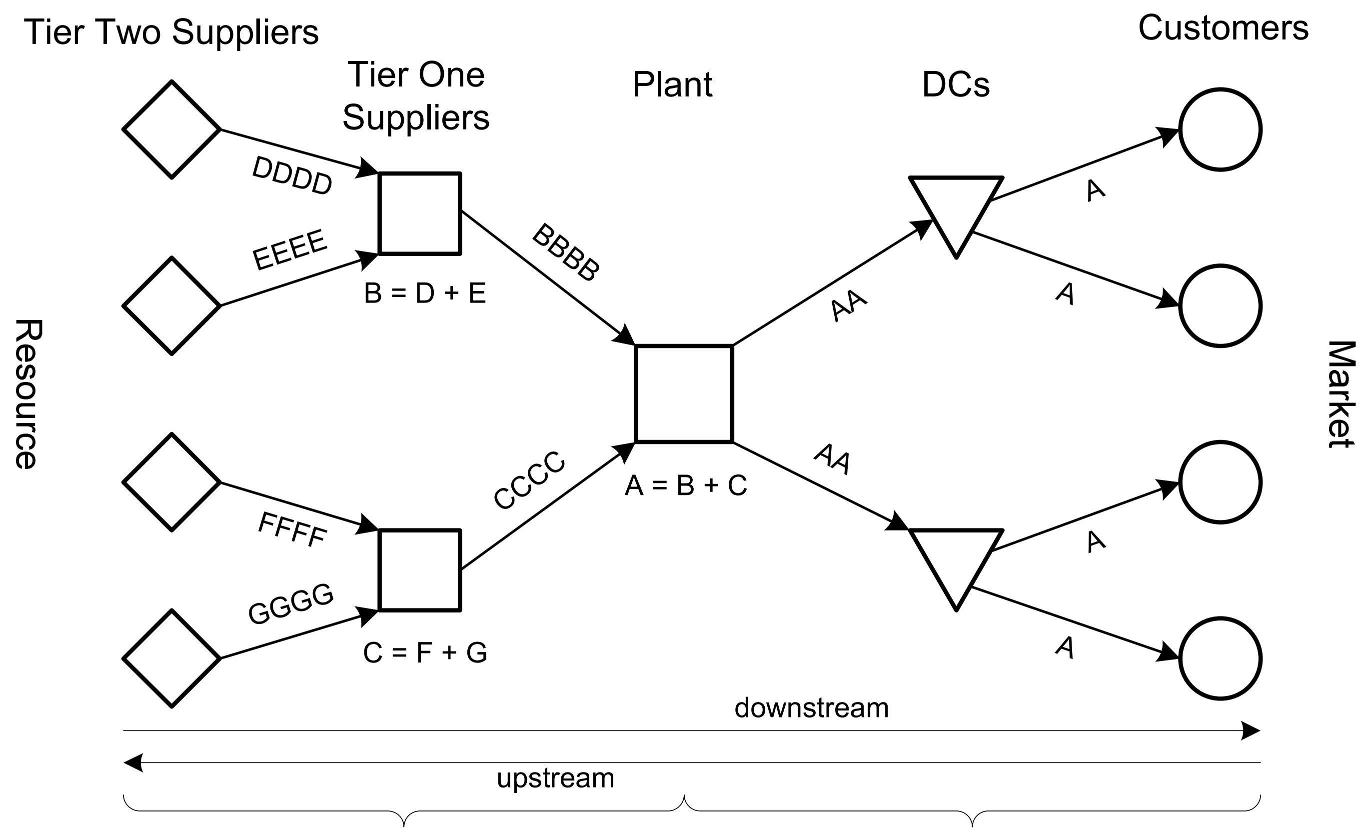
Location 2:
Single-Facility Location

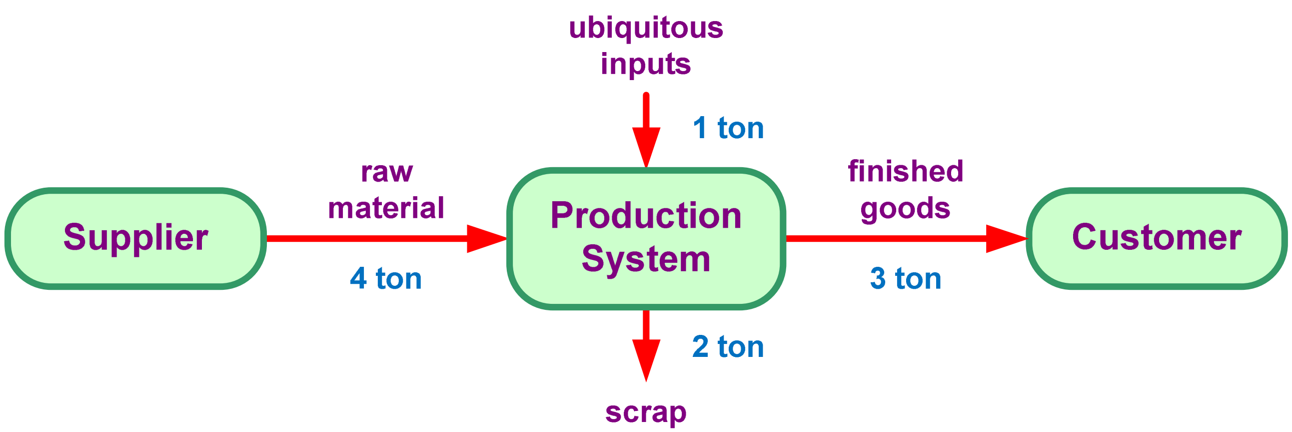
- **Monetary vs. physical weight:** A production process can be physically weight *losing* but monetarily weight *gaining*
- *Topic of this lecture:* What are the weights and how are they determined in a minisum weighted-distance location problem?

