Location 6: Aggregate Demand

- Locating an NF at the center of population of a region does not reduce the travel distance between the NF and the population to zero
  - Need to estimate the average distance from the center to the region
  - Will assume the population is uniformly distributed over a region
  - Useful for retail distribution where location of each EF is not known

Bottom-Up vs Top-Down Analysis

- Bottom-Up: HW3 Q2

$$\boxed{r \begin{array}{l} \nearrow TC_{\text{NEW}_1} \\ \searrow TC_{\text{NEW}_2} \end{array}}$$

$$\mathbf{P}_{3\times 2} = \text{lon-lat of EFs}$$

$$\mathbf{f} = \left[48, 24, 35\right] \quad \text{(TL/yr)}$$

$$r = 2 \quad \text{(\$/TL-mi)}$$

$$g = \frac{1}{3}\left[\frac{d_{RD}(\mathbf{P}_1,\mathbf{P}_2)}{d_{GC}(\mathbf{P}_1,\mathbf{P}_2)} + \frac{d_{RD}(\mathbf{P}_2,\mathbf{P}_3)}{d_{GC}(\mathbf{P}_2,\mathbf{P}_3)} + \frac{d_{RD}(\mathbf{P}_3,\mathbf{P}_1)}{d_{GC}(\mathbf{P}_3,\mathbf{P}_1)}\right]$$

$$TC(\mathbf{x}) = \sum_{i=1}^{3} f_i\, r\, g\, d_{GC}(\mathbf{x},\mathbf{P}_i) \quad \text{(outbound trans. costs)}$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

$$TC^* = TC(\mathbf{x}^*)$$

$$\mathbf{x}^{\text{cary}} = \text{lon-lat of Cary}$$

$$TC^{\text{cary}} = TC(\mathbf{x}^{\text{cary}})$$

$$\Delta TC = TC^{\text{cary}} - TC^*$$

- Top-Down: estimate *r* (circuity factor cancels, so not needed, see HW 4 Q2)

$$\boxed{TC_{\text{OLD}} \rightarrow r_{\text{nom}} \rightarrow TC_{\text{NEW}}}$$

$$TC^{\text{cary}} = \text{current known } TC$$

$$10 \text{ ton/TL} = \text{known tons per truckload}$$

$$\mathbf{f} = \left[480, 240, 350\right] \quad \text{(ton/yr)}$$

$$r_{\text{nom}} = \frac{TC^{\text{cary}}}{\sum_{i=1}^{3} f_i\, d_{GC}(\mathbf{x}^{\text{cary}},\mathbf{P}_i)} \quad \text{(\$/ton-mi)}$$

$$TC(\mathbf{x}) = \sum_{i=1}^{3} f_i\, r_{\text{nom}} d_{GC}(\mathbf{x},\mathbf{P}_i)$$

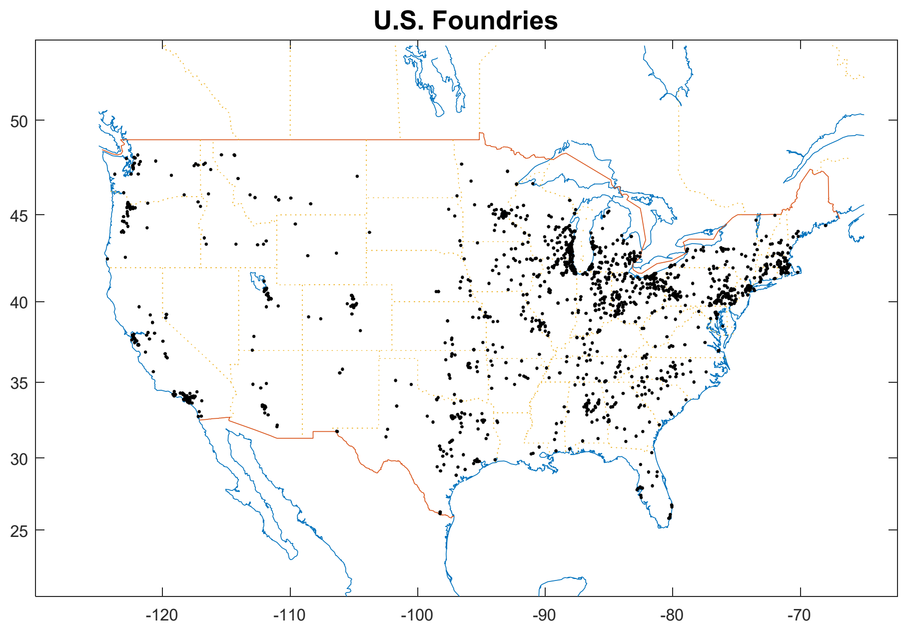
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

