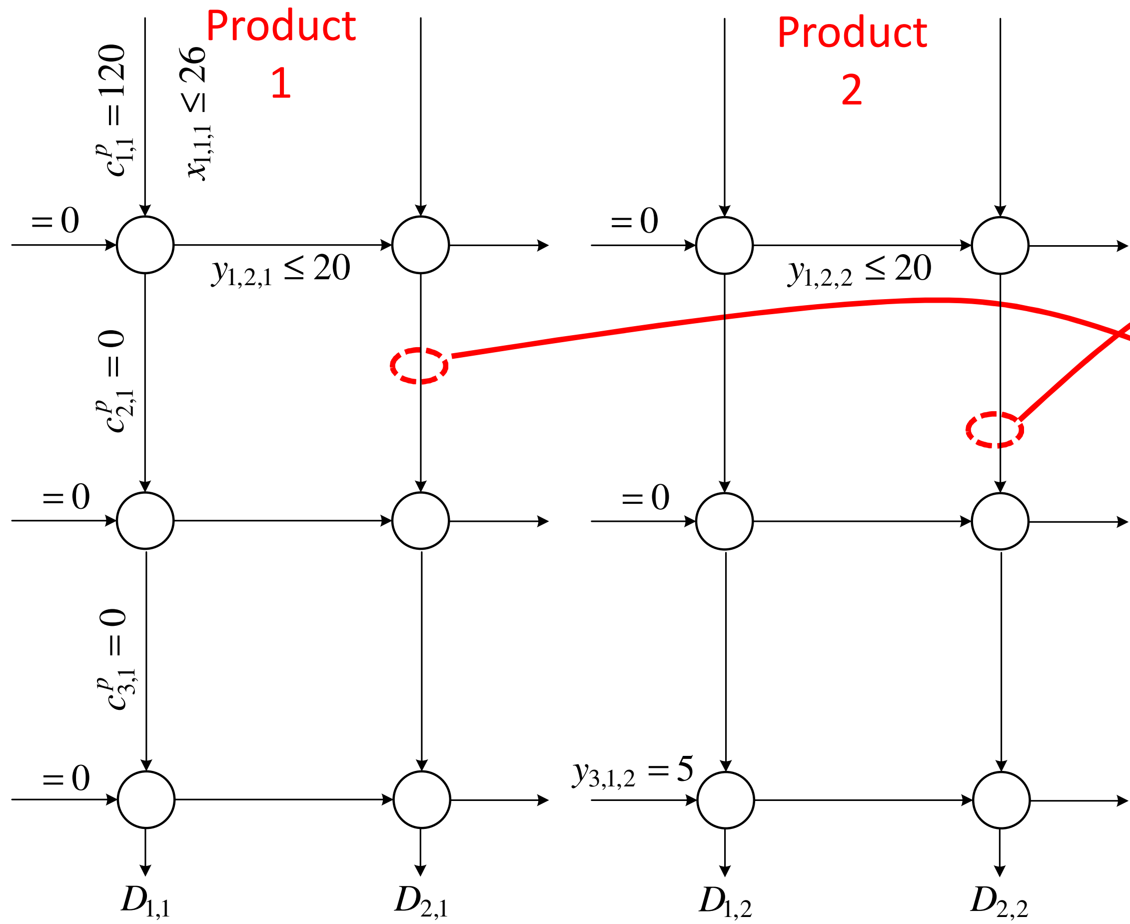
# Networks 4: Production-Inventory Planning: Multiple Products

- Adding constraints to a math program can never improve a solution
- Can ignore situations that would make a solution worse
  - Makes it easier to add constraints to implement a decision
  - Cf. setup constraints

# Coupling Constraints

- Otherwise separate product networks connected via sharing the **combined** use of a resource's capacity




