Location 3:
Geocoding and Great-Circle Distances

- How can the distances between facility locations be determined?
  - Not computationally feasible to repeatedly calculate the actual road distances between locations.
  - Most continuous locations examined during Nelder-Mead procedure are not connected to a road

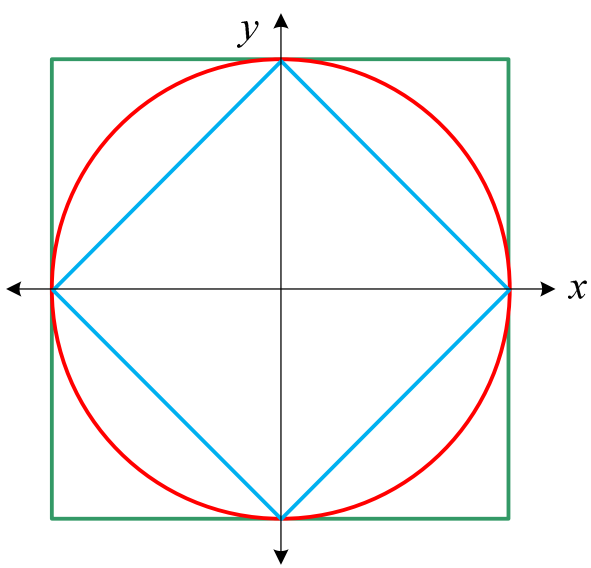
Metric Distances

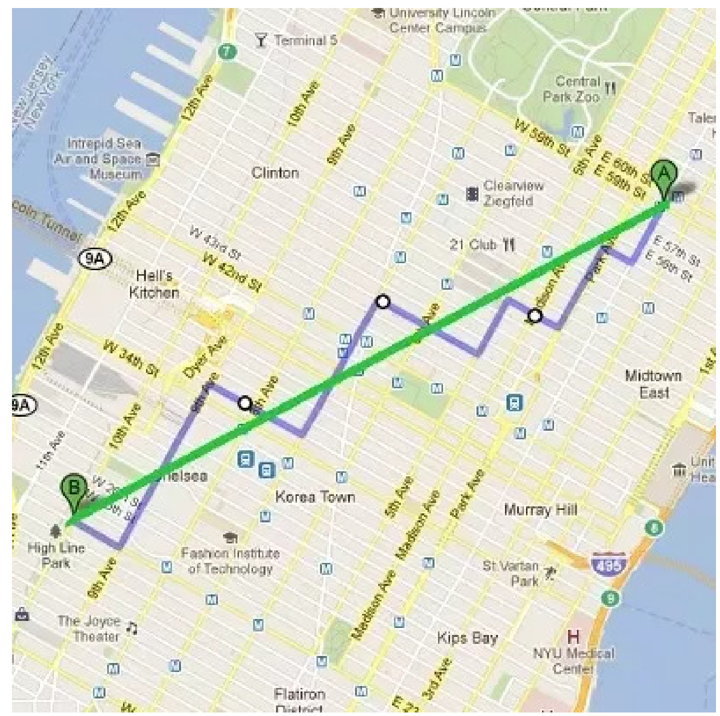
General l_p: $$d_p(P_1,P_2)=\left[\,|x_1-x_2|^p+|y_1-y_2|^p\,\right]^{\frac{1}{p}}, \quad p\geq 1$$

Rectilinear $(p{=}1)$: $$d_1(P_1,P_2)=|x_1-x_2|+|y_1-y_2|$$

Euclidean $(p{=}2)$: $$d_2(P_1,P_2)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Chebychev $(p\to\infty)$: $$d_\infty(P_1,P_2)=\max\left\{|x_1-x_2|,|y_1-y_2|\right\}$$





