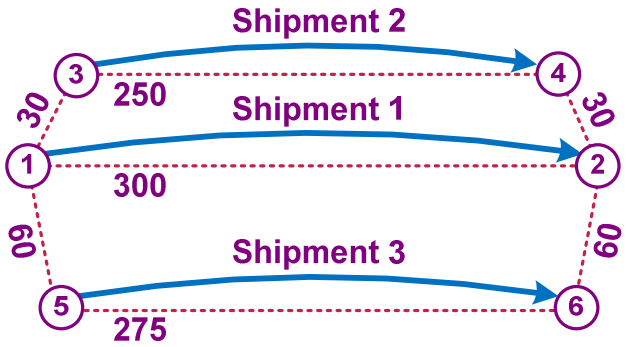
Routing 2:
Route-based Construction Procedures

- Two simple construction procedures, `mincostinsert` and `savings`, along with `twoopt` improvement, can be used for most routing applications
  - Can handle interleaved multi-stop routing, where each shipment has a different origin and destination

Multi-Stop Routing

- Each shipment might have a different origin and/or destination $\Rightarrow$ node/location sequence not adequate



$$L = (y_1, \ldots, y_n) = (1, 2, 3) \qquad n\text{-element shipment sequence}$$

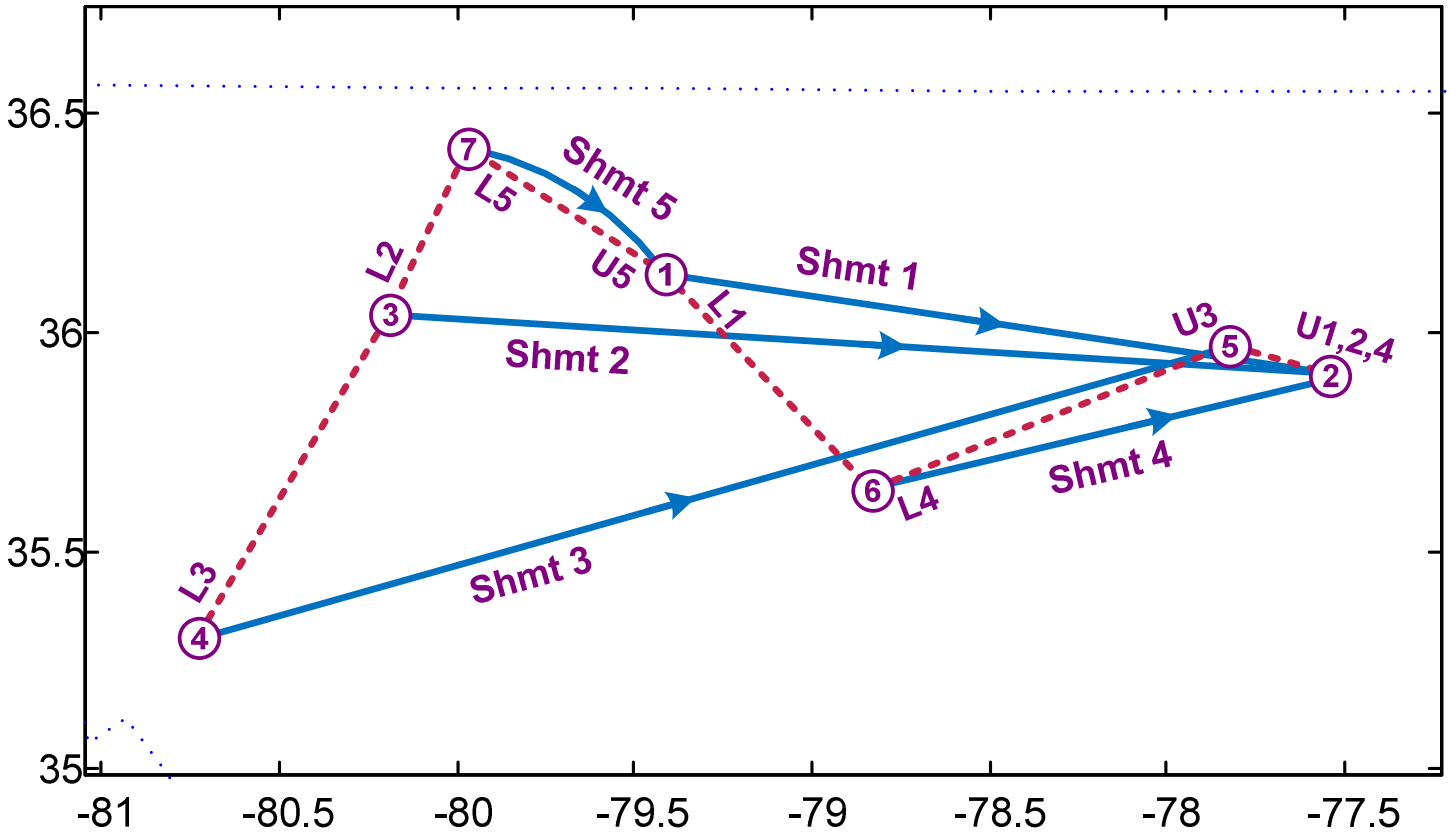
$$R = (z_1, \ldots, z_{2n}) = (3, 1, 2, 2, 1, 3) \qquad 2n\text{-element route sequence}$$