Couples each network

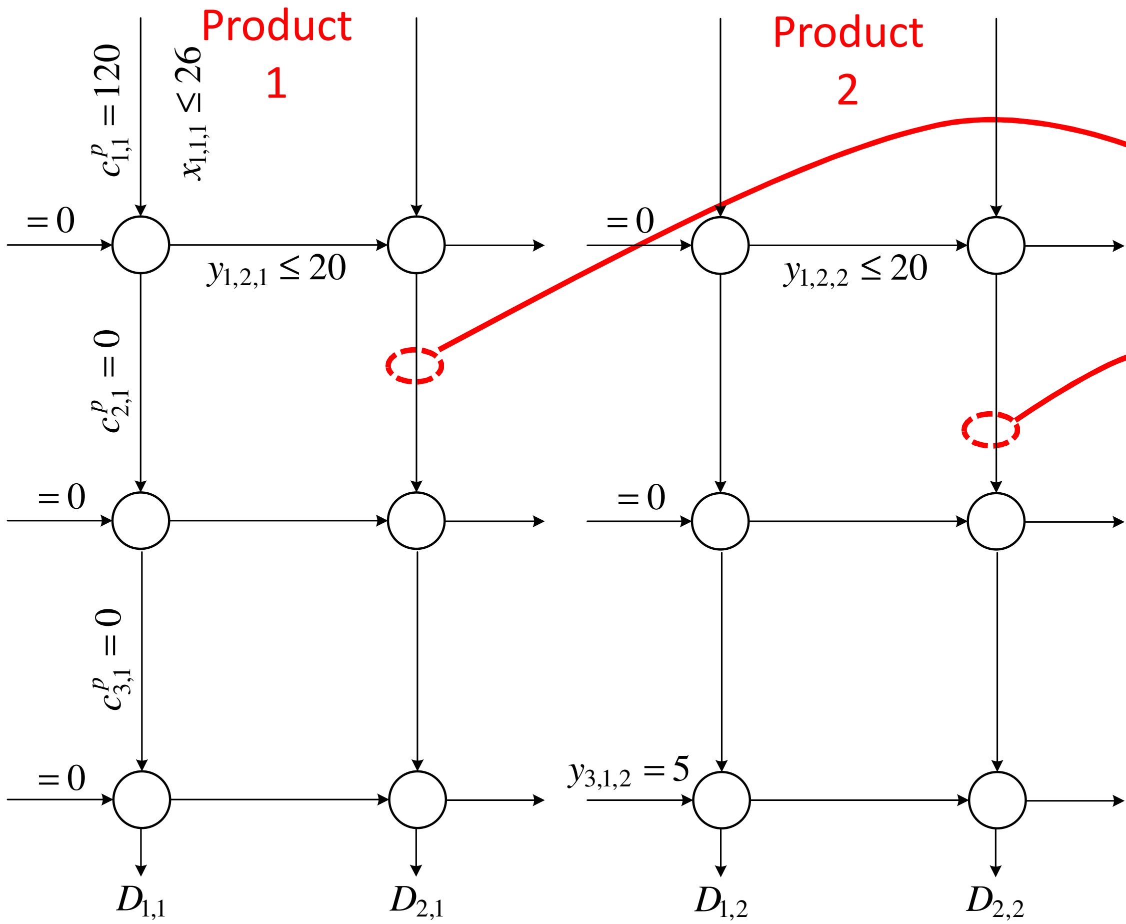
$$x_{2,2,1} + x_{2,2,2} \le u_{2,2}$$

Continuous variables

Shared capacity

# Linking Constraints

- Otherwise separate product networks connected via sharing the **exclusive** use of a resource's capacity




