ISE 754: Logistics Engineering

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Fall 2024

Topics and Schedule

1. Introduction
2. Location
3. Transport
   - Exam 1 (take home)
4. Networks
5. Routing
   - Exam 2 (take home)
6. Inventory
   - Final exam (in class)

Lecture		Date	Section	Topic	Assignment
1	19	AUG, MON	1. Introduction	1. Modeling, Guesstimation, and Fermi Problems	
2	21	AUG, WED		2. Basic Concepts in Julia	HW 1
3	26	AUG, MON	2. Location	1. Types of Location Problems	
4	28	AUG, WED		2. Single-Facility Location	HW 2
5	4	SEP, WED		3. Great-Circle Distances, Circuity, and Transport Cost	
6	9	SEP, MON		4. Geocoding and Allocation	HW 3
7	11	SEP, WED		5. Multifacility Location and Aggregate Demand Points	
8	16	SEP, MON		6. UFL Heuristics	HW 4
9	18	SEP, WED		7. Logistics Network Analysis	
10	23	SEP, MON		8. Discrete Location and MILP	HW 5
11	25	SEP, WED	3. Transport	1. Overview of Freight Transport	
12	30	SEP, MON		2. One-Time Truck Shipments	
13	2	OCT, WED		3. Periodic Truck Shipments	
14	7	OCT, MON		4. Crossdocking	HW 6
15	9	OCT, WED	4. Networks	1. Basic Graph Models	
16	16	OCT, WED		Review for Exam 1	Exam 1
17	21	OCT, MON		2. Shortest Path Problem and Road Networks	
18	23	OCT, WED		3. Production-Inventory Planning: Single Product	HW 7
19	28	OCT, MON		4. Production-Inventory Planning: Multiple Products	
20	30	OCT, WED	5. Routing	1. Traveling Salesman Problem	
21	4	NOV, MON		2. Route-based Construction/Improvement Procedures	
22	6	NOV, WED		3. Vehicle Routing	HW 8
23	11	NOV, MON	6. Inventory	1. Working, Economic, and Safety Stock	
24	13	NOV, WED		Review for Exam 2	Exam 2
25	18	NOV, MON		2. One-Time and Periodic Policies	
26	20	NOV, WED		3. Multi-Echelon Inventory Systems	HW 9
27	25	NOV, MON		(Safety Lecture)	
28	2	DEC, MON		Review for Final Exam	
	11	DEC, WED	Final Exam, 8:30 – 11:00am 136 MRC		

Introduction 1: Modeling, Guesstimation, and Fermi Problems

"When in doubt, estimate. In an emergency, guess. But be sure to go back and clean up the mess when the real numbers come along."

- **Guesstimation:** When faced with the need to estimate data or the performance of a system for which you have no easy way to model, then instead of just saying "I don't have a clue," you can often use guesstimation to get an estimate that is at least within an order of magnitude of the correct answer.
  - Think of this as your failsafe modeling technique.
- **Level of analysis:** use the simplest (*least costly*) analysis necessary to select between multiple alternatives, taking into account the *time* of the analyst

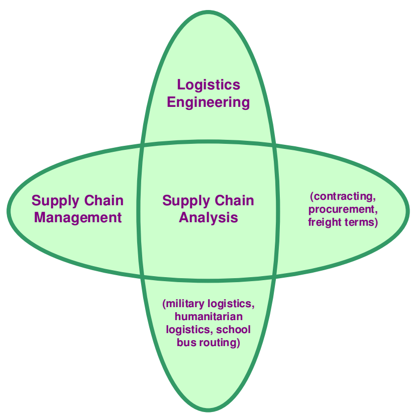
Production Systems

- Production Systems (my view for this class)
  - Manufacturing Systems: produce a good
    - can own/control (inanimate) thing ⇒ Inventory possible
    - provide form utility
  - Service Systems: produce a service
    - don't own the person or thing ⇒ Inventory not possible
    - provide general work utility
  - Logistics Systems
    - provide time/place utility
    - support manufacturing and service systems
- Production System Design:
  - Given design of good/service and process,
    - determine production capacity and logistics needed

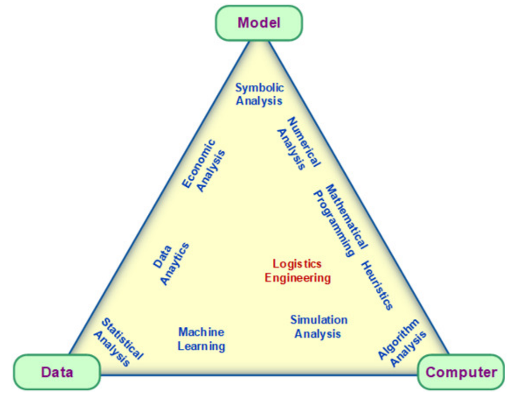
What Makes Production System Design Hard?

1. Things not always where you want them when you want them
   – where ⇒ transport and location ⇒ logistics
   – when ⇒ inventory ⇒ scheduling and production planning
2. Resources are lumpy
   ⇒ minimum effective size ⇒ fixed cost ⇒ economies of scale and scope
   – Babbage's Law: need worker's skill to match most difficult task
3. Things vary
   – both demand and production process variability cause problems
   – variability can be known or unknown
     - uncertainty/randomness = unknown variability
       – random demand, machine breakdowns
     - known variability can be due to
       – seasonal demand
       – bad control of production system

SCM vs Logistics Engineering



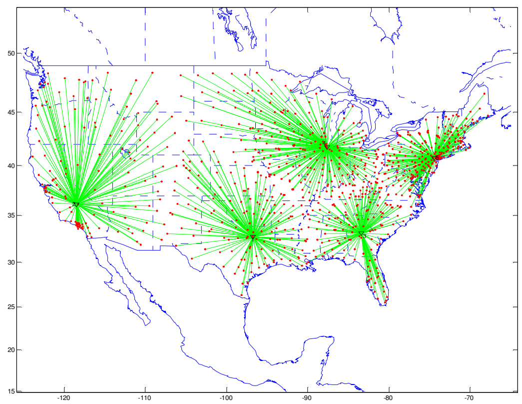
Analysis Triangle



Scope

- Strategic (years)
  – Logistics network design
  – Capacity planning
- Tactical (weeks-year)
  – Multi-echelon, multi-period, multi-product production and inventory models
  – Transportation planning
- Operational (minutes-week)
  – Vehicle routing

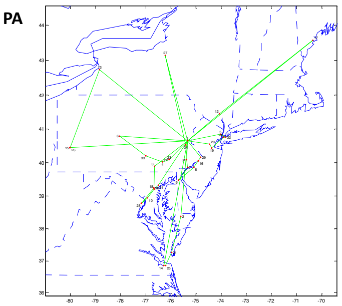
Strategic: Logistics Network Design



Optimal locations for five DCs serving 877 customers throughout the U.S.

Operational: Vehicle Routing

Eight routes served from DC in Harrisburg, PA



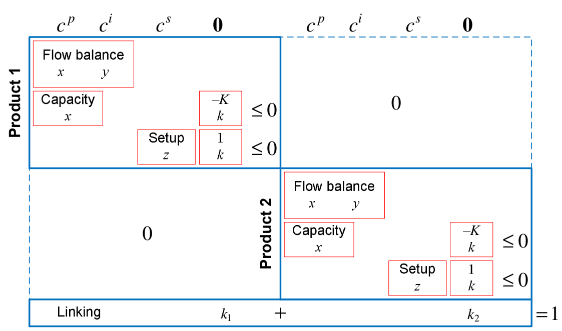
Route Summary Information

Route	Load Weight	Route Time	Customers in Route	Layover Required
1	12,122	18.36	4	1
2	4,833	16.05	2	1
3	9,642	17.26	3	1
4	25,957	13.77	6	0
5	12,512	9.90	2	0
6	15,156	13.70	5	0
7	29,565	11.30	6	0
8	32,496	8.84	5	0
		109.18		3

Detailed Route Information

						Start	L/D (hr)	Depart	Total Time (hr)	Zip Code
Route		1								
	1					23:29	0	23:29	0	18020
	14	2,328	7.51	7:00	399	7:00	0.59	7:35	8.1	23510
	25	4,697	0.19	7:46	6	7:46	0.68	8:27	0.87	23502
	21	3,682	1.12	9:34	37	9:34	0.64	10:13	1.76	23310
	2	1,415	2.51	12:43	93	12:43	0.56	13:17	3.07	21801
	1	0	4.57	17:51	196	17:51	0	17:51	4.57	18020
Total		12,122	15.89		731		2.47		18.36	

Tactical: Production-Inventory Model



Constraint matrix for a 2-product, multi-period model with setups

Levels of Modeling

0. Guesstimation (order of magnitude)
1. Mean value analysis (linear, ±20%)
2. Nonlinear models (incl. variance, ±5%)
3. Simulation models (complex interactions)
4. Prototypes/pilot studies
5. Build/do and then tweak it

Geometric Mean

- How many people can be crammed into a car?
  - Certainly more than one and less than 100: the average (50) seems to be too high, but the geometric mean (10) is reasonable

$$\text{Geometric Mean:} \quad X = \sqrt{LB \times UB} = \sqrt{1 \times 100} = 10$$

- Often it is difficult to directly estimate input parameter *X*, but is easy to estimate reasonable lower and upper bounds (LB and UB) for the parameter
  - Since the guessed LB and UB are usually orders of magnitude apart, use of the arithmetic mean would give too much weight to UB
  - Geometric mean gives a more reasonable estimate because it is a logarithmic average of LB and UB
  - What if the LB is 0?

Fermi Problems

- *Fermi Problems*, named after the physicist Enrico Fermi, are inspired guesses about quantities that seem almost impossible to determine given the limited data that you have available.
- Solving a Fermi Problem involves "reasonable" (i.e., +/– 10%) *guesstimation* of the input parameters needed and back-of-the-envelope type approximations.
  - Goal is to have an answer that is within an order of magnitude of the correct answer (or what is termed a *zeroth-order approximation*)
  - Works because over- and under-estimations of each parameter tend to cancel each other out as long as there is no consistent bias

Ex: How Many McDonald's Restaurants in U.S.?

- How many McDonald's restaurants in U.S.?

Parameter	LB		UB	Estimate	
Annual per capita demand	1	1 order/person-day x 350 day/yr =	350	18.71	(order/person-yr)
U.S. population				300,000,000	(person)
Operating hours per day				16	(hr/day)
Orders per store per minute (in-store + drive-thru)				1	(order/store-min)
Analysis					
Annual U.S. demand		(person) x (order/person-yr) =		5,612,486,080	(order/yr)
Daily U.S. demand		(order/yr)/365 day/yr =		15,376,674	(order/day)
Daily demand per store		(hrs/day) x 60 min/hr x (order/store-min) =		960	(order/store-day)
Est. number of U.S. stores		(order/day) / (order/store-day) =		16,017	(store)

- "Reasonable" guesstimates can be made for all of the parameters needed to make the estimation except for customer demand; as a result, the geometric mean of the estimated lower and upper bounds on demand is used as the estimate.
- The actual number of McDonald's restaurants in the U.S. as of 2013 is 14,267, which is around 10% below the estimate.
- A key assumption in the analysis is that the number of McDonald's restaurants in the U.S. has reached market saturation, allowing the entire U.S. population to be used as the customer base.

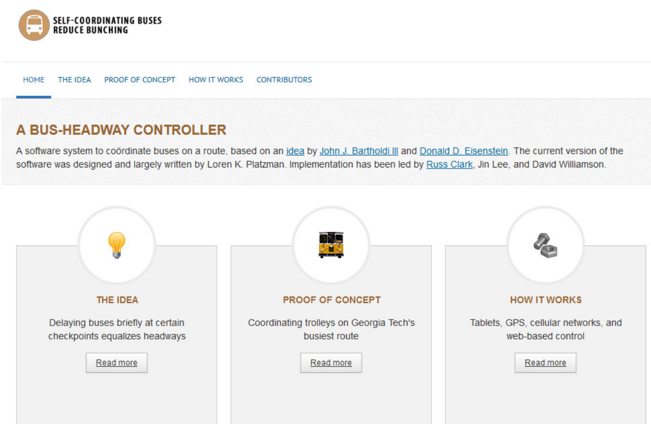
System Performance Estimation

- Often easy to estimate performance of a new system if can assume either perfect or no control
- Example: estimate waiting time for a bus
  - 8 min. avg. time (aka "headway") between buses
  - Customers arrive at random
    - assuming no web-based bus tracking
  - Perfect control (LB): wait time = half of headway
  - No control (*practical* UB): wait time = headway
    - assuming buses arrive at random (Poisson process)

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{8}{2} \times 8} = 5.67 \text{ min}$$

  - Bad control can result in higher values than no control

http://www.nextbuzz.gatech.edu/



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Ex: Waiting Time for a Bus

- If, during the morning rush, there are three buses operating on Wolfline Route 13 and it takes them 45 minutes, on average, to complete one circuit of the route, what is the estimated waiting time for a student who does not use Passio Go for real-time bus tracking?

Answer :

$$\text{Frequency (TH)} = \frac{WIP}{CT} = \frac{3 \text{ bus/circuit}}{45 \text{ min/circuit}} = \frac{1}{15} \text{ bus/min}, \quad \text{Headway} = \frac{1}{\text{Freq.}} = 15 \text{ min/bus}$$

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{15}{2} \times 15} = 10.61 \text{ min}$$

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Ex: Fermi Problem

- Estimate the average amount spent per trip to a grocery store. Total U.S. supermarket sales were recently determined to be \$649,087,000,000, but it is not clear whether this number refers to annual sales, or monthly, or weekly sales.

$$\textbf{Answer :} \quad \frac{\$6.5e11}{3e8} \approx \$2{,}000 \text{ / person-yr}, \quad LB = 1 \text{ trips/wk}, UB = 7 \text{ trips/wk}$$

$$\Rightarrow \sqrt{1(7)} \times 52 \approx 3 \times 50 \approx 150 \text{ trips/yr} \Rightarrow \frac{\$2{,}000}{150} \approx \$15 \text{ / person-trip} \Rightarrow \text{Annual}$$

Supermarket / Grocery Store Statistics	Data
Total number of grocery store employees	3,400,000
Total supermarket sales in 2015	\$649,087,000,000
Total supermarket sales in 2012	\$602,609,000,000
Total number of grocery stores / supermarkets	37,053
Median weekly sales per supermarket store	\$384,911
Average grocery store transaction amount	\$27.30
Average number of grocery store trips per week a consumers makes	2.2
Average number of items carried in a supermarket	38,718

Why is estimate so much less than reported value?

$$\frac{\frac{\$6.5e11}{27.50}}{\sqrt{1(7)} \times 52} \approx 2e8 \text{ no. customers}$$

$$\Rightarrow 100 \frac{2e8}{3e8} \approx 67\%$$

customers as percent total pop

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Question 1.1.1 (Practice)

When guesstimating the value for a quantity, which procedure is likely to be most effective?

a) Combine simple parameters that have no consistent bias into a single parameter to be estimated so that over- and under-estimations tend to cancel each other out.
b) Use mean value analysis so that the error in the estimate for the quantity is no more than ± 20% or, if possible, a nonlinear model, so the error is reduced to less than ± 5%
c) Break the problem into a series of simpler parameters to be guesstimated so that the errors in the estimation of each parameter tend to cancel each other out when they're combined.
d) Guesstimate a value for the quantity assuming perfect control and assuming no control, and then take the square root of their product as the overall estimate.

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Introduction 2: Basic Concepts in Julia

Why We Created Julia

14 February 2012, Jeff Bezanson, Stefan Karpinski, Viral B. Shah, Alan Edelman

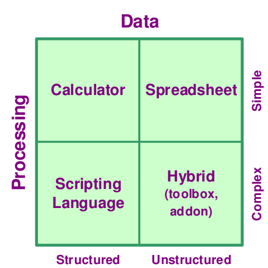
We are greedy: we want more.

We want a language that's open source, with a liberal license. We want the speed of C with the dynamism of Ruby. We want a language that's homoiconic, with true macros like Lisp, but with obvious, familiar mathematical notation like Matlab. We want something as usable for general programming as Python, as easy for statistics as R, as natural for string processing as Perl, as powerful for linear algebra as Matlab, as good at gluing programs together as the shell. Something that is dirt simple to learn, yet keeps the most serious hackers happy. We want it interactive and we want it compiled.

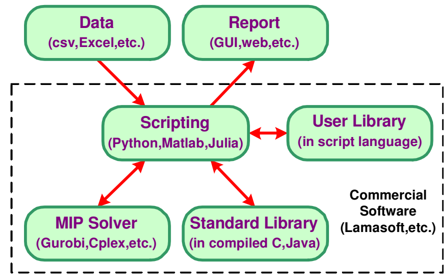
(Did we mention it should be as fast as C?)

Source: julialang.org/blog/2012/02/why-we-created-julia/

Computational Tools



Logistics Software Stack



- Julia (1.10) scripting language:
  - almost as fast as C and Java (but not FORTRAN)
  - does not require a standard library complied in C/C++/Java for speed (unlike Python, Matlab, and R)
  - uses multiple dispatch to make type-specific (fast) versions of same function
  - JuMP package with algebraic language (macros) to interface with MIP solvers

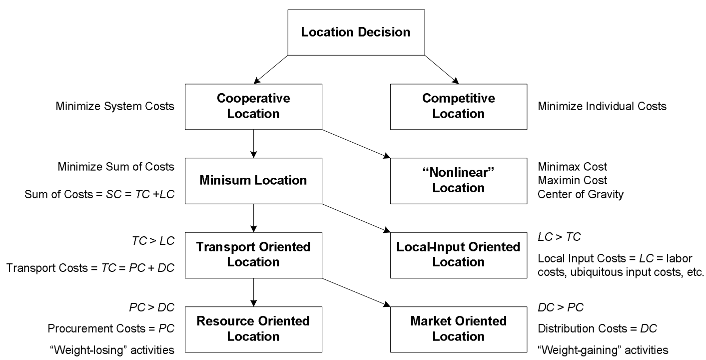
Location 1: Types of Location Problems

- For most private-industry-related applications, minimizing the sum of distances is the most appropriate objective for determining the optimal location
  - This is because transport cost (roughly speaking) increases directly proportional to distance (it's *linear*)
    - Truck drivers are paid by the mile
  - In many public or personal applications, costs increase faster than distance (they're *nonlinear*)
    - Most people would prefer 20 thirty-minute driving trips to one 10-hour trip

Why Are Cities Located Where They Are?

