Location 4:
Allocation and ALA

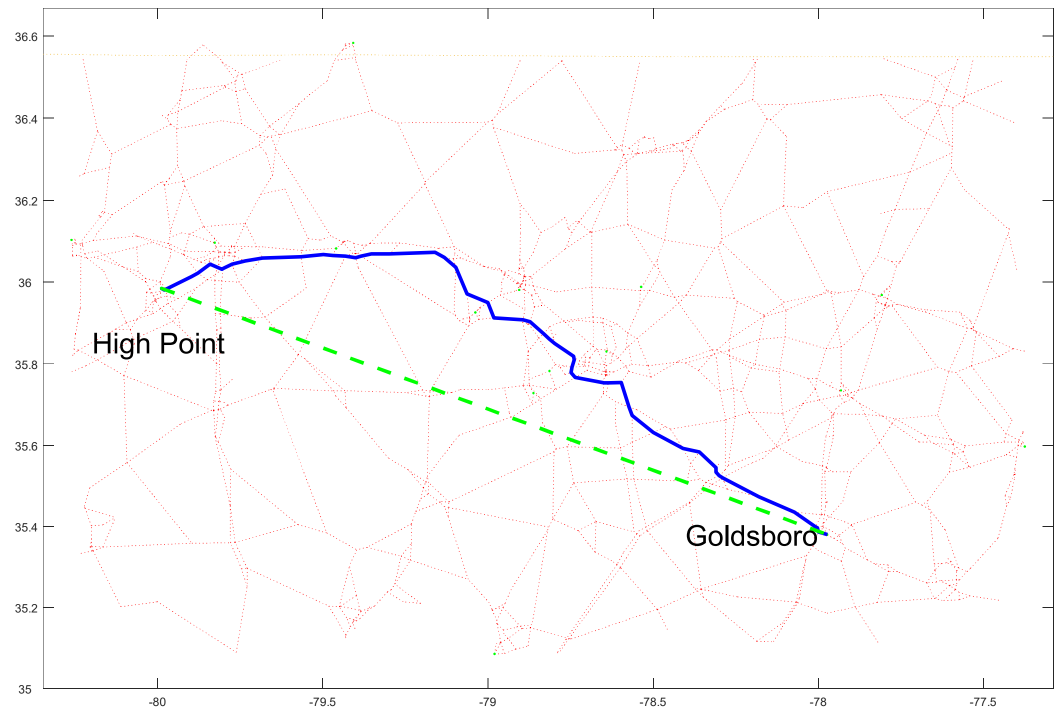
- When determining the location of NFs used for distribution (DCs):
  - Each EF (customer) is usually served by only one DC
  - Allocation of EFs to a DC is based primarily on DC's location
  - Requires solving both an allocation and location problem

Circuity Factor

$$\textit{Circuity Factor}: \quad g = \frac{1}{n}\sum \frac{d_{\text{road}_i}}{d_{GC_i}}, \quad \text{given } n \text{ samples, where usually } 1.1 \le g \le 1.5$$

$$d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2), \quad \text{estimated road distance from } P_1 \text{ to } P_2$$

From High Point to Goldsboro: Road = 143 mi, Great Circle = 121 mi, Circuity = 1.19



