Inventory 1: Working, Economic, and One-Time Safety Stock

- What Makes Production System Design Hard?
  1. Things not always where you want them when you want them $\Rightarrow$ Working stock
  2. Resources are lumpy $\Rightarrow$ Economic stock
  3. Things vary

     variability can be known or unknown

     uncertainty/randomness = unknown variability

     uncertainty/randomness $\Rightarrow$ Safety stock

Role/Position of Inventory

- Same units of inventory can serve multiple roles at each position in a production process

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)
- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)
- *Safety stock:* held to buffer effects of uncertainty
  - (decoupling, MRO (maintenance, repair, and operations))

Total Logistics Cost

- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$
\begin{aligned}
TLC &= TC + IC + PC \\
TC &= \text{transport cost} \\
IC &= \text{inventory cost} \\
&= IC_{\text{working}} + IC_{\text{economic}} + IC_{\text{safety}} \\
PC &= \text{purchase cost}
\end{aligned}
$$

Total logistics costs are any of the relevant costs associated with providing a logistics service, where a relevant cost is a cost that differs when comparing multiple alternatives and, as such, can be used in making a decision between the alternatives.

- *Economic* (*cycle*) *stock*: held to allow cheaper large shipments
- *Working* (*in-transit*) *stock*: goods in transit or awaiting transshipment
- *Safety stock*: held due to demand and transport uncertainty (e.g., shipment arriving earlier than needed "just in case")
- *Purchase cost*: can be different for different suppliers

Determining Optimal Inventory Levels

- **Deterministic world:** min *TLC* using average demand and transport lead times
  - *Working and economic*: optimal level balances *IC* with *TC* and *PC* since $\uparrow IC \Rightarrow \downarrow TC$ and $\downarrow PC$
  - *Safety*: optimal level = 0 $\Rightarrow$ *IC* = 0
- **Stochastic (real) world:** can't just min *TLC*
  - *Working and economic*: Can still min *TLC,* using (*detrended*) averages
  - *Safety*: max Total Profit (*TP* = *TR* – *TLC*), using actual (historical or synthetic) demand and transport lead times
    - If inv level = 0 (out of stock) when a demand occurs $\Rightarrow$ lost sale (0 revenue) or backorder (likely $\downarrow$ price)
    - Optimal safety stock level balances out-of-stock revenue $\downarrow$ with $\uparrow$ *IC* due to holding/disposing stock

Inventory Cost

- *Inventory Cost* (IC) of working, economic, and safety stock can be calculated in the same general way:

$$IC = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$
$$= vh\ (\$/\text{ton-yr}) \times q\ (\text{ton}) = (\$/\text{yr})$$
$$v = \text{unit value of inventory } (\$/\text{ton})$$
$$h = \text{inventory carrying rate, cost per dollar of inventory per year } (\$/\$\text{-yr} = 1/\text{yr})$$
$$q = \text{average annual inventory level (ton)}$$

- Value for v can be determined from the purchase price or production cost
- Rate h usually not known directly
  - Based on interest, warehousing, and obsolescence costs
- Determining q differs for working, economic, and safety stock

Inventory Carrying Rate

- **Inv. Carrying Rate** (h) = interest + warehousing + obsolescence
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate (*yr*) h = 0.3 $\Rightarrow h_{\mathrm{obs}} \approx$ 0.2 (mfg product)
  - *Low* FGI cost (*yr*): $h = h_{\mathrm{int}} + h_{\mathrm{wh}} + h_{\mathrm{obs}}$
  - *High* FGI cost (*hr*): $h \approx h_{\mathrm{obs}}$, can ignore interest & warehousing
    - $(h_{\mathrm{int}}+h_{\mathrm{wh}})/H$ = (0.05+0.06)/2000 = 0.000055 (H = oper. hr/yr)
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h_{obs}

$$h_{\mathrm{obs}} t_h v = x_h v \Rightarrow h_{\mathrm{obs}} t_h = x_h \Rightarrow \boxed{h_{\mathrm{obs}} = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h_{\mathrm{obs}}}$$