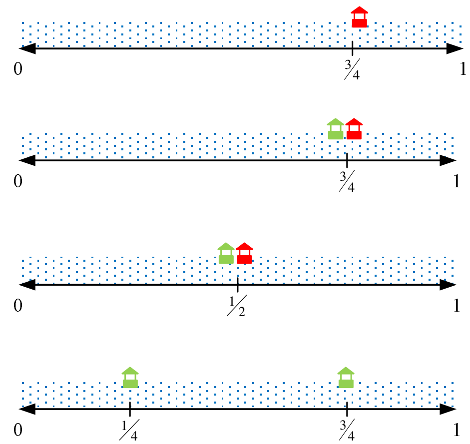
- Minimizing total logistics costs is often principle factor

Taxonomy of Location Problems

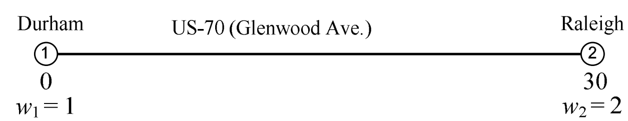


Location where cost of producing good is more than cost of distributing it, then have resource oriented location (think of production of metal ore…needs to be close to source of raw materials). Market orientation location has greater concern with distributing goods since production costs are not as high or transportation costs are high (think of distribution center for Amazon where its more beneficial to have your facility located closer to the market). [S. March]

Hotelling's Law



1-D Cooperative Location



$$\text{Min } TC = \sum w_i d_i$$

$$\text{Min } TC = \sum w_i d_i^2$$

$$a_1 = 0, \quad a_2 = 30$$

$$TC = \sum w_i d_i^2 = \sum w_i \left(x - a_i\right)^2$$

$$\frac{dTC}{dx} = 2\sum w_i \left(x - a_i\right) = 0 \Rightarrow$$

$$x\sum w_i = \sum w_i a_i \Rightarrow$$

Squared-Euclidean Distance ⇒ Center of Gravity: $x^* = \frac{\sum w_i a_i}{\sum w_i} = \frac{1(0) + 2(30)}{1+2} = 20$

Linear vs Nonlinear Location

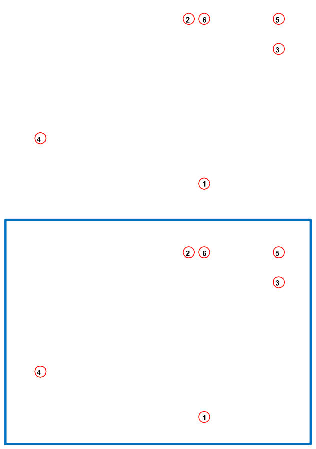
Linear: $\min \sum w_i d_i$ Nonlinear: $\begin{cases} \min \sum w_i d_i^2 & \text{center of gravity} \\ \min\{\max d_i\} & \text{minimax} \\ \max\{\min d_i\} & \text{maximin} \\ \min \sum k_i + w_i d_i & \text{fixed cost (affine)} \\ \min \sum w_i \sqrt{d_i} & \text{economy of scale} \end{cases}$

- Cost $\propto$ distance
- Private firm pays driver
- Easy to solve:
  - convex $\Rightarrow$ easy continuous location
  - LP $\Rightarrow$ easy discrete location

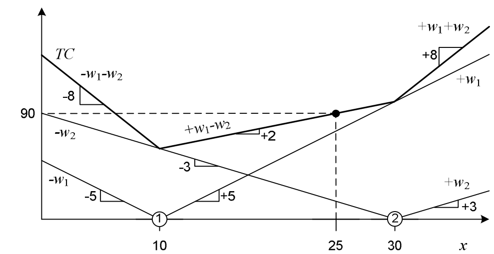
- Cost $\not\propto$ distance
- "Psychological cost"
  - Marchetti's constant: avg. commute 1 hr/day
- Public/personal
- More difficult to solve

Minimax and Maximin Location

- Minimax
  - Min max distance
  - Set covering problem
  - optimal point is usually halfway between two points that are furthest apart (points 4 and 5)
- Maximin
  - Max min distance
  - AKA obnoxious facility location



2-EF Minisum Location



$$TC(x) = \sum w_i d_i = \beta_1(x - x_1) + \beta_2(x - x_2), \quad \text{where } \beta_i = \begin{cases} w_i, & \text{if } x \ge x_i \\ -w_i, & \text{if } x < x_i \end{cases}$$

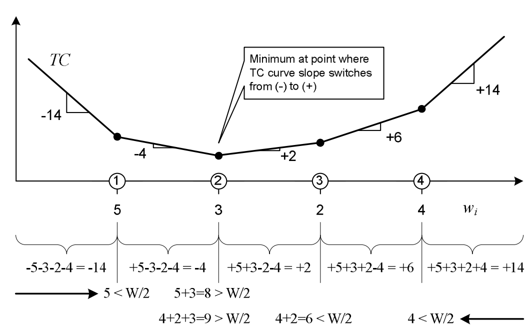
$$TC(25) = w_1(25-10) + (-w_2)(25-30)$$
$$= 5(15) + (-3)(-5) = 90$$

Note: This means of finding the optimal solution only works because *TC* is linear, need to use numerical optimization otherwise

Median Location: 1-D 4 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$



Median Location: 1-D 7 EFs

Median location: For each dimension x of X:

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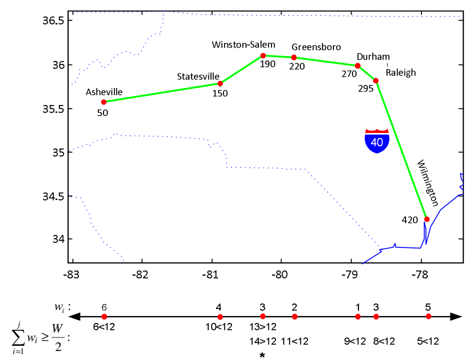
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$



Ex: 1-D Median Location

- Traveling north, I-95 passes through or near the following cities: Jacksonville, FL; Savannah, GA; Florence, SC; Lumberton, NC; Fayetteville, NC; Rocky Mount, NC; and Richmond, VA. A company wants to build a facility along I-95 to serve customers in these cities. If the weekly demand in truckloads of customers in each city is 12, 32, 6, 15, 24, 11, and 20, respectively, determine where the facility should be located to minimize the distance traveled to serve the customers assuming that I-95 will be used for all travel.

```
cities = ["Jacksonville, FL", "Savannah, GA", "Florence, SC", "Lumberton, NC",
          "Fayetteville, NC", "Rocky Mount, NC", "Richmond, VA"]
w = [12, 32, 6, 15, 24, 11, 20]
@show sum(w)
@show sum(w)/2
@show cumsum(w)
# Find index of city where cumulative demand first equals or exceeds half of total demand
idx = findfirst(x -> x >= sum(w)/2, cumsum(w))
```

```
sum(w) = 120
sum(w) / 2 = 60.0
cumsum(w) = [12, 44, 50, 65, 89, 100, 120]

4
```

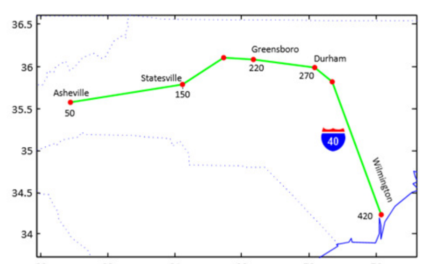
```
println("The optimal location for the facility is: ", cities[idx])
```

```
The optimal location for the facility is: Lumberton, NC
```


Question 2.1.1

Assuming 5, 15, 10, 10, and 20 trips per month are made to EFs in Asheville, Durham, Greensboro, Statesville, and Wilmington, respectively, determine the minisum location.

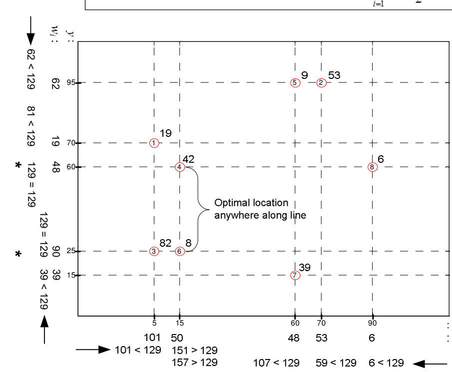
a) Asheville
b) Durham
c) Greensboro
d) Statesville
e) Wilmington



Median Location: 2-D Rectilinear Distance 8 EFs

Median location: For each dimension x of X:

1. Order EFs so that $|x_1| \le |x_2| \le \cdots \le |x_m|$
2. Locate x-dimension of NF at the first EFj where $\sum_{i=1}^{j} w_i \ge \frac{W}{2}$, where $W = \sum_{i=1}^{m} w_i$

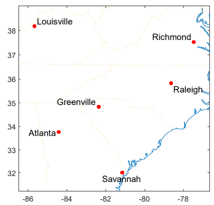


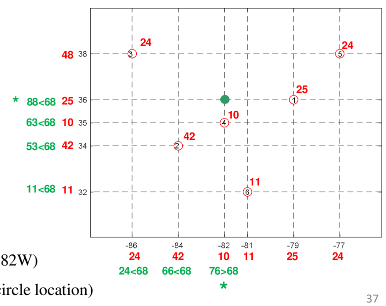
$$d_1(P_1, P_2) = |x_1 - x_2| + |y_1 - y_2|$$

$$d_2(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Ex: 2D Loc with Rect Approx to GC Dist

- It is expected that 25, 42, 24, 10, 24, and 11 truckloads will be shipped each year from your DC to six customers located in Raleigh, NC (36N,79W), Atlanta, GA (34N,84W), Louisville, KY (38N,86W), Greenville, SC (35N, 82W), Richmond, VA (38N,77W), and Savannah, GA (32N,81W). Assuming that all distances are rectilinear, where should the DC be located in order to minimize outbound transportation costs?



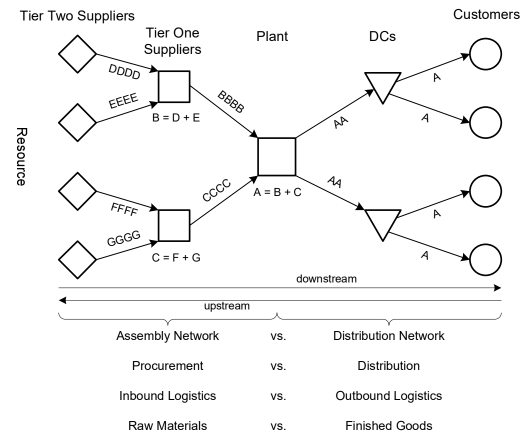


$W = \sum w_i = 136, \quad \frac{W}{2} = 68$

Answer: Optimal location (36N,82W)

(65 mi from opt great-circle location)

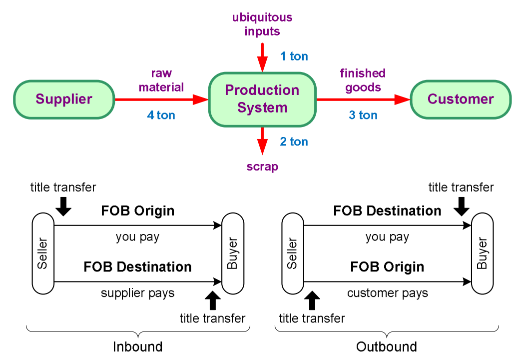
Logistics Network for a Plant



Location 2: Single-Facility Location

- **Monetary vs. physical weight:** A production process can be physically weight *losing* but monetarily weight *gaining*
- *Topic of this lecture:* What are the weights and how are they determined in a minisum weighted-distance location problem?

Basic Production System



FOB (free on board)

FOB and Location

- Choice of FOB terms (who directly pays for transport) usually does not impact location decisions:

$$\text{Procurement cost} = \text{Landed cost at supplier} + \color{red}{\text{Inbound transport cost}}$$

$$\text{Production cost} = \text{Procurement cost} + \text{Local resource cost (labor, etc.)}$$

$$\text{Total delivered cost} = \text{Production cost} + \color{red}{\text{Outbound transport cost}}$$

$$\text{Transport cost (TC)} = \text{Inbound transport cost} + \text{Outbound transport cost}$$

  - *Purchase price* from supplier and *sale price* to customer adjusted to reflect who is paying transport cost
  - Usually determined by who can provide the transport at the lowest cost
    - Savings in lower transport cost allocated (bargained) between parties

Monetary vs. Physical Weight

$$\min TC(X) = \sum_{i=1}^{m} w_i\, d(X,P_i) = \sum_{i=1}^{m} \underbrace{f_i\, r_i}_{w_i}\, d(X,P_i)$$

where
TC = total transport cost ($/yr)
w_i = monetary weight ($/mi-yr)
f_i = physical weight rate (ton/yr)
r_i = transport rate ($/ton-mi)
$d(X,P_i)$ = distance between NF at X and EF$_i$ at P_i (mi)
NF = new facility to be located
EF = existing facility
m = number of EFs

(Montetary) Weight Gaining:	$\Sigma w_{\text{in}} < \Sigma w_{\text{out}}$
Physically Weight Losing:	$\Sigma f_{\text{in}} > \Sigma f_{\text{out}}$

Minisum Location: TC vs. TD

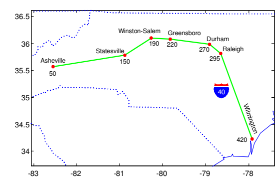
- Assuming local input costs are
  - same at every location or
  - insignificant as compared to transport costs,

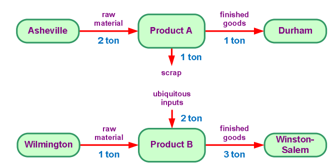
  the minisum transport-oriented single-facility location problem is to locate NF to minimize TC
- Can minimize total distance (TD) if transport rate is same:

$$\min TD(X) = \sum_{i=1}^{m} w_i\, d(X,P_i) = \sum_{i=1}^{m} \underbrace{f_i\, r_i}_{w_i}\, d(X,P_i)$$

where
TD = total transport distance (mi/yr)
w_i = monetary weight (trip/yr)
f_i = trips per year (trip/yr)
r_i = transport rate = 1
$d(X,P_i)$ = per-trip distance between NF and EF$_i$ (mi/trip)

Ex: Single Supplier and Customer Locations

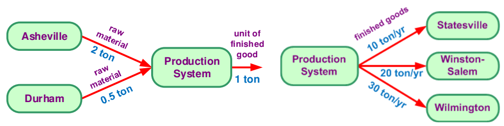




- The cost per ton-mile (i.e., the cost to ship one ton, one mile) for both raw materials and finished goods is the same (e.g., $0.10).
  1. Where should the plant for each product be located?
  2. How would location decision change if customers paid for distribution costs (FOB Origin) instead of the producer (FOB Destination)?
     - In particular, what would be the impact if there were competitors located along I-40 producing the same product?
  3. Which product is weight gaining and which is weight losing?
  4. If both products were produced in a single shared plant, why is it now necessary to know each product's annual demand (f_i)?

$$TC(X) = \sum_{i=1}^{m} \underbrace{f_i\, r_i}_{w_i}\, d(X,P_i)$$

Ex: 1-D Location with Procurement and Distribution Costs



Assume: all scrap is disposed of locally

A product is to be produced in a plant that will be located along I-40. Two tons of raw materials from a supplier in Ashville and a half ton of a raw material from a supplier in Durham are used to produce each ton of finished product that is shipped to customers in Statesville, Winston-Salem, and Wilmington. The demand of these customers is 10, 20, and 30 tons, respectively, and it costs \$0.33 per ton-mile to ship raw materials to the plant and \$1.00 per ton-mile to ship finished goods from the plant. Determine where the plant should be located so that procurement and distribution costs (i.e., transportation costs to and from the plant) are minimized, and whether the plant is weight gaining or weight losing.

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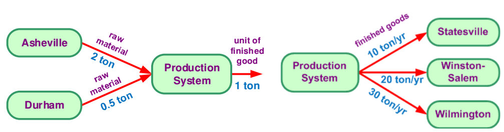
Ex: 1-D Location with Procurement and Distribution Costs



Assume: all scrap is disposed of locally

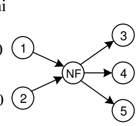
$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

$$\overbrace{\underset{(\$/\text{mi-yr})}{w_i}}^{\text{monetary weight}} = \overbrace{\underset{(\text{ton/yr})}{f_i}}^{\text{physical weight}} \times \underset{(\$/\text{ton-mi})}{r_i}$$

$r_{\text{in}} = \$0.33/\text{ton-mi}$ $\qquad r_{\text{out}} = \$1.00/\text{ton-mi}$

$f_1 = BOM_1 \sum f_{\text{out}} = 2(60) = 120, \quad w_1 = f_1 r_{\text{in}} = 40$

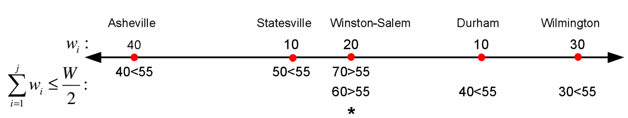
$f_2 = BOM_2 \sum f_{\text{out}} = 0.5(60) = 30, \quad w_2 = f_2 r_{\text{in}} = 10$



$f_3 = 10, \quad w_3 = f_3 r_{\text{out}} = 10$

$f_4 = 20, \quad w_4 = f_4 r_{\text{out}} = 20$

$f_5 = 30, \quad w_5 = f_5 r_{\text{out}} = 30$



(Montetary) Weight Gaining: $\Sigma w_{\text{in}} = 50 < \Sigma w_{\text{out}} = 60$

Physically Weight Losing: $\Sigma f_{\text{in}} = 150 > \Sigma f_{\text{out}} = 60$

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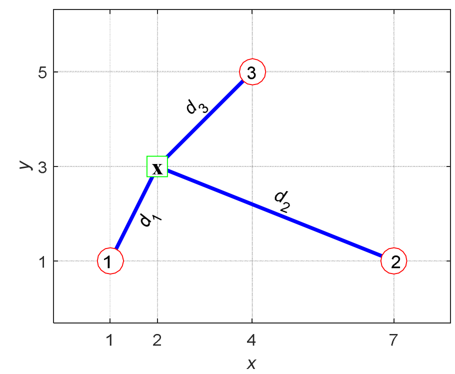
2-D Euclidean Distance



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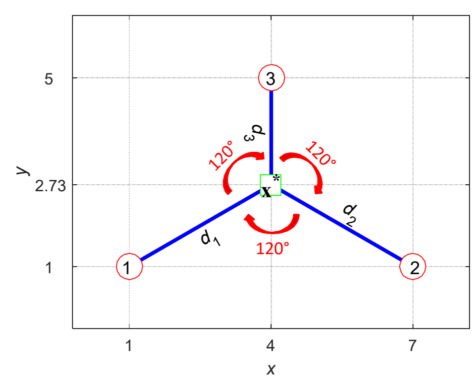
Minisum Distance Location



Fermat's Problem (1629):

Given three points, find fourth (Steiner point) such that sum to others is minimized

(Solution: Optimal location corresponds to all angles = 120°)

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Minisum Weighted-Distance Location

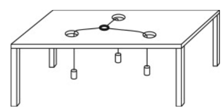
- Solution for 2-D+ and non-rectangular distances:
  - *Majority Theorem:* Locate NF at EF*j* if $w_j \geq \frac{W}{2}, \quad \text{where } W = \sum_{i=1}^{m} w_i$
  - Mechanical (Varigon frame)
  - 2-D rectangular approximation
  - Numerical: nonlinear unconstrained optimization
    - Analytical/estimated gradient (quasi-Newton)
    - Direct, gradient-free (Nelder-Mead)

$m = \text{number of EFs}$

$$TC(\mathbf{x}) = \sum_{i=1}^{m} w_i d_i(\mathbf{x})$$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$

$$TC^* = TC(\mathbf{x}^*)$$



Varignon Frame

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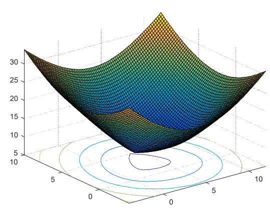
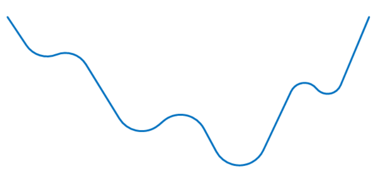
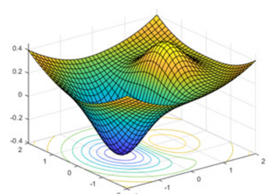
Convex vs Nonconvex Optimization







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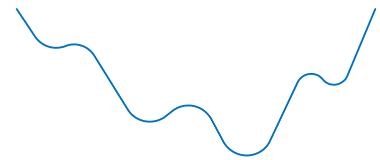
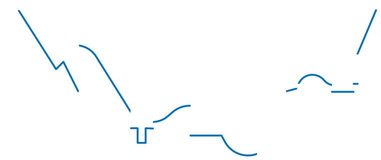
Gradient vs Direct Methods

- Numerical nonlinear unconstrained optimization:
  - Analytical/estimated gradient
    - quasi-Newton



  - Direct, gradient-free
    - Nelder-Mead



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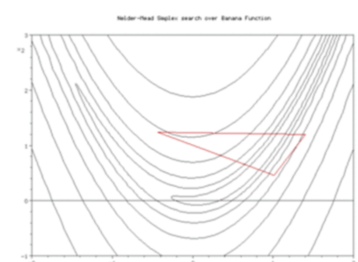
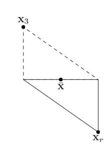
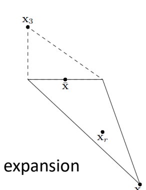
Nelder-Mead Simplex Method



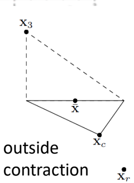


reflection

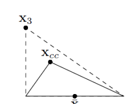


expansion

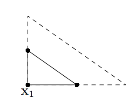
- AKA amoeba method
- Simplex is triangle in 2-D (dashed line in figures)



outside contraction



inside contraction



a shrink

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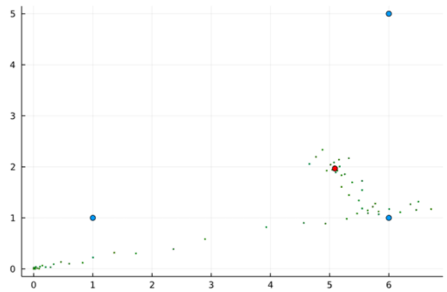
Ex: Nelder-Mead in Action

- Starting from point (0,0), the optimal point close to (5,2) is found by Nelder-Mead that minimizes the sum of the distances to the three points at (1,1), (6,1), and (6,5)
  - Each green point represent an evaluation of the objective function (each evaluation sums the distance from that point the three points)

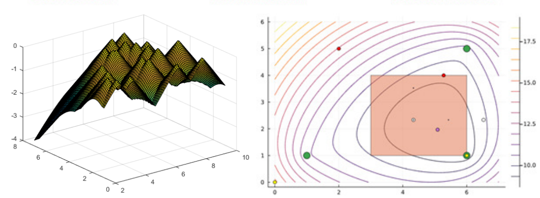


Location 3: Geocoding and Great-Circle Distances

- How can the distances between facility locations be determined?
  - Not computationally feasible to repeatedly calculate the actual road distances between locations.
  - Most continuous locations examined during Nelder-Mead procedure are not connected to a road

Picking a Starting Point

- Numerical nonlinear optimization techniques require specifying a starting point (`x0`): `optimize(x -> sum(d2.([x], pt)), x0)`
  - If convex or unimodal, any starting point will lead to the global optimum
  - Otherwise, different starting points can lead to different local optima



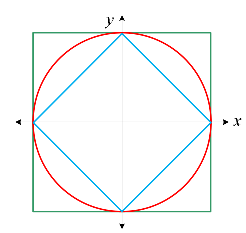
Metric Distances

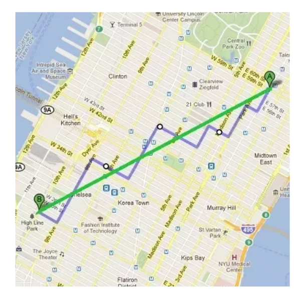
General l_p: $d_p(P_1,P_2)=\left[\left|x_1-x_2\right|^p+\left|y_1-y_2\right|^p\right]^{\frac{1}{p}}, \quad p\ge 1$

Rectilinear $(p=1)$: $d_1(P_1,P_2)=\left|x_1-x_2\right|+\left|y_1-y_2\right|$

Euclidean $(p=2)$: $d_2(P_1,P_2)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Chebychev $(p\to\infty)$: $d_\infty(P_1,P_2)=\max\left\{\left|x_1-x_2\right|,\left|y_1-y_2\right|\right\}$





Chebychev Distances

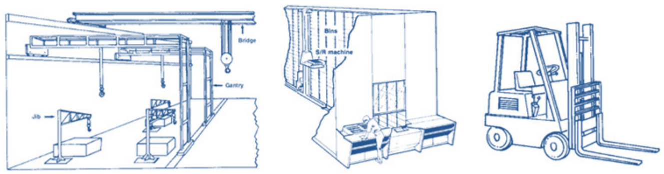
Proof

Without loss of generality, let $P_1 = (x, y)$, for $x, y \geq 0$, and $P_2 = (0, 0)$. Then $d_\infty(P_1,P_2) = \max\{x, y\}$ and $d_p(P_1,P_2) = \left[x^p + y^p\right]^{1/p}$.