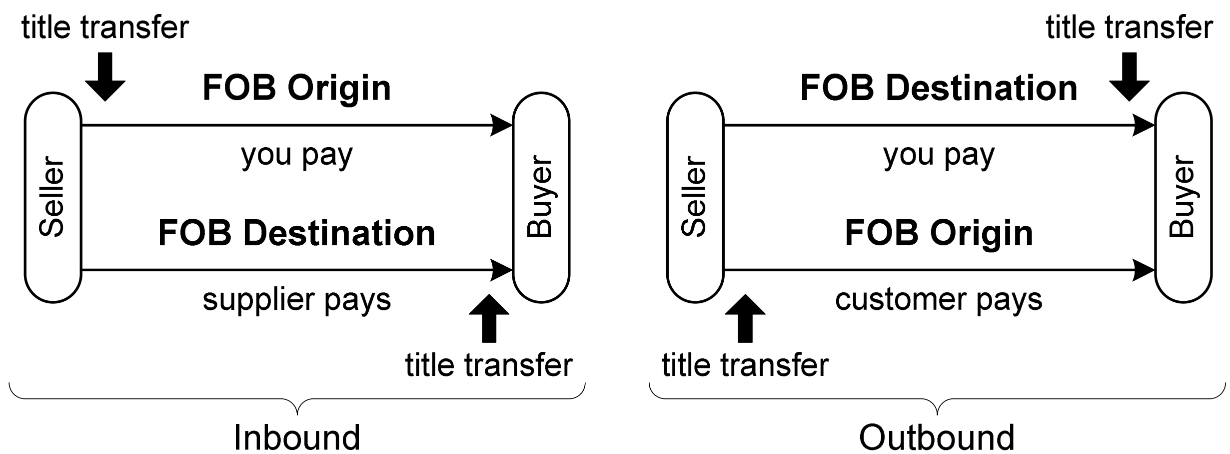
Logistics Network for a Plant



Assembly Network	vs.	Distribution Network
Procurement	vs.	Distribution
Inbound Logistics	vs.	Outbound Logistics
Raw Materials	vs.	Finished Goods

Basic Production System





FOB (free on board)

FOB and Location

- Choice of FOB terms (who directly pays for transport) usually does not impact location decisions:

Procurement cost	=	Landed cost at supplier	+	Inbound transport cost
Production cost	=	Procurement cost	+	Local resource cost (labor, etc.)
Total delivered cost	=	Production cost	+	Outbound transport cost
Transport cost (TC)	=	Inbound transport cost	+	Outbound transport cost

  – *Purchase price* from supplier and *sale price* to customer adjusted to reflect who is paying transport cost
  – Usually determined by who can provide the transport at the lowest cost
    - Savings in lower transport cost allocated (bargained) between parties

Monetary vs. Physical Weight

$$\min TC(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} d(X, P_i)$$

where TC = total transport cost ($/yr)

w_i = monetary weight ($/mi-yr)

f_i = physical weight rate (ton/yr)

r_i = transport rate ($/ton-mi)

$d(X, P_i)$ = distance between NF at X and EF_i at P_i (mi)

NF = new facility to be located

EF = existing facility

m = number of EFs

(Montetary) Weight Gaining: $\Sigma w_{\text{in}} < \Sigma w_{\text{out}}$

Physically Weight Losing: $\Sigma f_{\text{in}} > \Sigma f_{\text{out}}$

Minisum Location: TC vs. TD

- Assuming local input costs are
  - same at every location or
  - insignificant as compared to transport costs,

  the minisum transport-oriented single-facility location problem is to locate NF to minimize TC
- Can minimize total distance (TD) if transport rate is same:

$$\min TD(X) = \sum_{i=1}^{m} w_i \, d(X, P_i) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} \, d(X, P_i)$$

where

$$TD = \text{total transport distance (mi/yr)}$$

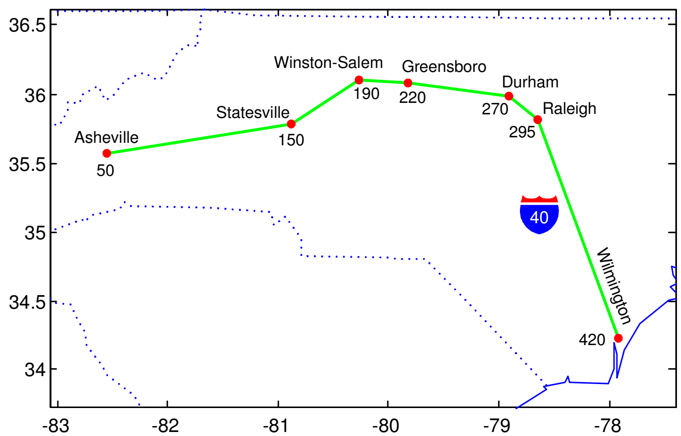
$$w_i = \text{monetary weight (trip/yr)}$$