$$X = (x_1, \ldots, x_{2n}) = (5, 1, 3, 4, 2, 6) \qquad 2n\text{-element location (node) sequence}$$

$$c_{ij} = \text{cost between locations } i \text{ and } j$$

$$c(R) = \sum_{i=1}^{2n-1} c_{x_i, x_{i+1}} = 60 + 30 + 250 + 30 + 60 = 430, \qquad \text{total cost of route } R$$

5-Shipment Example



Route sequence: $R = (3, 2, 5, 5, 1, 4, 3, 1, 2, 4)$

Location sequence: $X = (4, 3, 7, 1, 1, 6, 5, 2, 2, 2)$

Route Sequencing Procedures

- Online procedure: add a shipment to an existing route as it becomes available
  - Insert and Improve: for each shipment, insert where it has the least increase in cost for route and then improve (`mincostinsert` → `twoopt`)
- Offline procedure: consider all shipments to decide order in which each added to route
  - Savings and Improve: using all shipments, determine insert ordering based on "savings," then improve final route (`savings` → `twoopt`)

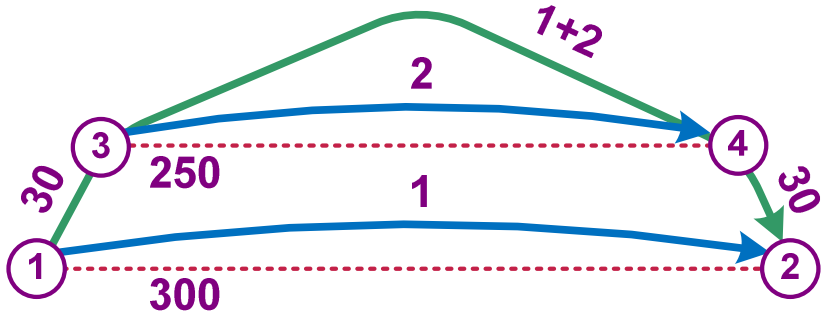
Min Cost Insert

		1			1			
1		•			•	2	2	×
2	2	•			•	2		c_2
3		•	2	2	•			c_3^*
4		•		2	•	2		c_4
5	2	•	2		•			c_5

$$\frac{n(n+3)}{2}-1 \quad \text{evaluations}$$

		1			2			2			1	
1	3	•	3		•			•			•	
2	3	•			•	3		•			•	
3	3	•			•			•	3		•	
4	3	•			•			•			•	3
5		•	3	3	•			•			•	
6		•		3	•	3		•				
7		•		3	•			•	3			
⋮		⋮			⋮			⋮			⋮	

Pairwise Savings



$$s_{ij} = \text{pairwise savings between shipments } i \text{ and } j$$

$$= c_i + c_j - c_{ij} > 0$$

$$s_{1,2} = 300 + 250 - 310$$

$$= 240$$