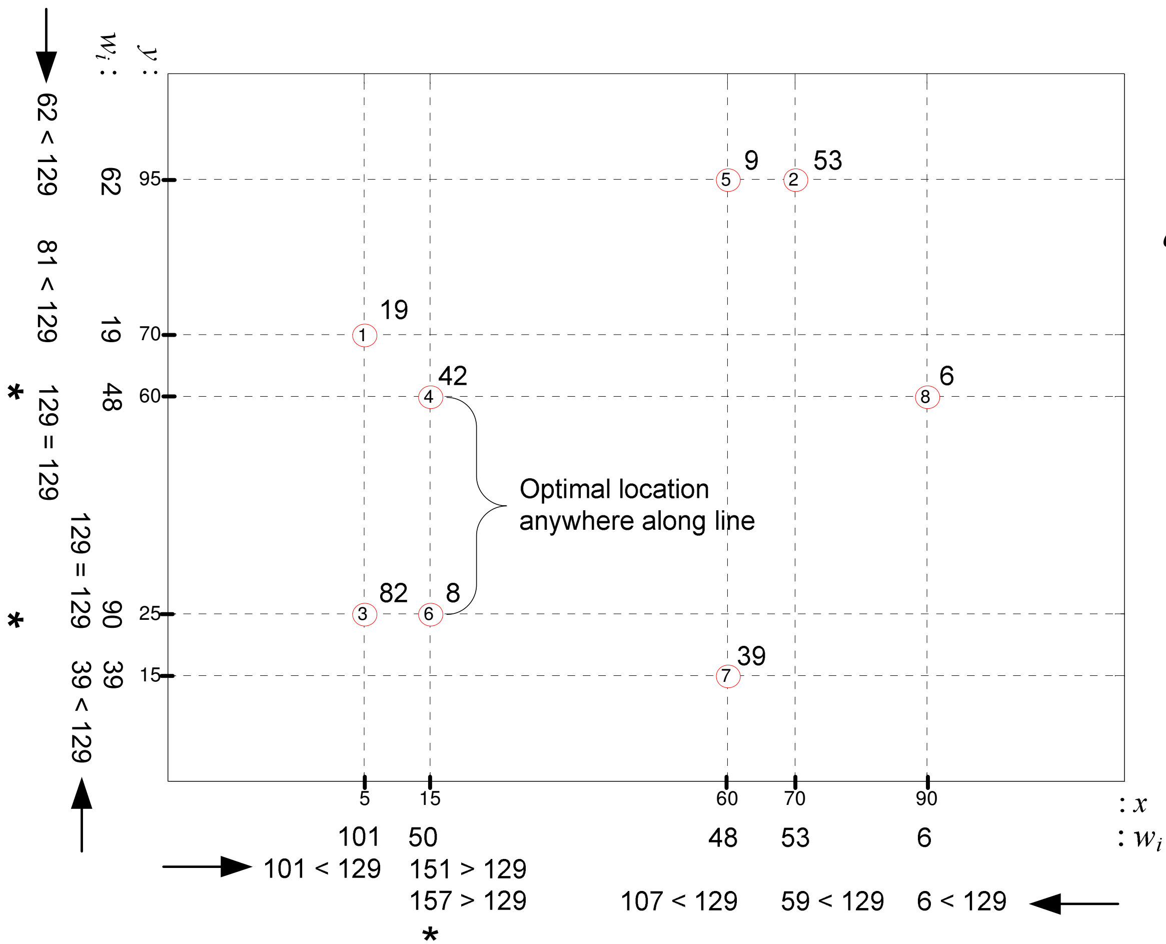
```
println("The optimal location for the facility is: ", cities[idx])
```

```
The optimal location for the facility is: Lumberton, NC
```

Median Location: 2-D Rectilinear Distance 8 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

