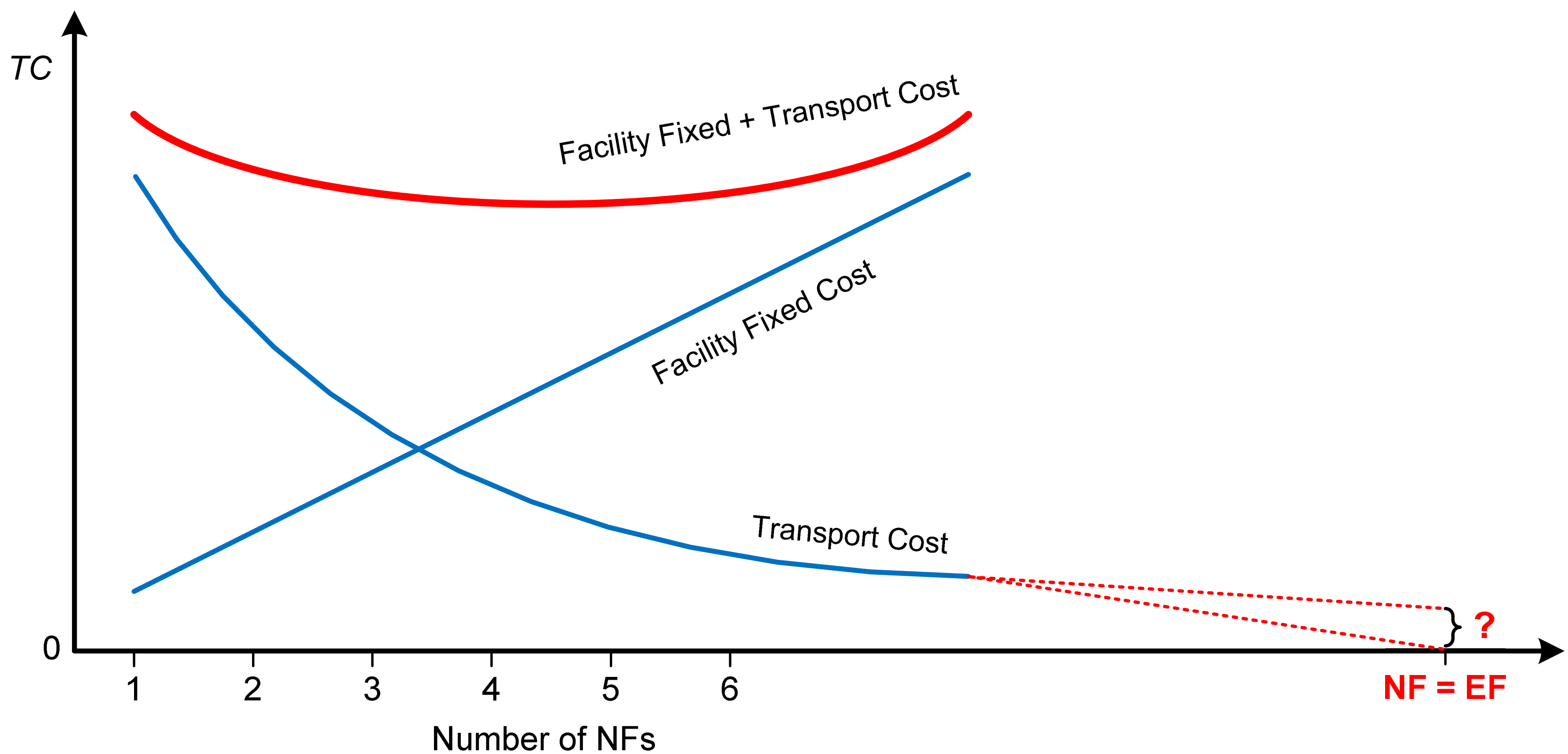
Location 7: Logistics Network Design

- What makes logistics network design hard?
  - Resources are lumpy (both production and transport)
    - ⇒ minimum effective size for each facility
    - ⇒ fixed production cost (cost that does not depend on facility size)
    - ⇒ economies of scale (more of one thing) and scope (many things)
  - *Fixed production* + *variable transport* costs can be used by the UFL model to determine the number and location of NFs
  - Don’t need to identify actual fixed production costs:
    - Will, instead, used intercept term of linear regression fit