Estimating Circuity Factors

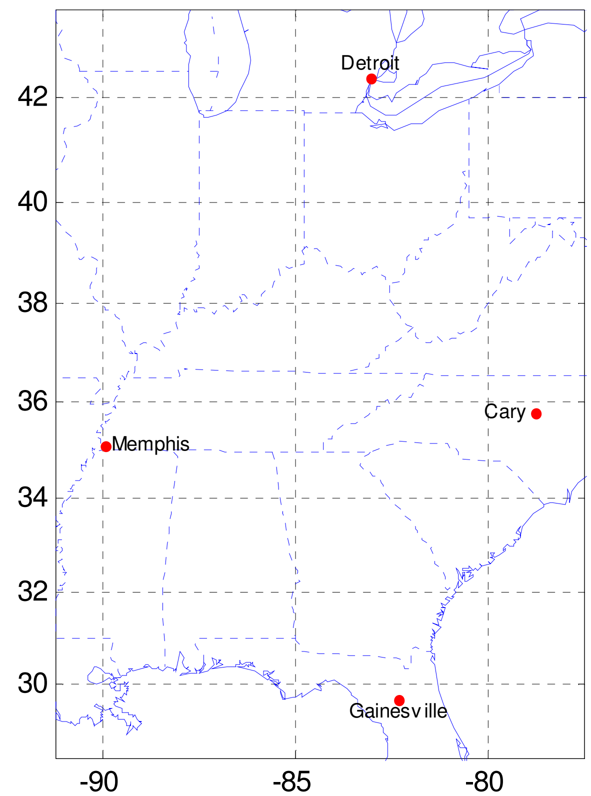
- Circuity factor depends on both the trip density and directness of travel network
  - Good default value for road travel is 1.2
  - Circuity factor of high density areas usually lower because there are more direct roads
  - Should use actual road network, not an estimated circuity factor, if
    - “Few” distances needed (just use Google Maps)
    - Short distances, since there are less direct roads
    - Obstacles (water, mountains) limit direct road travel
  - Circuity factors for rail travel are higher than road travel due to less dense network
  - Note: just 5-10 road sample pairs are needed to provide a reasonable estimate of circuity as long as samples independent (don’t overlap)
    - This is because the overestimates tend to cancel the underestimates

Circuity Factors and Location

- *HW 3:* Find NF location for EFs at Detroit, Gainesville, and Memphis, then determine increase in *TC* if NF instead in Cary
- Since a circuity factor just multiplies distances by the same constant amount, how does it affect the location decision?
  - Does not impact the actual location found
  - Does impact TC since transport rate (r) is in $/ton-mi
    - $\Rightarrow$ can increase the benefit/cost associated with using a good/bad location

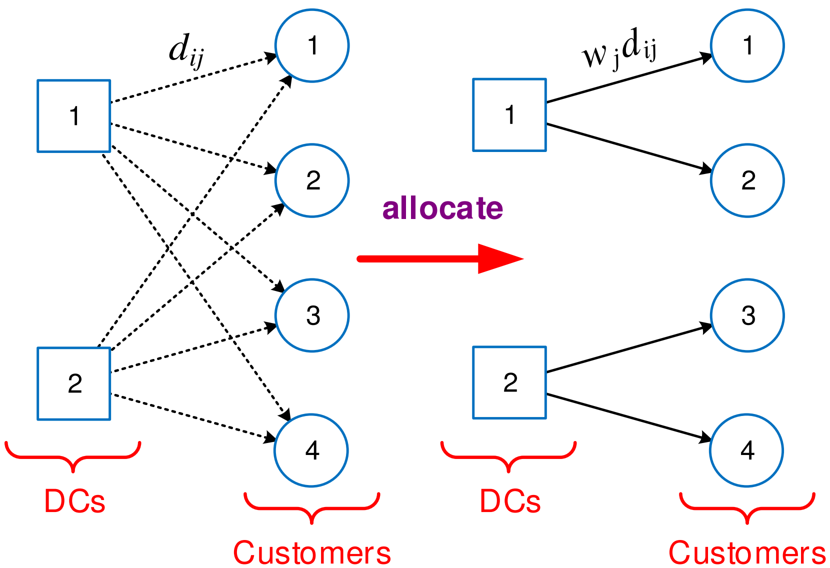
$$TC(X) = \sum_{i=1}^{m} \underbrace{f_i\, r_i}_{w_i}\, d(X, P_i)$$

$$d(X, P_i) = d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2)$$



Allocation

- **Example:** given n DCs and m customers, with customer j receiving w_j TLs per week, determine the total distance per week assuming each customer is served by its *closest* DC



$$w = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$$

$$D = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 45 & 35 & 25 & 15 \end{bmatrix}$$

$$TD = 2(10) + 4(20) + 6(25) + 8(15)$$
$$= 370$$

Pseudocode

- Different ways of representing how allocation and TD can be calculated
  - High-level pseudocode most concise, but leaves out many implementation details (sets don't specify order, initial starting points)
  - Low-level pseudocode includes more implementation details, which can hide/obscure the core idea, and are usually not essential