$$x_{2,2,1} \le u_{2,1} z_{2,2,1}$$

$$x_{2,2,2} \le u_{2,2} z_{2,2,2}$$

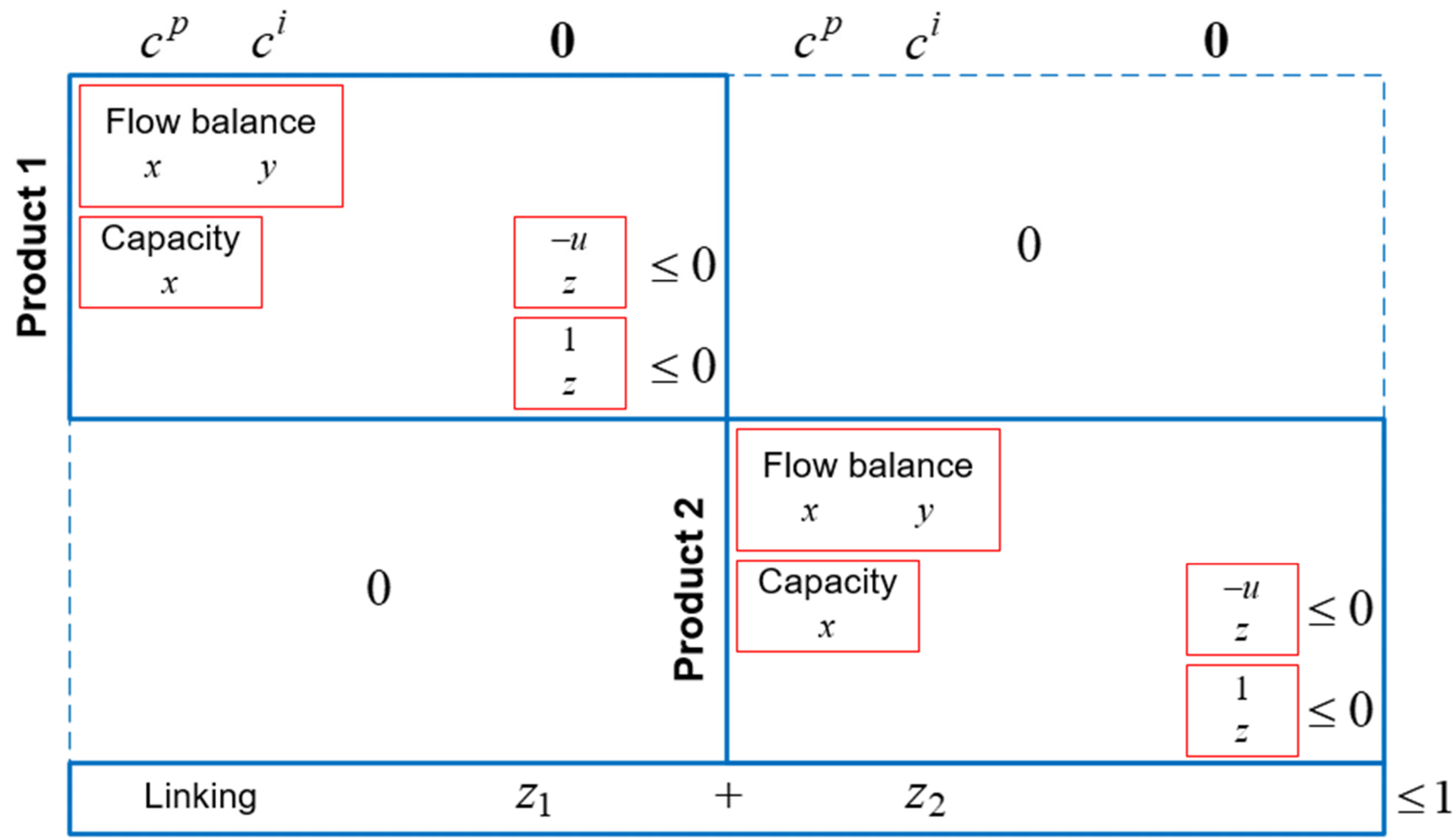
Links each network

$$z_{2,2,1} + z_{2,2,2} \le 1$$

Binary variables

At most one

# Multiple Products with Exclusive Shared Resources



# Multiple Products with Exclusive Shared Resources

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{m}\sum_{j=1}^{t}\sum_{k=1}^{g} c_{ik}^{p} x_{ijk} + \sum_{i=1}^{m}\sum_{j=2}^{t+1}\sum_{k=1}^{g} c_{ik}^{i} y_{ijk} \\
\text{subject to} \quad & -x_{ijk} + x_{(i+1)jk} - y_{ijk} + y_{i(j+1)k} = 0, \qquad i = 1,\ldots,m-1; j = 1,\ldots,t; k = 1,\ldots,g \quad (a) \\
& -x_{mjk} - y_{mjk} + y_{m(j+1)k} = d_{jk}, \qquad j = 1,\ldots,t; k = 1,\ldots,g \quad (b) \\
& x_{ijk} \le u_{ik} z_{ijk}, \qquad i = 1,\ldots,m; j = 1,\ldots,t; k = 1,\ldots,g \quad (c) \\
& \sum_{k=1}^{g} z_{ijk} \le 1, \qquad i = 1,\ldots,m; j = 1,\ldots,t \quad (d) \\
& y_{i1k} = y_{i}^{0}, \qquad i = 1,\ldots,m; k = 1,\ldots,g \\
& y_{i(t+1)k} = y_{ik}^{t+1}, \qquad i = 1,\ldots,m; k = 1,\ldots,g \\
& x, y \ge 0; z \in \{0,1\},
\end{aligned}
$$