Transport Cost if NF at every EF

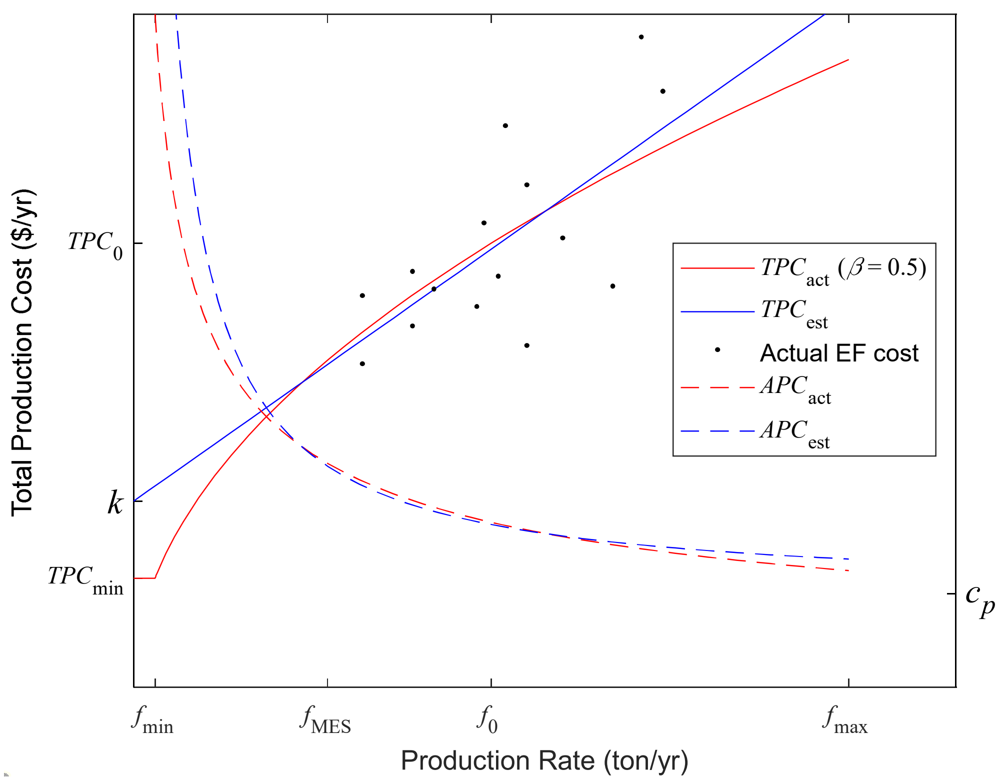


Note: d_a^0 and $d_a > 0$ even when $d = 0$

$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

Fixed Cost and Economies of Scale

- Cost data from existing facilities can be used to fit linear estimate
  - Economies of scale in production $\Rightarrow k > 0$ and $\beta < 1$



$$TPC_{act} = \max_{f < f_{max}} \left\{ TPC_{min}, TPC_0 \left(\frac{f}{f_0} \right)^{\beta} \right\}$$

$$\beta = \begin{cases} 0.62, & \text{Hand tool mfg.} \\ 0.48, & \text{Construction} \\ 0.41, & \text{Chemical processing} \\ 0.23, & \text{Medical centers} \end{cases}$$