If $x = y$, then $\lim\limits_{p\to\infty}\left[x^p + y^p\right]^{1/p} = \lim\limits_{p\to\infty}\left[2x^p\right]^{1/p} = \lim\limits_{p\to\infty}\left[2^{1/p}x\right] = x$.

If $x < y$, then $\lim\limits_{p\to\infty}\left[x^p + y^p\right]^{1/p} = \lim\limits_{p\to\infty}\left[\left((x/y)^p + 1\right)y^p\right]^{1/p} = \lim\limits_{p\to\infty}\left((x/y)^p + 1\right)^{1/p} y = 1 \cdot y = y$.

A similar argument can be made if $x > y$. ■



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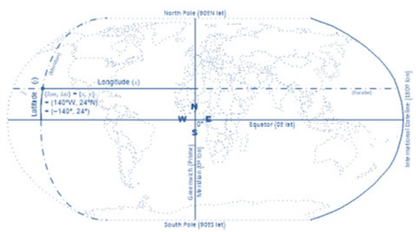
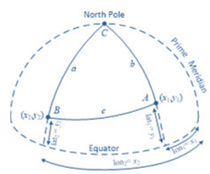
Great Circle Distances





$(lon_1, lat_1) = (x_1, y_1), \quad (lon_2, lat_2) = (x_2, y_2)$

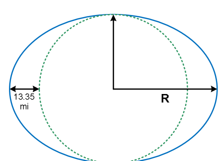
$d_{rad} = $ (great circle distance in radians of a sphere)

$= \cos^{-1}\left[\sin y_2 \sin y_1 + \cos y_2 \cos y_1 \cos(x_1 - x_2)\right]$

$R = $ (radius of earth at equator) − (bulge from north pole to equator)

$= 3{,}963.34 - 13.35 \sin\left(\dfrac{y_1 + y_2}{2}\right)$ mi, $\quad = 6{,}378.388 - 21.476 \sin\left(\dfrac{y_1 + y_2}{2}\right)$ km

$d_{GC} = $ distance (x_1, y_1) to $(x_2, y_2) = \boxed{d_{rad} \cdot R}$



$$x_{\deg} = \begin{cases} DD + \dfrac{MM}{60} + \dfrac{SS}{3{,}600}, & \text{if } E \text{ or } N \\ -DD - \dfrac{MM}{60} - \dfrac{SS}{3{,}600}, & \text{if } W \text{ or } S \end{cases}$$

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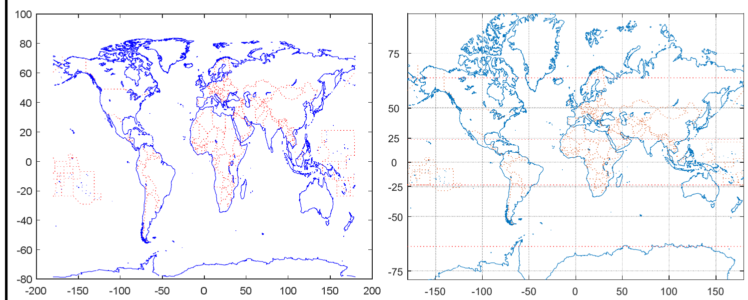
Mercator Projection



$x_{\text{rad}} = \dfrac{x_{\deg}}{180}\pi$ and $x_{\deg} = \dfrac{x_{\text{rad}} \cdot 180}{\pi}$

$x_{\text{proj}} = x$

$y_{\text{proj}} = \sinh^{-1}(\tan y)$

$y = \tan^{-1}(\sinh y_{\text{proj}})$

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Location 4: Allocation and ALA

- When determining the location of NFs used for distribution (DCs):
  - Each EF (customer) is usually served by only one DC
  - Allocation of EFs to a DC is based primarily on DC's location
  - Requires solving both an allocation and location problem

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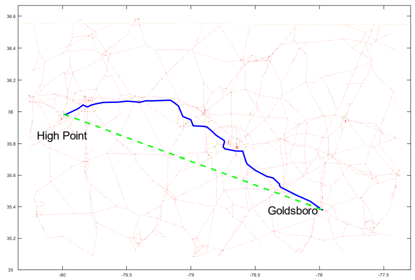
Circuity Factor

$$\textit{Circuity Factor}: \quad g = \frac{1}{n}\sum \frac{d_{\text{road}_i}}{d_{GC_i}}, \quad \text{given } n \text{ samples, where usually } 1.1 \le g \le 1.5$$

$$d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2), \quad \text{estimated road distance from } P_1 \text{ to } P_2$$

From High Point to Goldsboro: Road = 143 mi, Great Circle = 121 mi, Circuity = 1.19



Estimating Circuity Factors

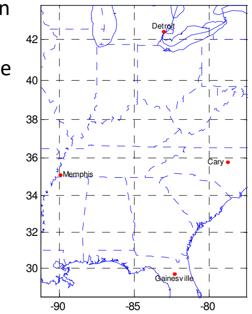
- Circuity factor depends on both the trip density and directness of travel network
  - Good default value for road travel is 1.2
  - Circuity factor of high density areas usually lower because there are more direct roads
  - Should use actual road network, not an estimated circuity factor, if
    - "Few" distances needed (just use Google Maps)
    - Short distances, since there are less direct roads
    - Obstacles (water, mountains) limit direct road travel
  - Circuity factors for rail travel are higher than road travel due to less dense network
  - Note: just 5-10 road sample pairs are needed to provide a reasonable estimate of circuity as long as samples independent (don't overlap)
    - This is because the overestimates tend to cancel the underestimates

Circuity Factors and Location

- *HW 3:* Find NF location for EFs at Detroit, Gainesville, and Memphis, then determine increase in *TC* if NF instead in Cary
- Since a circuity factor just multiplies distances by the same constant amount, how does it affect the location decision?
  - Does not impact the actual location found
  - Does impact TC since transport rate (r) is in \$/ton-mi
    - $\Rightarrow$ can increase the benefit/cost associated with using a good/bad location

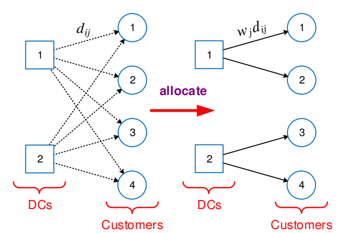
$$TC(X) = \sum_{i=1}^{m} \underbrace{f_i r_i}_{w_i} d(X, P_i)$$

$$d(X, P_i) = d_{\text{road}} \approx g \cdot d_{GC}(P_1, P_2)$$



Allocation

- **Example:** given n DCs and m customers, with customer j receiving w_j TLs per week, determine the total distance per week assuming each customer is served by its *closest* DC



$$w = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$$

$$D = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 45 & 35 & 25 & 15 \end{bmatrix}$$

$$TD = 2(10) + 4(20) + 6(25) + 8(15)$$
$$= 370$$

Pseudocode

- Different ways of representing how allocation and TD can be calculated
  - High-level pseudocode most concise, but leaves out many implementation details (sets don't specify order, initial starting points)
  - Low-level pseudocode includes more implementation details, which can hide/obscure the core idea, and are usually not essential

Low-level Pseudocode

```
TD = 0
for j = 1:m
   dj = D(1,j)
   for i = 2:n
      if D(i,j) < dj
         dj = D(i,j)
      end
   end
   TD = TD + w(j)*dj
end
```

High-level Pseudocode

$$N = \{1,\ldots,n\}, \quad n = |N|$$

$$M = \{1,\ldots,m\}, \quad m = |M|$$

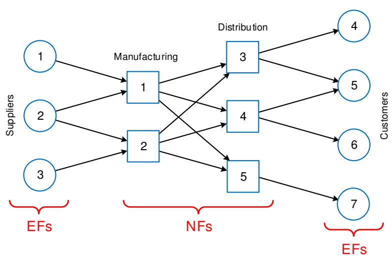
$$\alpha = [\alpha_j] = \arg\min_{i \in N} d_{ij}$$

$$TD = \sum_{j \in M} w_j d_{\alpha_j, j}$$

Julia

```
α = [argmin(c) for c in eachcol(D)]
W = sparse(α, 1:m, w, n, m)
TD = sum(W .* D)
```


Minisum Multifacility Location



$n =$ no. of NFs, $\quad m =$ no. of EFs

$\mathbf{X}_{n\times d} =$ NF locations, $\quad \mathbf{P}_{m\times d} =$ EF locations

$$\mathbf{V}_{n\times n} = \begin{array}{c|ccccc} \text{NF-NF} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & + & + & + \\ 2 & 0 & 0 & + & + & + \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

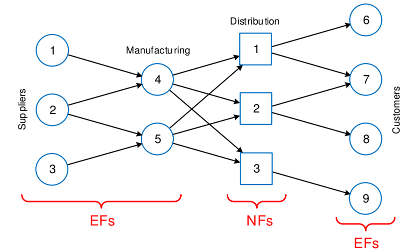
$$\mathbf{W}_{n\times m} = \begin{array}{c|ccccccc} \text{NF-EF} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & + & + & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & + & + & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & + & + & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & + & + & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & + \end{array}$$

$$TC(\mathbf{X}) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(\mathbf{X}_j, \mathbf{X}_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} TC(\mathbf{X})$$

$$TC^* = TC(\mathbf{X}^*)$$

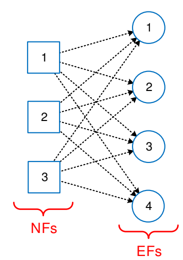
Multiple Single-Facility Location



$$TC(\mathbf{X}) = \sum_{j=1}^{n}\sum_{k=1}^{n} v_{jk} d(\mathbf{X}_j, \mathbf{X}_k) + \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$= \sum_{j=1}^{n} TC(\mathbf{X}_j)$$

Facility Location–Allocation Problem



- Determine both the location of *n* NFs and the allocation of flow requirements of *m* EFs that minimize TC

$$w_{ji} = r_{ji} f_{ji} = (1) f_{ji} = \text{flow between NF}j \text{ and EF}i$$

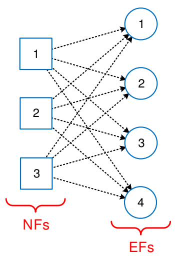
$$w_i = \text{total flow requirememts of EF}i$$

$$TC(\mathbf{X}, \mathbf{W}) = \sum_{j=1}^{n}\sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$\mathbf{X}^*, \mathbf{W}^* = \arg\min_{\mathbf{X},\mathbf{W}} \left\{ TC(\mathbf{X}, \mathbf{W}) : \sum_{j=1}^{n} w_{ji} = w_i,\ w_{ji} \ge 0 \right\}$$

$$TC^* = TC(\mathbf{X}^*, \mathbf{W}^*)$$

Integrated Formulation



- If there are no capacity constraints on NFs, it is optimal to always satisfy all the flow requirements of an EF from its closest NF
- Requires search of (n x d)-dimensional TC that combines location with allocation

$$\alpha_i(\mathbf{X}) = \arg\min_j d(\mathbf{X}_j, \mathbf{P}_i)$$

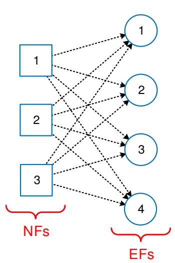
$$TC(\mathbf{X}) = \sum_{i=1}^{m} w_i d(\mathbf{X}_{\alpha_i(\mathbf{X})}, \mathbf{P}_i)$$

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} TC(\mathbf{X})$$

$$TC^* = TC(\mathbf{X}^*)$$

```
function TCint(X)
    D = Dgc(X, P)
    α = [argmin(c) for c in eachcol(D)]
    n, m = size(D)
    return sum(sparse(α, 1:m, w, n, m) .* D)
end
```


Alternating Formulation



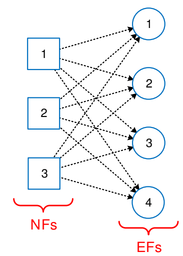
- Alternate between finding locations and finding allocations until no further TC improvement
- Requires n d-dimensional location searches together with separate allocation procedure
- Separating location from allocation allows other types of location and/or allocation procedures to be used:
  - Allocation with NF with capacity constraints (solved as minimum cost network flow problem)
  - Location with some NFs at fixed locations

$$allocate(\mathbf{X}) = \left[w_{ji}\right] = \begin{cases} w_i, & \text{if } \arg\min_k d(\mathbf{X}_k, \mathbf{P}_i) = j \\ 0, & \text{otherwise} \end{cases}$$

$$TC(\mathbf{X}, \mathbf{W}) = \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ji} d(\mathbf{X}_j, \mathbf{P}_i)$$

$$locate(\mathbf{W}, \mathbf{X}) = \arg\min_{\mathbf{X}} TC(\mathbf{X}, \mathbf{W})$$

ALA: Alternate Location–Allocation



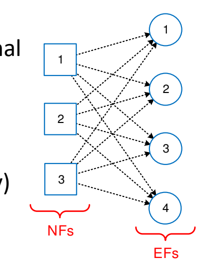
procedure $ala(\mathbf{X})$
$TC \leftarrow \infty$, $done \leftarrow$ false
repeat
  $\mathbf{W}' \leftarrow allocate(\mathbf{X})$
  $\mathbf{X}' \leftarrow locate(\mathbf{W}', \mathbf{X})$
  $TC' \leftarrow TC(\mathbf{X}', \mathbf{W}')$
  if $TC' < TC$
    $TC \leftarrow TC'$, $\mathbf{X} \leftarrow \mathbf{X}'$, $\mathbf{W} \leftarrow \mathbf{W}'$
  else
    $done \leftarrow$ true
  endif
until $done$ = true
return $\mathbf{X}$, $\mathbf{W}$

```
function ala(X°)
    TC°, done = Inf, false
    while !done
        W = alloc(X°)
        X′ = loc(W, X°)
        TC′ = TC(W, X′)
        println(TC′)
        if TC′ < TC°
            TC°, X° = TC′, X′
        else
            done = true
        end
    end
    return X°, TC°
end
```

- Edge case: What if a NF is not allocated to any EFs?
  - Can happen if initial NF locations are chosen randomly
  - One way to handle: Randomly relocate unallocated NFs to EFs

Integrated vs. Alternate Formulations

- Both only give a local optimal solution (not convex)
- Alternate more flexible and can be faster
  - solving n d-dimensional location problems and simple allocation
    - ⇒ Nelder-Mead works well for 2-D
  - Integrated solving an (n x d)-dimensional problem
    - can be more computationally difficult
- When might integrated be better?
  - if there are no allocation (e.g., capacity) or location constraints on the NFs



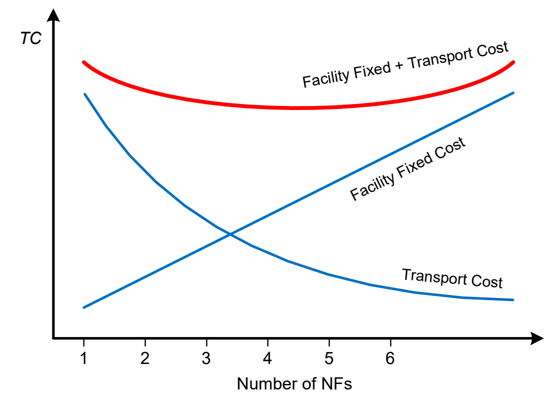
Best Retail Warehouse Locations

Number of Locations	Average Transit Time (days)	Warehouse Location		
1	2.20	Bloomington, IN		
2	1.48	Ashland, KY	Palmdale, CA	
3	1.29	Allentown, PA	Palmdale, CA	McKenzie, TN
4	1.20	Edison, NJ	Palmdale; CA	Chicago, IL
		Meridian, MS		
5	1.13	Madison, NJ	Palmdale, CA	Chicago, IL
		Dallas, TX	Macon, GA	
6	1.08	Madison, NJ	Pasadena, CA	Chicago, IL
		Dallas, TX	Macon, GA	Tacoma, WA
7	1.07	Madison, NJ	Pasadena, CA	Chicago, IL
		Dallas, TX	Gainesville, GA	Tacoma, WA
		Lakeland, FL		
8	1.05	Madison, NJ	Pasadena, CA	Chicago, IL
		Dallas, TX	Gainesville, GA	Tacoma, WA
		Lakeland, FL	Denver, CO	
9	1.04	Madison, NJ	Alhambra, CA	Chicago, IL
		Dallas, TX	Gainesville, GA	Tacoma, WA
		Lakeland. FL	Denver, CO	Oakland, CA
10	1.04	Newark, NJ	Alhambra, CA	Rockford, IL
		Palistine, TX	Gainesville, GA	Tacoma, WA
		Lakeland, FL	Denver, CO	Oakland. CA
		Mansfield, OH		

Location 5:
UFL Heuristics

- Unlike the ALA, the *Uncapacitated Facility Location* (UFL) problem determines both the number of NFs and their locations
- Heuristics for the UFL usually provide a solution that is within ± 3% of the optimal
  - Optimal determined using MILP formulation of the UFL
  - Since data used for instances has significant errors, heuristics effectively solve the problem
  - Similar to ALA, MILP formulation is only needed if additional constraints are added to the problem

Optimal Number of NFs



Uncapacitated Facility Location (UFL)

- NFs can only be located at discrete set of sites
  - Allows inclusion of fixed cost of locating NF at site $\Rightarrow$ opt number NFs
  - Variable costs are usually transport cost from NF to/from EF
  - Total of $2^n - 1$ potential solutions (all nonempty subsets of sites)

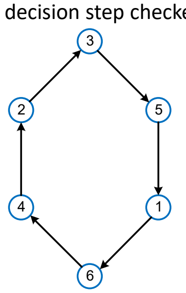
$$M = \{1,\dots,m\}, \quad \text{existing facilites (EFs)}$$
$$N = \{1,\dots,n\}, \quad \text{sites available to locate NFs}$$
$$M_i \subseteq M, \quad \text{set of EFs served by NF at site } i$$
$$c_{ij} = \text{variable cost to serve EF } j \text{ from NF at site } i$$
$$k_i = \text{fixed cost of locating NF at site } i$$
$$Y \subseteq N, \quad \text{sites at which NFs are located}$$
$$Y^* = \arg\min_Y \left\{ \sum_{i\in Y} k_i + \sum_{i\in Y} \sum_{j\in M_i} c_{ij} : \bigcup_{i\in Y} M_i = M \right\}$$
$$= \text{min cost set of sites where NFs located}$$
$$\left|Y^*\right| = \text{number of NFs located}$$

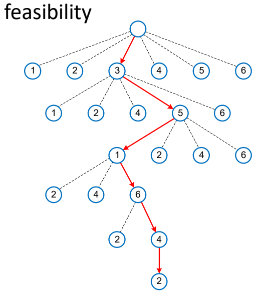
Heuristic Solutions

- Most problems in logistics engineering don't admit optimal solutions, only
  - Within some bound of optimal (provable bound, opt. gap)
  - Best known solution (stop when need to have solution)
- Heuristics - computational effort split between
  - Construction: construct a feasible solution
  - Improvement: find a better feasible solution
- Easy construction:
  - any random point or permutation is feasible
  - can then be improved ⇒ *construct-then-improve* multiple times
- Hard construction:
  - almost no chance of generating a random feasible solution due to constraints on what is a feasible solution
  - need to include randomness at decision points as solution is generated in order to construct multiple different solutions (which "might" then be able to be improved)

Heuristic Construction Procedures

- Easy construction:
  - any random permutation is feasible and can then be improved
- Hard construction:
  - almost no chance of generating a random solution in a single step that is feasible, need to include randomness at decision points as a solution is constructed
  - each decision step checked for feasibility

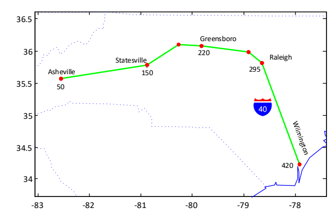




UFL Solution Techniques

- Being uncapacitated allows simple heuristics to be used to solve
  - ADD construction: add one NF at a time
  - DROP construction: drop one NF at a time
  - XCHG improvement: move one NF at a time to unoccupied sites
  - HYBRID algorithm combination of ADD and DROP construction with XCHG improvement, repeating until no change in Y
    - Use as default heuristic for UFL
    - See Daskin [2013] for more details
- UFL can be solved as a MILP
  - Easy MILP, LP relaxation usually optimal (for strong formulation)
  - MILP formulation allows constraints to easily be added
    - e.g., capacitated facility location, fixed number of NFs, some NF at fixed location
  - Will model UFL as MILP mainly to introduce MILP, will use UFL HYBRID algorithm to solve most problems