  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as t_{CT}

Working Stock

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)

		Position		
		Raw Material	Work in Process	Finished Goods
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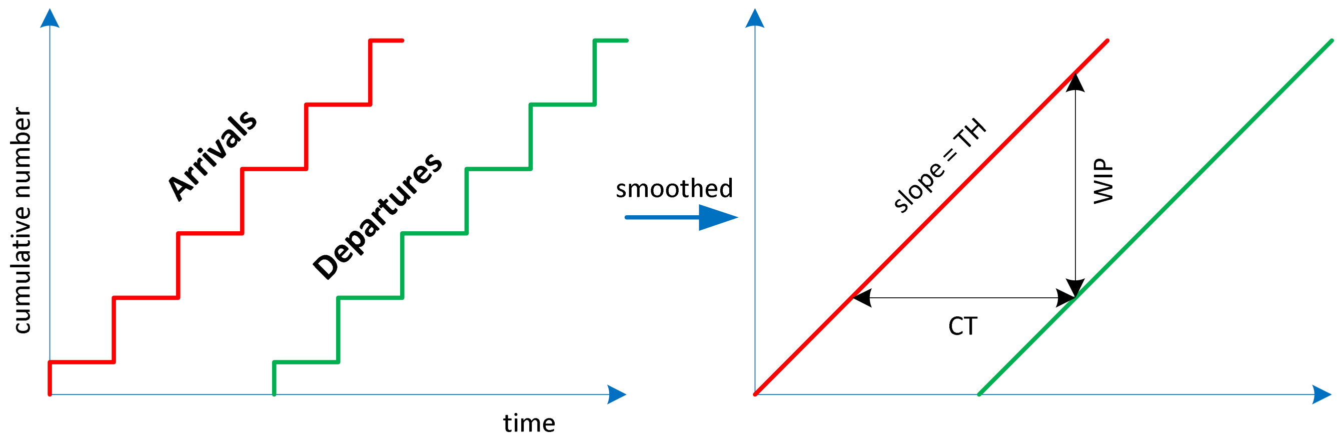
- Was ignored for truck transport since transit time were just a few days
  - – compared to weeks between shipments
- Important for comparing international transport alternatives
  - – ocean (weeks) vs. air cargo (hours)

Average Working Stock = WIP



$$\textbf{Little's Law:}\quad TH(r) = \frac{WIP(q)}{CT(t)},\quad CT = \frac{WIP}{TH},\quad \boxed{WIP = TH \cdot CT}$$

where $TH = r = \dfrac{q}{t} = throughput$

= average rate at which work is produced (units per hour)

$WIP = q = work\text{-}in\text{-}process$

= average number of units of work in a production system

$CT = t = cycle\ time$

= average time each unit of work is in a production system

Ex: Little's Law

- If it takes, on average, nine semesters for an undergraduate ISE student to graduate and 40 students, on average, graduate each semester, how many students are in the department?

$$
\begin{aligned}
WIP &= TH \cdot CT \\
&= 40(9) = 360 \text{ students}
\end{aligned}
$$

  – Note: Little's Law only works if WIP is not changing; taking the average is a good estimate of the steady state working inventory.