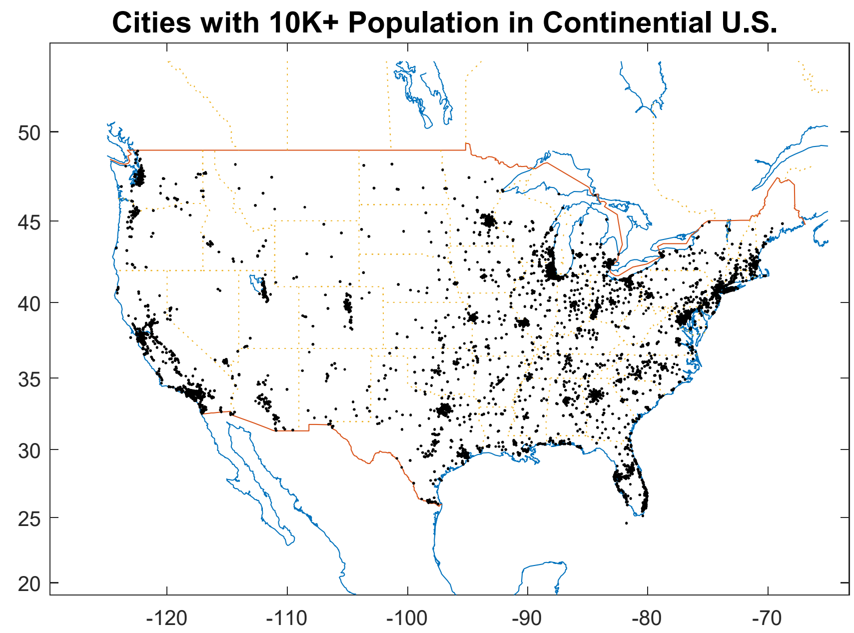
$$TC^* = TC(\mathbf{x}^*)$$

$$\Delta TC = TC^{\text{cary}} - TC^*$$

Actual vs Population-Inferred Demand

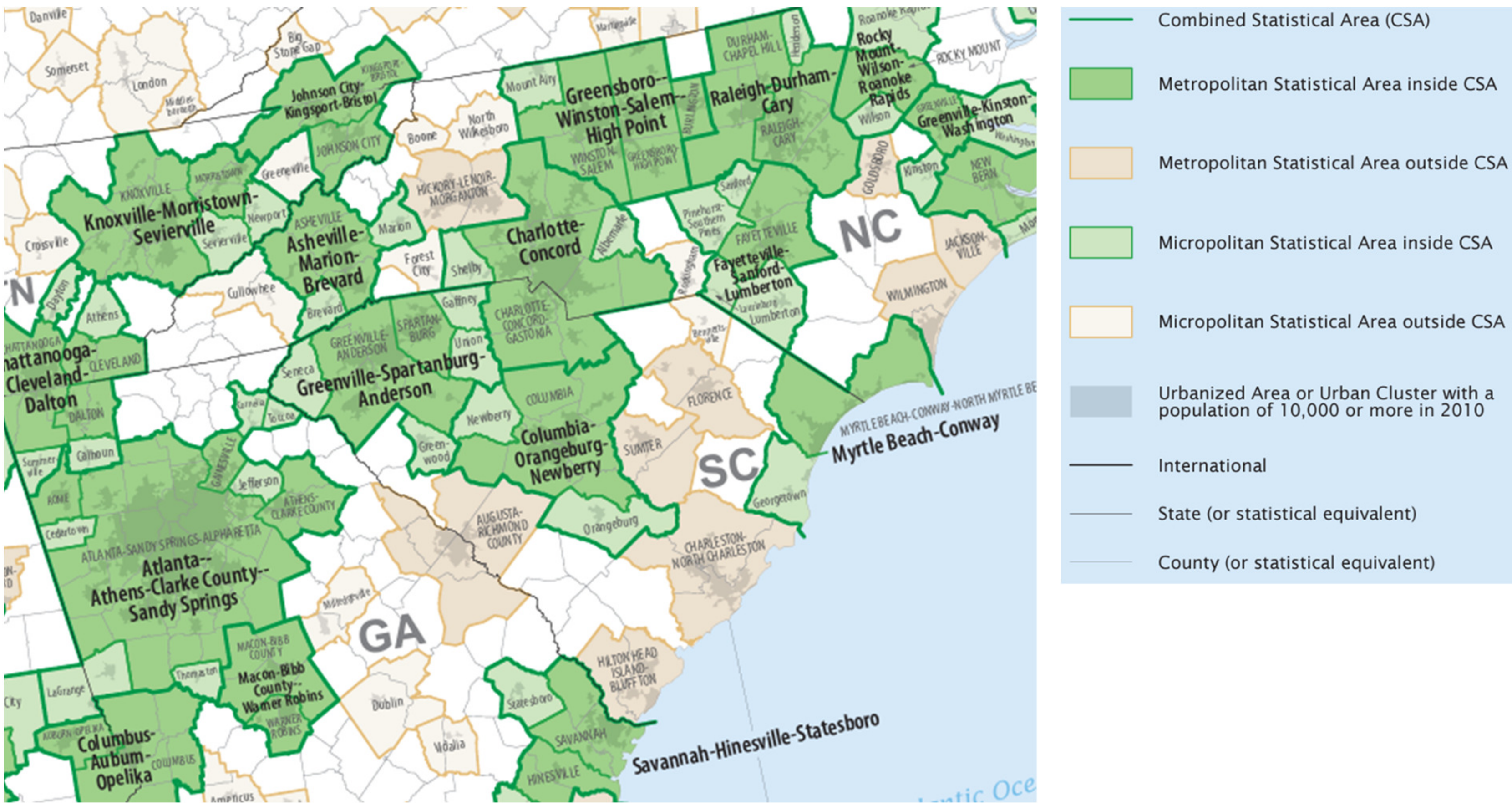
- Actual demand:
  - Know location of each customer along with their demand
    - Example: U.S. foundries, concentrated in Great Lakes
- Population-inferred demand:
  - Assume demand proportional to geographical dispersion of population
    - Example: Any logistics network for retail





U.S. Geographic Statistical Areas

- Defined by Office of Management and Budget (OMB)
  - Each consists of one or more counties



Aggregate Demand Point Data Sources

- Aggregate demand point: centroid of population + area + population
- Good rule of thumb: use at least 10x number of NFs (≈ 100 pts provides minimum coverage for locating ≈ 10 NFs)

1. City data: ONLY USE FOR LABELING!, not as aggregate demand points
2. 3-digit ZIP codes: ≈ 1000 pts covering U.S., = 20 pts NC
3. County data: ≈ 3000 pts covering U.S., = 100 pts NC
   - Grouped by state or CBSA
   - CBSA (Core-Based Statistical Area) defined by set of counties (918 in U.S.)
   - CSA (Combined Statistical Area) defined by set of CBSAs (180 in U.S.)
   - FIPS code = 5-digit state-county FIPS code
= 2-digit state code + 3-digit county code
= 37183 = 37 NC FIPS + 183 Wake FIPS
4. 5-digit ZIP codes: > 35K pts U.S., ≈ 1000 pts NC
5. Census Tract: > 84K pts U.S., ≈ 2700 pts NC
6. Census Block Group: > 240K pts U.S., ≈ 1000 pts Raleigh-Durham-Cary, NC CSA
   - Grouped by state, county, CBSA, or CSA
   - Finest resolution aggregate demand data source
     - Each group is composed of several census blocks, but blocks don't have area info