$$TPC_{est} = k + c_p f$$

$$APC_{act} = \frac{TPC_{act}}{f} = \frac{TPC_0}{f_0^{\beta}} f^{\beta - 1}$$

$$APC_{est} = \frac{k}{f} + c_p$$

$$k = \text{fixed cost}$$

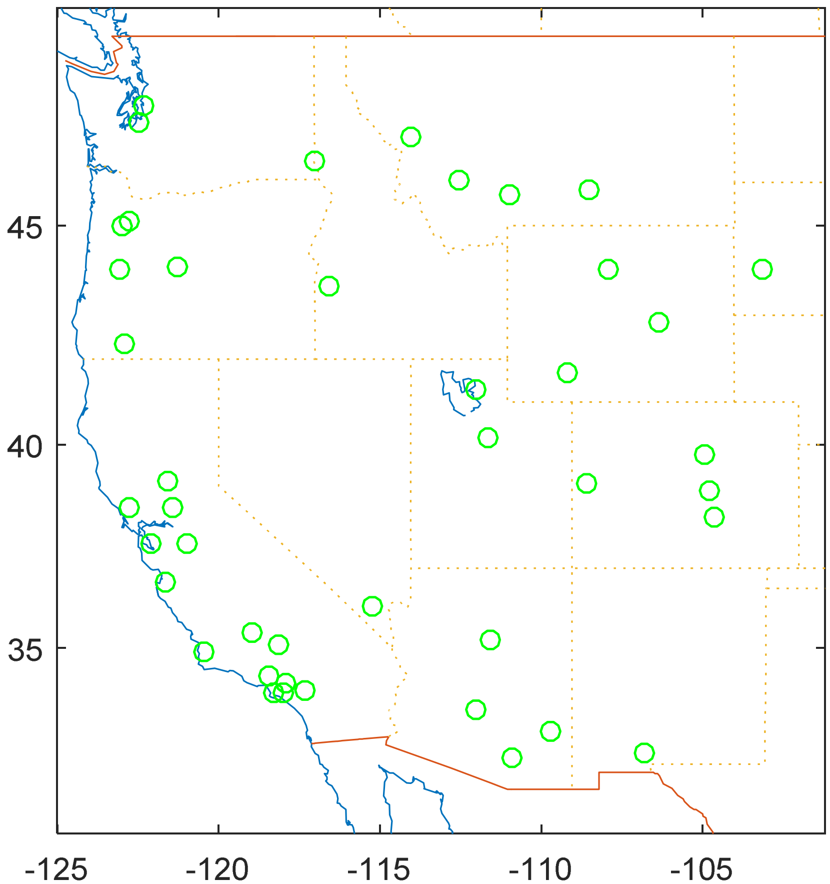
$$c_p = \text{constant unit production cost}$$

$$f_{min}/f_{max} = \text{min/max feasible scale}$$

$$f_{MES} = \textit{Minimum Efficient Scale}$$

$$TPC_0/f_0 = \text{base cost/rate}$$

Ex: Popco Bottling Company



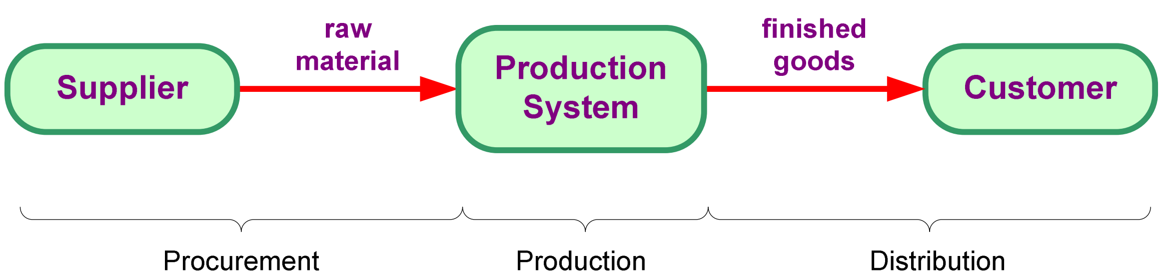
- **Problem:** Popco currently has 42 bottling plants (green circles) across the western U.S. and wants to know if they should consider reducing or adding plants to improve their profitability.
- **Solution:** Formulate as an UFL to determine the number of plants that minimize Popco's production, procurement, and distribution costs.