Low-level Pseudocode

```
TD = 0
for j = 1:m
   dj = D(1,j)
   for i = 2:n
      if D(i,j) < dj
         dj = D(i,j)
      end
   end
   TD = TD + w(j)*dj
end
```

High-level Pseudocode

$$N = \{1,\ldots,n\}, \quad n = |N|$$

$$M = \{1,\ldots,m\}, \quad m = |M|$$

$$\alpha = [\alpha_j] = \arg\min_{i \in N} d_{ij}$$

$$TD = \sum_{j \in M} w_j d_{\alpha_j, j}$$

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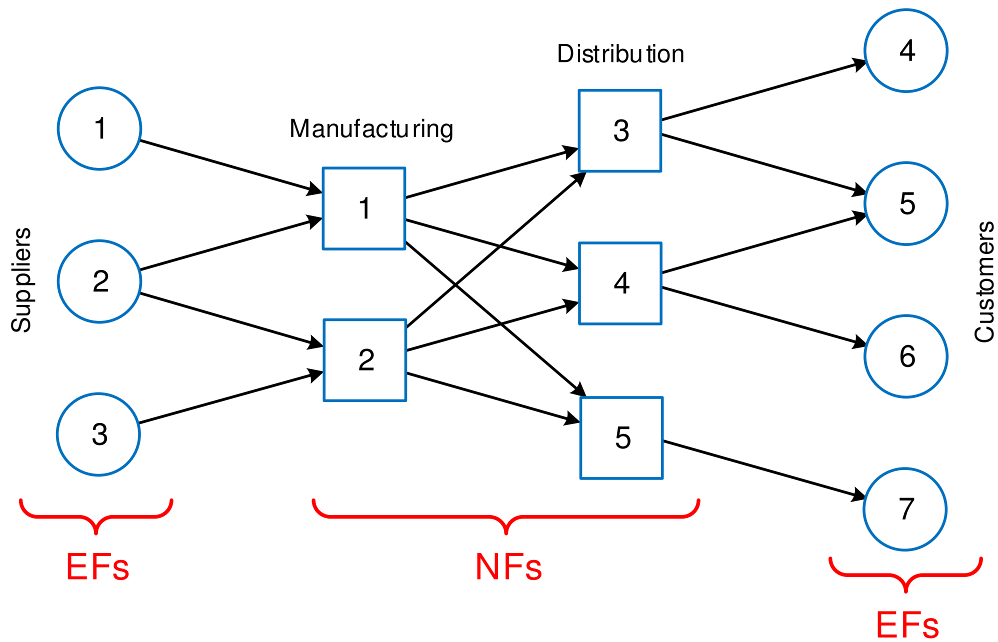
```
α = [argmin(c) for c in eachcol(D)]
W = sparse(α, 1:m, w, n, m)
TD = sum(W .* D)
```

Minisum Multifacility Location



$$n = \text{no. of NFs}, \quad m = \text{no. of EFs}$$

$$\mathbf{X}_{n\times d} = \text{NF locations}, \quad \mathbf{P}_{m\times d} = \text{EF locations}$$

$$\mathbf{V}_{n\times n} = \begin{array}{c|ccccc} \text{NF-NF} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & + & + & + \\ 2 & 0 & 0 & + & + & + \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