Ex: UFL ADD



$k = [150 \quad 200 \quad 150 \quad 150 \quad 200]$

$c_{ij} = w_j d_{ij} = f_j r d_{ij} = (1)(1) d_{ij} = d_{ij}$

	c_{ij}	1	2	3	4	5
Asheville:	1	0	100	170	245	370
Statesville:	2	100	0	70	145	270
Greensboro:	3	170	70	0	75	200
Raleigh:	4	245	145	75	0	125
Wilmington:	5	370	270	200	125	0

$Y = \{\ \}$

Y	1	2	3	4	5	c_{Yj}	k_Y	$c_{Yj} + k_Y$
1	0	100	170	245	370	885	150	1,035
2	100	0	70	145	270	585	200	785
3	170	70	0	75	200	515	150	665
4	245	145	75	0	125	590	150	740
5	370	270	200	125	0	965	200	1,165

$Y = \{3\}$

Y	1	2	3	4	5	c_{Yj}	k_Y	$c_{Yj} + k_Y$
3,1	0	70	0	75	200	345	300	645
3,2	100	0	0	75	200	375	350	725
3,4	170	70	0	0	125	365	300	665
3,5	170	70	0	75	0	315	350	665

$Y = \{3,1\}$

Y	1	2	3	4	5	c_{Yj}	k_Y	$c_{Yj} + k_Y$
3,1,2	0	0	0	75	200	275	500	775
3,1,4	0	70	0	0	125	195	450	645
3,1,5	0	70	0	75	0	145	500	645

$Y^* = \{3,1\}$

UFLADD: Add Construction Procedure

procedure $ufladd(\mathbf{k}, \mathbf{C})$
$Y \leftarrow \{\}$
$TC \leftarrow \infty$, $done \leftarrow$ false
repeat
  $TC' \leftarrow \infty$
  for $i' \in \{1,\dots,n\} \setminus Y$
    $TC'' \leftarrow \sum_{h \in Y \cup i'} k_h + \sum_{j=1}^{m} \min_{h \in Y \cup i'} c_{hj}$
    if $TC'' < TC'$
      $TC' \leftarrow TC''$, $i \leftarrow i'$
    endif
  endfor
  if $TC' < TC$
    $TC \leftarrow TC'$, $Y \leftarrow Y \cup i$
  else
    $done \leftarrow$ true
  endif
until $done =$ true
return Y, TC

```
function ufladd(k, C)
    fTC(y) = sum(k[y]) + sum(minimum(C[y, :], dims=1))
    y = Int[]
    TC°, done = Inf, false
    while !done
        TC, i = Inf, nothing                  # Stops if y = all NF
        for i′ = setdiff(1:size(C, 1), y)  # since i′ = []
            TC′ = fTC(vcat(y, i′))
            if TC′ < TC
                TC, i = TC′, i′
            end
        end
        if TC < TC°                         # TC = Inf if y = all NF
            TC°, y = TC, push!(y, i)
        else
            done = true
        end
    end
    return y, TC°
end
```

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Modified UFLADD

procedure $ufladd(\mathbf{k}, \mathbf{C}, \boxed{Y, p})$
$\cancel{Y \leftarrow \{\}}$
$TC \leftarrow \infty$, $done \leftarrow$ false
repeat
  $TC' \leftarrow \infty$
  for $i' \in \{1,\dots,n\} \setminus Y$
    $TC'' \leftarrow \sum_{h \in Y \cup i'} k_h + \sum_{j=1}^{m} \min_{h \in Y \cup i'} c_{hj}$
    if $TC'' < TC'$
      $TC' \leftarrow TC''$, $i \leftarrow i'$
    endif
  endfor
  if $\boxed{(p = \{\} \text{ and } TC' < TC) \text{ or } (p \neq \{\} \text{ and } |Y| < p)}$
    $TC \leftarrow TC'$, $Y \leftarrow Y \cup i$
  else
    $done \leftarrow$ true
  endif
until $done =$ true
return Y, TC

- *Y* input can be used to start UFLADD with *Y* NFs
  - Used in hybrid heuristic
- *p* input can be used to keep adding until number of NFs = *p*
  - Used in p-median heuristic

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UFLXCHG: Exchange Improvement Procedure

procedure $uflxchg(\mathbf{k}, \mathbf{C}, Y)$
$TC \leftarrow \sum_{i \in Y} k_i + \sum_{j=1}^{m} \min_{i \in Y} c_{ij}$
$TC' \leftarrow \infty$, $done \leftarrow$ false
while $|y| > 1$ and $done =$ false
  for $i' \in y$
    for $j' \in \{1,\dots,n\} \setminus Y$
      $Y' \leftarrow Y \setminus i' \cup j'$
      $TC'' \leftarrow \sum_{i \in Y'} k_i + \sum_{j=1}^{m} \min_{i \in Y'} c_{ij}$
      if $TC'' < TC'$
        $TC' \leftarrow TC''$, $i \leftarrow i'$, $j \leftarrow j'$
      endif
    endfor
  endfor
  if $TC' < TC$
    $TC \leftarrow TC'$, $Y \leftarrow Y \setminus i \cup j$
  else
    $done \leftarrow$ true
  endif
endwhile
return Y, TC

```
function uflxchg(k, C, y::Vector{Int})
    if k isa Number
        k = fill(k, size(C, 1))
    end
    fTC(y) = sum(k[y]) + sum(minimum(C[y, :], dims=1))
    N = 1:size(C, 1)
    TC° = fTC(y)
    done = false
    while length(y) > 1 && !done      # No exchange if 1 NF
        TC, i, j = Inf, nothing, nothing
        for i′ in  y
            for j′ in setdiff(N, y)
                swap!(y, i′, j′)      # Swap i' in y with j'
                TC′ = fTC(y)
                if TC′ < TC
                    TC, i, j = TC′, i′, j′
                end
                revert!(y, i′, j′)   # Restore original y
            end
        end
        if TC < TC°
            TC° = TC
            swap!(y, i, j)
        else
            done = true
        end
    end
    return y, TC°
end
```

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UFL: Hybrid Algorithm

procedure $ufl(\mathbf{k}, \mathbf{C})$
$Y', TC' \leftarrow ufladd(\mathbf{k}, \mathbf{C})$
$done \leftarrow$ false
repeat
  $Y, TC \leftarrow uflxchg(\mathbf{k}, \mathbf{C}, Y')$
  if $Y \neq Y'$
    $Y', TC' \leftarrow ufladd(\mathbf{k}, \mathbf{C}, Y)$
    $Y'', TC'' \leftarrow ufldrop(\mathbf{k}, \mathbf{C}, Y)$
    if $TC'' < TC'$
      $TC' \leftarrow TC''$, $Y' \leftarrow Y''$
    endif
    if $TC' \geq TC$
      $done \leftarrow$ true
    endif
  else
    $done \leftarrow$ true
  endif
until $done =$ true
return Y, TC

```
function ufl(k, C)
    y′, TC′ = ufladd(k, C)
    y, TC = y′, TC′
    done = false
    while !done
        y, TC = uflxchg(k, C, y′)
        if Set(y) !== Set(y′)
            y′, TC′ = ufladd(k, C, y)
            y′′, TC′′ = ufldrop(k, C, y)
            if TC′′ < TC′
                y′, TC′ = y′′, TC′′
            end
            if TC′ >= TC
                done = true
            end
        else
            done = true
        end
    end
    return y, TC
end
```

Note: Drop starts from *y*

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P-Median Location Problem

- Similar to UFL, except
  - Number of NF has to equal *p* (discrete version of ALA)
  - No fixed costs

$p = \text{number of NFs}$

$$Y^* = \arg\min_Y \left\{ \sum_{i \in Y} \sum_{j \in M_i} c_{ij} : \bigcup_{i \in Y} M_i = M, |Y| = p \right\}$$

  - Since usually *n* » *p*, only UFLADD used to construct the solution
    - Starting from *n* NFs, UFLDROP would take too long
    - Same is true for UFL Hybrid

```
function pmedian(p, C)
    y = ufladd(0, C, p = p)[1]
    return uflxchg(0, C, y)
end
```


Bottom-Up vs Top-Down Analysis

- Bottom-Up: HW3 Q2

$$\boxed{r \begin{array}{l} \nearrow TC_{\text{NEW}_1} \\ \searrow TC_{\text{NEW}_2} \end{array}}$$

$$\mathbf{P}_{3\times 2} = \text{lon-lat of EFs}$$
$$\mathbf{f} = [48, 24, 35] \quad \text{(TL/yr)}$$
$$r = 2 \quad \text{(\$/TL-mi)}$$
$$g = \frac{1}{3}\left[\frac{d_{RD}(\mathbf{P}_1, \mathbf{P}_2)}{d_{GC}(\mathbf{P}_1, \mathbf{P}_2)} + \frac{d_{RD}(\mathbf{P}_2, \mathbf{P}_3)}{d_{GC}(\mathbf{P}_2, \mathbf{P}_3)} + \frac{d_{RD}(\mathbf{P}_3, \mathbf{P}_1)}{d_{GC}(\mathbf{P}_3, \mathbf{P}_1)} \right]$$
$$TC(\mathbf{x}) = \sum_{i=1}^{3} f_i \, r \, g \, d_{GC}(\mathbf{x}, \mathbf{P}_i) \quad \text{(outbound trans. costs)}$$
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$
$$TC^* = TC(\mathbf{x}^*)$$
$$\mathbf{x}^{\text{cary}} = \text{lon-lat of Cary}$$
$$TC^{\text{cary}} = TC(\mathbf{x}^{\text{cary}})$$
$$\Delta TC = TC^{\text{cary}} - TC^*$$

- Top-Down: estimate *r* (circuity factor cancels, so not needed, see HW 4 Q2)

$$\boxed{TC_{\text{OLD}} \rightarrow r_{\text{nom}} \rightarrow TC_{\text{NEW}}}$$

$$TC^{\text{cary}} = \text{current known } TC$$
$$10 \text{ ton/TL} = \text{known tons per truckload}$$
$$\mathbf{f} = [480, 240, 350] \quad \text{(ton/yr)}$$
$$r_{\text{nom}} = \frac{TC^{\text{cary}}}{\sum_{i=1}^{3} f_i \, d_{GC}(\mathbf{x}^{\text{cary}}, \mathbf{P}_i)} \quad \text{(\$/ton-mi)}$$
$$TC(\mathbf{x}) = \sum_{i=1}^{3} f_i \, r_{\text{nom}} d_{GC}(\mathbf{x}, \mathbf{P}_i)$$
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} TC(\mathbf{x})$$
$$TC^* = TC(\mathbf{x}^*)$$
$$\Delta TC = TC^{\text{cary}} - TC^*$$

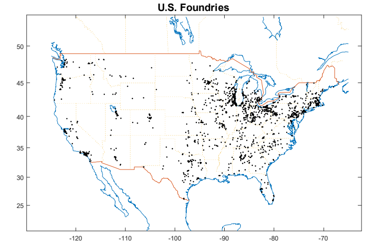
Location 6: Aggregate Demand

- Locating an NF at the center of population of a region does not reduce the travel distance between the NF and the population to zero
  - Need to estimate the average distance from the center to the region
  - Will assume the population is uniformly distributed over a region
  - Useful for retail distribution where location of each EF is not known

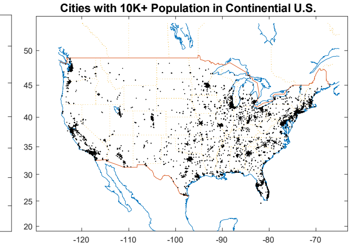
Actual vs Population-Inferred Demand

- Actual demand:
  - Know location of each customer along with their demand
    - Example: U.S. foundries, concentrated in Great Lakes
- Population-inferred demand:
  - Assume demand proportional to geographical dispersion of population
    - Example: Any logistics network for retail

U.S. Foundries

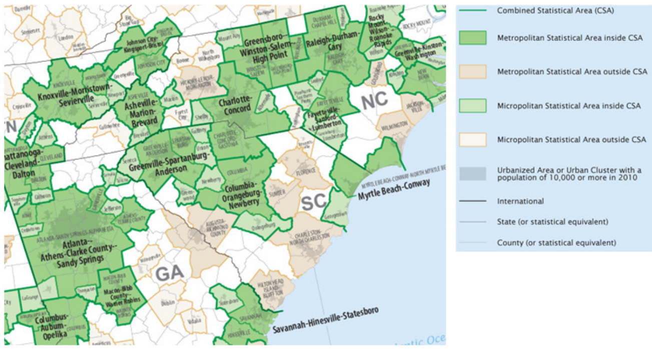


Cities with 10K+ Population in Continental U.S.



U.S. Geographic Statistical Areas

- Defined by Office of Management and Budget (OMB)
  - Each consists of one or more counties



Aggregate Demand Point Data Sources

- Aggregate demand point: centroid of population + area + population
- Good rule of thumb: use at least 10x number of NFs (≈ 100 pts provides minimum coverage for locating ≈ 10 NFs)

1. City data: ONLY USE FOR LABELING!, not as aggregate demand points
2. 3-digit ZIP codes: ≈ 1000 pts covering U.S., = 20 pts NC
3. County data: ≈ 3000 pts covering U.S., = 100 pts NC
   - Grouped by state or CBSA
   - CBSA (Core-Based Statistical Area) defined by set of counties (918 in U.S.)
   - CSA (Combined Statistical Area) defined by set of CBSAs (180 in U.S.)
   - FIPS code = 5-digit state-county FIPS code
     = 2-digit state code + 3-digit county code
     = 37183 = 37 NC FIPS + 183 Wake FIPS
4. 5-digit ZIP codes: > 35K pts U.S., ≈ 1000 pts NC
5. Census Tract: > 84K pts U.S., ≈ 2700 pts NC
6. Census Block Group: > 240K pts U.S., ≈ 1000 pts Raleigh-Durham-Cary, NC CSA
   - Grouped by state, county, CBSA, or CSA
   - Finest resolution aggregate demand data source
     - Each group is composed of several census blocks, but blocks don't have area info

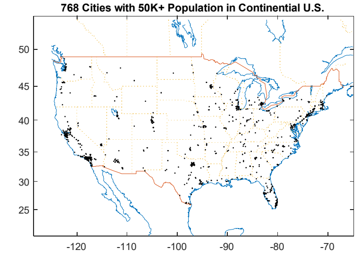
City vs CSA Population Data

Rank	City; State	2010 population	2012 population
1	New York City; New York	8,175,133	**8,336,697**
2	Los Angeles; California	3,792,621	**3,857,799**
3	Chicago; Illinois	2,695,598	**2,714,856**
4	Houston; Texas	2,099,451	**2,160,821**
5	Philadelphia; Pennsylvania	1,526,006	**1,547,607**
6	Phoenix; Arizona	1,445,632	**1,488,750**
7	San Antonio; Texas	1,327,407	**1,382,951**
8	San Diego; California	1,307,402	**1,338,348**
9	Dallas; Texas	1,197,816	**1,241,162**
10	San Jose; California	945,942	**982,765**
11	Austin; Texas	790,390	**842,592**
12	Jacksonville; Florida	821,784	**836,507**
13	Indianapolis; Indiana	820,445	**834,852**
14	San Francisco; California	805,235	**825,863**
15	Columbus; Ohio	787,033	**809,798**
16	Fort Worth; Texas	741,206	**777,992**
17	Charlotte; North Carolina	731,424	**775,202**
18	Detroit; Michigan	713,777	**701,475**
19	El Paso; Texas	649,121	**672,538**
20	Memphis; Tennessee	646,889	**655,155**

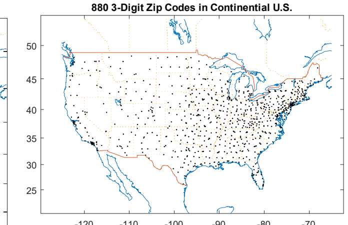
Metropolitan Area	2010 Population	City
New York-Northern NJ-Long Island, NY-NJ-PA	18,897,109	New York
Los Angeles-Long Beach-Santa Ana, CA	12,828,837	Los Angeles
Chicago-Joliet-Naperville, IL-IN-WI	9,461,105	Chicago
Dallas-Fort Worth-Arlington, TX	6,371,773	Dallas
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	5,965,343	Philadelphia
Houston-Sugar Land-Baytown, TX	5,946,800	Houston
Washington-Arlington-Alexandria, DC-VA-MD-WV	5,582,170	Washington
Miami-Fort Lauderdale-Pompano Beach, FL	5,564,635	Miami
Atlanta-Sandy Springs-Marietta, GA	5,268,860	Atlanta
Boston-Cambridge-Quincy, MA-NH	4,552,402	Boston
San Francisco-Oakland-Fremont, CA	4,335,391	San Francisco
Detroit-Warren-Livonia, MI	4,296,250	Detroit
Riverside-San Bernardino-Ontario, CA	4,224,851	Riverside
Phoenix-Mesa-Glendale, AZ	4,192,887	Phoenix
Seattle-Tacoma-Bellevue, WA	3,439,809	Seattle
Minneapolis-St. Paul-Bloomington, MN-WI	3,279,833	Minneapolis
San Diego-Carlsbad-San Marcos, CA	3,095,313	San Diego
St. Louis, MO-IL	2,812,896	St. Louis
Tampa-St. Petersburg-Clearwater, FL	2,783,243	Tampa
Baltimore-Towson, MD	2,710,489	Baltimore

Resolution of Aggregate Data Points

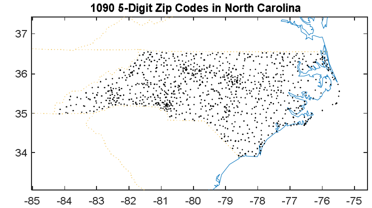
768 Cities with 50K+ Population in Continental U.S.



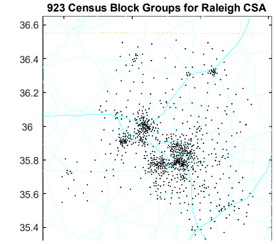
880 3-Digit Zip Codes in Continental U.S.