$m$ = number of production stages

$t$ = number of periods of production

$g$ = number of products

$c_{ik}^{p}$ = production cost (dollar/ton) in stage $i$ for product $k$

$x_{ijk}$ = production (ton) at stage $i$ in period $j$ for product $k$

$c_{ik}^{i}$ = inventory cost (dollar/ton) in stage $i$ for product $k$

$y_{ijk}$ = stage-$i$ inventory (ton) from period $j-1$ to $j$ for product $k$

$z_{ijk}$ = production indicator at stage $i$ in period $j$ for product $k$

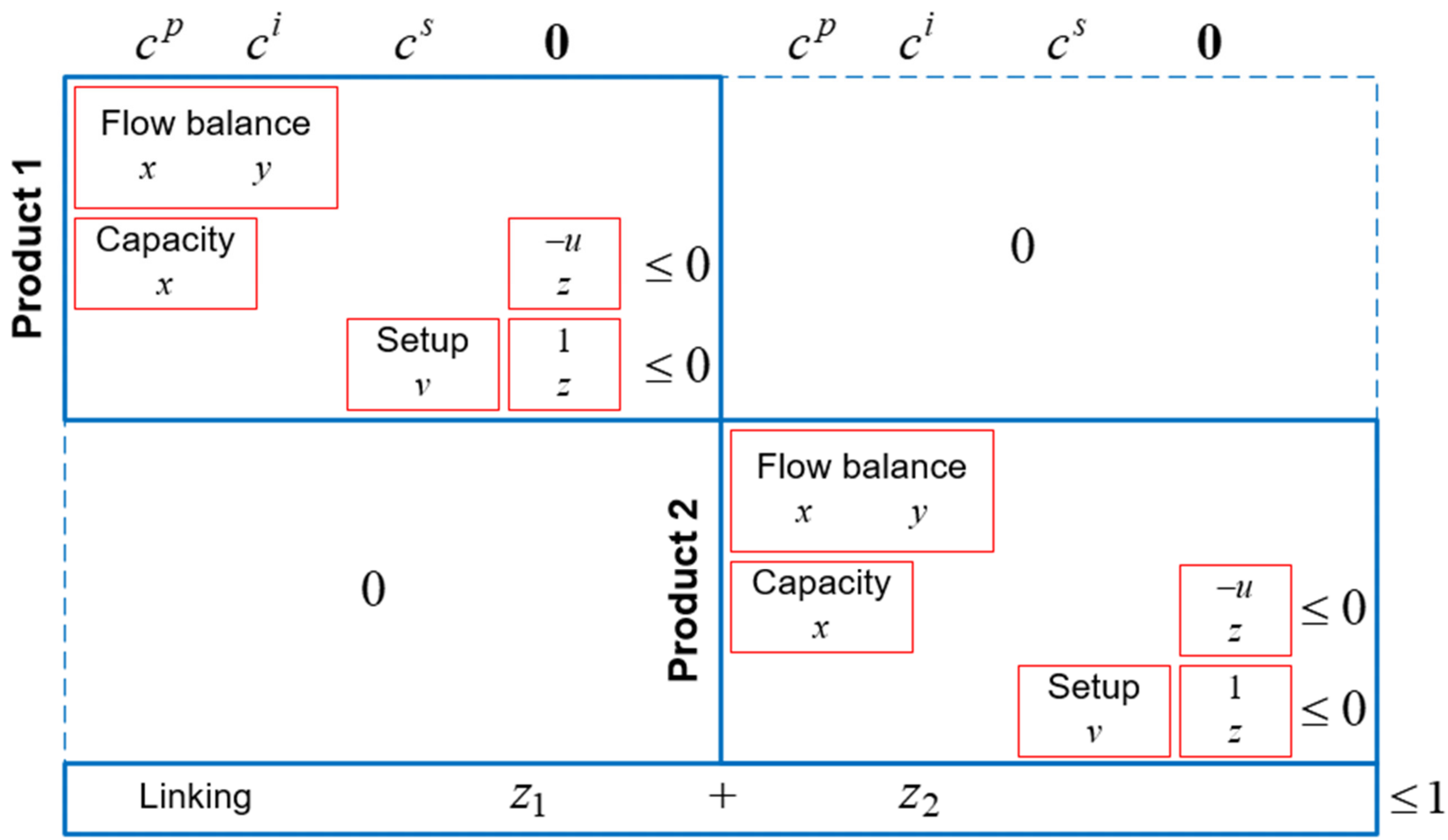
$d_{jk}$ = demand (ton) in period $j$ for product $k$