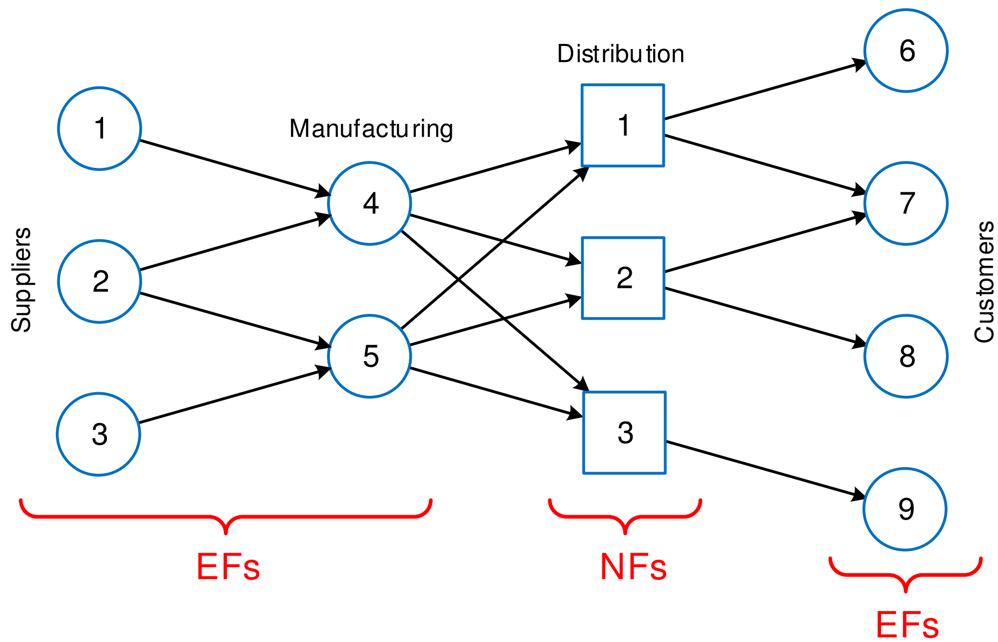
$$\mathbf{W}_{n\times m} = \begin{array}{c|ccccccc} \text{NF-EF} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & + & + & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & + & + & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & + & + & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & + & + & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & + \end{array}$$

$$TC(\mathbf{X}) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(\mathbf{X}_j, \mathbf{X}_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} TC(\mathbf{X})$$

$$TC^* = TC(\mathbf{X}^*)$$

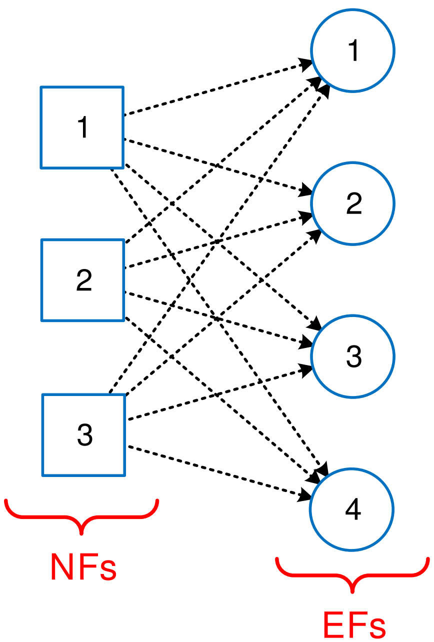
Multiple Single-Facility Location



$$TC(\mathbf{X}) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(\mathbf{X}_j, \mathbf{X}_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$= \sum_{j=1}^{n} TC(\mathbf{X}_j)$$

Facility Location–Allocation Problem



- Determine both the location of *n* NFs and the allocation of flow requirements of *m* EFs that minimize TC

$$w_{ji} = r_{ji} f_{ji} = (1) f_{ji} = \text{flow between NF}j \text{ and EF}i$$

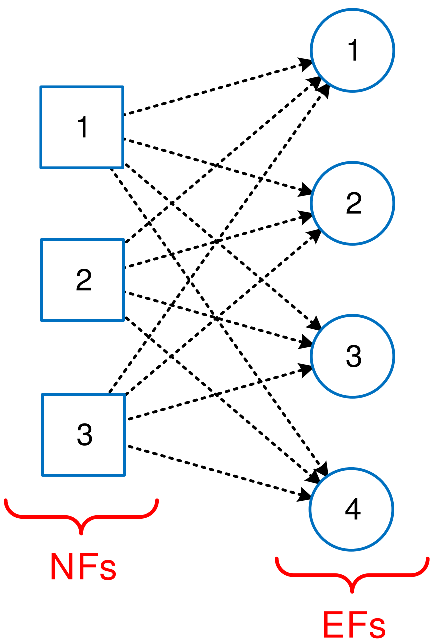
$$w_i = \text{total flow requirememts of EF}i$$