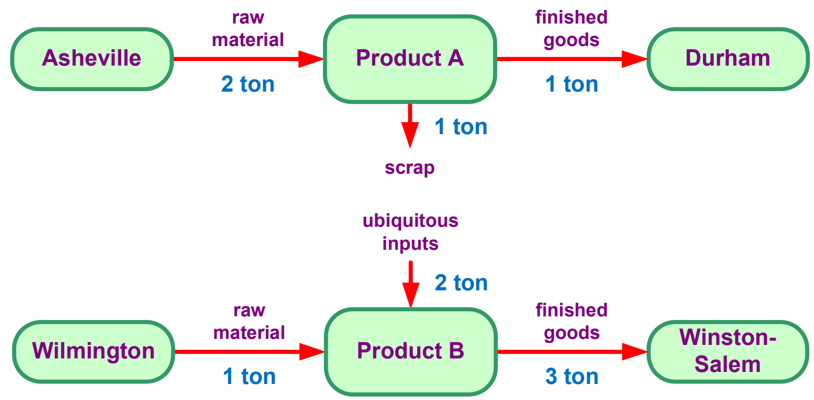
$$f_i = \text{trips per year (trip/yr)}$$

$$r_i = \text{transport rate} = 1$$

$$d(X, P_i) = \text{per-trip distance between NF and EF}_i \text{ (mi/trip)}$$

Ex: Single Supplier and Customer Locations

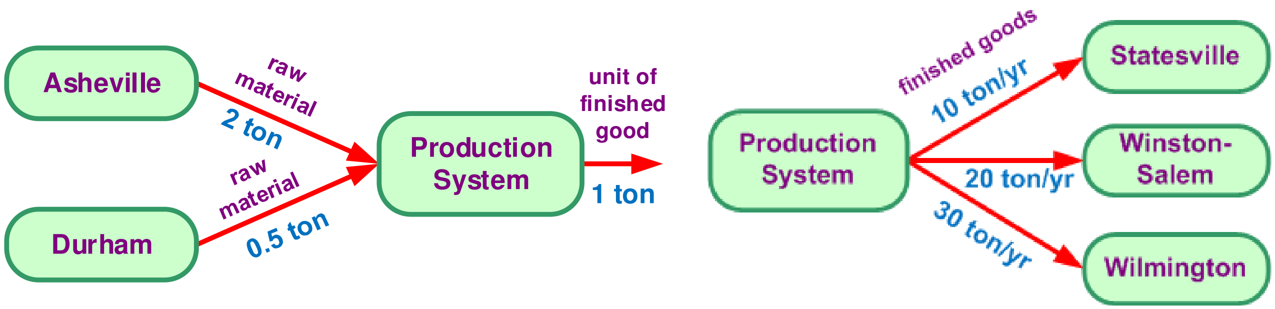




- The cost per ton-mile (i.e., the cost to ship one ton, one mile) for both raw materials and finished goods is the same (e.g., $0.10).
  1. Where should the plant for each product be located?
  2. How would location decision change if customers paid for distribution costs (FOB Origin) instead of the producer (FOB Destination)?
     - In particular, what would be the impact if there were competitors located along I-40 producing the same product?
  3. Which product is weight gaining and which is weight losing?
  4. If both products were produced in a single shared plant, why is it now necessary to know each product's annual demand (f_i)?

$$TC(X) = \sum_{i=1}^{m} \underbrace{f_i \, r_i}_{w_i} \, d(X, P_i)$$

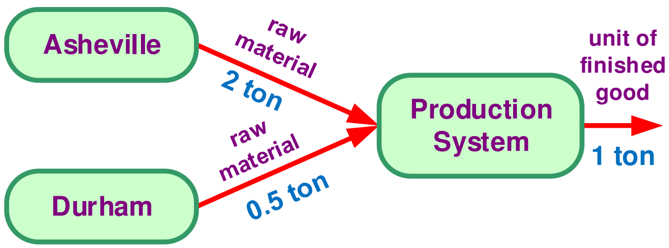
Ex: 1-D Location with Procurement and Distribution Costs

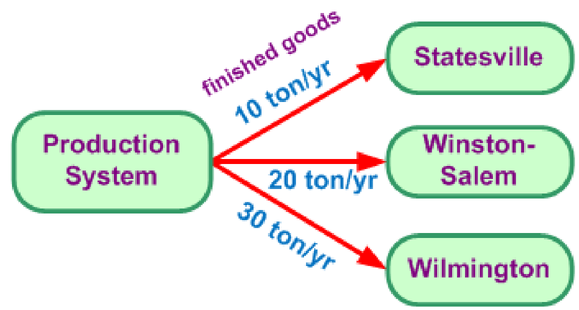


Assume: all scrap is disposed of locally

A product is to be produced in a plant that will be located along I-40. Two tons of raw materials from a supplier in Ashville and a half ton of a raw material from a supplier in Durham are used to produce each ton of finished product that is shipped to customers in Statesville, Winston-Salem, and Wilmington. The demand of these customers is 10, 20, and 30 tons, respectively, and it costs $0.33 per ton-mile to ship raw materials to the plant and $1.00 per ton-mile to ship finished goods from the plant. Determine where the plant should be located so that procurement and distribution costs (i.e., transportation costs to and from the plant) are minimized, and whether the plant is weight gaining or weight losing.

