Routing 3: Vehicle Routing

- Constraints turn a single TSP tour into several routes
- When a potential route violates a constraint, e.g.,
  - Total route distance
  - Total route time
  - Vehicle capacity
  - Delivery/pickup time windows
- Its cost can be set to infinity so that it is never selected

Vehicle Routing Problem

- VRP = TSP + vehicle constraints
- Constraints:
  - Capacity (weight, cube, etc.)
  - Maximum TC (HOS: 11 hr max)
  - Time windows (with/without delay at customer)
    - VRP uses absolute windows that can be checked by simple scanning
    - Project scheduling uses relative windows solved by shortest path with negative arcs
  - Maximum number of routes/vehicles (hard)
- Criteria:
  1. Number of routes/vehicles
  2. TC (time or distance)
- VRP solution can be one time or periodic
  - One time (operational) VRP minimizes TC
  - Periodic (tactical) VRP minimizes TLC (sometimes called a “milk run”)

Clark-Wright (Offline) Savings Procedure

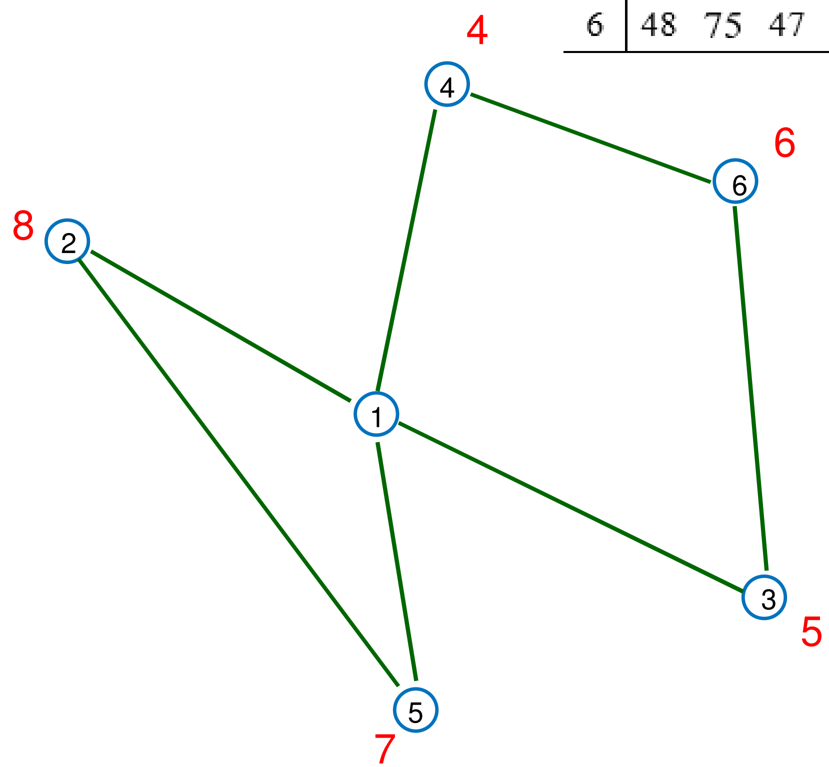
- First (1964), and still best, offline routing procedure if only have vehicle capacity constraints
- Pairs of shipments ordered in terms of their decreasing (positive) pairwise savings
- Given savings pair i-j, without exceeding capacity constraint, either:
  1. Create new route if i and j not in any existing route
  2. Add i to route only if j at beginning or end of route
  3. Combine routes only if i and j are endpoints of each route

Ex: Clark-Wright Savings Procedure

- Node 1 is depot, nodes 2-6 customers
- Customer demands 8, 3, 4, 7, 6, resp.
- Vehicle capacity is 15
- Symmetric costs

