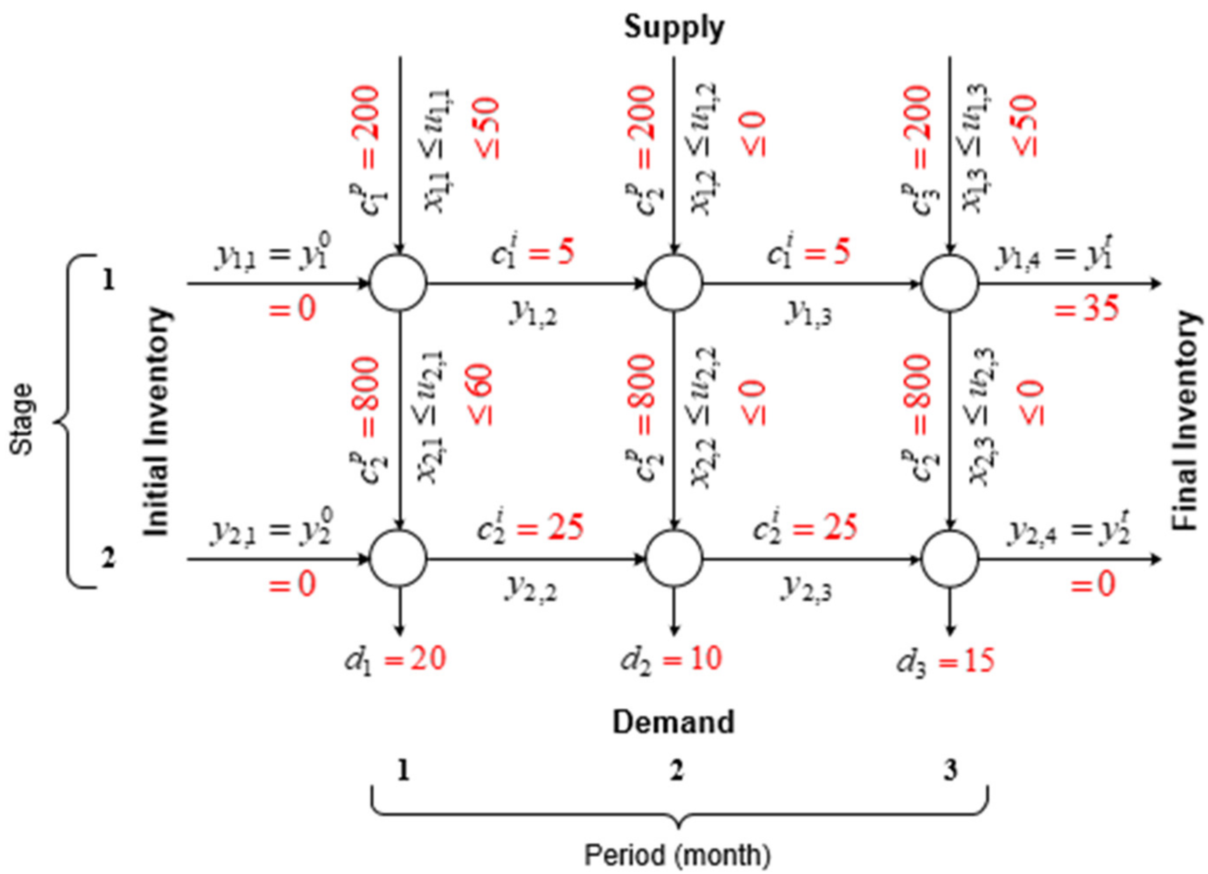
Networks 3: Production-Inventory Planning: Single Product

- Production-inventory planning models are one of the main uses of mathematical programming in industry
  - They provide a means to make complex decisions over a rolling planning horizon
  - Decisions are complex because each has impacts across multiple time periods and multiple stages in the production process
  - Models are resolved each time period, using the latest demand forecasts

Production and Inventory: One Product

- Production-inventory planning models are the main use of math programming in industry. They provide a means to make complex decisions over a rolling planning horizon.



- Inventory cost at each stage based on accumulated prod (mfg and trans) expenditures:

$$h = 0.3 \frac{\$}{\$\text{-yr}} = 0.3$$

$$\frac{h}{t} = \frac{0.3}{12} \frac{\$}{\$\text{-month}} = 0.025$$

$$c_m^i = \frac{h}{t} \sum_{j=1}^{m} c_j^p$$

$$c_1^i = \frac{0.3}{12} 200 = 5$$

$$c_2^i = \frac{0.3}{12}(200 + 800)$$
$$= 25$$

Production and Inventory: One Product

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{t} c_i^p x_{ij} + \sum_{i=1}^{m}\sum_{j=2}^{t+1} c_i^i y_{ij}$$

$$\begin{aligned}
\text{subject to} \quad -x_{ij} + x_{(i+1)j} - y_{ij} + y_{i(j+1)} &= 0, \\
-x_{m,j} - y_{m,j} + y_{m(j+1)} &= d_j,
\end{aligned}$$

Flow balance

Capacity

$$x_{ij} \le u_{ij},$$

Initial/Final inventory

$$\begin{aligned}
y_{i,1} &= y_i^0, \\
y_{i(t+1)} &= y_i^{t+1}
\end{aligned}$$

Use var. LB & UB instead of constraints

$$x, y \ge 0,$$

where,

$$\begin{aligned}
m &= \text{number of production stages} \\
t &= \text{number of periods of production} \\
c_i^p &= \text{production cost (dollar/ton) in stage } i \\
x_{ij} &= \text{production (ton) at stage } i \text{ in period } j \\
c_i^i &= \text{inventory cost (dollar/ton) in stage } i \\
y_{ij} &= \text{stage-}i\text{ inventory (ton) from period } j-1 \text{ to } j \\
d_j &= \text{demand (ton) in period } j \\
u_{ij} &= \text{production capacity (ton) of stage } i \text{ in period } j \\
y_i^0 &= \text{initial inventory (ton) of stage } i \\
y_i^t &= \text{final inventory (ton) of stage } i.
\end{aligned}$$