Ex: 1-D Location with Procurement and Distribution Costs





$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

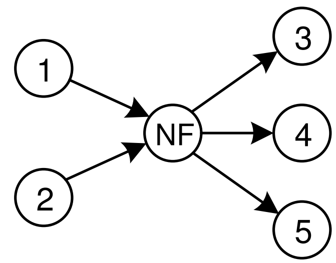
$$\overbrace{\underset{(\$/\text{mi-yr})}{w_i}}^{\text{monetary weight}} = \overbrace{\underset{(\text{ton/yr})}{f_i}}^{\text{physical weight}} \times \underset{(\$/\text{ton-mi})}{r_i}$$

Assume: all scrap is disposed of locally

$r_{\text{in}} = \$0.33/\text{ton-mi}$ $\qquad$ $r_{\text{out}} = \$1.00/\text{ton-mi}$

$$f_1 = BOM_1 \sum f_{\text{out}} = 2(60) = 120, \quad w_1 = f_1 r_{\text{in}} = 40$$

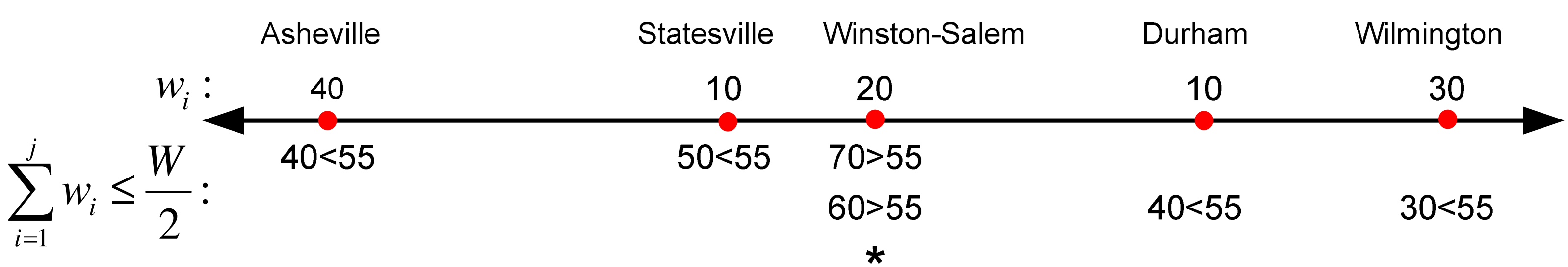
$$f_2 = BOM_2 \sum f_{\text{out}} = 0.5(60) = 30, \quad w_2 = f_2 r_{\text{in}} = 10$$



$$f_3 = 10, \quad w_3 = f_3 r_{\text{out}} = 10$$

$$f_4 = 20, \quad w_4 = f_4 r_{\text{out}} = 20$$

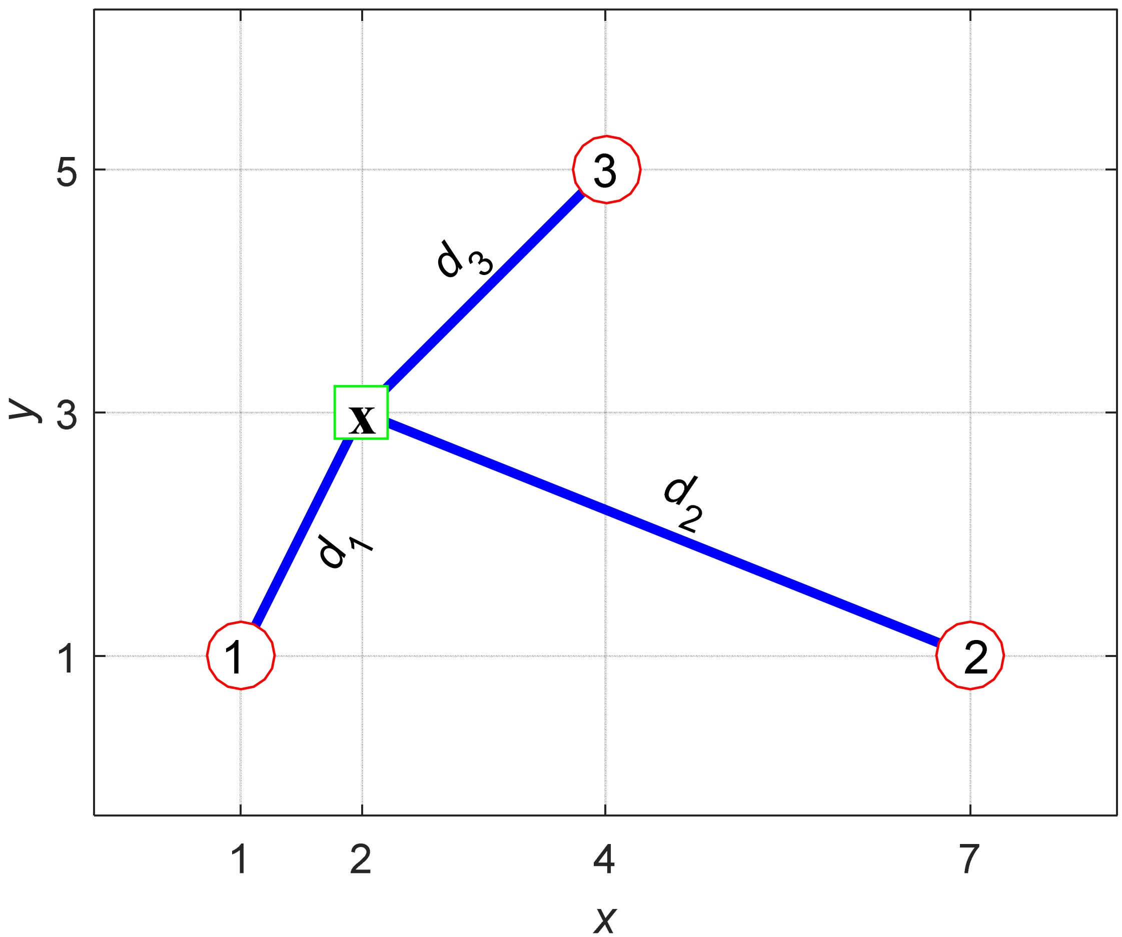
$$f_5 = 30, \quad w_5 = f_5 r_{\text{out}} = 30$$