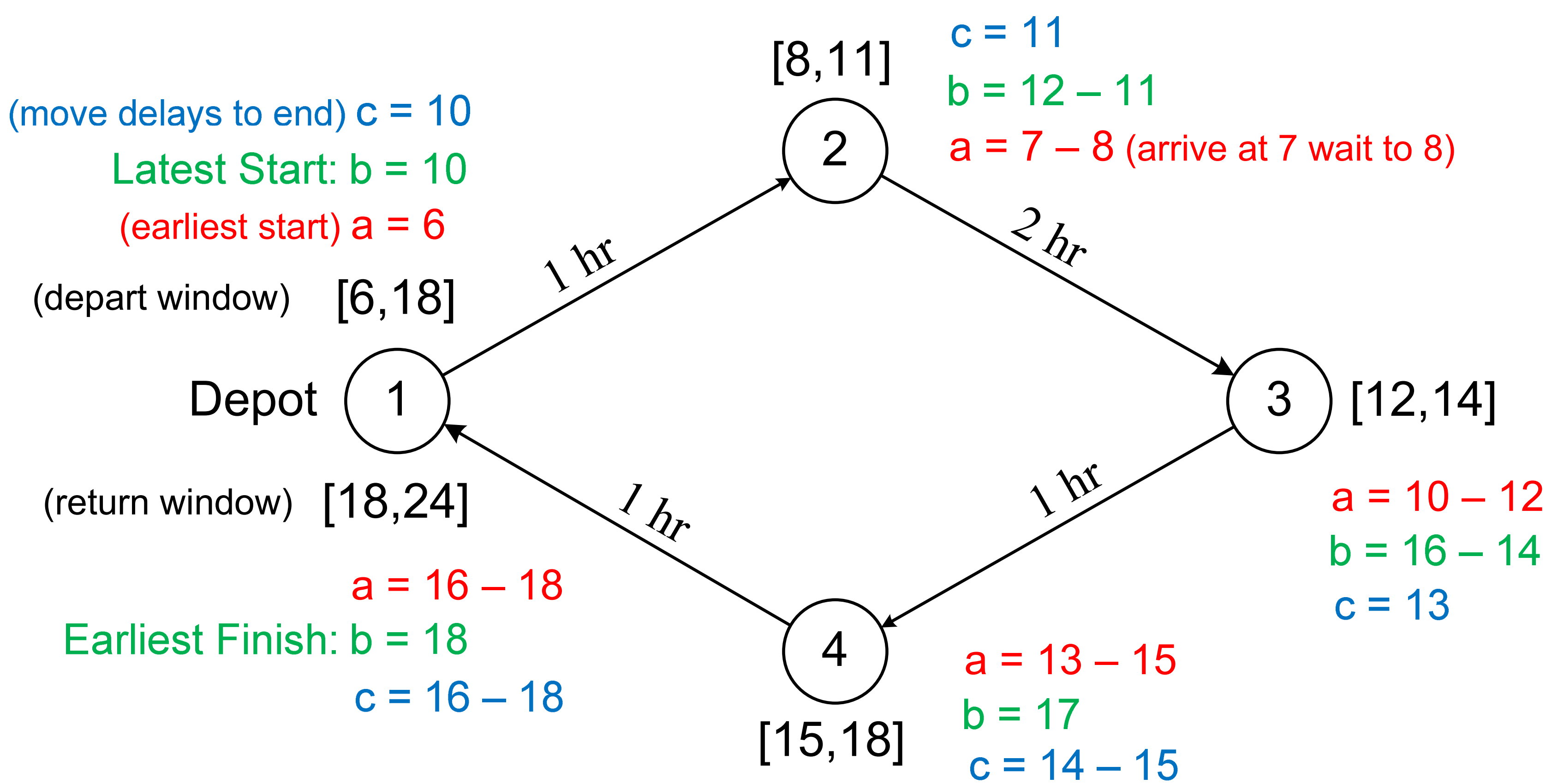
	1	2	3	4	5	6
1	0	40	48	38	33	48
2	40	0	87	46	65	75
3	48	87	0	67	41	47
4	38	46	67	0	70	34
5	33	65	41	70	0	69
6	48	75	47	34	69	0

i	j	s_{ij}
2	3	$40+48-87=1$
~~2~~	~~4~~	$40+38-46=32$
2	5	8
~~2~~	~~6~~	13
~~3~~	~~4~~	19
~~3~~	~~5~~	40
3	6	49
4	5	1
4	6	52
~~5~~	~~6~~	12