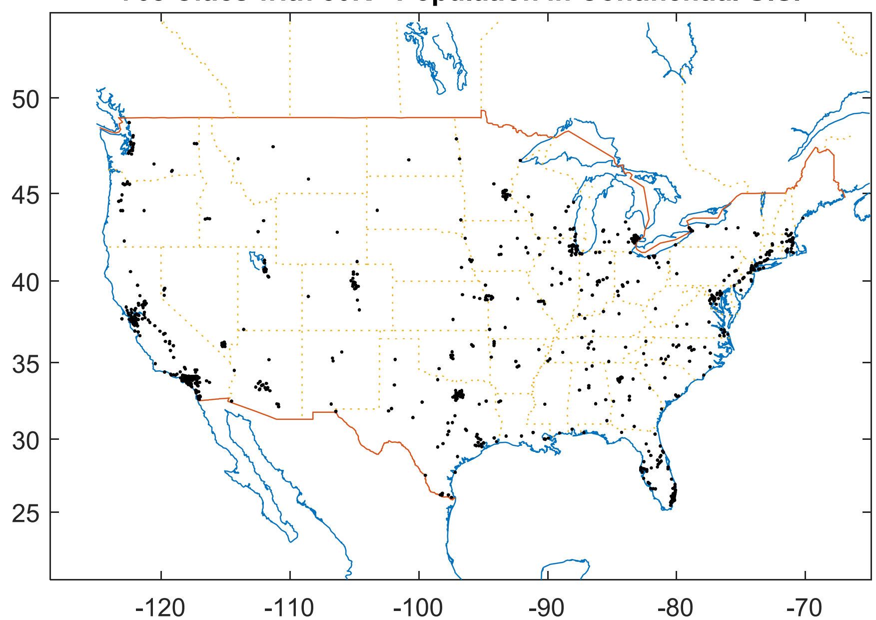
City vs CSA Population Data

Rank	City; State	2010 population	2012 population
1	New York City; New York	8,175,133	**8,336,697**
2	Los Angeles; California	3,792,621	**3,857,799**
3	Chicago; Illinois	2,695,598	**2,714,856**
4	Houston; Texas	2,099,451	**2,160,821**
5	Philadelphia; Pennsylvania	1,526,006	**1,547,607**
6	Phoenix; Arizona	1,445,632	**1,488,750**
7	San Antonio; Texas	1,327,407	**1,382,951**
8	San Diego; California	1,307,402	**1,338,348**
9	Dallas; Texas	1,197,816	**1,241,162**
10	San Jose; California	945,942	**982,765**
11	Austin; Texas	790,390	**842,592**
12	Jacksonville; Florida	821,784	**836,507**
13	Indianapolis; Indiana	820,445	**834,852**
14	San Francisco; California	805,235	**825,863**
15	Columbus; Ohio	787,033	**809,798**
16	Fort Worth; Texas	741,206	**777,992**
17	Charlotte; North Carolina	731,424	**775,202**
18	Detroit; Michigan	713,777	**701,475**
19	El Paso; Texas	649,121	**672,538**
20	Memphis; Tennessee	646,889	**655,155**

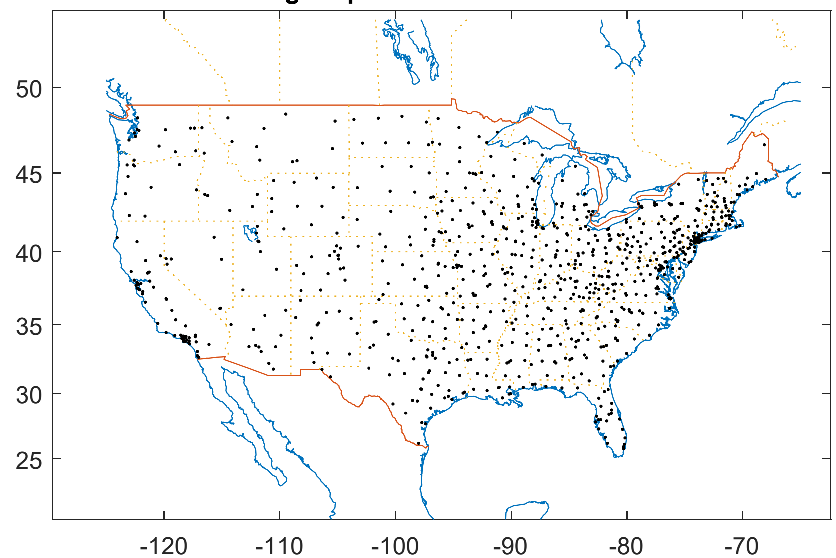
Metropolitan Area	2010 Population	City
New York-Northern NJ-Long Island, NY-NJ-PA	18,897,109	New York
Los Angeles-Long Beach-Santa Ana, CA	12,828,837	Los Angeles
Chicago-Joliet-Naperville, IL-IN-WI	9,461,105	Chicago
Dallas-Fort Worth-Arlington, TX	6,371,773	Dallas
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	5,965,343	Philadelphia
Houston-Sugar Land-Baytown, TX	5,946,800	Houston
Washington-Arlington-Alexandria, DC-VA-MD-WV	5,582,170	Washington
Miami-Fort Lauderdale-Pompano Beach, FL	5,564,635	Miami
Atlanta-Sandy Springs-Marietta, GA	5,268,860	Atlanta
Boston-Cambridge-Quincy, MA-NH	4,552,402	Boston
San Francisco-Oakland-Fremont, CA	4,335,391	San Francisco
Detroit-Warren-Livonia, MI	4,296,250	Detroit
Riverside-San Bernardino-Ontario, CA	4,224,851	Riverside
Phoenix-Mesa-Glendale, AZ	4,192,887	Phoenix
Seattle-Tacoma-Bellevue, WA	3,439,809	Seattle
Minneapolis-St. Paul-Bloomington, MN-WI	3,279,833	Minneapolis
San Diego-Carlsbad-San Marcos, CA	3,095,313	San Diego
St. Louis, MO-IL	2,812,896	St. Louis
Tampa-St. Petersburg-Clearwater, FL	2,783,243	Tampa
Baltimore-Towson, MD	2,710,489	Baltimore