Chebychev Distances

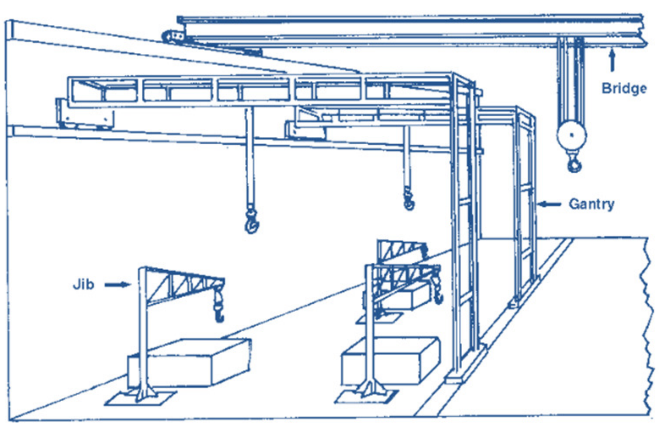
Proof

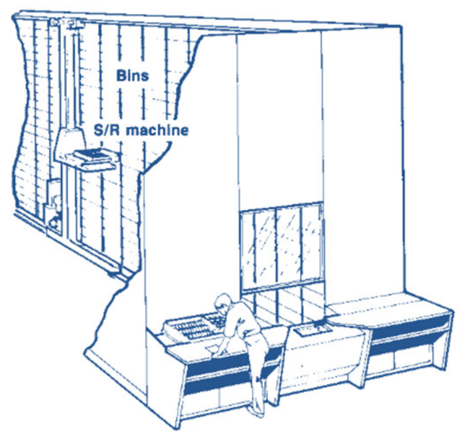
Without loss of generality, let $P_1 = (x, y)$, for $x, y \geq 0$, and $P_2 = (0, 0)$. Then $d_\infty(P_1, P_2) = \max\{x, y\}$ and $d_p(P_1, P_2) = \left[x^p + y^p\right]^{1/p}$.

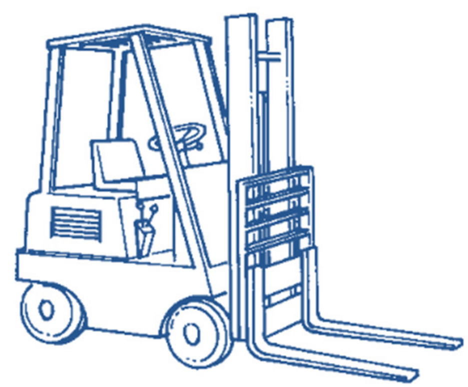
If $x = y$, then $\lim\limits_{p\to\infty}\left[x^p + y^p\right]^{1/p} = \lim\limits_{p\to\infty}\left[2x^p\right]^{1/p} = \lim\limits_{p\to\infty}\left[2^{1/p}x\right] = x$.

If $x < y$, then $\lim\limits_{p\to\infty}\left[x^p + y^p\right]^{1/p} = \lim\limits_{p\to\infty}\left[\left((x/y)^p + 1\right)y^p\right]^{1/p} = \lim\limits_{p\to\infty}\left((x/y)^p + 1\right)^{1/p} y = 1 \cdot y = y$.

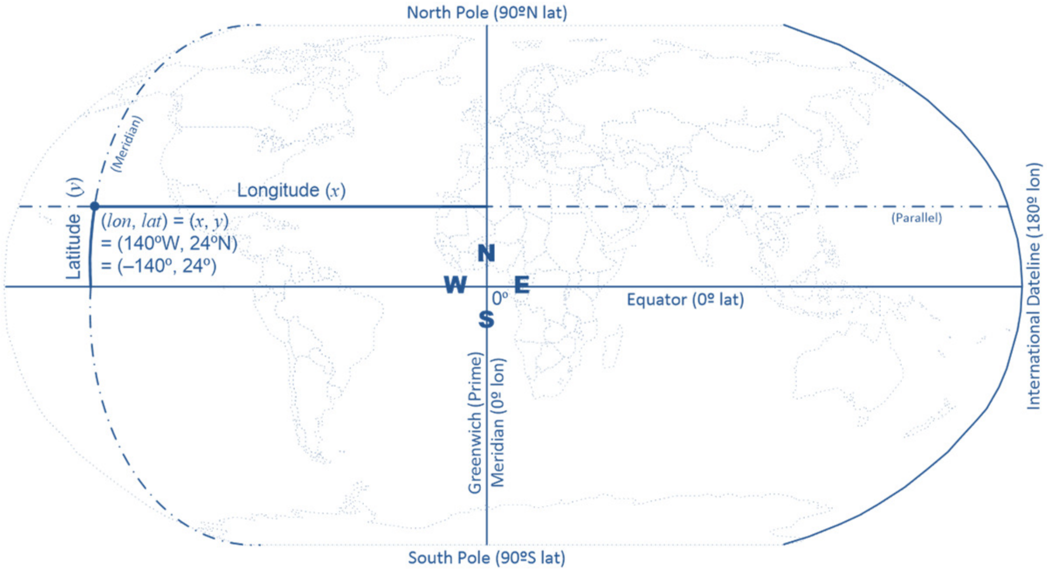
A similar argument can be made if $x > y$. ■