Ex: VRP with Time Windows

[0,24] hr; Loading/unloading time = 0; Capacity = ∞; LB = 5 hr

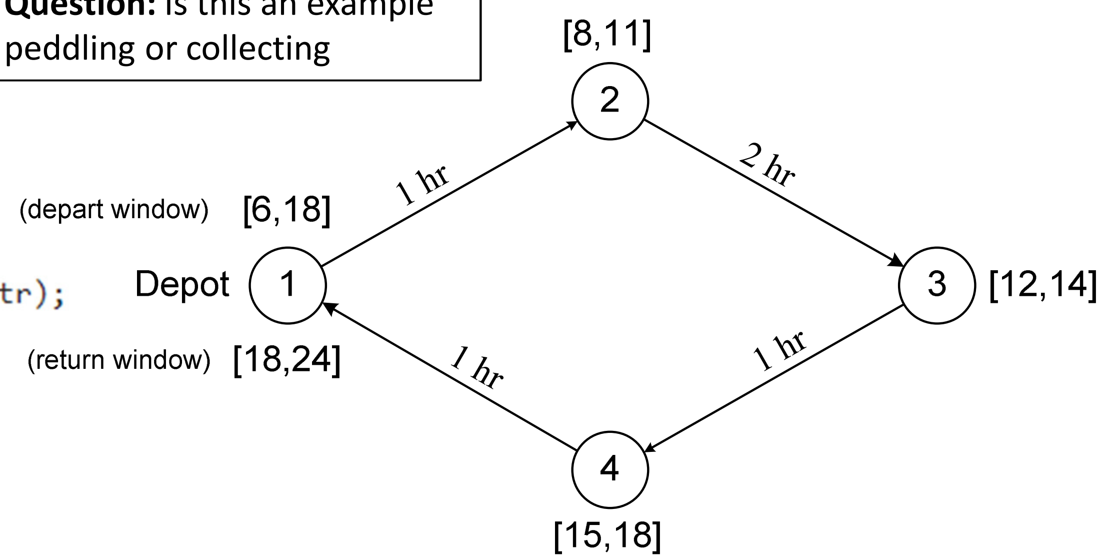


Earliest Finish – Latest Start = 18 – 10 = 8 hr = 5 travel + 3 delay

Ex: VRP with Time Windows

```
T = zeros(4);
T(1,2) = 1; T(2,3) = 2;
T(3,4) = 1; T(4,1) = 1;
sh = vec2struct('b',1,'e',[2 3 4]);
sh = vec2struct(sh,'tU',0,'temin',...
   [8 12 15],'temax',[11 14 18]);
tr = struct('b',1,'e',1,'tbmin',6,...
   'tbmax',18,'temin',18,'temax',24);
[TC,~,out] = rteTC([1 2 3 1 2 3],sh,T,tr);
sdisp(sh),sdisp(out,false),TC
```