Resolution of Aggregate Data Points

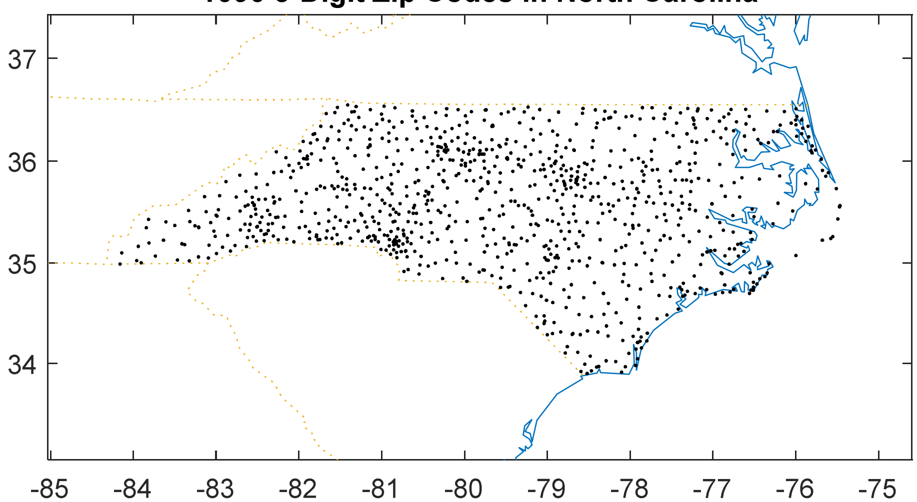
768 Cities with 50K+ Population in Continential U.S.



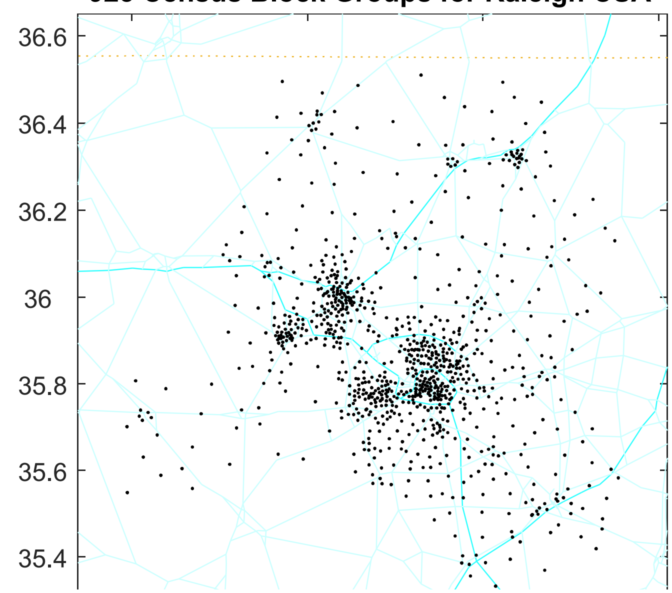
880 3-Digit Zip Codes in Continential U.S.