1090 5-Digit Zip Codes in North Carolina



923 Census Block Groups for Raleigh CSA

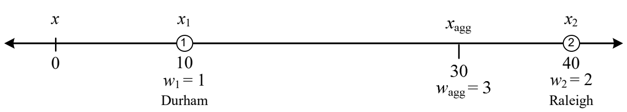


Demand Point Aggregation

- *Existing facility* (EF): actual physical location of demand source
  - Each EF has a well-defined weight w_i and location x_i
- *Aggregate demand point*: single location representing multiple demand sources in a region
  - Need to determine aggregate weight w_{agg} and location x_{agg}
  - Also, need measure a of extent of region, (length, 1-D; area, 2-D), since *assuming demand is uniformly spread over region*

Centroid as Aggregate Location

- Calculation of aggregate location depends on objective



- For minisum location, would like for any location x:

$$(w_1 + w_2)\, d(x, x_{agg}) = w_1 d(x, x_1) + w_2 d(x, x_2), \quad \text{let } x = 0,\ x_1, x_2 > 0$$

$$(w_1 + w_2)\, x_{agg} = w_1 x_1 + w_2 x_2$$

$$x_{agg} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2} \Rightarrow \quad \text{centroid}$$

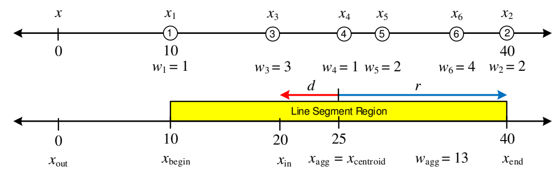
Note: if $x_1 < x < x_2$, then x_{agg} not centroid

- For squared distance: $(w_1 + w_2)\, x_{agg}^2 = w_1 x_1^2 + w_2 x_2^2$

$$x_{agg} = \sqrt{\frac{w_1 x_1^2 + w_2 x_2^2}{w_1 + w_2}} \Rightarrow \quad \text{not centroid}$$

1-D Average Distance

- Define region enclosing multiple points, with total weight of points spread uniformly across region



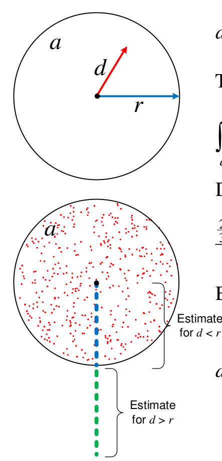
- Calculation of average distance d_a from x to all points in region differs if inside/outside region
  - d_a^0 (average distance if x at centroid) used to approximate d_a
  - **Note:** d_a^0 and $d_a > 0$ even when $d = 0$

$$d = \left| x - x_{centroid} \right|, \quad r = \frac{\left| x_{end} - x_{begin} \right|}{2}$$

$$d_a = \begin{cases} \dfrac{r}{2} + \dfrac{d^2}{2r}, & \text{if } d < r \\ d, & \text{otherwise} \end{cases}$$

$$d_a^0 = \max\left\{ d, \frac{r}{2} \right\} \approx d_a$$

2-D Average Distance



$$a = \pi r^2 \Rightarrow r = \sqrt{\frac{a}{\pi}}$$

Total distance centroid to all points (x, y) in a:

$$\iint_a \sqrt{x^2 + y^2}\, dx dy = \int_0^r \int_0^{2\pi} s \cdot s\, d\theta ds = 2\pi \int_0^r s^2 ds = \frac{2}{3}\pi s^3 \Big|_{s=0}^{r} = \frac{2}{3}\pi r^3$$

Dividing total distance by a gives approx. average distance:

$$\frac{\frac{2}{3}\pi r^3}{a} = \frac{2}{3} r \Rightarrow d_a^0 = \max\left\{ d, \frac{2r}{3} \right\} \approx d_a$$

Empirical estimate, where $d_a > d$ even when $d > r$:

$$d_a = \begin{cases} \dfrac{2r}{3} + \dfrac{d}{48} + \dfrac{9d^2}{20r}, & \text{if } d < r \\ d + \dfrac{3r^2}{23d}, & \text{otherwise} \end{cases}$$

Location 7:
Logistics Network Design

- What makes logistics network design hard?
  - Resources are lumpy (both production and transport)
    - ⇒ minimum effective size for each facility
    - ⇒ fixed production cost (cost that does not depend on facility size)
    - ⇒ economies of scale (more of one thing) and scope (many things)
  - *Fixed production* + *variable transport* costs can be used by the UFL model to determine the number and location of NFs
  - Don't need to identify actual fixed production costs:
    - Will, instead, used intercept term of linear regression fit

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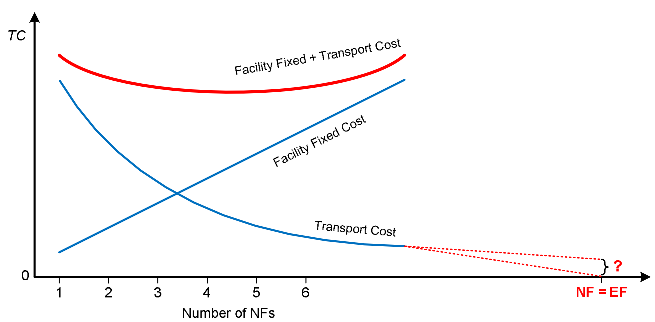
Transport Cost if NF at every EF



Note: d_a^0 and $d_a > 0$ even when $d = 0$

$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

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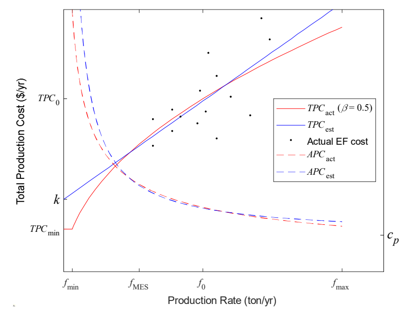
Fixed Cost and Economies of Scale

- Cost data from existing facilities can be used to fit linear estimate
  - Economies of scale in production $\Rightarrow k > 0$ and $\beta < 1$



$$TPC_{\text{act}} = \max_{f < f_{\max}} \left\{ TPC_{\min}, TPC_0 \left(\frac{f}{f_0} \right)^{\beta} \right\}$$

$$\beta = \begin{cases} 0.62, & \text{Hand tool mfg.} \\ 0.48, & \text{Construction} \\ 0.41, & \text{Chemical processing} \\ 0.23, & \text{Medical centers} \end{cases}$$

$$TPC_{\text{est}} = k + c_p f$$

$$APC_{\text{act}} = \frac{TPC_{\text{act}}}{f} = \frac{TPC_0}{f_0^{\beta}} f^{\beta - 1}$$

$$APC_{\text{est}} = \frac{k}{f} + c_p$$

k = fixed cost

c_p = constant unit production cost

$f_{\min}/f_{\max}$ = min/max feasible scale

f_{MES} = *Minimum Efficient Scale*

TPC_0/f_0 = base cost/rate

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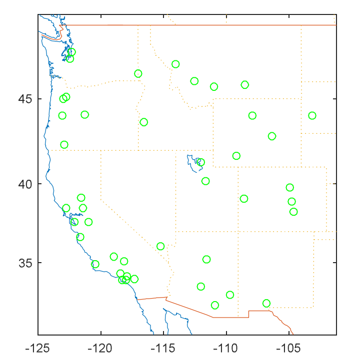
Ex: Popco Bottling Company



- **Problem:** Popco currently has 42 bottling plants (green circles) across the western U.S. and wants to know if they should consider reducing or adding plants to improve their profitability.
- **Solution:** Formulate as an UFL to determine the number of plants that minimize Popco's production, procurement, and distribution costs.

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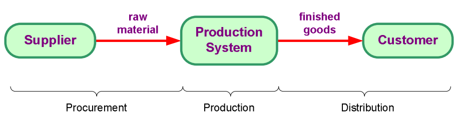
Ex: Popco Bottling Company

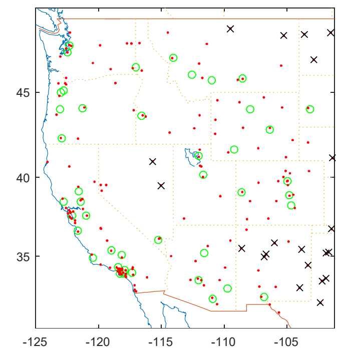
- Following representative information is available for each of N current plants (DC) i:

$$xy_i = \text{location}$$
$$f_i^{DC} = \text{aggregate production (tons)}$$
$$TPC_i = \text{total production and procurement cost}$$
$$TDC_i = \text{total distribution cost}$$

- Assuming plants are (monetarily) weight gaining since they are bottling plants, so UFL can ignore inbound procurement costs related to location



Ex: Popco Bottling Company



2. Allocate all 3-digit ZIP codes to closest plant (up to 200 mi max) to serve as aggregate customer demand points.

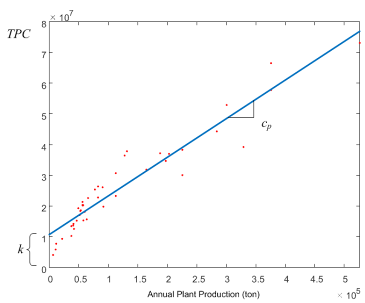
$$M_i = \left\{ j : \arg\min_h d_{hj} = i \text{ and } d_{ij} \le d_{\max} \right\}$$

$$d_{\max} = 200 \text{ mi}$$

$$M = \bigcup_{i \in N} M_i$$

```
for r in eachrow(z3)
    d = Dgc([r.LON r.LAT], hcat(DC.LON, DC.LAT)) * 1.2
    idx = argmin(d)[2]
    if d[idx] <= 200
        r.IDX = idx
    end
end
filter!(r -> r.IDX != 0, z3)
```


Ex: Popco Bottling Company



- Difficult to estimate fixed cost of each new facility because this cost must not include any cost related to quantity of product produced at facility.

1. Use plant (DC) production costs to find UFL fixed costs via linear regression
   - variable production costs c_p do not change and can be cut

```
ŷ(p, x) = p[1] .+ p[2]*x
loss(p, x, y) = sum((y .- ŷ(p, x)).^2)
k, cp = optimize(p -> loss(p, DC.fDC, DC.TPC), [0., 1.]).minimizer
```

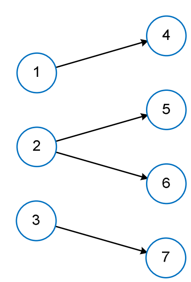
```
2-element Vector{Float64}:
   1.0603652532813296e7
 126.15062860391504
```

$$TPC = \sum_{i \in N} TPC_i = \sum_{i \in N} \left(k + c_p f_i^{DC} \right)$$

$$(\text{only keep } k \text{ for UFL})$$

Ex: Popco Bottling Company

3. Allocate each plant's demand (tons of product) to each of its customers based on its population.



$$f_{j \in M_i} = f_i^{DC} \frac{q_j}{\sum_{h \in M_i} q_h}$$

$$q_j = \text{population of EF}j$$

$$f_5 = f_2^{DC} \frac{q_5}{q_5 + q_6}$$

```
gdf = groupby(z3, :IDX)
z3 = transform(gdf, :POP => sum => :DCPOP)
z3 = leftjoin(z3, DC[!, [:IDX, :fDC]], on=:IDX)
z3.f = map(r -> r.fDC * r.POP / r.DCPOP, eachrow(z3))
```

Ex: Popco Bottling Company

4. Estimate a nominal transport rate (\$/ton-mi) using the ratio of total distribution cost (\$) to the sum of the product of the demand (ton) at each customer and its distance to its plant (mi).

$$r_{\text{nom}} = \frac{\sum_{i \in N} TDC_i}{\sum_{i \in N} \sum_{j \in M_j} f_j d_{ij}^a}$$

```
dDC2r(r) = Daa([DC[r.IDX,:LON] DC[r.IDX, :LAT]],[r.LON r.LAT], r.ALAND)[1]
rnom = sum(DC.TDC) / sum(map(r -> r.f * dDC2r(r), eachrow(z3)))
```

```
2.8625376133920297
```


Ex: Popco Bottling Company

5. Calculate UFL variable transportation cost c_{ij} (\$) for each possible NF site i (all customer and plant locations) and EF site j (all customer locations) as the product of customer j demand (ton), distance from site i to j (mi), and the nominal transport rate (\$/ton-mi).

$$\mathbf{C} = \left[c_{ij} \right]_{\substack{i \in M \cup N \\ j \in M}} = \left[r_{\text{nom}} f_j d_{ij}^a \right]_{\substack{i \in M \cup N \\ j \in M}}$$

```
# Include demand + DC locations for NF sites
NF = hcat(vcat(z3.LON, DC.LON), vcat(z3.LAT, DC.LAT))
D = Daa(NF, hcat(z3.LON, z3.LAT), z3.ALAND)
C = rnom*z3.f'.*D
```


Ex: Popco Bottling Company

6. Solve as UFL, where *TC* returned includes all new distribution costs and the fixed portion of production costs.

$$TC = \overbrace{\sum_{i \in Y} k_i}^{\text{fixed cost}} + \overbrace{\sum_{i \in Y} \sum_{j \in M_i} c_{ij}}^{\text{transport cost}}$$

$$n = \left| M \cup N \right|, \quad \text{number of potential NF sites}$$

$$m = \left| M \right|, \quad \text{number of EF sites}$$

```
y, TC = ufl(fill(k, size(C, 1)), C)
```

```
  Add: 7.645948563275278e8
 Xchg: 7.35417248086411e8
  Add: 7.35417248086411e8
 Drop: 7.3515904304392 93e8
 Xchg: 7.34021255286948e8
  Add: 7.34021255286948e8
 Drop: 7.34021255286948e8
```

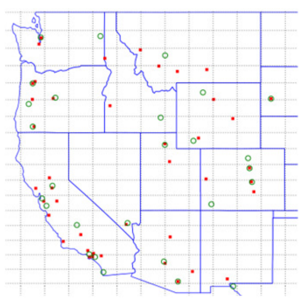
```
% reduction is TC = 17.9607898179728 2
```

3×3 DataFrame

Row	_	Orig	New
	String	Float64	Float64
1	No. of NFs	42.0	27.0
2	TDC	4.49367e8	4.47723e8
3	TC	8.9472e8	7.34021e8

Ex: Popco Bottling Company

- **Results:** Original 42 plants (red) reduced to 27 (green circles), *TDC* stayed about the same but *TC* decreased by 18%
- **Note:** The new plants are a mix of
  - Original plants
  - New plants very close to a closed original plant (why?)
  - New plants far from any of the original plants
- **Question:** If an original plant is kept, will it have sufficient capacity (since fewer plants)
- **Question:** If Popco was planning to expand to serve the entire continental U.S., how could the current results be utilized



Location 8: Discrete Location and MILP

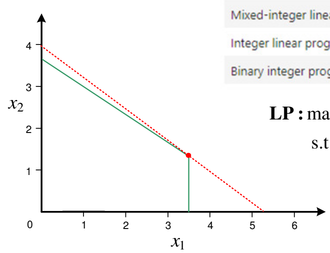
- What computer technology has had the biggest impact on industry since 1990?
  1. Data science/analytics
  2. AI (visual/voice recognition, ChatGPT, etc)
  3. MILP solver improvements
- MILP biggest impact has been on tactical decisions
  – Production planning
  – Scheduling

Speedup from 1990-2014:
- – 320,000 × computer speed
- – 580,000 × algorithm improvements
- ⇒ 10 days of 24/7 processing → 1 sec

Mixed-Integer Programming

- Mixed-integer programming used when
  – decision variables need to be integer-valued
  – decisions need to be made as part of the solution procedure, and can only be implemented using discrete (typically binary) decision variables

Mixed-integer program	MIP	NLP + some integer variables
Mixed-integer linear program	MILP	LP + some integer variables
Integer linear program	ILP	LP + all integer variables
Binary integer program	BIP	LP + all binary variables



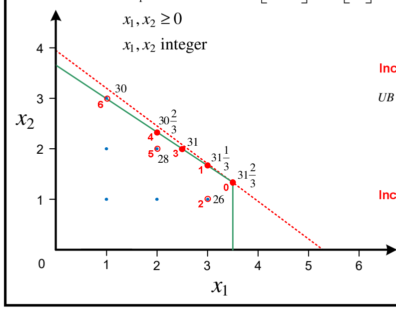
$$\begin{aligned}\textbf{LP}: \max\ & 6x_1 + 8x_2 \\ \text{s.t.}\ & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\ \le 7 \\ & x_1, x_2 \ge 0\end{aligned}$$

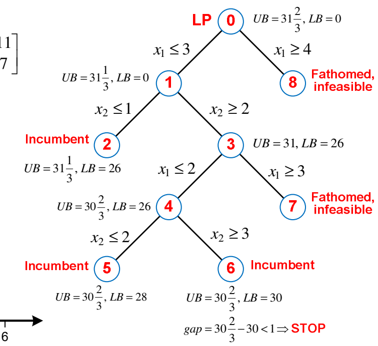
$$\begin{aligned}\text{LP:}\quad & \max\ \mathbf{c}'\mathbf{x} \\ & \text{s.t.}\ \ \mathbf{Ax} \le \mathbf{b} \\ & \qquad\ \ \mathbf{x} \ge 0 \\ \text{MILP:}\quad & \text{some } x_i \text{ integer} \\ \text{ILP:}\quad & \mathbf{x} \text{ integer} \\ \text{BLP:}\quad & \mathbf{x} \in \{0,1\}\end{aligned}$$

Solving a ILP: Branch and Bound

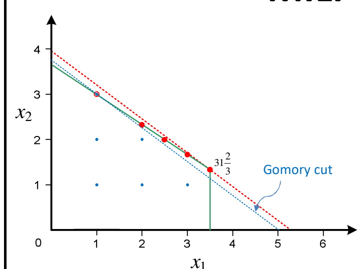
- For maximization ILP problems:
  - LP solutions provide *UBs*
  - Feasible ILP (incumbent) solutions provide *LBs*
- Stop when *gap* = *UB* – *LB* < some threshold

$$\begin{aligned}\max\ & 6x_1 + 8x_2 \\ \text{s.t.}\ & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\ \le 7 \\ & x_1, x_2 \ge 0 \\ & x_1, x_2 \text{ integer}\end{aligned} \qquad \mathbf{c} = \begin{bmatrix}6 & 8\end{bmatrix},\ \mathbf{A} = \begin{bmatrix}2 & 3 \\ 2 & 0\end{bmatrix}, \mathbf{b} = \begin{bmatrix}11 \\ 7\end{bmatrix}$$





MILP Solvers



Cplex	IBM, first commercial solver
Gurobi	Developed by Robert Bixby
FICO Xpress	Used by LLamasoft
SAS/OR	Part of SAS system (not supported in `JuMP`)
Cbc	COIN-OR open-source solver
GLPK	Free Software Foundation GNU open-source solver

- **Pre-solve**: eliminate variables
  $2x_1 + 2x_2 \le 1,\ x_1, x_2 \ge 0$ and integer
  $\Rightarrow x_1 = x_2 = 0$
- **Cutting planes**: cuts off LP solutions (Gomory cut)
- **Heuristics**: find good initial incumbent solution (Hybrid UFL)
- **Parallel**: use separate cores to solve nodes in B&B tree
- **Speedup** from 1990-2014:
  – 320,000 × computer speed
  – 580,000 × algorithm improvements
  ⇒ 10 days of 24/7 processing → 1 sec
- **Speedup** from 2001-2020:
  – 20 × computer speed
  – 50 × algorithm improvements
  ⇒ 1000 × speedup

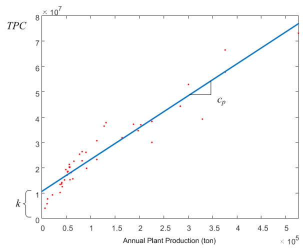
Total Logistics Costs

- All costs impacting location decision termed *total logistics costs*
- When production costs are represented by a line with intercept (k) and slope (c_p) terms
  - only cost associated with adding another NF is k
  - c_p the same for any number of NFs

$$
\begin{aligned}
&\text{Total production cost:} && TPC = k + c_p f \\
&\text{Total transport cost:} && TC = f\,r\,d \\
&\text{Total logistics cost:} && TLC = k + f\,r\,d
\end{aligned}
$$

where

$$
\begin{aligned}
k &= \text{fixed production cost (\$/yr)} \\
c_p &= \text{unit production cost (\$/ton)} \\
f &= \text{production rate (ton/yr)} \\
r &= \text{transport rate (\$/ton-mi)} \\
d &= \text{transport distance (mi)}
\end{aligned}
$$



Uncapacitated Facility Location (UFL)

- NFs can only be located at discrete set of sites
  - Allows inclusion of fixed cost of locating NF at site $\Rightarrow$ opt number NFs
  - Variable costs are usually transport cost from NF to/from EF
  - Total of $2^n - 1$ potential solutions (all nonempty subsets of sites)

$$
\begin{aligned}
M &= \{1,\dots,m\}, \quad \text{existing facilites (EFs)} \\
N &= \{1,\dots,n\}, \quad \text{sites available to locate NFs} \\
M_i &\subseteq M, \quad \text{set of EFs served by NF at site } i \\
c_{ij} &= \text{variable cost to serve EF } j \text{ from NF at site } i \\
k_i &= \text{fixed cost of locating NF at site } i \\
Y &\subseteq N, \quad \text{sites at which NFs are located} \\
Y^* &= \arg\min_Y \left\{ \sum_{i\in Y} k_i + \sum_{i\in Y} \sum_{j\in M_i} c_{ij} : \bigcup_{i\in Y} M_i = M \right\} \\
&= \text{min cost set of sites where NFs located} \\
|Y^*| &= \text{number of NFs located}
\end{aligned}
$$

UFL Solution Techniques

- Being uncapacitated allows simple heuristics to be used to solve
  - ADD construction: add one NF at a time
  - DROP construction: drop one NF at a time
  - XCHG improvement: move one NF at a time to unoccupied sites
  - HYBRID algorithm combination of ADD and DROP construction with XCHG improvement, repeating until no change in Y
    - Use as default heuristic for UFL
    - See Daskin [2013] for more details
- UFL can be solved as a MILP
  - Easy MILP, LP relaxation usually optimal (for strong formulation)
  - MILP formulation allows constraints to easily be added
    - e.g., capacitated facility location, fixed number of NFs, some NF at fixed location

MILP Formulation of UFL

$$
\begin{aligned}
\min \quad & \sum_{i\in N} k_i y_i + \sum_{i\in N} \sum_{j\in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i\in N} x_{ij} = 1, \qquad j \in M \\
& \cancel{m y_i \ge \sum_{j\in M} x_{ij}, \qquad i \in N} \\
& 0 \le x_{ij} \le 1, \qquad i \in N, j \in M \\
& y_i \in \{0,1\}, \qquad i \in N
\end{aligned}
$$

Weak Formulation

- Every EF needs to be assigned to a NF
- NF needs to be established at every site that has EFs assigned to it
  - Max m EFs can be assigned to each NF
- x_{ij}'s always 0 or 1 in UFL
  - But easier to solve if not binary

$$\boxed{y_i \ge x_{ij}, \qquad i \in N, j \in M}$$

Strong Formulation

where

$$
\begin{aligned}
k_i &= \text{fixed cost of NF at site } i \in N = \{1,\dots,n\} \\
c_{ij} &= \text{variable cost from } i \text{ to serve EF } j \in M = \{1,\dots,m\} \\
y_i &= \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases} \\
x_{ij} &= \text{fraction of EF } j \text{ demand served from NF at site } i.
\end{aligned}
$$

UFL Costs in MILP

$$TLC = \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} = \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} \underbrace{f_i \, r \, d_{ij}}_{c_{ij}} x_{ij}$$

where TLC = total fixed and variable cost ($/yr)

k_i = fixed production cost ($/yr)

c_{ij} = variable transport cost ($/yr)

f_i = demand rate (ton/yr)

r = transport rate ($/ton-mi)

d_{ij} = distance between NF at site i and EF at site j (mi)

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Capacitated Facility Location (CFL)

$$\begin{aligned}
\min \quad & \sum_{i \in N} k_i y_i + \sum_{i \in N} \sum_{j \in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} x_{ij} = 1, \quad j \in M \\
& K_i y_i \ge \sum_{j \in M} f_j x_{ij}, \quad i \in N \\
& 0 \le x_{ij} \le 1, \quad i \in N, j \in M \\
& y_i \in \{0,1\}, \quad i \in N
\end{aligned}$$

where

k_i = fixed cost of NF at site $i \in N = \{1,\ldots,n\}$

c_{ij} = variable cost from i to serve EF $j \in M = \{1,\ldots,m\}$

K_i = capacity of NF at site $i \in N = \{1,\ldots,n\}$

f_j = demand EF $j \in M = \{1,\ldots,m\}$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

x_{ij} = fraction of EF j demand served from NF at site i.