Ex: Popco Bottling Company

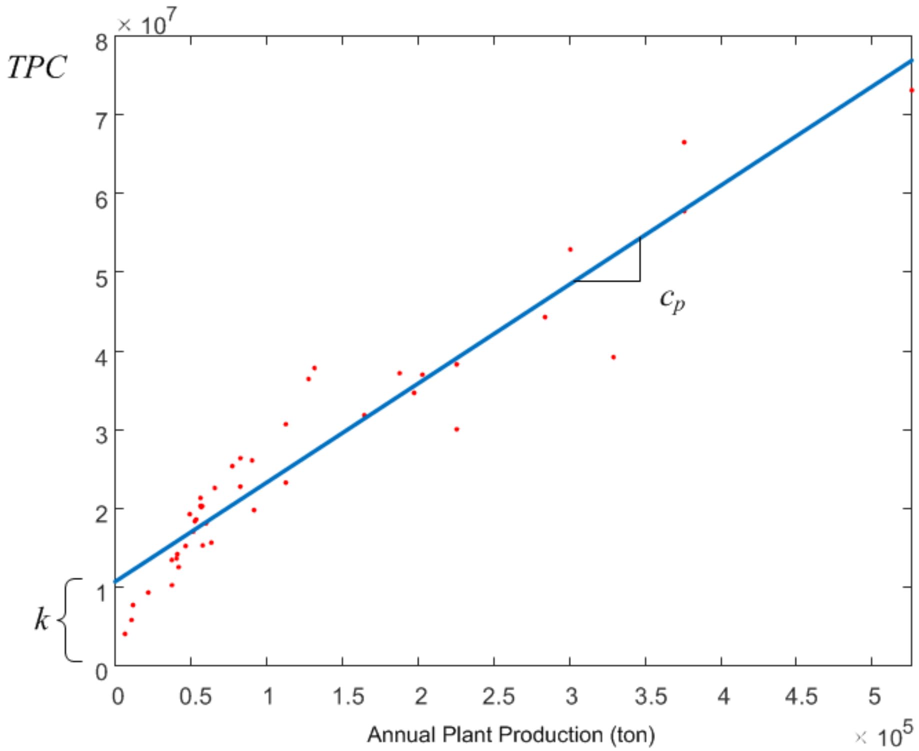
- Following representative information is available for each of *N* current plants (DC) *i*:

$$
\begin{aligned}
xy_i &= \text{location} \\
f_i^{DC} &= \text{aggregate production (tons)} \\
TPC_i &= \text{total production and procurement cost} \\
TDC_i &= \text{total distribution cost}
\end{aligned}
$$

- Assuming plants are (monetarily) weight gaining since they are bottling plants, so UFL can ignore inbound procurement costs related to location



Ex: Popco Bottling Company




```
ŷ(p, x) = p[1] .+ p[2]*x
loss(p, x, y) = sum((y .- ŷ(p, x)).^2)
k, cp = optimize(p -> loss(p, DC.fDC, DC.TPC), [0., 1.]).minimizer
```

```
2-element Vector{Float64}:
   1.0603652532813296e7
 126.15062860391504
```