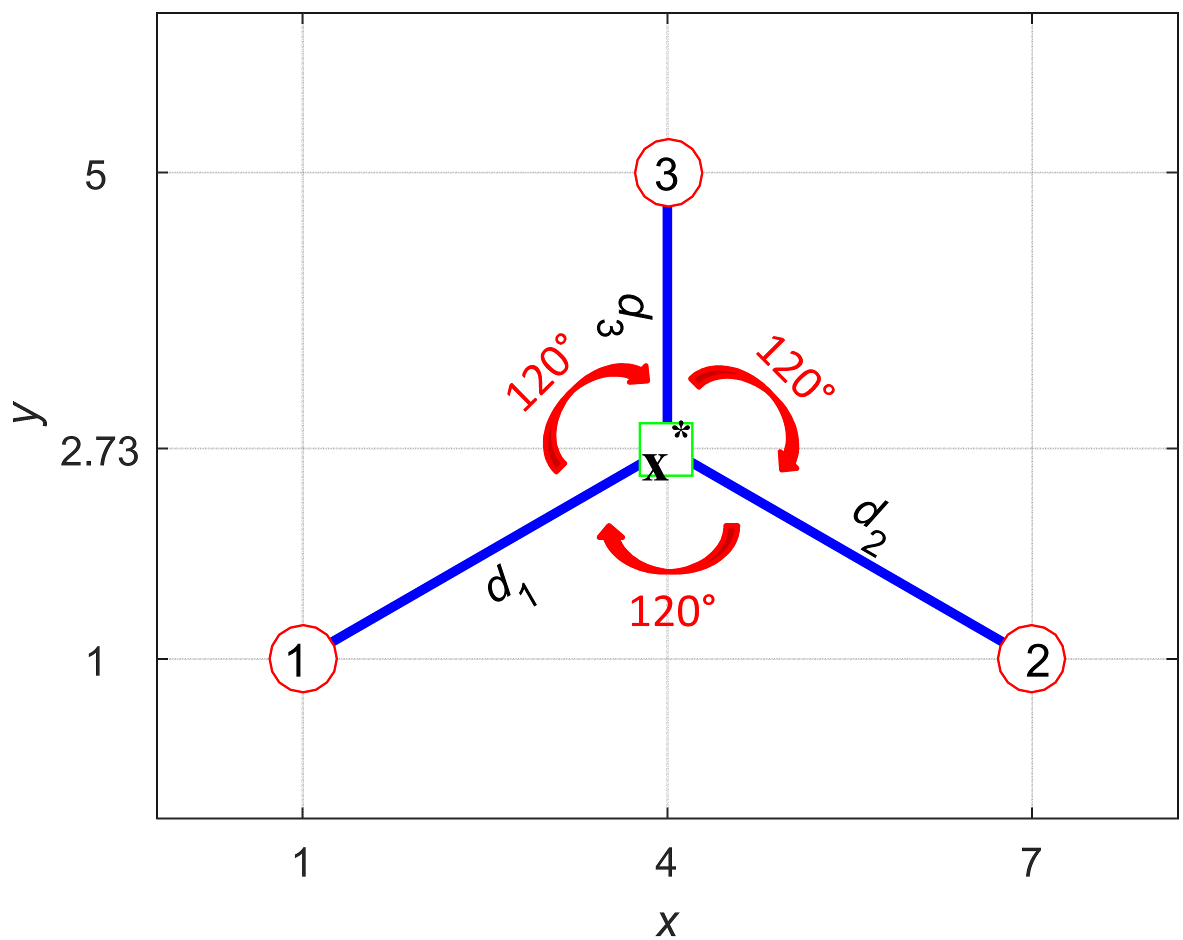
(Montetary) Weight Gaining: $\Sigma w_{\text{in}} = 50 < \Sigma w_{\text{out}} = 60$

Physically Weight Losing: $\Sigma f_{\text{in}} = 150 > \Sigma f_{\text{out}} = 60$

2-D Euclidean Distance



Minisum Distance Location



Fermat's Problem (1629):

Given three points, find fourth (Steiner point) such that sum to others is minimized

(Solution: Optimal location corresponds to all angles = 120°)

Minisum Weighted-Distance Location

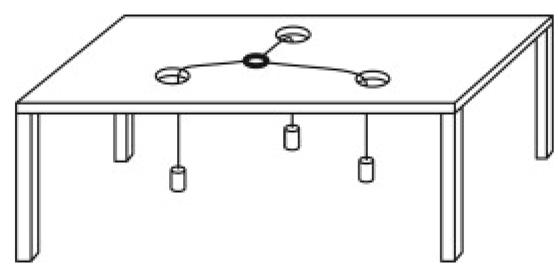
- Solution for 2-D+ and non-rectangular distances:
  - *Majority Theorem:* Locate NF at EF*j* if $w_j \geq \frac{W}{2}, \quad \text{where } W = \sum_{i=1}^{m} w_i$
  - Mechanical (Varigon frame)
  - 2-D rectangular approximation
  - Numerical: nonlinear unconstrained optimization
    - Analytical/estimated gradient (quasi-Newton)
    - Direct, gradient-free (Nelder-Mead)