- CFL does not have simple and effective heuristics, unlike UFL
- Other types of constraints:
  - Fix NF i at site j: set *LB* and *UB* of x_{ij} to 1
  - Convert UFL to p-Median: set all k to 0 and add constraint sum$\{y_i\} = p$

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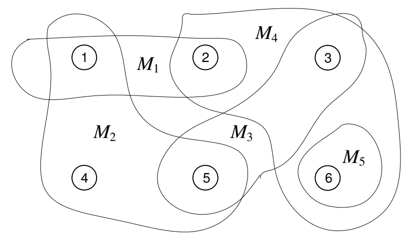
(Weighted) Set Covering

$M = \{1,\ldots,m\}$, objects to be covered

$M_i \subseteq M, i \in N = \{1,\ldots,n\}$, subsets of M

c_i = cost of using M_i in cover

$I^* = \arg\min_I \left\{ \sum_{i \in I} c_i : \bigcup_{i \in I} M_i = M \right\}$, min cost covering of M



$M = \{1,\ldots,6\}$

$i \in N = \{1,\ldots,5\}$

$M_1 = \{1,2\}, M_2 = \{1,4,5\}, M_3 = \{3,5\}$

$M_4 = \{2,3,6\}, M_5 = \{6\}$

$c_i = 1,$ for all $i \in N$

$I^* = \arg\min_I \left\{ \sum_{i \in I} c_i : \bigcup_{i \in I} M_i = M \right\}$

$= \{2,4\}$

$\sum_{i \in I^*} c_i = 2$

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(Weighted) Set Covering

$M = \{1,\ldots,m\}$, objects to be covered

$M_i \subseteq M, i \in N = \{1,\ldots,n\}$, subsets of M

c_i = cost of using M_i in cover

$I^* = \arg\min_I \left\{ \sum_{i \in I} c_i : \bigcup_{i \in I} M_i = M \right\}$, min cost covering of M

$$\begin{aligned}
\min \quad & \sum_{i \in N} c_i x_i \\
\text{s.t.} \quad & \sum_{i \in N} a_{ji} x_i \ge 1, \quad j \in M \\
& x_i \in \{0,1\}, \quad i \in N
\end{aligned}$$

```
model = Model(Cbc.Optimizer)   # Unweighted set cover
@variable(model, x[N], Bin )
@objective(model, Min, sum(x[i] for i ∈ N) )
@constraint(model, [j ∈ M], sum(A[j,i]*x[i] for i ∈ N) >= 1 )
```

where

$$x_i = \begin{cases} 1, & \text{if } M_i \text{ is in cover} \\ 0, & \text{otherwise} \end{cases}$$

$$a_{ji} = \begin{cases} 1, & \text{if } j \in M_i \\ 0, & \text{otherwise.} \end{cases}$$

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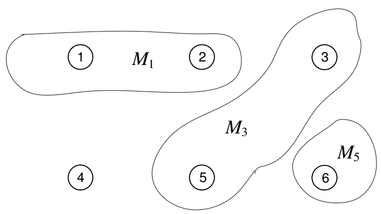
Set Packing

- Maximize the number of mutually disjoint sets
  - Dual of Set Covering problem
  - Not all objects required in a packing
  - Limited logistics engineering application (c.f. bin packing)



$$\begin{aligned} \max \quad & \sum_{i \in N} x_i \\ \text{s.t.} \quad & \sum_{i \in N} a_{ji} x_i \le 1, \quad && j \in M \\ & x_i \in \{0,1\}, && i \in N \end{aligned}$$

Topics

1. Introduction
2. Location
3. **Transport**
   - Exam 1 (take home)
4. Networks
5. Routing
   - Exam 2 (take home)
6. Inventory
   - Final exam (in class)

Transport 1: Overview of Freight Transport

(Slides in a separate file)

Transport 2: One-Time Truck Shipments

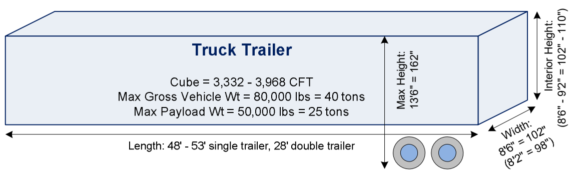
- **Load density:** freight capacity is determined by both the *weight* and *cube* of a load
- Trucking is the only transport mode that most shippers need to have detailed knowledge of
  - Only trucks used for shipping/receiving at most facilities
    - Trucks transport from facility to railhead, port, airport
  - Other modes handled by specialized freight brokers

Trucking

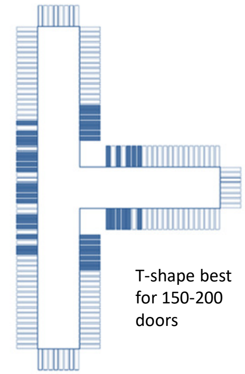
U.S. For-Hire Trucking Services

	TL	LTL	PX
Minimum payload	10,000 lb	150 lb	2 lb
Average payload[12]	30,000 lb	1000 lb	10 lb
Maximum payload	50,000 lb	10,000 lb	70 (UPS) – 150 lb
Average length of haul	294 mi	752 mi	894 mi
Average value	$775/ton	$7002/ton	$37,538/ton

Truck enclosed van semi-trailer (interior dimensions in parenthesis)



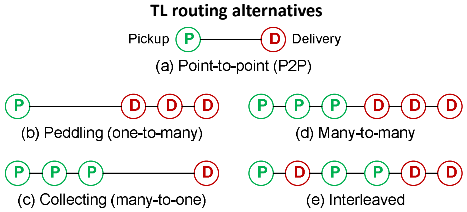
LTL Terminal



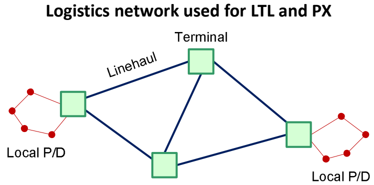
T-shape best for 150-200 doors

Trucking Operations

TL routing alternatives



Logistics network used for LTL and PX



One-Time vs Periodic Shipments

- **One-Time Shipments** (*operational* decision): know shipment size q
  - Know when and how much to ship, need to determine if TL and/or LTL to be used
  - Must contact carrier or have agreement to know charge
    - Can/should estimate charge before contacting carrier
- **Periodic Shipments** (*tactical* decision): know demand rate f, must determine size q
  - Need to determine how often and how much to ship
  - Analytical transport charge formula allow "optimal" size (and shipment frequency) to be estimated
    - U.S. Bureau of Labor Statistic's *Producer Price Index* (PPI) for TL and LTL used to estimate transport charges

Transport Charges

- **Spot price:** current no-contract charge
- **Contract price:** negotiated multi-shipment charge
  - Charge for each single origin and destination (*lane*)
  - Charge for any origin/destination (OD) based discount off of *tariff*
- **Estimated price:** based on analytical formula
  - Only used for planning purposes (e.g., prior to negotiation)
  - Gives estimate of average charge across all OD pairs
    - Mix of spot and contract
    - Why are spot prices usually higher than contract? Are they always?
  - *Design constants* used for missing data
  - Can determine "optimal" shipment size/frequency

Logistics Engineering Design Constants

1. Circuity Factor: 1.2 (g)
   – 1.2 × GC distance ≈ actual road distance
2. Local vs. Intercity Transport:
   – Local: < 50 mi ⇒ use actual road distances
   – Intercity: > 50 mi ⇒ can estimate road distances
   - 50-250 mi ⇒ return possible (11 HOS)
   - > 250 mi ⇒ always one-way transport
   - > 500-750 mi ⇒ intermodal rail possible
3. Inventory Carrying Cost (h) = funds + storage + obsolescence
   – 16% average (no product information, per U.S. Total Logistics Costs)
   - (16% ≈ 5% funds + 6% storage + 5% obsolescence)

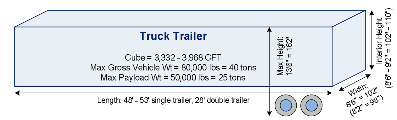
   – 5-10% low-value product (construction)
   – 25-30% general durable manufactured goods
   – 50+% computer/electronic equipment
   – >> 100% perishable goods (produce)

Logistics Engineering Design Constants

4. $\dfrac{\text{Value}}{\text{Transport Cost}} \gg 1: \; \$1\ \text{ft}^3 \approx \dfrac{\$2{,}620 \text{ Shanghai-LA/LB shipping cost}}{2{,}400\ \text{ft}^3\ 40' \text{ ISO container capacity}}$

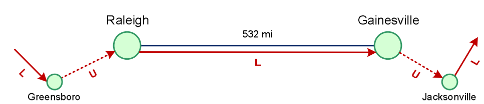
5. TL Weight Capacity: 25 tons (K_{wt})
   – (40 ton max per regulation) –
   (15 ton tare for tractor-trailer)
   = 25 ton max payload
   – Weight capacity = 100% of physical capacity

6. TL Cube Capacity: 2,750 ft³ (K_{cu})
   – Trailer physical capacity = 3,332 ft³
   – Effective capacity =
   3,332 × 0.80 ≈ 2,750 ft³
   – Cube capacity = 80% of physical capacity



Logistics Engineering Design Constants

7. TL Revenue per Loaded Truck-Mile: \$2/mi in 2004 (r)
   – TL revenue for the carrier is your TL cost as a shipper



15%, average deadhead travel

\$1.60, cost per mile in 2004

$\dfrac{\$1.60}{1-0.15} = \$1.88$, cost per loaded-mile

6.35%, average operating margin for trucking

$\dfrac{\$1.88}{1-0.0635} \approx \$2.00$, revenue per loaded-mile

TL Cost per Mile in 2004

- Bottom-up estimate of TL cost per mile in 2004:

Interest Rate		Annual	Source
Prime Rate		4.75%	(http://www.forecasts.org/prime.htm)
Increase over Prime		2.00%	
Nominal Interest Rate		6.75%	
Current Inflation Rate		2.70%	(http://www.forecasts.org/inflation.htm)
Real Interest Rate	(*i*)	4.05%	
Lease			
Economic Life	(*N*, yr)	7.25	= 754,000/103,945, Avg mi until replace/Avg mi [1]
Investment Cost	(*IV*, $)	132,576	[2]
Salvage Percentage		20%	
Salvage Value	(*SV*, $)	26,515	
Effective Investment Cost	(IV^{eff}, $)	112,695	
Cost Cap Recovery	(*K*, $/yr)	18,240	
Costing			
Annual Mileage	(mi/yr)	103,945	[1]
Fuel Efficiency	(mi/gal)	4.5	p. 10 in [3]
Fuel Cost per Gallon	($/gal)	1.780	(https://www.eia.gov/petroleum/gasdiesel/)
Fuel Cost	($/mi)	0.39555556	
Annual Fuel Cost	($/yr)	41,116	
Tire,Repair,Insurance	($/yr)	58,367	$0.34/mi in 1988 [3] * 103945 * (2.18/1) [4]
Driver Salary with Benefits	($/yr)	49,058	$34,343 mean wage/(1 - 0.3) [5]
Operating Cost	($/yr)	148,541	
Operating Cost per Unit	($/mi)	1.43	
Annual Investment Cost	($/yr)	18,240	
Cost per Mile	($/mi)	0.18	
Total Annual Cost	($/yr)	166,781	
Cost per Mile	($/mi)	1.60	

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Truck Shipment Example: One-Time

1. Assuming that the product is to be shipped P2P TL, what is the maximum payload for each trailer used for the shipment?

$$q_{\max}^{wt} = K_{wt} = 25 \text{ ton}$$

$$K_{cu} = 2750 \text{ ft}^3$$

$$s = \frac{40 \text{ lb/unit}}{9 \text{ ft}^3\text{/unit}} = 4.4444 \text{ lb/ft}^3$$

$$K_{cu} = \frac{q_{\max}^{cu}}{\left(\dfrac{s}{2000}\right)} \Rightarrow q_{\max}^{cu} = \frac{sK_{cu}}{2000}$$

$$q_{\max} = \min\left\{q_{\max}^{wt}, q_{\max}^{cu}\right\} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\}$$

$$= \min\left\{25, \frac{4.4444(2750)}{2000}\right\} = 6.1111 \text{ ton}$$

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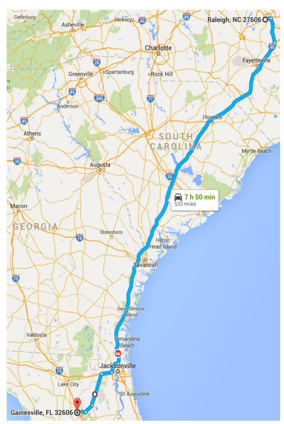
Truck Shipment Example

- Product shipped in cartons from Raleigh, NC (27606) to Gainesville, FL (32606)
- Each identical carton weighs 40 lb and occupies 9 ft³ (its *cube*)
  - Don't know linear dimensions of each unit for TL and LTL
- Cartons can be stacked on top of each other in a trailer
- Additional info/data is presented only when it is needed to determine answer



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Truck Shipment Example: One-Time

2. On Jan 10, 2018, 300 cartons of the product were shipped. How many truckloads were required for this shipment?

$$q = 300\frac{40}{2000} = 6 \text{ ton}, \quad \left\lceil \frac{q}{q_{\max}} \right\rceil = \left\lceil \frac{6}{6.1111} \right\rceil = 1 \text{ truckload}$$

3. Before contacting the carrier to negotiate (and using Jan 2018 PPI), what would have been the estimated TL transport charge for this shipment?

$$d = 532 \text{ mi}$$

$$r_{TL} = \frac{PPI_{TL}^{\text{Jan 2018}}}{PPI_{TL}^{2004}} \times r_{2004} = \frac{PPI_{TL}}{102.7} \times \$2.00/\text{mi}$$

$$= \frac{131.0}{102.7} \times \$2.00/\text{mi} = \$2.5511/\text{mi}$$

$$c_{TL} = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d = \left\lceil \frac{6}{6.1111} \right\rceil (2.5511)(532) = \$1,357.20$$

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Truck Shipment Example: One-Time

Series Id: PCU484121484121
Series Title: PPI industry data for General freight trucking, long-distance TL, not seasonally adjusted
Industry: General freight trucking, long-distance TL
Product: General freight trucking, long-distance TL
Base Date: 200312

https://data.bls.gov/multi-screen?survey=pc

Download: xlsx

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2003												100.0
2004	100.3	101.1	101.2	101.5	101.9	102.5	102.6	103.0	104.0	104.7	105.4	104.8
2005	105.5	106.2	106.9	107.5	108.3	108.4	108.3	108.6	110.0	111.3	111.9	111.0
2006	110.3	110.4	110.4	110.8	112.1	112.4	112.5	113.2	113.4	113.0	112.5	112.3
2007	113.0	112.5	112.4	112.9	113.1	112.8	113.0	113.3	113.8	114.0	115.1	115.8
2008	116.0	115.9	116.5	117.8	120.5	123.0	124.0	124.0	121.8	121.3	117.8	115.1
2009	113.2	112.1	110.4	109.7	109.8	110.1	111.4	111.0	111.7	110.8	111.5	110.9
2010	110.8	111.0	111.9	112.2	113.2	113.5	113.4	113.7	113.8	114.4	115.8	116.1
2011	116.5	117.4	119.3	121.0	121.7	121.4	121.3	121.2	122.0	122.0	123.2	123.3
2012	124.0	124.6	126.2	126.7	127.0	125.8	125.6	126.8	127.4	127.2	126.9	127.0
2013	126.7	127.2	128.0	127.5	127.8	127.6	127.6	127.6	127.1	127.2	127.6	127.4
2014	127.9	128.2	128.7	129.5	130.6	130.8	130.3	130.4	130.4	129.7	129.8	128.9
2015	126.7	126.0	126.0	126.2	126.3	127.1	126.9	126.2	125.9	125.5	125.8	124.8
2016	124.6	123.4	123.2	123.6	122.8	122.7	123.0	123.0	123.3	124.1	124.1	124.2
2017	124.4	124.7	124.2	124.3	124.0	124.2	124.2	125.9	126.6	126.6	128.5	130.3
2018	131.0	132.0	132.0	132.3	133.6	135.4	136.8	137.5	138.6	139.8	140.3	140.4
2019	139.9	138.6	138.2	136.7	137.3	137.7	136.8	136.1	136.6	136.6	140.1	141.0
2020	136.3	136.1	134.3	131.8	128.2	132.0	134.0	135.6	139.0	142.2	146.6	147.6
2021	146.1	151.3	154.7	159.3	162.1	159.3	159.929	164.691	169.568	173.847	182.629	187.738
2022	198.275	208.028	211.126	209.823	210.733	203.350	204.018	200.068	197.526	197.549	195.947	200.778
2023	187.156	194.459	202.307	184.407	192.615	190.471	178.666	173.268	182.025	184.002	187.206	177.712
2024	178.925	178.320	172.822	172.961	181.573(P)	182.584(P)	188.840(P)	189.015(P)				

P : Preliminary. All indexes are subject to monthly revisions up to four months after original publication.

Truck Shipment Example: One-Time

Series Id: PCU484122484122
Series Title: PPI industry data for General freight trucking, long-distance LTL, not seasonally adjusted
Industry: General freight trucking, long-distance LTL
Product: General freight trucking, long-distance LTL
Base Date: 200312

https://data.bls.gov/multi-screen?survey=pc

Download: xlsx

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2003												100.0
2004	101.1	101.5	101.9	101.4	102.2	103.4	104.8	105.1	106.3	107.1	108.1	108.0
2005	108.1	107.5	108.6	109.3	110.5	111.8	113.3	113.2	114.6	116.1	115.3	113.7
2006	114.7	114.3	115.4	116.6	119.2	119.2	119.5	120.3	119.7	117.5	117.6	117.9
2007	117.1	116.5	117.8	121.5	122.4	122.7	121.7	121.2	120.9	122.0	123.7	124.0
2008	123.5	124.9	127.8	130.1	131.0	133.3	132.2	130.7	129.4	126.8	123.5	121.2
2009	126.3	126.6	125.4	125.7	126.0	125.3	125.9	126.1	125.4	125.0	125.4	123.4
2010	122.2	123.5	125.9	127.0	127.1	126.4	126.3	126.4	126.8	127.8	131.5	131.0
2011	131.8	134.1	134.1	137.1	139.1	137.3	137.5	138.3	140.1	140.9	141.0	140.6
2012	141.4	141.7	144.9	145.5	144.2	143.7	145.0	147.4	149.9	150.1	149.5	150.3
2013	148.7	149.9	151.0	148.8	148.1	151.2	150.7	151.7	152.4	153.0	152.3	154.4
2014	153.8	153.4	154.5	158.5	158.9	158.8	160.5	160.9	158.0	157.3	156.8	158.8
2015	157.0	154.5	156.1	153.8	156.5	155.1	154.8	153.4	153.2	152.1	156.2	156.6
2016	156.1	158.7	158.3	160.3	160.2	160.7	161.2	161.1	159.5	163.1	164.3	165.6
2017	166.4	166.8	166.7	167.0	168.0	167.9	169.7	169.9	173.3	173.5	175.1	175.5
2018	177.4	178.5	178.4	178.9	181.0	183.1	183.2	183.3	184.0	185.3	184.8	183.9
2019	188.5	188.8	187.8	186.2	189.9	190.3	188.1	190.1	187.8	188.7	189.3	185.2
2020	193.6	191.1	189.7	188.4	187.7	186.1	190.3	189.9	185.5	189.6	187.7	187.1
2021	196.7	196.8	202.5	203.4	203.7	210.7	208.302	207.759	209.299	211.703	217.984	218.148
2022	222.856	220.690	239.535	243.598	251.869	258.802	248.576	242.100	242.930	237.312	242.376	233.568
2023	240.539	236.298	233.331	228.176	231.510	230.664	230.773	240.812	242.879	244.315	240.901	236.187
2024	242.245	244.230	245.484	246.943	244.791(P)	241.732(P)	247.454(P)	243.762(P)				

P : Preliminary. All indexes are subject to monthly revisions up to four months after original publication.