$$TC(\mathbf{X},\mathbf{W}) = \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j,\mathbf{P}_i)$$

$$\mathbf{X}^*,\mathbf{W}^* = \arg\min_{\mathbf{X},\mathbf{W}} \left\{ TC(\mathbf{X},\mathbf{W}) : \sum_{j=1}^{n} w_{ji} = w_i,\ w_{ji} \geq 0 \right\}$$

$$TC^* = TC(\mathbf{X}^*,\mathbf{W}^*)$$

Integrated Formulation



- If there are no capacity constraints on NFs, it is optimal to always satisfy all the flow requirements of an EF from its closest NF
- Requires search of (n x d)-dimensional TC that combines location with allocation

$$\alpha_i(\mathbf{X}) = \arg\min_j d(\mathbf{X}_j, \mathbf{P}_i)$$

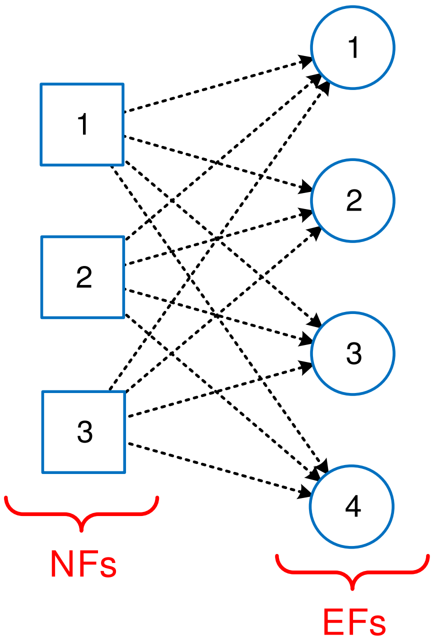
$$TC(\mathbf{X}) = \sum_{i=1}^{m} w_i d(\mathbf{X}_{\alpha_i(\mathbf{X})}, \mathbf{P}_i)$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} TC(\mathbf{X})$$

$$TC^* = TC(\mathbf{X}^*)$$

```
function TCint(X)
    D = Dgc(X, P)
    α = [argmin(c) for c in eachcol(D)]
    n, m = size(D)
    return sum(sparse(α, 1:m, w, n, m) .* D)
end
```

Alternating Formulation



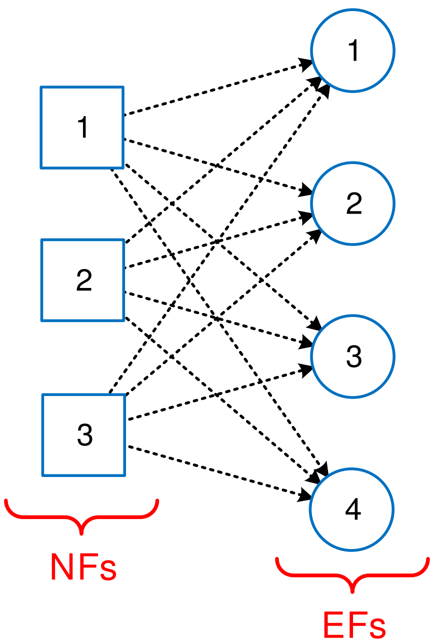
- Alternate between finding locations and finding allocations until no further TC improvement
- Requires *n* *d*-dimensional location searches together with separate allocation procedure
- Separating location from allocation allows other types of location and/or allocation procedures to be used:
  - Allocation with NF with capacity constraints (solved as minimum cost network flow problem)
  - Location with some NFs at fixed locations

$$allocate(\mathbf{X}) = \left[w_{ji} \right] = \begin{cases} w_i, & \text{if } \arg\min_k d(\mathbf{X}_k, \mathbf{P}_i) = j \\ 0, & \text{otherwise} \end{cases}$$

$$TC(\mathbf{X}, \mathbf{W}) = \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$locate(\mathbf{W}, \mathbf{X}) = \arg\min_{\mathbf{X}} TC(\mathbf{X}, \mathbf{W})$$