$$m = \text{number of EFs}$$

$$TC(\mathbf{x}) = \sum_{i=1}^{m} w_i d_i(\mathbf{x})$$

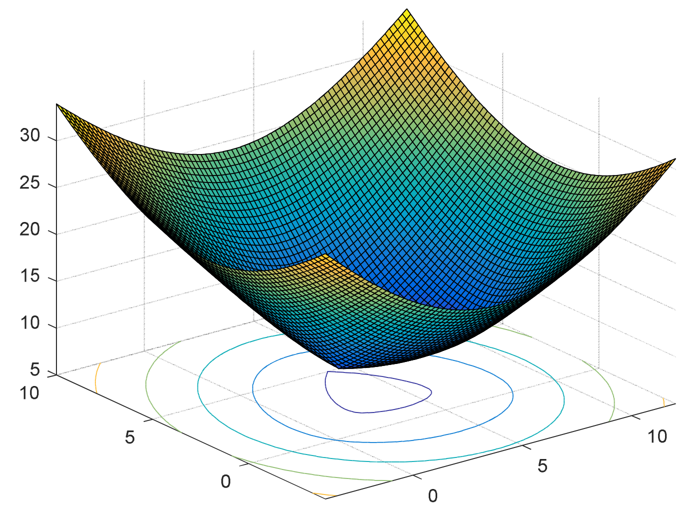
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