1090 5-Digit Zip Codes in North Carolina



923 Census Block Groups for Raleigh CSA

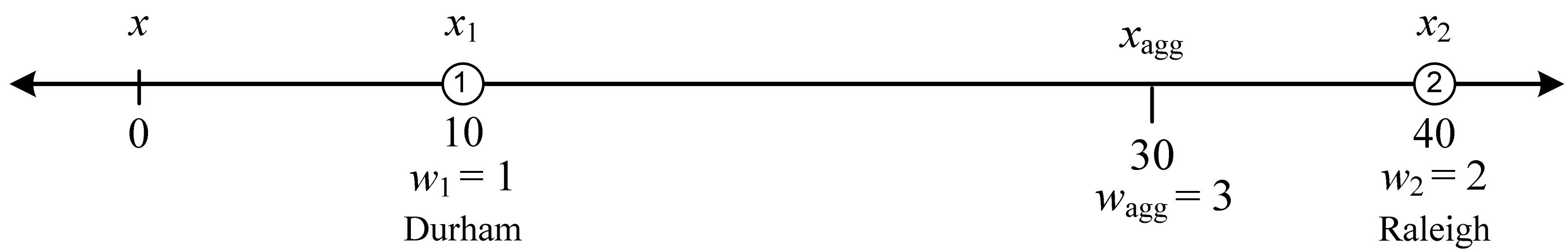


Demand Point Aggregation

- *Existing facility* (EF): actual physical location of demand source
  - Each EF has a well-defined weight w_i and location x_i
- *Aggregate demand point*: single location representing multiple demand sources in a region
  - Need to determine aggregate weight w_{agg} and location x_{agg}
  - Also, need measure a of extent of region, (length, 1-D; area, 2-D), since *assuming demand is uniformly spread over region*

Centroid as Aggregate Location

- Calculation of aggregate location depends on objective



- For minisum location, would like for any location x:

$$\left(w_1 + w_2\right) d(x, x_{agg}) = w_1 d(x, x_1) + w_2 d(x, x_2), \quad \text{let } x = 0,\ x_1, x_2 > 0$$

$$\left(w_1 + w_2\right) x_{agg} = w_1 x_1 + w_2 x_2$$

$$x_{agg} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2} \Rightarrow \quad \text{centroid}$$

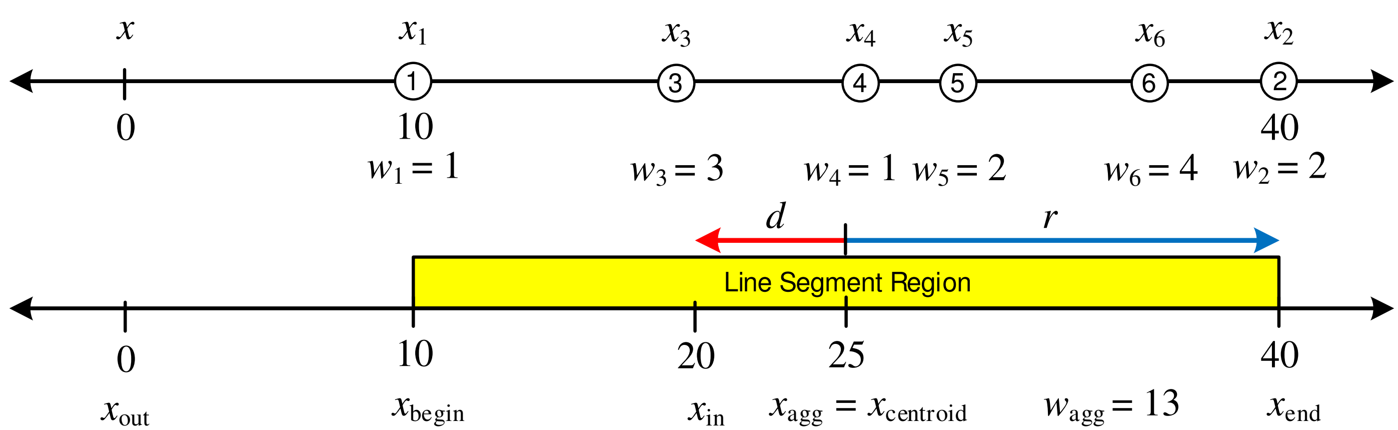
$$\textbf{Note:}\ \text{if } x_1 < x < x_2,\ \text{then } x_{agg}\ \text{not centroid}$$