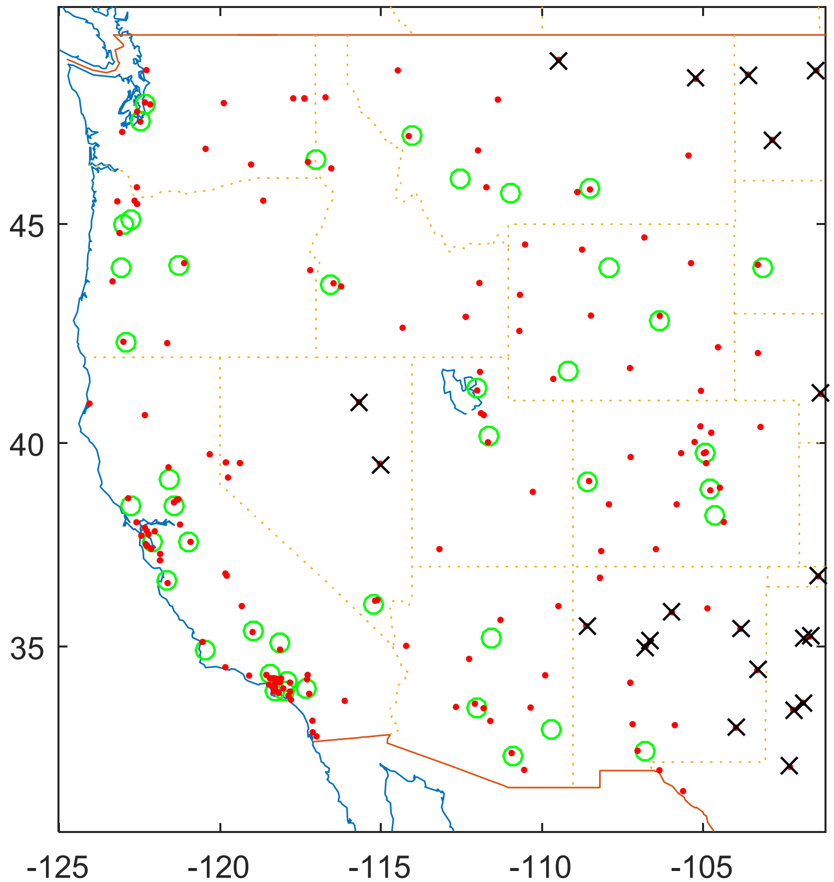
- Difficult to estimate fixed cost of each new facility because this cost must not include any cost related to quantity of product produced at facility.

1. Use plant (DC) production costs to find UFL fixed costs via linear regression
   - variable production costs c_p do not change and can be cut

$$TPC = \sum_{i \in N} TPC_i = \sum_{i \in N} \left(k + c_p f_i^{DC} \right)$$

$$\left(\text{only keep } k \text{ for UFL}\right)$$

Ex: Popco Bottling Company



2. Allocate all 3-digit ZIP codes to closest plant (up to 200 mi max) to serve as aggregate customer demand points.

$$M_i = \left\{ j : \arg\min_h d_{hj} = i \text{ and } d_{ij} \le d_{\max} \right\}$$

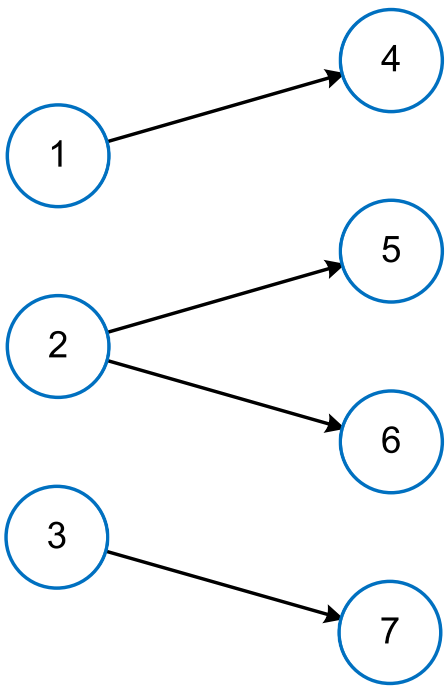
$$d_{\max} = 200 \text{ mi}$$

$$M = \bigcup_{i \in N} M_i$$

```
for r in eachrow(z3)
    d = Dgc([r.LON r.LAT], hcat(DC.LON, DC.LAT)) * 1.2
    idx = argmin(d)[2]
    if d[idx] <= 200
        r.IDX = idx
    end
end
filter!(r -> r.IDX != 0, z3)
```