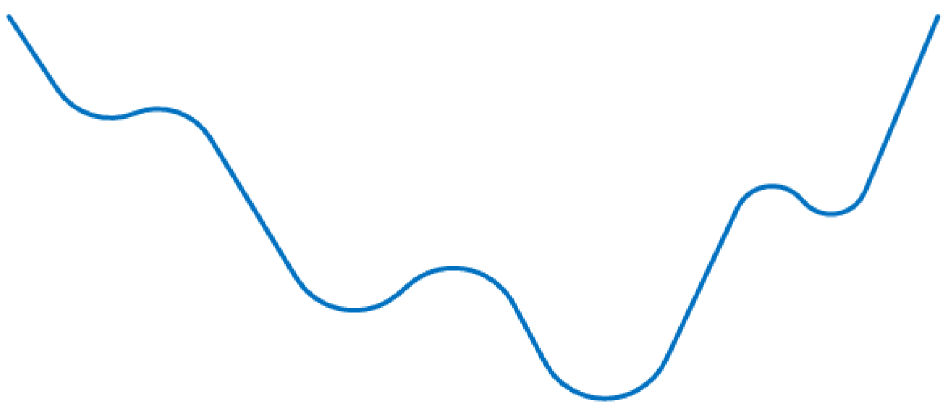
$$TC^* = TC(\mathbf{x}^*)$$

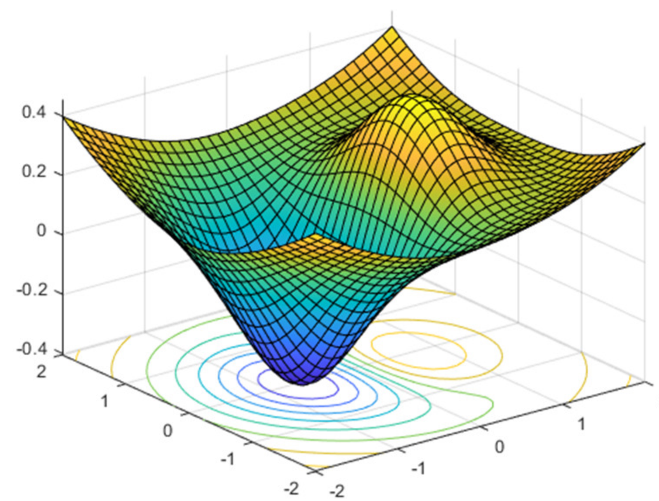


Varignon Frame

Convex vs Nonconvex Optimization

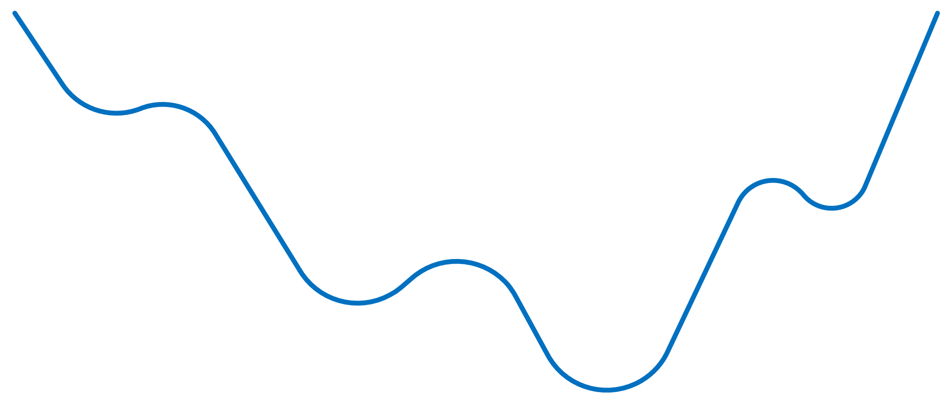




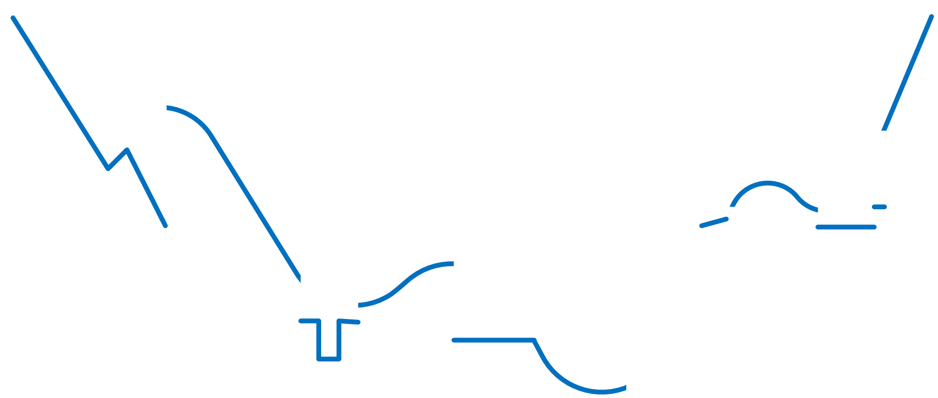


Gradient vs Direct Methods

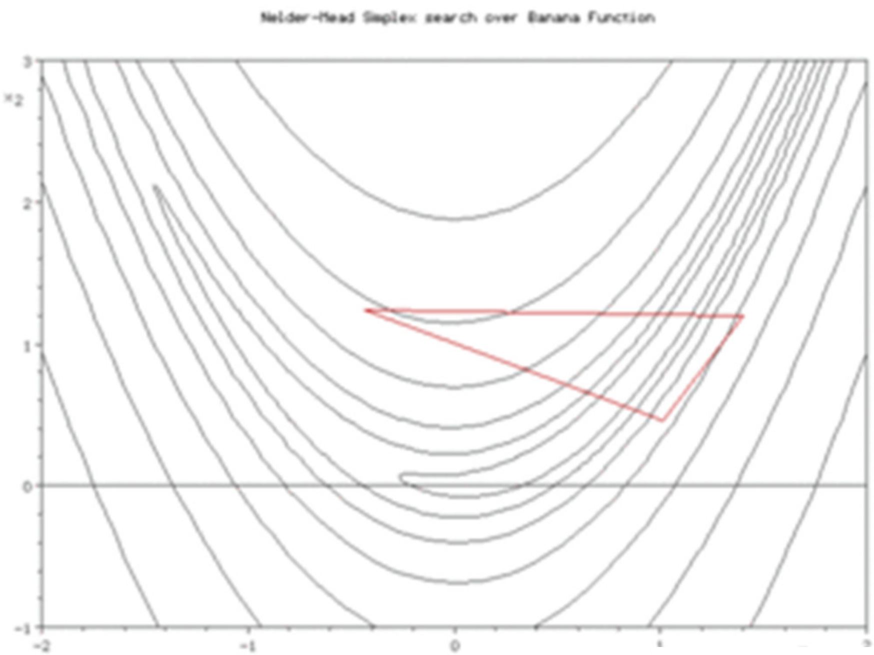
- Numerical nonlinear unconstrained optimization:
  - Analytical/estimated gradient
    - quasi-Newton