ALA: Alternate Location–Allocation



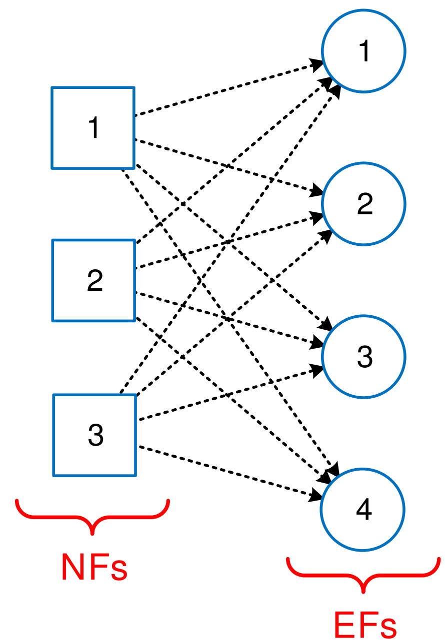
procedure $ala(\mathbf{X})$
$TC \leftarrow \infty$, $done \leftarrow$ false
repeat
  $\mathbf{W}' \leftarrow allocate(\mathbf{X})$
  $\mathbf{X}' \leftarrow locate(\mathbf{W}', \mathbf{X})$
  $TC' \leftarrow TC(\mathbf{X}', \mathbf{W}')$
  if $TC' < TC$
    $TC \leftarrow TC'$, $\mathbf{X} \leftarrow \mathbf{X}'$, $\mathbf{W} \leftarrow \mathbf{W}'$
  else
    $done \leftarrow$ true
  endif
until $done =$ true
return $\mathbf{X}, \mathbf{W}$

```
function ala(X°)
    TC°, done = Inf, false
    while !done
        W = alloc(X°)
        X′ = loc(W, X°)
        TC′ = TC(W, X′)
        println(TC′)
        if TC′ < TC°
            TC°, X° = TC′, X′
        else
            done = true
        end
    end
    return X°, TC°
end
```

- Edge case: What if a NF is not allocated to any EFs?
  - Can happen if initial NF locations are chosen randomly
  - One way to handle: Randomly relocate unallocated NFs to EFs

Integrated vs. Alternate Formulations

- Both only give a local optimal solution (not convex)
- Alternate more flexible and can be faster
  - solving *n* *d*-dimensional location problems and simple allocation
    - $\Rightarrow$ Nelder-Mead works well for 2-D
  - Integrated solving an (*n* x *d*)-dimensional problem
    - can be more computationally difficult
- When might integrated be better?
  - if there are no allocation (e.g., capacity) or location constraints on the NFs



Best Retail Warehouse Locations

Number of Locations	Average Transit Time (days)	Warehouse Location		
1	2.20	Bloomington, IN		
2	1.48	Ashland, KY	Palmdale, CA	
3	1.29	Allentown, PA	Palmdale, CA	McKenzie, TN
4	1.20	Edison, NJ Meridian, MS	Palmdale; CA	Chicago, IL
5	1.13	Madison, NJ Dallas, TX	Palmdale, CA Macon, GA	Chicago, IL
6	1.08	Madison, NJ Dallas, TX	Pasadena, CA Macon, GA	Chicago, IL Tacoma, WA
7	1.07	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA	Chicago, IL Tacoma, WA
8	1.05	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA
9	1.04	Madison, NJ Dallas, TX Lakeland. FL	Alhambra, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA Oakland, CA
10	1.04	Newark, NJ Palistine, TX Lakeland, FL Mansfield, OH	Alhambra, CA Gainesville, GA Denver, CO	Rockford, IL Tacoma, WA Oakland. CA