Ex: Popco Bottling Company

3. Allocate each plant's demand (tons of product) to each of its customers based on its population.



$$f_{j \in M_i} = f_i^{DC} \frac{q_j}{\sum_{h \in M_i} q_h}$$

$$q_j = \text{population of EF}j$$

$$f_5 = f_2^{DC} \frac{q_5}{q_5 + q_6}$$

```
gdf = groupby(z3, :IDX)
```

```
z3 = transform(gdf, :POP => sum => :DCPOP)
```

```
z3 = leftjoin(z3, DC[!, [:IDX, :fDC]], on=:IDX)
```

```
z3.f = map(r -> r.fDC * r.POP / r.DCPOP, eachrow(z3))
```

Ex: Popco Bottling Company

4. Estimate a nominal transport rate (\$/ton-mi) using the ratio of total distribution cost (\$) to the sum of the product of the demand (ton) at each customer and its distance to its plant (mi).

$$r_{\text{nom}} = \frac{\sum_{i \in N} TDC_i}{\sum_{i \in N} \sum_{j \in M_j} f_j \, d_{ij}^a}$$

```
dDC2r(r) = Daa([DC[r.IDX,:LON] DC[r.IDX, :LAT]],[r.LON r.LAT], r.ALAND)[1]
rnom = sum(DC.TDC) / sum(map(r -> r.f * dDC2r(r), eachrow(z3)))
```

```
2.8625376133920297
```

Ex: Popco Bottling Company

5. Calculate UFL variable transportation cost c_{ij} (\$) for each possible NF site *i* (all customer and plant locations) and EF site *j* (all customer locations) as the product of customer *j* demand (ton), distance from site *i* to *j* (mi), and the nominal transport rate (\$/ton-mi).

$$\mathbf{C} = \left[c_{ij} \right]_{\substack{i \in M \cup N \\ j \in M}} = \left[r_{\text{nom}} f_j d_{ij}^a \right]_{\substack{i \in M \cup N \\ j \in M}}$$

```
# Include demand + DC locations for NF sites
NF = hcat(vcat(z3.LON, DC.LON), vcat(z3.LAT, DC.LAT))
D = Daa(NF, hcat(z3.LON, z3.LAT), z3.ALAND)
C = rnom*z3.f'.*D
```

Ex: Popco Bottling Company

6. Solve as UFL, where *TC* returned includes all new distribution costs and the fixed portion of production costs.

$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

$$n = \left| M \cup N \right|, \quad \text{number of potential NF sites}$$

$$m = \left| M \right|, \quad \text{number of EF sites}$$

```
y, TC = ufl(fill(k, size(C, 1)), C)
```

```
  Add: 7.645948563275278e8
 Xchg: 7.35417248086411e8
  Add: 7.35417248086411e8
 Drop: 7.351590430439293e8
 Xchg: 7.34021255286948e8
  Add: 7.34021255286948e8
 Drop: 7.34021255286948e8
```

```
% reduction is TC = 17.96078981797282
```

3×3 DataFrame

Row	_ (String)	Orig (Float64)	New (Float64)
1	No. of NFs	42.0	27.0
2	TDC	4.49367e8	4.47723e8
3	TC	8.9472e8	7.34021e8

Ex: Popco Bottling Company

- **Results:** Original 42 plants (red) reduced to 27 (green circles), *TDC* stayed about the same but *TC* decreased by 18%
- **Note:** The new plants are a mix of
  - Original plants
  - New plants very close to a closed original plant (why?)
  - New plants far from any of the original plants
- **Question:** If an original plant is kept, will it have sufficient capacity (since fewer plants)
- **Question:** If Popco was planning to expand to serve the entire continental U.S., how could the current results be utilized