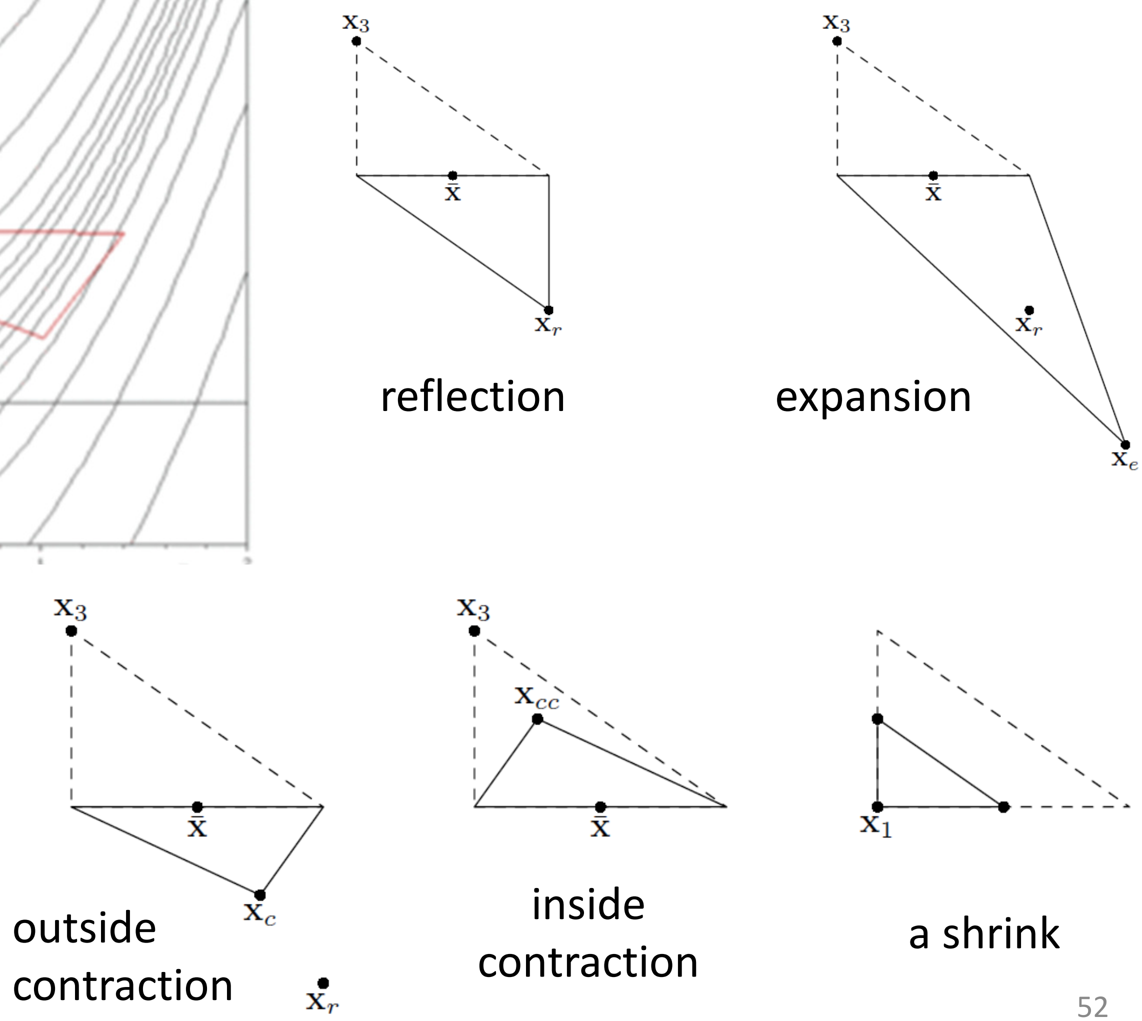


  - Direct, gradient-free
    - Nelder-Mead



Nelder-Mead Simplex Method

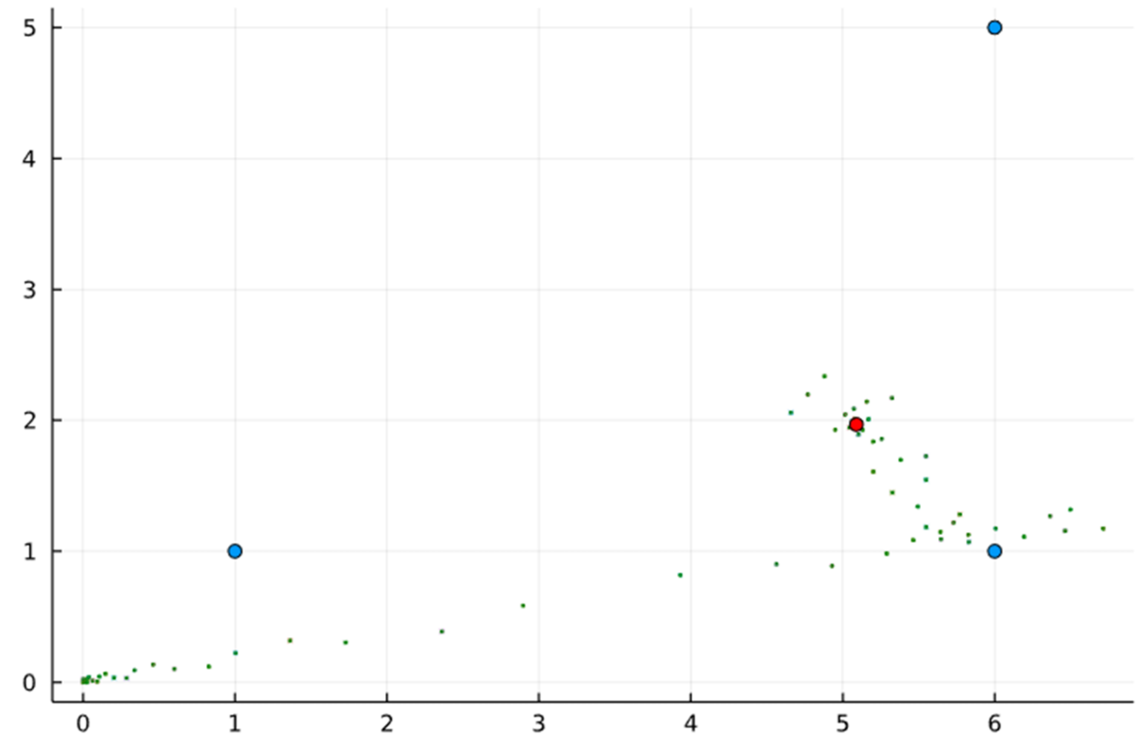




- AKA amoeba method
- Simplex is triangle in 2-D (dashed line in figures)

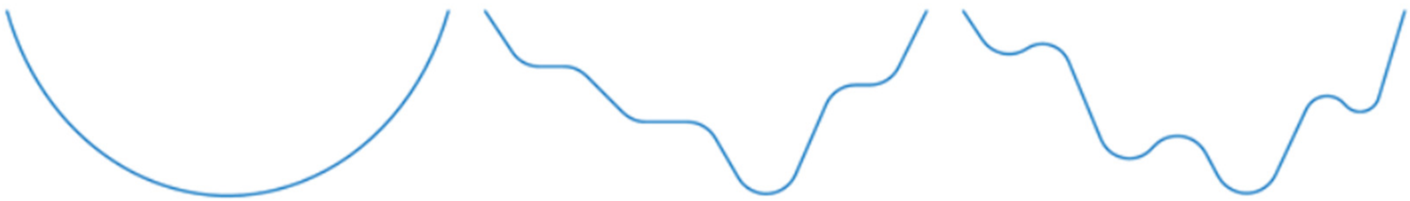
Ex: Nelder-Mead in Action

- Starting from point (0,0), the optimal point close to (5,2) is found by Nelder-Mead that minimizes the sum of the distances to the three points at (1,1), (6,1), and (6,5)
  - Each green point represent an evaluation of the objective function (each evaluation sums the distance from that point the three points)



Picking a Starting Point

- Numerical nonlinear optimization techniques require specifying a starting point (`x0`): `optimize(x -> sum(d2.([x], pt)), x0)`
  - If convex or unimodal, any starting point will lead to the global optimum
  - Otherwise, different starting points can lead to different local optima



Convex Unimodal | Nonconvex Unimodal | Nonconvex Multimodal