Question: Is this an example peddling or collecting

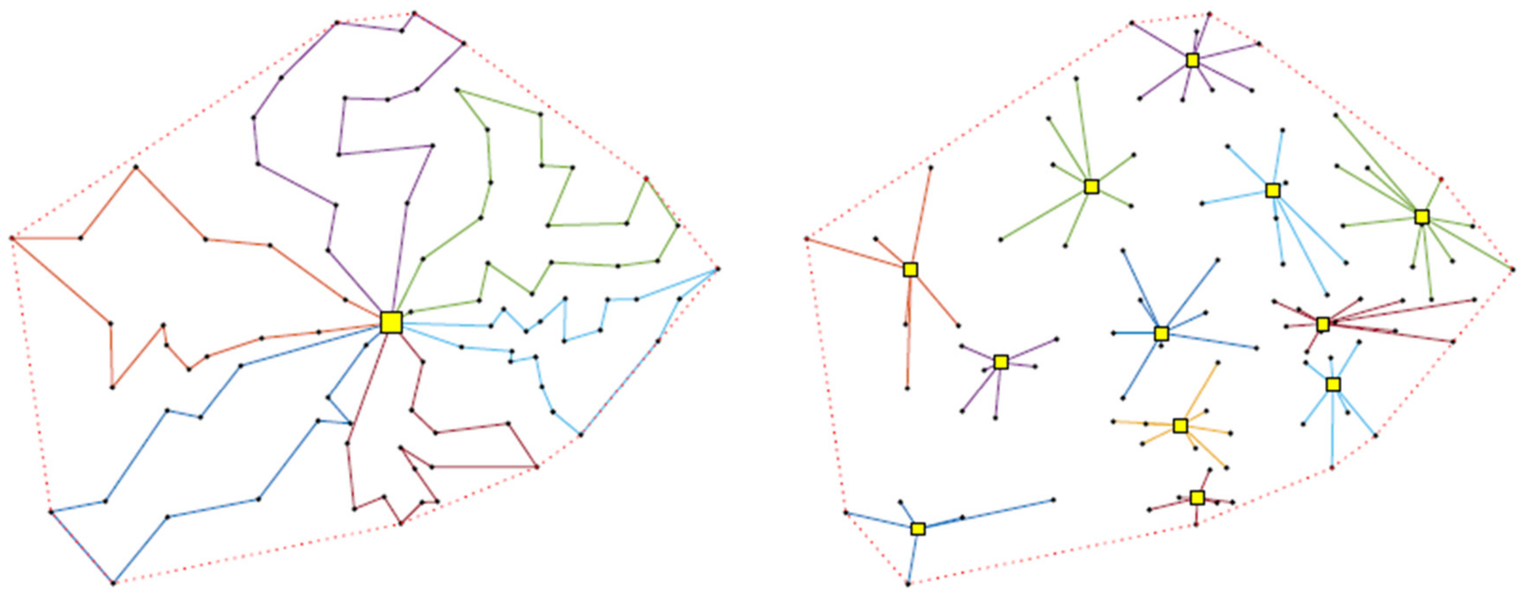



```
sh:  b  e  tU  temin  temax
--:-----------------------
 1:  1  2   0      8     11
 2:  1  3   0     12     14
 3:  1  4   0     15     18

out:  Rte  Loc  Cost  Arrive  Wait  TWmin  Start  LU  Depart  TWmax  Total
---:----------------------------------------------------------------------
  1:    0    1     0       0     0      6     10   0      10     18      0
  2:    1    1     0      10     0      6     10   0      10     18      0
  3:    2    1     0      10     0      6     10   0      10     18      0
  4:    3    1     0      10     0      6     10   0      10     18      0
  5:    1    2     1      11     0      8     11   0      11     11      1
  6:    2    3     2      13     0     12     13   0      13     14      2
  7:    3    4     1      14     1     15     15   0      15     18      2
  8:    0    1     1      16     2     18     18   0      18     24      3
TC =
```

Tactical Routing

- Most routing procedures are for operational decisions
  - Given actual demands, determine actual route
  - For tactical decisions, can only estimate routing cost



(a) Multi-stop deliveries from a single DC. (b) Point-to-point deliveries from several DCs.

Figure 3. Multi-stop versus point-to-point delivery for the same set of customers (DC shown as yellow squares).