Ex: In-Transit Inventory

- Currently, a factory in Los Angeles imports 12 containers per year from a supplier in Shanghai, each with 15 tons of raw material. Each ton of RM costs \$7000 and loses 10% of its value after three months. Each container spends 15 days in transit, and its transport cost is \$2620. What is the annual in-transit inventory cost for RM?

$$\text{Shipment size: } q = 15 \text{ ton}/L, \quad \text{Shipment frequency: } n = 12\ L/\text{yr}$$

$$CT:\ t = \frac{15}{365.25}\ \text{yr}, \quad TH:\ nq = 12(15) = 180 \text{ ton/yr}$$

$$WIP = TH \cdot CT:\ q_I = nqt = 12(15)\frac{15}{365.25} = 7.39 \text{ ton}$$

$$h_{\text{obs}} = \frac{x_h}{t_h} = \frac{0.1}{0.25} = 0.4, \quad h = 0.05 + 0.06 + h_{\text{obs}} = 0.51$$

$$IC_w = vhq_I = 7000(0.51)7.39 = \$26,390.14/\text{yr} \quad \left[\$/\text{ton}\left(1/\text{yr}\right)\text{ton} = \$/\text{yr}\right]$$

Ex: Supplier Selection

- Cont prev Ex: A domestic supplier has been identified with a cost of $7500 per ton. 12 TLs per year, each containing 15 tons of RM would be shipped 200 miles in one day. TL revenue per loaded mile is $2, should the domestic supplier be used?

$$\begin{aligned} TLC_1 &= TC + IC_w + PC = nc_L + vhq_I + nvq \\ &= 12(2620) + 7000(0.51)7.39 + 12(7000)15 \\ &= 31,440 + 26,390 + 1,260,000 = \$1,317,830/\text{yr} \end{aligned}$$

$$r_{TL} = \$2/\text{mi}, \quad d = 200 \text{ mi}, \quad c_L = r_{TL} d = \$400$$

$$q_I = nqt = 12(15)\frac{1}{365.25} = 0.4928 \text{ ton}$$

$$\begin{aligned} TLC_2 &= TC + IC_w + PC = nc_L + vhq_I + nvq \\ &= 12(400) + 7500(0.51)0.4928 + 12(7500)15 \\ &= 4,800 + 1,885 + 1,350,000 = \$1,356,685/\text{yr} \end{aligned}$$

Economic Stock

- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)

		Position		
		Raw Material	Work in Process	Finished Goods
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	Safety Stock			

- *Cycle* (economic) inventory: held to allow cheaper large shipments/orders or production batches
- *Anticipation* (economic) inventory: product that is cheaper to purchase or produced early
  – Product on sale, discounted, or discontinued
  – Production that occurs throughout the year ahead of a single peak selling period (e.g., Christmas decorations)

Cycle Inventory

- *Cycle* (economic) inventory:
  - Inventory held between periodic shipments/orders or production batches to serve steady/constant demand
  - Average annual cycle inventory level: sum of the average level at the origin and destination

$$IC_{\text{cycle}} = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$

$$= vh\ (\$/\text{ton-yr}) \times \alpha q\ (\text{ton}) = (\$/\text{yr})$$

q = average shipment/order or production batch size (ton)

α = average inter-shipment inventory fraction at Origin and Destination

v = unit value of inventory ($/ton)

h = inventory carrying rate, cost per dollar of inventory per year ($/$-yr = 1/yr)

Note: q_I = average annual inventory level

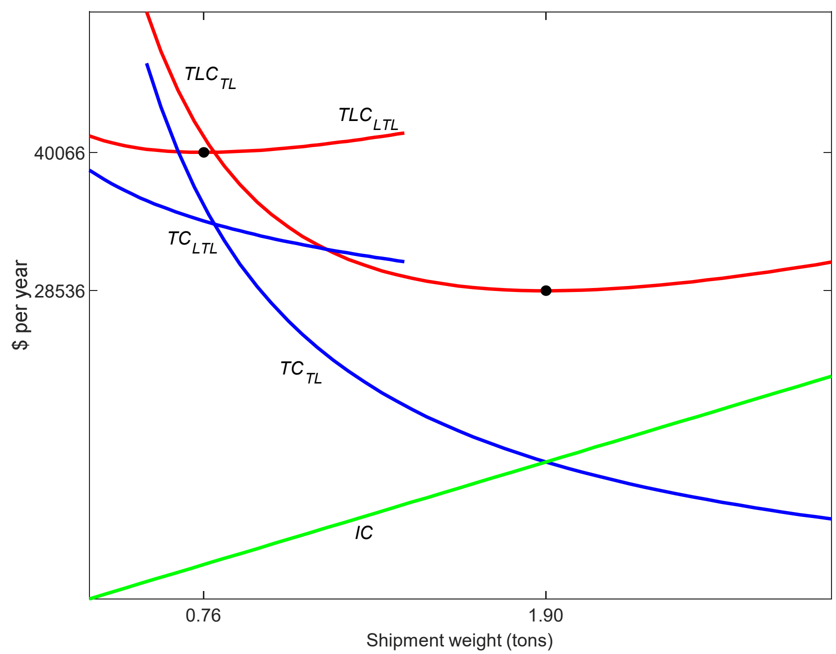
$$= \begin{cases} \alpha q, & \text{cycle (economic) inventory} \\ nqt, & \text{in-transit (working) inventory} \end{cases}$$