Truck Shipment Example: One-Time

4. Using the Jan 2018 PPI LTL rate estimate, what was the transport charge to ship 15 cartons LTL?

$$q = 15\frac{40}{2000} = 0.3 \text{ ton}$$

$$r_{LTL} = PPI_{LTL}\left[\frac{\frac{s^2}{8}+14}{\left(q^{\frac{1}{7}}d^{\frac{15}{29}}-\frac{7}{2}\right)(s^2+2s+14)}\right]$$

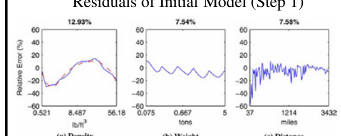
$$= 177.4\left[\frac{\frac{4.44^2}{8}+14}{\left(0.3^{\frac{1}{7}}532^{\frac{15}{29}}-\frac{7}{2}\right)(4.44^2+2(4.44)+14)}\right] = \$3.7770 \text{ / ton-mi}$$

$$c_{LTL} = r_{LTL}\,q\,d = 3.7770(0.3)(532) = \$602.81$$

LTL Rate Estimation Formula

- Top-down regression using representation LTL tariff rates:

Residuals of Initial Model (Step 1)



$$\min \sum_{q\in Q}\sum_{s\in S}\sum_{d\in D} w_q w_s w_d \left|\frac{r_{LTL}(q,s,d)}{r_{\text{tariff}}(q,s,d)} - 1\right|$$

$w_i =$ weighting factors represent relative likelihood of shipment weights, densities, and distances

$$r^{(N)}_{LTL}(q,s,d) = \left(\frac{\beta_1}{\beta_2 + q^{\beta_3}s^{\beta_4}d^{\beta_5}}\right)\left(\frac{\beta_6 s^3+\beta_7 s^2+\beta_8 s+\beta_9}{s^2+\beta_{10}s+\beta_{11}}\right)$$

$$r_{LTL}(q,s,d) = PPI_{LTL}\left[\frac{(s^2/8)+14}{(q^{1/7}d^{15/29}-(7/2))(s^2+2s+14)}\right]$$

Residuals of Full Model (Step 3)

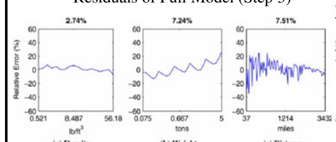


Table 2. Analysis steps with associated parameter values and weighted absolute relative errors (WAREs).

	Initial	Residual fit	Full	Simplify	Normalise	Simplify	Round
Step	1	2	3	4	5	6	7
WARE	20.49%	–	11.37%	11.66%	11.66%	11.66%	11.93%
1	100.1316	–	–	–	–	–	–
2	−1.6073	–	−3.0846	−3.5254	−3.5254	−3.5257	−7/2
3	0.2749	–	0.1567	0.1538	0.1538	0.1537	1/7
4	0.4575	–	−0.0644	–	–	–	–
5	0.4859	–	0.5044	0.5176	0.5176	0.5175	15/29
6	–	−0.0101	−0.2557	–	–	–	–
7	–	0.3859	33.7447	23.5443	0.1214	0.1208	1/8
8	–	−2.2739	−286.5365	−3.0893	−0.0159	–	–
9	–	−0.0294	5579.0010	2763.8200	14.2564	14.0210	14
10	–	−4.0998	6.9150	1.7135	1.7135	1.6699	2
11	–	15.2667	27.4923	14.5298	14.5298	14.3123	14

Truck Shipment Example: One-Time

5. What would the shipment size have to be so that the TL and LTL charges are equal?

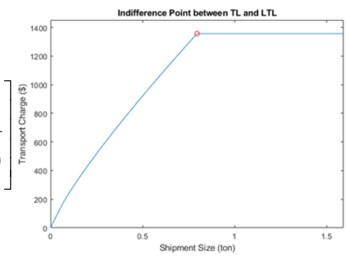
$$c_{TL}(q) = \left\lceil \frac{q}{q_{\max}} \right\rceil r_{TL} d$$

$$r_{LTL}(q) = PPI_{LTL}\left[\frac{\frac{s^2}{8}+14}{\left(q^{\frac{1}{7}} d^{\frac{15}{29}} - \frac{7}{2}\right)\left(s^2+2s+14\right)} \right]$$

$$c_{LTL}(q) = r_{LTL}(q)\, q d$$

$$q_I = \arg\min_q \left(\left| c_{TL}(q) - c_{LTL}(q) \right| \right)$$

$$= 0.7960 \text{ ton}$$



Truck Shipment Example: One-Time

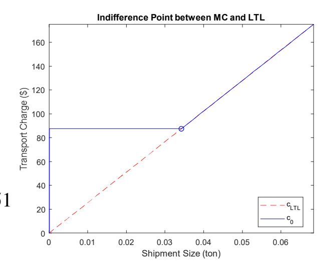
6. What are the TL and LTL minimum charges?

$$MC_{TL} = \left(\frac{r_{TL}}{2}\right)45 = \$57.40$$

$$MC_{LTL} = \left(\frac{PPI_{LTL}}{104.2}\right)\left(45 + \frac{d^{\frac{28}{19}}}{1625}\right)$$

$$= \left(\frac{177.4}{104.2}\right)\left(45 + \frac{532^{\frac{28}{19}}}{1625}\right) = \$87.51$$

$$q_I = 0.0342 \times 2000 = 68.43 \text{ lb}$$



- Why do these charges not depend on size of the shipment?
- Why does only the LTL minimum charge depend of the distance of the shipment?

Truck Shipment Example: One-Time

- Independent Transport Charge ($):

$$c_0(q) = \min\left\{ \max\left\{ c_{TL}(q), MC_{TL} \right\}, \max\left\{ c_{LTL}(q), MC_{LTL} \right\} \right\}$$

Truck Shipment Example: One-Time

7. What is the most likely freight class for this LTL shipment?

- Load density is the main factor in determining the class:

$$s = \frac{40 \text{ lb/unit}}{9 \text{ ft}^3\text{/unit}} = 4.4444 \text{ lb/ft}^3$$

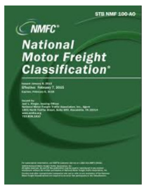
$\Rightarrow$ Class 200

Class-Density Relationship

Class	Load Density (lb/ft³) Minimum	Load Density (lb/ft³) Average	Max Physical Weight (tons)	Max Effective Cube (ft³)
500	–	0.52	0.72	*2,750*
400	1	1.49	2.06	*2,750*
300	2	2.49	3.43	*2,750*
250	3	3.49	4.80	*2,750*
200	4	4.49	6.17	*2,750*
175	5	5.49	7.55	*2,750*
150	6	6.49	8.92	*2,750*
125	7	7.49	10.30	*2,750*
110	8	8.49	11.67	*2,750*
100	9	9.72	13.37	*2,750*
92.5	10.5	11.22	15.43	*2,750*
85	12	12.72	17.49	*2,750*
77.5	13.5	14.22	19.55	*2,750*
70	15	18.01	24.76	*2,750*
65	22.5	25.50	*25*	1,961
60	30	32.16	*25*	1,555
55	35	39.68	*25*	1,260
50	50	56.18	*25*	890

Truck Shipment Example: One-Time

- The *National Motor Freight Classification* (NMFC) can be used to determine the product class
- Based on:
  1. Load density
  2. Special handling
  3. Stowability
  4. Liability



Item	Description	Class	NMFC	Sub
Abietic Acid	Abietic Acid, in drums	55	42605	-
Accordions	Accordions, in boxes	125	138820	-
Acetonitrile	Acetonitrile, in boxes or drums. See item 60000 for class dependent upon released value	85	42645	-
Acetylene	in steel cylinders	70	85520	-
Acid Fish Scrap	Fish Scrap, NOI, dry, not ground, pulverized nor screened, or Acid Fish Scrap, in bags	77.5	69980	-
Aircraft Parts	metal, struts, skins, panels	200	11790	01
Aluminum Channel	U channel	60	13340	-
Aluminum Table Set	aluminum table SU	200	82105	01
Ambulance Stretcher	stretcher	200	56920	06
Arches Support	Iron Steel	60	52460	-
Architectural Details	6 - 8 lbs per cubic foot	125	56290	05
Architectural Details	2 - 4 lbs per cubic ft	250	56290	03
Assembled Furniture	Bathroom cabinet set up	300	39220	01
Assembled Furniture	Highboys, dressers, wooden set up	125	80120	01
Assembled Furniture	Wood furniture 4-6 Lbs per cu ft	150	82270	04
Assembled Furniture	Chairs wooden setup w/out upholstery	300	80770	01
Assembled Furniture	Chairs wooden setup w/out upholstery KD	125	80770	03
Assembled Furniture	Couch w/ back & arms put together	175	80865	03
Assembled Furniture	Chairs put together w/ upholstery	200	79255	01
Assembled Furniture	Metal cabinets in boxes	110	39270	06
Assembled Furniture	18 gauge steel cabinet	70	39340	-
Assembled Furniture	Benches, cabinets, tables for workstations	125	23410	-
Assembled Furniture	Buffets, china cabinets put together	125	80080	-
Assembled Furniture	Cabinets of metal or plastic for storage	92.5	39235	-
Assembled Furniture	Tanning bed	150	109050	-
Assembled Furniture	Mattresses, in packages or boxes	200	79550	-
Athletic / Sporting Goods	Gym equipment, playground, sports items. Density Item			
Attachments: Backhoe	NOI: Attachments, backhoe (Backhoes), tractor or truck, on lift truck skids or pallets:	175	114217	01
Attachments: Backhoe	Attachments, backhoe (Backhoes), tractor or truck, on lift truck skids or pallets: Each shipped with all components secured to a single pallet, platform or skid, weighing 1100 pounds or more and having a density of 8 pounds or greater per cubic foot	100	114217	02

Truck Shipment Example: One-Time

8. Using the same LTL shipment, what is the transport cost found using the undiscounted CzarLite tariff?

$$q = 0.3, \quad class = 200$$

$$disc = 0, \quad MC = 95.23$$

$$i = \arg\left\{ q_i^B \,\middle|\, q_{i-1}^B \le q < q_i^B \right\}$$

$$= \arg\left\{ q_2^B \,\middle|\, q_1^B \le q < q_2^B \right\}$$

$$= \arg\left\{ q_2^B \,\middle|\, 0.25 \le 0.3 < 0.5 \right\} = 2$$

Freight Class	Rate Breaks (i) 1	2	3	4	5	6	7	8	9&10
500	341.42	314.14	245.80	201.48	158.60	112.37	55.66	55.66	55.66
400	273.88	251.99	197.19	161.61	127.22	91.12	45.10	45.10	45.10
300	206.34	189.85	148.56	121.76	95.85	69.47	34.43	34.43	34.43
250	172.56	158.77	124.23	101.83	80.15	58.03	28.79	28.79	28.79
200	138.78	127.69	99.92	81.89	64.47	47.19	23.40	23.40	23.40
175	121.37	111.68	87.39	71.62	56.38	41.27	20.39	20.39	20.39
50	41.57	38.26	29.94	24.54	19.12	16.10	11.52	8.85	8.14
Tons (q_i^B)	0.25	0.5	1	2.5	5	10	15	20	∞

$$c_{\text{tariff}} = (1 - disc)\max\left\{ MC, \min\left\{ OD(class, i)\, 20q,\ OD(class, i+1)\, 20q_i^B \right\} \right\}$$

$$= (1-0)\max\left\{ 95.23, \min\left\{ OD(200,2)\,20(0.3),\ OD(200,3)\,20(0.5) \right\} \right\}$$

$$= \max\left\{ 95.23, \min\left\{ (127.69)\,20(0.3),\ (99.92)20(0.5) \right\} \right\}$$

$$= \max\left\{ 95.23, \min\left\{ 766.14, 999.20 \right\} \right\} = \$766.14$$

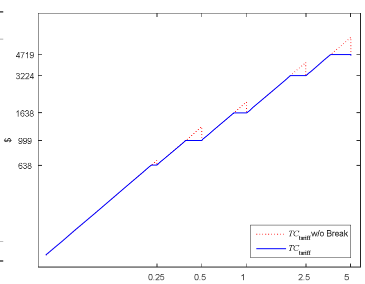
Truck Shipment Example: One-Time

- CzarLite tariff table for O-D pair 27606-32606

$$cwt = \text{hundredweight} = 100 \text{ lb} = \frac{100}{2000} = \frac{1}{20} \text{ ton}$$

Tariff (in \$/cwt) from Raleigh, NC (27606) to Gainesville, FL (32606)
(532 mi, CzarLite DEMOCZ02 04-01-2000, minimum charge = \$95.23)

Freight Class	Rate Breaks (i) 1	2	3	4	5	6	7	8	9&10
500	341.42	314.14	245.80	201.48	158.60	112.37	55.66	55.66	55.66
400	273.88	251.99	197.19	161.61	127.22	91.12	45.10	45.10	45.10
300	206.34	189.85	148.56	121.76	95.85	69.47	34.43	34.43	34.43
250	172.56	158.77	124.23	101.83	80.15	58.03	28.79	28.79	28.79
200	138.78	127.69	99.92	81.89	64.47	47.19	23.40	23.40	23.40
175	121.37	111.68	87.39	71.62	56.38	41.27	20.39	20.39	20.39
150	104.49	96.13	75.22	61.66	48.53	35.96	17.75	17.75	17.75
125	87.59	80.60	63.07	51.69	40.69	30.24	15.00	15.00	15.00
110	77.57	71.37	55.85	45.77	36.04	28.61	14.40	14.40	14.40
100	71.23	65.55	51.29	42.04	33.09	27.58	14.03	10.80	9.90
92	66.48	61.18	47.88	39.24	30.89	25.75	13.68	10.52	9.66
85	61.74	56.80	44.45	36.43	28.68	23.91	13.20	10.15	9.32
77	56.99	52.44	41.04	33.63	26.48	22.07	12.60	9.68	8.89
70	52.77	48.55	37.99	31.14	24.51	20.43	12.00	9.23	8.47
65	50.07	46.08	36.05	29.56	23.04	19.39	11.87	9.14	8.39
60	47.44	43.64	34.15	28.00	21.82	18.37	11.76	9.04	8.30
55	44.75	41.17	32.22	26.40	20.59	17.32	11.64	8.96	8.22
50	41.57	38.26	29.94	24.54	19.12	16.10	11.52	8.85	8.14
Tons (q_i^B)	0.25	0.5	1	2.5	5	10	15	20	∞



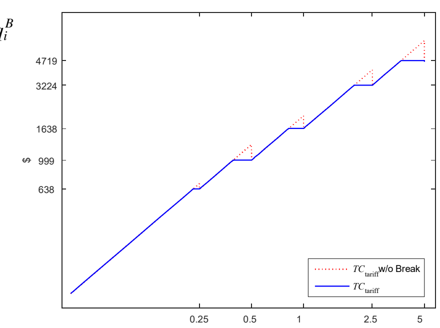
Truck Shipment Example: One-Time

- What is the weight break between the rate breaks at 0.25 and 0.5 tons?

$$q_i^W = \frac{OD(class, i+1)}{OD(class, i)} q_i^B$$

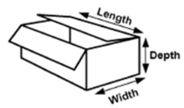
$$= \frac{99.92}{127.69}(0.5)$$

$$= 0.3913 \text{ ton}$$



Truck Shipment Example: One-Time

- Additional information needed for online one-time (spot) LTL rate quotes using Coyote.com website
  - Shipment weight in pounds:



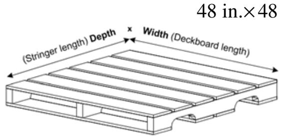
$$2000\, q_{LTL} = 2000\,(0.3) = 600 \text{ lb}$$

  - Carton dimensions:

$$cu = 9 \text{ ft}^3 = 9 \times 12^3 \text{in}^3 \Rightarrow l \times w \times d = 15{,}552 \text{ in}^3 = 24 \times 24 \times 27 \text{ in}^3$$

  - Stack cartons on pallet to make it easier to transload: (Why was this not as necessary for TL?)

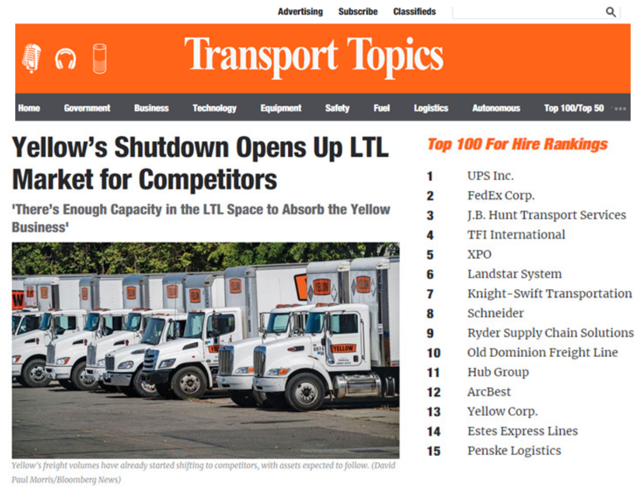
$$48 \text{ in.} \times 48 \text{ in. pallet} \Rightarrow 8 \text{ carton/pallet} \Rightarrow \left\lceil \frac{15}{8} \right\rceil = 2 \text{ pallets}$$



$$\Rightarrow \text{height} = 2 \times 27 + 5 = 59 \text{ in.}$$

(5 in. height of unloaded pallet)

Misc.



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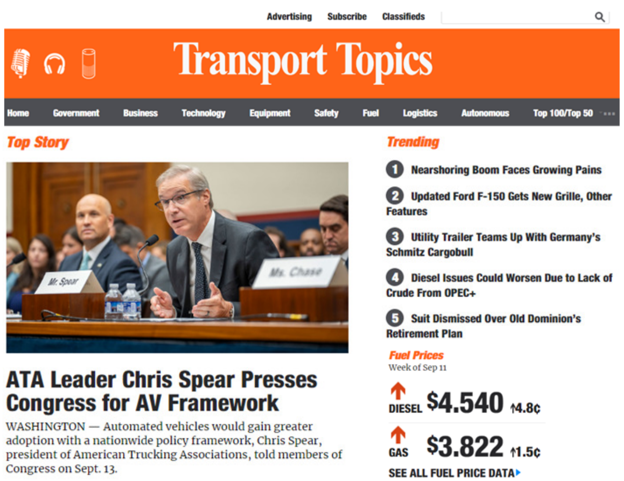

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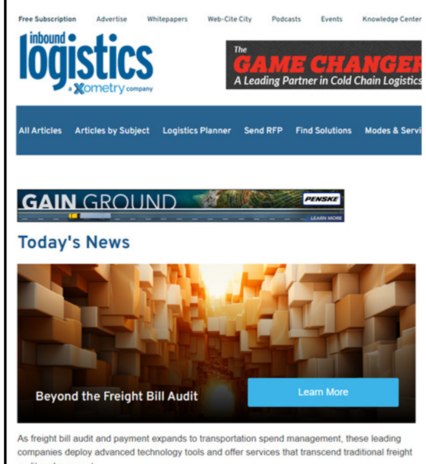

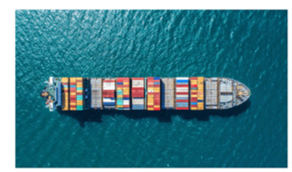

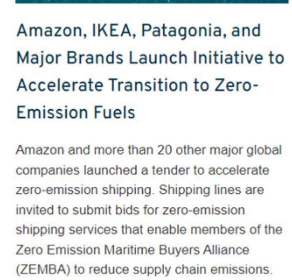

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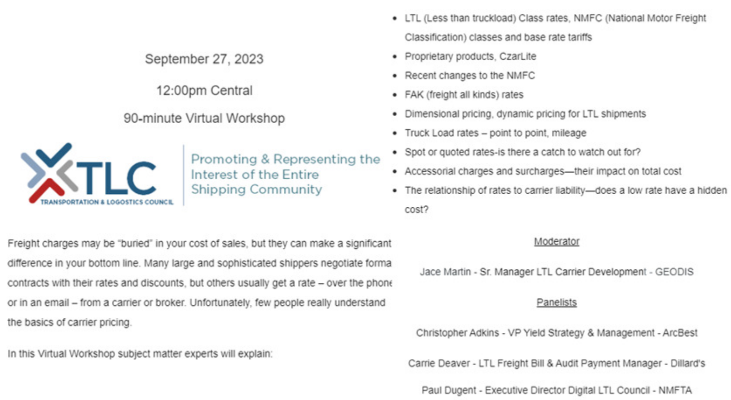

September 27, 2023

12:00pm Central

90-minute Virtual Workshop

TLC TRANSPORTATION & LOGISTICS COUNCIL

Promoting & Representing the Interest of the Entire Shipping Community

Freight charges may be "buried" in your cost of sales, but they can make a significant difference in your bottom line. Many large and sophisticated shippers negotiate forma contracts with their rates and discounts, but others usually get a rate – over the phone or in an email – from a carrier or broker. Unfortunately, few people really understand the basics of carrier pricing.

In this Virtual Workshop subject matter experts will explain:

- LTL (Less than truckload) Class rates, NMFC (National Motor Freight Classification) classes and base rate tariffs
- Proprietary products, CzarLite
- Recent changes to the NMFC
- FAK (freight all kinds) rates
- Dimensional pricing, dynamic pricing for LTL shipments
- Truck Load rates – point to point, mileage
- Spot or quoted rates-is there a catch to watch out for?
- Accessorial charges and surcharges—their impact on total cost
- The relationship of rates to carrier liability—does a low rate have a hidden cost?