$u_{ik}$ = production capacity (ton) of stage $i$ in period $j$ for product $k$

$y_{ik}^{0}$ = initial inventory (ton) of stage $i$ for product $k$

$y_{ik}^{t+1}$ = final inventory (ton) of stage $i$ for product $k$.

# Multiple Products with Setup Costs



# Setup Constraints

$z_{mtg} \in \{0,1\}$, production indicator

| $z_{mtg}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

$v_{mtg} \in \{0,1\}$, setup indicator

| $v_{mtg}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

| | $-v_t$ | $+ \; z_t$ | $- \; z_{t-1}$ | $\leq 0$ | |
|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 1 | −1 | |
| Don't want (not feasible) | 0 | 1 | 0 | 1 | |
| | 0 | 1 | 1 | 0 | |
| | 1 | 0 | 0 | −1 | Feasible, but not min cost |
| | 1 | 0 | 1 | −2 | Feasible, but not min cost |
| Want (feasible) | 1 | 1 | 0 | 0 | |
| | 1 | 1 | 1 | −1 | Feasible, but not min cost |

# Multiple Products with Setup Costs

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{t}\sum_{k=1}^{g} c_{ik}^{p} x_{ijk} + \sum_{i=1}^{m}\sum_{j=2}^{t+1}\sum_{k=1}^{g} c_{ik}^{i} y_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{t}\sum_{k=1}^{g} c_{ik}^{s} v_{ijk}$$

$$\begin{aligned}
\text{subject to} \quad & -x_{ijk} + x_{(i+1)jk} - y_{ijk} + y_{i(j+1)k} = 0, && i = 1,\ldots,m-1; j = 1,\ldots,t; k = 1,\ldots,g \quad (a)\\
& -x_{mjk} - y_{mjk} + y_{m(j+1)k} = d_{jk}, && j = 1,\ldots,t; k = 1,\ldots,g \quad (b)\\
& x_{ijk} \le u_{ik} z_{ijk}, && i = 1,\ldots,m; j = 1,\ldots,t; k = 1,\ldots,g \quad (c)\\
& -v_{i1k} + z_{i1k} \le z_{ik}^{0}, && i = 1,\ldots,m; k = 1,\ldots,g \quad (d)\\
& -v_{ijk} + z_{ijk} - z_{i(j-1)k} \le 0, && i = 1,\ldots,m; j = 2,\ldots,t; k = 1,\ldots,g \quad (e)\\
& \sum_{k=1}^{g} z_{ijk} \le 1, && i = 1,\ldots,m; j = 1,\ldots,t \quad (f)\\
& y_{i1k} = y_{i}^{0}, && i = 1,\ldots,m; k = 1,\ldots,g\\
& y_{i(t+1)k} = y_{ik}^{t+1}, && i = 1,\ldots,m; k = 1,\ldots,g\\
& x, y \ge 0; v, z \in \{0,1\},
\end{aligned}$$