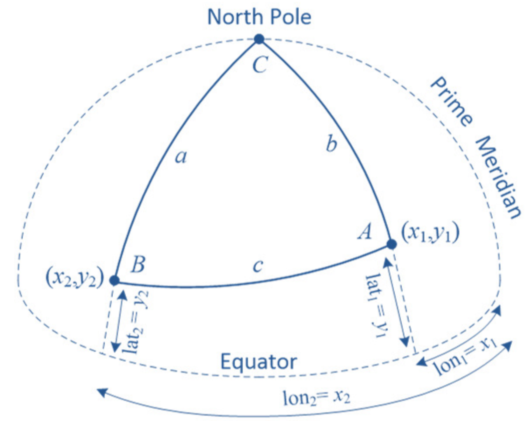


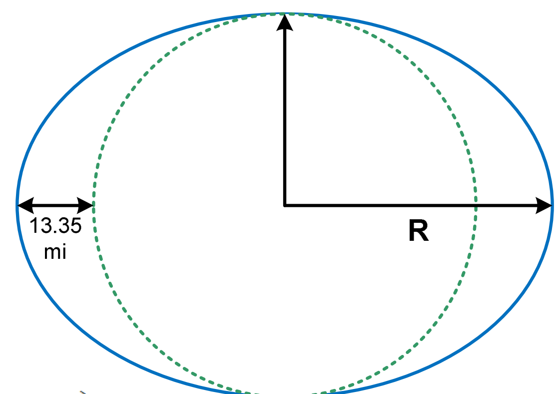




Great Circle Distances







$$(lon_1, lat_1) = (x_1, y_1), \quad (lon_2, lat_2) = (x_2, y_2)$$

$$d_{rad} = (\text{great circle distance in radians of a sphere})$$

$$= \cos^{-1}\left[\sin y_2 \sin y_1 + \cos y_2 \cos y_1 \cos(x_1 - x_2)\right]$$

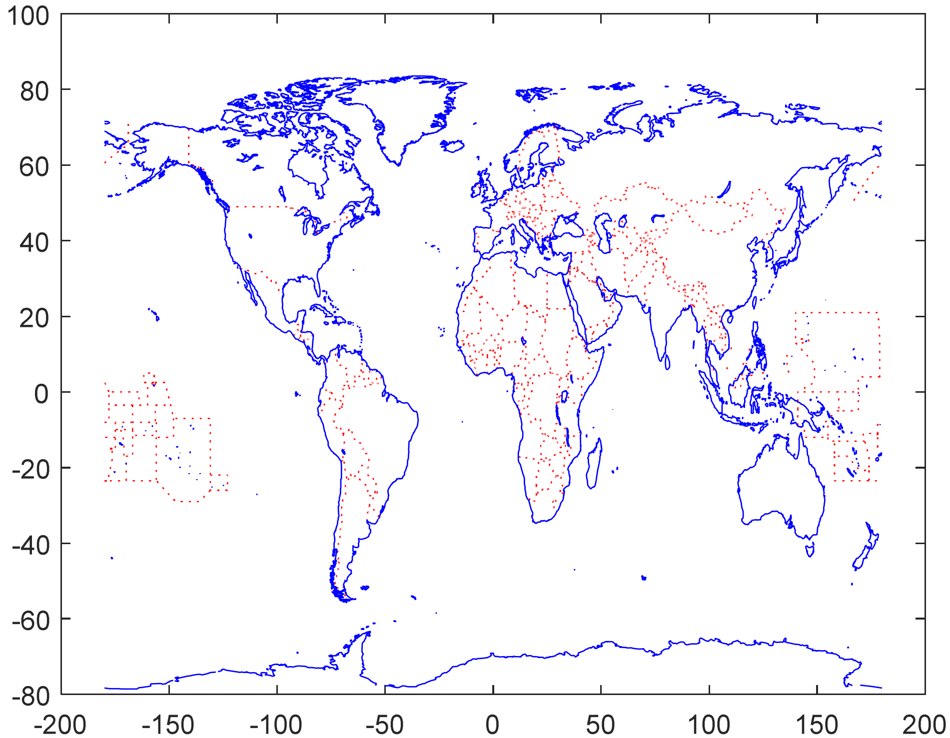
$$R = (\text{radius of earth at equator}) - (\text{bulge from north pole to equator})$$