Optimal-Size Truck Shipments

- As shown previously (see Periodic Truck Shipments):

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha vhq = \frac{f}{q} rd + \alpha vhq$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q_{TL}^{*} = \sqrt{\frac{f r_{TL} d}{\alpha vh}}$$



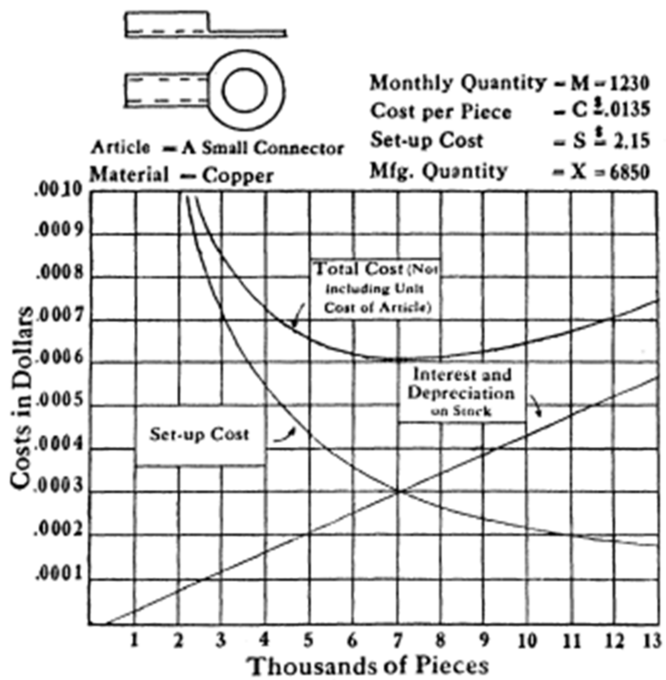
Economic Order Quantity (EOQ)

- Finds the optimal quantity (termed the EOQ) that minimizes the number of shipments/orders or production batches and the cost of holding them as inventory
- Early (1913) paper "How Many Parts to Make at Once" in *Factory, The Magazine of Management* by Ford W. Harris that introduced simple square-root formula to determine how many parts to make

$$TLC(q) = \frac{f}{q}k + \frac{1}{2}vhq, \quad \alpha = \frac{1}{2}$$

$$EOQ: \; q^* = \sqrt{\frac{2fk}{vh}}$$

$$k = \begin{cases} \text{fixed production cost} \\ \text{setup cost} \\ \text{transport cost} \\ \text{ordering cost} \end{cases}$$



Safety Stock

- *Safety stock:* held to buffer effects of uncertainty

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *One-time* safety stock: Uncertain demand and not able to carry inventory
  - Unmet demand is a lost sale
  - Excess product is disposed of
- *Periodic* safety stock: Uncertain demand and/or transport lead time, and able to carry inventory
  - Unmet demand is either a lost sale or backordered
  - Excess product is carried over to the next period