where,

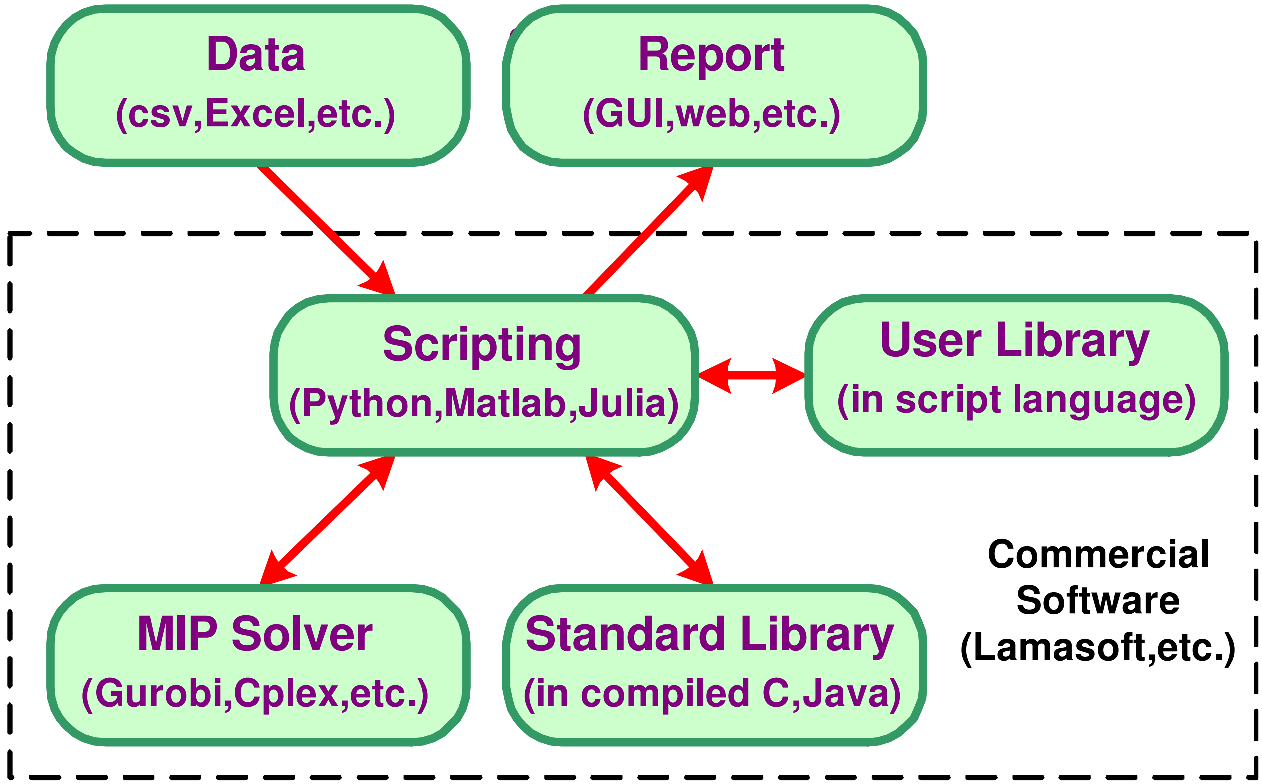
$m$ = number of production stages
$t$ = number of periods of production
$g$ = number of products
$c_{ik}^{p}$ = production cost (dollar/ton) in stage $i$ for product $k$
$x_{ijk}$ = production (ton) at stage $i$ in period $j$ for product $k$
$c_{ik}^{i}$ = inventory cost (dollar/ton) in stage $i$ for product $k$
$y_{ijk}$ = stage-$i$ inventory (ton) from period $j-1$ to $j$ for product $k$
$c_{ik}^{s}$ = setup cost (dollar) in stage $i$ for product $k$

$v_{ijk}$ = setup indicator at stage $i$ in period $j$ for product $k$
$z_{ijk}$ = production indicator at stage $i$ in period $j$ for product $k$
$d_{jk}$ = demand (ton) in period $j$ for product $k$
$u_{ik}$ = production capacity (ton) of stage $i$ in period $j$ for product $k$
$z_{ik}^{0}$ = initial setup at stage $i$ for product $k$
$y_{ik}^{0}$ = initial inventory (ton) of stage $i$ for product $k$
$y_{ik}^{t+1}$ = final inventory (ton) of stage $i$ for product $k$.

# Discussion

1. Indicator variables require MILP instead of LP:
2. Lumpy resources require indicator variables to include fixed costs (setup, prod scale economies, etc)
3. Shared resources:
   - Coupling constraints used to ensures total output doesn't exceed available capacity (can still be LP)
   - Linking constraints used to ensure only one activity per period
4. Demand over planning horizon can be a mix of firm and forecasted orders from MPS/MRP/ERP
5. What is a realistic or typical number of demand periods?
   - Length should include all significant planning decisions (e.g., scheduled maintenance)
6. How is safety stock taken into consideration?

# Work Flow of Logistics Software Stack




- **Flow:** *Data → Model → Solver → Output → Report*
  - reports are run on a regular period-to-period, *rolling-horizon* basis as part of normal operations management
  - model only changed when logistics network changes