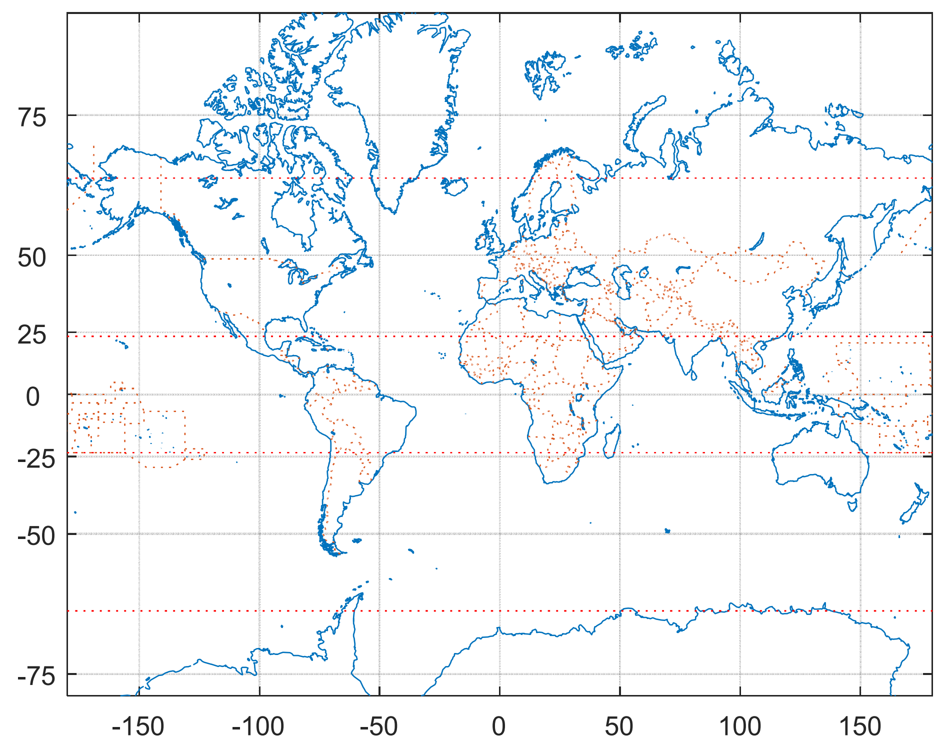
$$= 3{,}963.34 - 13.35 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ mi}, \quad = 6{,}378.388 - 21.476 \sin\left(\frac{y_1 + y_2}{2}\right) \text{ km}$$

$$d_{GC} = \text{distance } (x_1, y_1) \text{ to } (x_2, y_2) = \boxed{d_{rad} \cdot R}$$

$$x_{\deg} = \begin{cases} DD + \dfrac{MM}{60} + \dfrac{SS}{3{,}600}, & \text{if } E \text{ or } N \\ -DD - \dfrac{MM}{60} - \dfrac{SS}{3{,}600}, & \text{if } W \text{ or } S \end{cases}$$

Mercator Projection





$$x_{\text{rad}} = \frac{x_{\text{deg}}}{180}\pi \quad \text{and} \quad x_{\text{deg}} = \frac{x_{\text{rad}} \cdot 180}{\pi}$$

$$x_{\text{proj}} = x$$

$$y_{\text{proj}} = \sinh^{-1}\left(\tan y\right)$$

$$y = \tan^{-1}\left(\sinh y_{\text{proj}}\right)$$