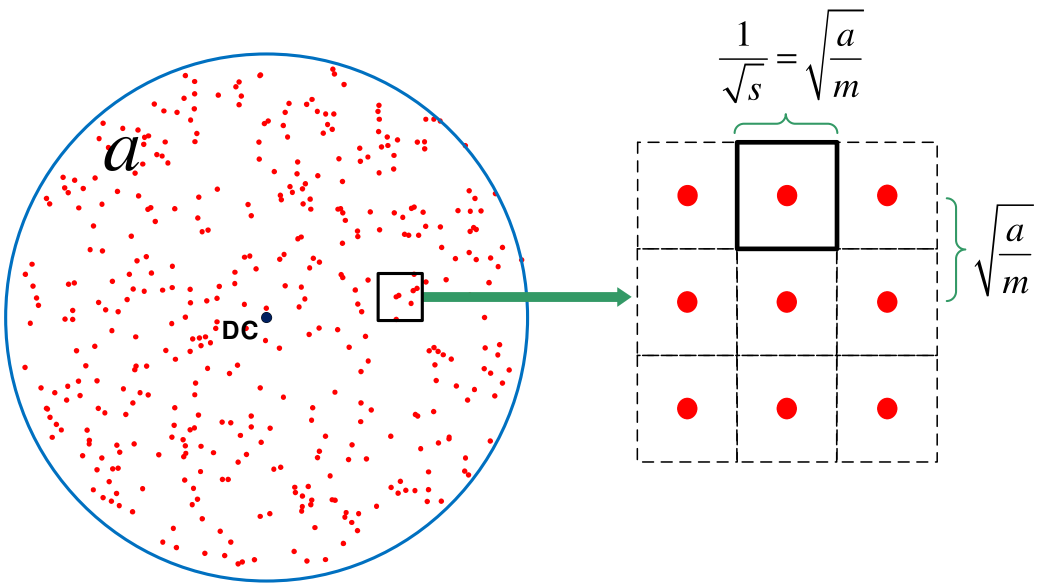
M.G. Kay, "Challenges and Opportunities Associated with Using Autonomous Vehicles and Drones for Home Delivery," MHI Whitepaper, 2020

Expected Route Distance

Given m customers in an area a, the density is $s = m/a$, and the expected distance between customers is $s^{-0.5} = \sqrt{a/m}$, resulting in an estimated total route distance that is proportional to $\varphi m\sqrt{a/m}$. Use known route distance for $m = 2$ to determine φ:

For $a = \pi$ and $m = 2$, $\varphi 2\sqrt{\dfrac{\pi}{2}} = \dfrac{2}{3}\left(2+\sqrt{2}\right) \Rightarrow \varphi = \dfrac{2\left(\sqrt{2}+1\right)}{3\sqrt{\pi}} \approx 0.9$, so that

$\boxed{\hat{d}_m^{TSP} = 0.9\sqrt{ma}}$, for routes passing through the center (DC) of a circular region.



:	m	Simulated	Estimate
1:	2	2.26	2.26
2:	5	3.79	3.57
3:	10	5.20	5.04
4:	20	7.02	7.13
5:	50	10.95	11.28
6:	100	15.50	15.95
7:	200	22.09	22.56
8:	500	35.07	35.67
9:	1,000	49.87	50.44

Ex: Estimate Number of Deliveries

- Assuming that, on average, a vehicle travels at 30 mph (including stops at red lights) and, after reaching a customer, it takes two minutes to drop off a delivery. If the service area is fifty square miles, estimate the number of deliveries that the vehicle can make in eight hours assuming that it has no capacity constraints.

$$\varphi = 0.9, \quad a = 50 \text{ mi}^2, \quad u = \frac{2}{60} \text{ hr}, \quad t = 8 \text{ hr}, \quad v = 30 \text{ mph}$$

$$um + \frac{\varphi\sqrt{ma}}{v} = t \Rightarrow m = \frac{a\varphi^2 + 2tuv^2 - \sqrt{a\varphi^2\left(a\varphi^2 + 4tuv^2\right)}}{2u^2v^2} = 159.6018$$

```
dTSP = @(m,a) 0.9*sqrt(m*a);
m = fminsearch(@(m) abs(u*m + dTSP(m,a)/v - t), 1)   % m0 = 1
m =
  159.6018
```

Question: Should the estimate be 159 or 160 deliveries?
Note: Rounding to 160 (closest integer) results in total time > 8 hrs