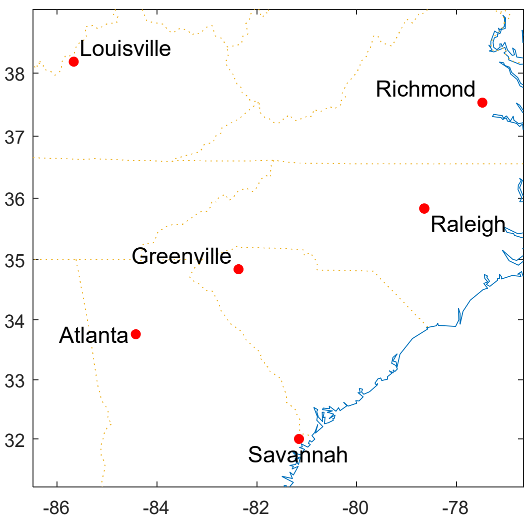


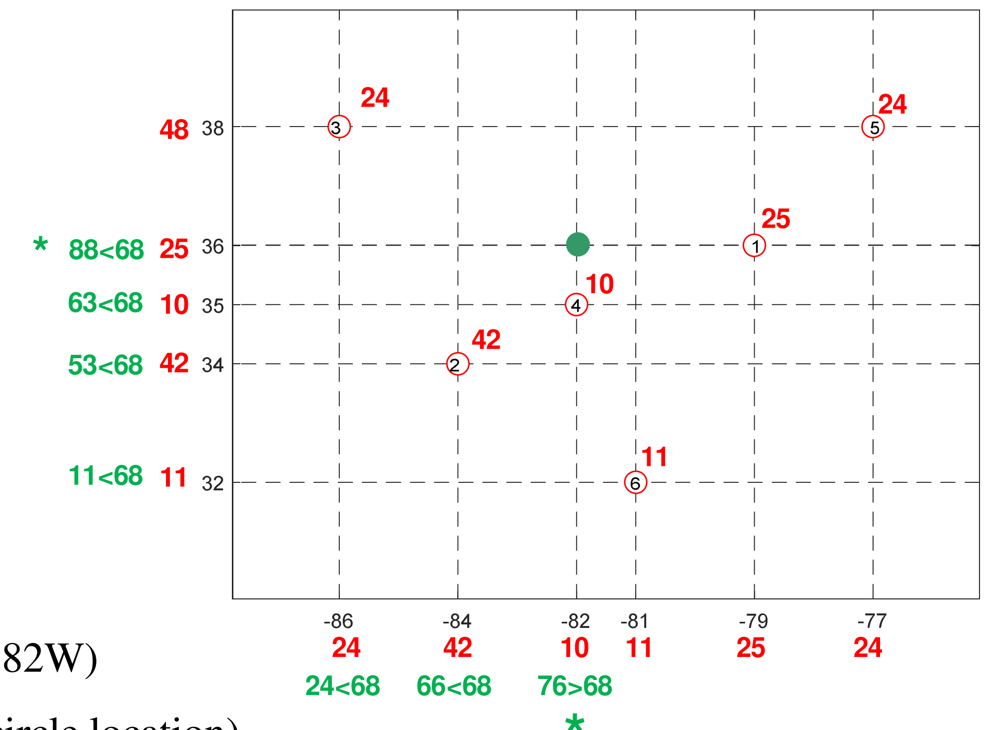
$$d_1(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$$

$$d_2(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Ex: 2D Loc with Rect Approx to GC Dist

- It is expected that 25, 42, 24, 10, 24, and 11 truckloads will be shipped each year from your DC to six customers located in Raleigh, NC (36N,79W), Atlanta, GA (34N,84W), Louisville, KY (38N,86W), Greenville, SC (35N, 82W), Richmond, VA (38N,77W), and Savannah, GA (32N,81W). Assuming that all distances are rectilinear, where should the DC be located in order to minimize outbound transportation costs?





$$W = \sum w_i = 136, \quad \frac{W}{2} = 68$$

Answer : Optimal location (36N,82W)

(65 mi from opt great-circle location)