- For squared distance: $\left(w_1 + w_2\right) x_{agg}^2 = w_1 x_1^2 + w_2 x_2^2$

$$x_{agg} = \sqrt{\frac{w_1 x_1^2 + w_2 x_2^2}{w_1 + w_2}} \Rightarrow \quad \text{not centroid}$$

1-D Average Distance

- Define region enclosing multiple points, with total weight of points spread uniformly across region



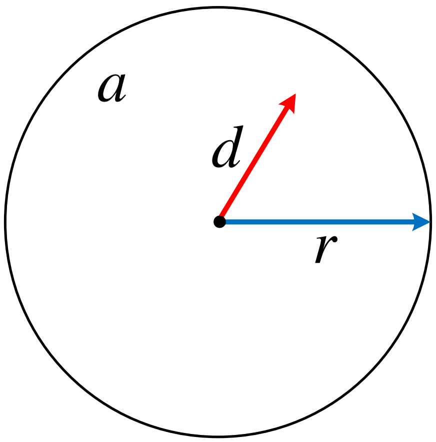
- Calculation of average distance d_a from x to all points in region differs if inside/outside region
  - d_a^0 (average distance if x at centroid) used to approximate d_a
  - **Note:** d_a^0 and $d_a > 0$ even when $d = 0$

$$d = \left| x - x_{\text{centroid}} \right|, \quad r = \frac{\left| x_{\text{end}} - x_{\text{begin}} \right|}{2}$$

$$d_a = \begin{cases} \dfrac{r}{2} + \dfrac{d^2}{2r}, & \text{if } d < r \\ d, & \text{otherwise} \end{cases}$$

$$d_a^0 = \max\left\{ d, \frac{r}{2} \right\} \approx d_a$$

2-D Average Distance



$$a = \pi r^2 \Rightarrow r = \sqrt{\frac{a}{\pi}}$$

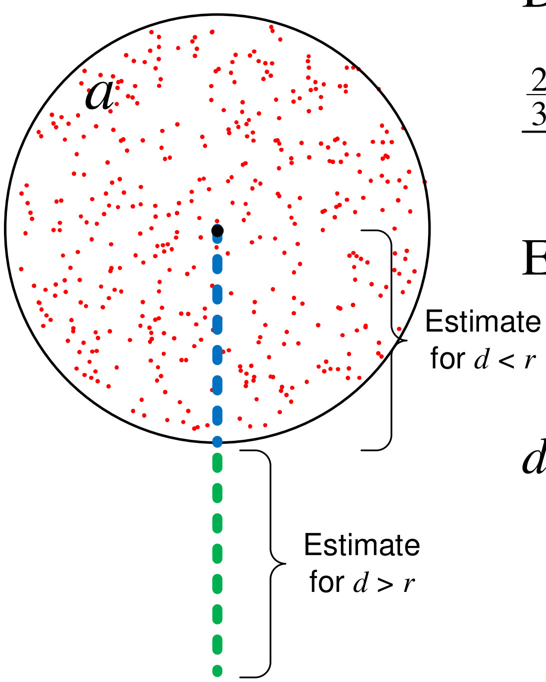
Total distance centroid to all points (x, y) in a:

$$\iint_a \sqrt{x^2 + y^2}\, dx dy = \int_0^r \int_0^{2\pi} s \cdot s\, d\theta ds = 2\pi \int_0^r s^2 ds = \frac{2}{3}\pi s^3 \Big|_{s=0}^{r} = \frac{2}{3}\pi r^3$$

Dividing total distance by a gives approx. average distance:

$$\frac{\frac{2}{3}\pi r^3}{a} = \frac{2}{3} r \Rightarrow d_a^0 = \max\left\{d, \frac{2r}{3}\right\} \approx d_a$$

Empirical estimate, where $d_a > d$ even when $d > r$:



$$d_a = \begin{cases} \frac{2r}{3} + \frac{d}{48} + \frac{9d^2}{20r}, & \text{if } d < r \\ d + \frac{3r^2}{23d}, & \text{otherwise} \end{cases}$$