Moderator

Jace Martin - Sr. Manager LTL Carrier Development - GEODIS

Panelists

Christopher Adkins - VP Yield Strategy & Management - ArcBest

Carrie Deaver - LTL Freight Bill & Audit Payment Manager - Dillard's

Paul Dugent - Executive Director Digital LTL Council - NMFTA

Transport 3: Periodic Truck Shipments

- **One-Time** (*operational* decision): know shipment size q
  - Know when and how much to ship
- **Periodic** (*tactical*): must determine size q
  - Need to determine how often and how much to ship

Truck Shipment Example: Periodic

9. Continuing with the example: assuming a constant annual demand for the product of 20 tons, what is the number of full truckloads per year?

$$f = 20 \text{ ton/yr}$$

$$q = q_{\max} = 6.1111 \text{ ton/ TL} \quad (\text{full truckload} \Rightarrow q \equiv q_{\max})$$

$$n = \frac{f}{q} = \frac{20}{6.1111} = 3.2727 \text{ TL/yr}, \quad \text{average shipment frequency}$$

- Why should this number not be rounded to an integer value?

Truck Shipment Example: Periodic

10. What is the shipment interval?

$$t = \frac{1}{n} = \frac{q}{f} = \frac{6.1111}{20} = 0.3056 \text{ yr/TL}, \quad \text{average shipment interval}$$

- How many days are there between shipments?

$$365.25 \text{ day/yr}$$

$$t \times 365.25 = \frac{365.25}{n} = 111.6042 \text{ day/TL}$$

Truck Shipment Example: Periodic

11. What is the annual full-truckload transport cost?

$$d = 532 \text{ mi}, \quad r_{TL} = \frac{PPI_{TL}^{\text{Jan 2018}}}{102.7} \times \$2.00/\text{mi} = \frac{131.0}{102.7} \times \$2.00/\text{mi} = \$2.5511/\text{mi}$$

$$r_{FTL} = \frac{r_{TL}}{q_{\max}} = \frac{2.5511}{6.1111} = \$0.4175/\text{ton-mi}$$

$$TC_{FTL} = f\, r_{FTL}\, d = n\, r_{TL} d \quad (= w\, d,\ w = \text{monetary weight in \$/mi})$$

$$= 3.2727\,(2.5511)\,532 = \$4,441.73/\text{yr}$$

- What would be the cost if the shipments were to be made at least every three months?

$$t_{\max} = \frac{3}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 4 \text{ TL/yr} \quad \Rightarrow \quad q = \frac{f}{\max\{n, n_{\min}\}}$$

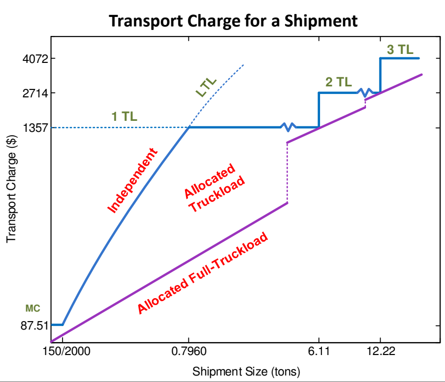
$$TC'_{FTL} = \max\{n, n_{\min}\}\, r_{TL} d$$

$$= \max\{3.2727, 4\}\, 2.5511(532) = \$5,428.78/\text{yr}$$

Truck Shipment Example: Periodic

- Independent and allocated full-truckload charges:

$$q \le q_{\max} \Rightarrow [UB, LB] = [c_0(q),\, q\, r_{FTL} d]$$



Truck Shipment Example: Periodic

- Same units of inventory can serve multiple roles at each position in a production process

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)
- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)
- *Safety stock:* held to buffer effects of uncertainty
  - (decoupling, MRO (maintenance, repair, and operations))

Truck Shipment Example: Periodic

- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$TLC = TC + IC + PC$$
$$TC = \text{transport cost}$$
$$IC = \text{inventory cost}$$
$$= IC_{\text{working}} + IC_{\text{economic}} + IC_{\text{safety}}$$
$$PC = \text{purchase cost}$$

> *Total logistics costs* are any of the relevant costs associated with providing a logistics service, where a relevant cost is a cost that differs when comparing multiple alternatives and, as such, can be used in making a decision between the alternatives.

- *Economic* (*cycle*) *stock*: held to allow cheaper large shipments
- *Working* (*in-transit*) *stock*: goods in transit or awaiting transshipment
- *Safety stock*: held due to transport uncertainty
  (e.g., shipment arriving earlier than needed "just in case")
- *Purchase cost*: can be different for different suppliers

Truck Shipment Example: Periodic

12. Since demand is constant throughout the year, one half of a shipment is stored at the destination, on average. Assuming that the production rate is also constant, one half of a shipment will also be stored at the origin, on average. Assuming each ton of the product is valued at \$25,000, what is a "reasonable estimate" for the total annual cost for this cycle inventory?

$$IC_{\text{cycle}} = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$
$$= vh\ (\$/\text{ton-yr}) \times \alpha q\ (\text{ton}) = (\$/\text{yr})$$

v = unit value of shipment (\$/ton)

h = inventory carrying rate, cost per dollar of inventory per year (\$/\$-yr = 1/yr)

α = average inter-shipment inventory fraction at Origin and Destination

q = shipment size (ton)

Truck Shipment Example: Periodic

- **Inv. Carrying Rate** (h) = interest + warehousing + obsolescence
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate (*yr*) h = 0.3 $\Rightarrow h_{\text{obs}} \approx$ 0.2 (mfg product)
  - *Low* FGI cost (*yr*): $h = h_{\text{int}} + h_{\text{wh}} + h_{\text{obs}}$
  - *High* FGI cost (*hr*): $h \approx h_{\text{obs}}$, can ignore interest & warehousing
    - $(h_{\text{int}}+h_{\text{wh}})/H$ = (0.05+0.06)/2000 = 0.000055 (H = oper. hr/yr)
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h_{obs}

$$h_{\text{obs}} t_h v = x_h v \Rightarrow h_{\text{obs}} t_h = x_h \Rightarrow \boxed{h_{\text{obs}} = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h_{\text{obs}}}$$

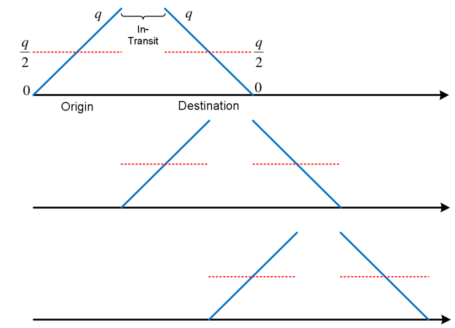
  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as t_{CT}

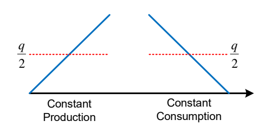
Truck Shipment Example: Periodic

- Average annual inventory level $= \frac{q}{2} + \frac{q}{2} = \left(\frac{1}{2} + \frac{1}{2}\right) q = (1)q \Rightarrow \alpha = 1$

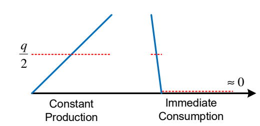


Truck Shipment Example: Periodic

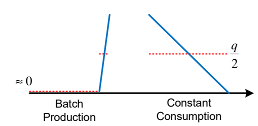
- Inter-shipment inventory fraction alternatives: $\alpha = \alpha_O + \alpha_D$



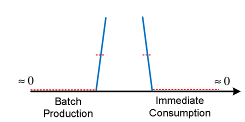
$$\alpha = \frac{1}{2} + \frac{1}{2} = 1$$



$$\alpha = \frac{1}{2} + 0 = \frac{1}{2}$$



$$\alpha = 0 + \frac{1}{2} = \frac{1}{2}$$



$$\alpha = 0 + 0 = 0$$

Truck Shipment Example: Periodic

- "Reasonable estimate" for the total annual cost for the cycle inventory:

$$
\begin{aligned}
IC_{\text{cycle}} &= \alpha v h q \\
&= (1)(25{,}000)(0.3)6.1111 \\
&= \$45{,}833.33 / \text{yr}
\end{aligned}
$$

where

$$\alpha = \frac{1}{2} \text{ at Origin } + \frac{1}{2} \text{ at Destination} = 1$$

$v = \$25{,}000 =$ unit value of shipment (\$/ton)

$h = 0.3 =$ estimated carrying rate for manufactured products (1/yr)

$q = q_{\max} = 6.111 =$ FTL shipment size (ton)

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Truck Shipment Example: Periodic

13. What is the annual total logistics cost (TLC) for these full-truckload TL shipments?

$$
\begin{aligned}
TLC_{FTL} &= TC_{FTL} + IC_{\text{cycle}} \\
&= n\, r_{TL} d + \alpha v h q \\
&= 3.2727\,(2.5511)\,532 + (1)(25{,}000)(0.3)6.1111 \\
&= 4{,}441.73 + 45{,}833.33 \\
&= \$50{,}275.06 / \, yr
\end{aligned}
$$

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Truck Shipment Example: Periodic

14. What is minimum possible annual total logistics cost for TL shipments, where the shipment size can now be less than a full truckload?

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha v h q = \frac{f}{q} r d + \alpha v h q$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q_{TL}^{*} = \sqrt{\frac{f\, r_{TL} d}{\alpha v h}} = \sqrt{\frac{20(2.5511)532}{(1)25000(0.3)}} = 1.9024 \text{ ton}$$

$$
\begin{aligned}
TLC_{TL}(q_{TL}^{*}) &= \frac{f}{q_{TL}^{*}} r_{TL} d + \alpha v h q_{TL}^{*} \\
&= \frac{20}{1.9024}(2.5511)532 + (1)25000\,(0.3)1.9024 \\
&= 14{,}268.12 + 14{,}268.12 \\
&= \$28{,}536.25 / \text{yr}
\end{aligned}
$$

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Truck Shipment Example: Periodic

- Including the minimum charge and maximum payload restrictions:

$$q_{TL}^{*} = \min\left\{ \sqrt{\frac{f \max\{r_{TL} d, MC_{TL}\}}{\alpha v h}}, q_{\max} \right\} \approx \sqrt{\frac{f\, r_{TL} d}{\alpha v h}}$$

- What is the TLC if this size shipment could be made as an allocated full-truckload?

$$
\begin{aligned}
TLC_{AllocFTL}(q_{TL}^{*}) &= \frac{f}{q_{TL}^{*}}\left(q_{TL}^{*} r_{FTL} d\right) + \alpha v h q_{TL}^{*} = f \frac{r_{TL}}{q_{\max}} d + \alpha v h q_{TL}^{*} \\
&= 20 \frac{2.5511}{6.1111} 532 + (1)25000\,(0.3)1.9024 \\
&= 4{,}441.73 + 14{,}268.12 \\
&= \$18{,}709.85 / \text{yr} \quad \left(\text{vs. } \$28{,}536.25 \text{ as independent P2P TL}\right)
\end{aligned}
$$

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Truck Shipment Example: Periodic

15. What is the optimal LTL shipment size?

$$TLC_{LTL}(q) = TC_{LTL}(q) + IC(q) = \frac{f}{q}c_{LTL}(q) + \alpha vhq$$

$$q^*_{LTL} = \arg\min_q TLC_{LTL}(q) = 0.7622 \text{ ton}$$

- Must be careful in picking bounds for optimization since the LTL formula is only valid for a limited range of values:

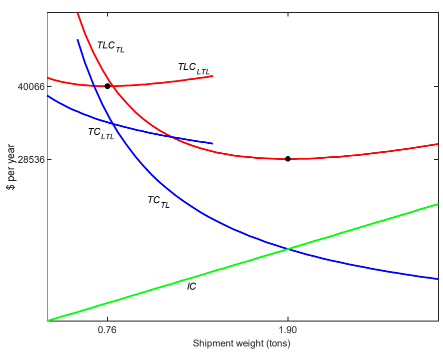
$$r_{LTL} = PPI_{LTL}\left[\frac{\frac{s^2}{8}+14}{\left(q^{\frac{1}{7}}d^{\frac{15}{29}}-\frac{7}{2}\right)(s^2+2s+14)}\right], \quad \begin{cases} 37 \le d \le 3354 \text{ (dist)} \\ \frac{150}{2{,}000} \le q \le \frac{10{,}000}{2{,}000} \text{ (wt)} \\ 2000\frac{q}{s} \le 650 \text{ ft}^3 \text{ (cube)} \end{cases}$$

$$\frac{150}{2000} \le q \le \min\left\{\frac{10{,}000}{2{,}000}, \frac{650s}{2000}\right\} \Rightarrow 0.075 \le q \le 1.44$$

Truck Shipment Example: Periodic

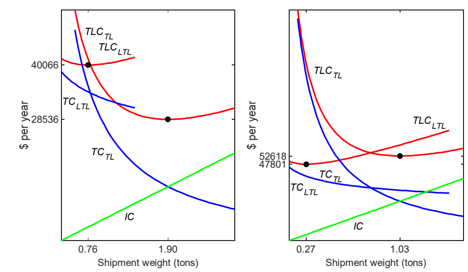
16. Should the product be shipped TL or LTL?

$$TLC_{LTL}(q^*_{LTL}) = TC_{LTL}(q^*_{LTL}) + IC(q^*_{LTL}) = 34{,}349.19 + 5{,}716.40 = \$40{,}065.59 / \text{yr}$$



Truck Shipment Example: Periodic

17. If the value of the product increased to $85,000 per ton, should the product be shipped TL or LTL?



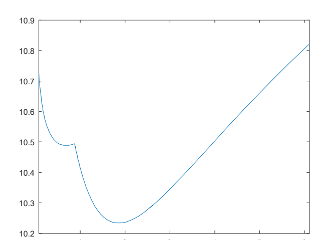
(a) $25000 value per ton

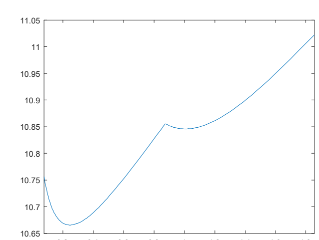
(b) $85000 value per ton

Truck Shipment Example: Periodic

- Better to pick from separate optimal TL and LTL because independent charge has two local minima:

$$q^*_0 = \arg\min_q \{TLC_{TL}(q), TLC_{LTL}(q)\} \qquad q^*_0 \overset{!}{=} \arg\min_q \left\{\frac{f}{q}c_0(q) + \alpha vhq\right\}$$





Truck Shipment Example: Periodic

18. On Jan 10, 2018, what was the optimal independent shipment size to ship 80 tons per year of a Class 60 product valued at \$5000 per ton between Raleigh and Gainesville?

$$s = 32.16 \text{ lb/ft}^3$$

$$q_0^* = \arg\min_q \{TLC_{TL}(q), TLC_{LTL}(q)\} = 8.5079 \text{ ton}$$

$$TLC_{TL}(q_0^*) = \$25{,}523.60 / \text{yr} < TLC_{LTL}(q_0^*)$$

Class-Density Relationship

Class	Load Density (lb/ft³) Minimum	Average	Max Physical Weight (tons)	Max Effective Cube (ft³)
500	–	0.52	0.72	*2,750*
400	1	1.49	2.06	*2,750*
300	2	2.49	3.43	*2,750*
65	22.5	25.50	*25*	1,961
60	30	32.16	*25*	1,555
55	35	39.68	*25*	1,260
50	50	56.18	*25*	890

Truck Shipment Example: Periodic

19. What is the optimal shipment size if both shipments will always be shipped together on the same truck (with same shipment interval)?

$$d_1 = d_2, \quad h_1 = h_2, \quad \alpha_1 = \alpha_2$$

$$f_{\text{agg}} = f_1 + f_2 = 20 + 80 = 100 \text{ ton}$$

$$s_{\text{agg}} = \frac{(\text{aggregate weight, in lb})}{(\text{aggregate cube, in ft}^3)} = \frac{f_{\text{agg}}}{\dfrac{f_1}{s_1} + \dfrac{f_2}{s_2}} = \frac{100}{\dfrac{20}{4.44} + \dfrac{80}{32.16}} = 14.31 \text{ lb/ft}^3$$

$$v_{\text{agg}} = \frac{f_1}{f_{\text{agg}}} v_1 + \frac{f_2}{f_{\text{agg}}} v_2 = \frac{20}{100} 85{,}000 + \frac{80}{100} 5000 = \$21{,}000 / \text{ton}$$

$$q_{TL}^* = \sqrt{\frac{f_{\text{agg}} r d}{\alpha v_{\text{agg}} h}} = \sqrt{\frac{100(2.5511)532}{(1)21000(0.3)}} = 4.6414 \text{ ton}$$

Truck Shipment Example: Periodic

- Summary of results:

```
         :   f      s        v     qmax      TLC        q      t
---------:-------------------------------------------------------
       1:   20    4.44   85,000    6.11   47,801.01   0.27    5.00
       2:   80   32.16    5,000   25.00   25,523.60   8.51   38.84
     1+2:                                 73,324.60
Aggregate:  100   14.31   21,000   19.68   58,481.90   4.64   16.95
```


Transport 4: Transshipment

- FTL $\Rightarrow q = q_{\max}$, while TL $\Rightarrow q \le q_{\max}$
- As a result, FTL has two very useful features:
    1. Most TL shipments should be close to FTL
        - Not a good long-term strategy to ship half-empty trucks
        - If necessary, aggregate with other shipments to make it FTL
    2. Monetary transport weights (w) and inventory cost (*IC*) are independent of NF location
        - Only transport cost (*TC*) needed for location analysis
        - Unlike TL and LTL, which require *TLC*

Ex: FTL vs Interval Constraint

- On average, 200 tons of components are shipped 750 miles from your fabrication plant to your assembly plant each year. The components are produced and consumed at a constant rate throughout the year. Currently, full truckloads of the material are shipped. What would be the impact on total annual logistics costs if TL shipments were made every two weeks? The revenue per loaded truck-mile is \$2.00; a truck's cubic and weight capacities are 3,000 ft³ and 24 tons, respectively; each ton of the material is valued at \$5,000 and has a density of 10 lb per ft³; the material loses 30% of its value after 18 months; and in-transit inventory costs can be ignored.

$$f = 200, \quad d = 750, \quad \alpha = \frac{1}{2} + \frac{1}{2} = 1, \quad r_{TL} = 2, \quad K_{cu} = 3000, \quad K_{wt} = 24, \quad v = 5000, \quad s = 10$$

$$h_{\text{obs}} = \frac{x_h}{t_h} = \frac{0.3}{1.5} = 0.2 \Rightarrow h = 0.05 + 0.06 + 0.2 = 0.31, \quad q_{FTL} = q_{\max} = \min\left\{K_{wt}, \frac{sK_{cu}}{2000}\right\} = 15$$

$$n_{FTL} = \frac{f}{q_{FTL}} = 13.33, TLC_{FTL} = n_{FTL} r_{TL} d + \alpha v h q_{FTL} = \$43,250, \quad \text{2-wk TL} \Rightarrow \text{LTL not considered}$$

$$t_{\max} = \frac{2(7)}{365.25} \text{ yr/TL} \Rightarrow n_{\min} = \frac{1}{t_{\max}} = 26.09 \text{ TL/yr}, \quad q_{2\text{wk}} = \frac{f}{\max\{n_{\text{FTL}}, n_{\min}\}} = 7.67 \text{ ton/TL}$$

$$TLC_{2\text{wk}} = n_{\min} r_{TL} d + \alpha v h q_{2\text{wk}} = \$51,016 \quad \Rightarrow \Delta TLC = TLC_{2\text{wk}} - TLC_{FTL} = \$7,766 \text{ per year increase}$$

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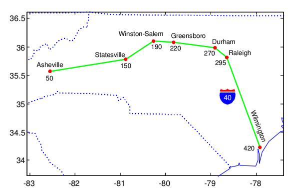
Ex: FTL Location

- Where should a DC be located in order to minimize transportation costs, given:
  1. FTLs containing mix of products A and B shipped P2P from DC to customers in Winston-Salem, Durham, and Wilmington
  2. Each customer receives 20, 30, and 50% of total demand
  3. 100 tons/yr of A shipped FTL P2P to DC from supplier in Asheville
  4. 380 tons/yr of B shipped FTL P2P to DC from Statesville
  5. Each carton of A weighs 30 lb, and occupies 10 ft³
  6. Each carton of B weighs 120 lb, and occupies 4 ft³
  7. Revenue per loaded truck-mile is \$2
  8. Each truck's cubic and weight capacity is 2,750 ft³ and 25 tons, respectively



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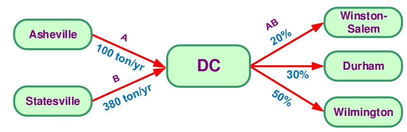
Ex: FTL Location



$$\underset{(\$/\text{yr})}{TC} = \sum \underset{(\$/\text{mi-yr})}{w_i} \times \underset{(\text{mi})}{d_i}$$

$$\underset{(\$/\text{mi-yr})}{w_i} = \underset{(\text{ton/yr})}{f_i} \times \underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \underset{(\text{TL/yr})}{n_i} \times \underset{(\$/\text{TL-mi})}{r_i}$$

$$\underset{(\$/\text{ton-mi})}{r_{FTL,i}} = \frac{r}{q_{\max}}, \quad n = \frac{f}{q_{\max}}$$

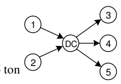
$$r = \$2/\text{TL-mi}, \quad f_{\text{agg}} = f_A + f_B = 100 + 380 = 480 \text{ ton/yr}, \quad s_{\text{agg}} = \frac{f_{\text{agg}}}{\frac{f_A}{s_A} + \frac{f_B}{s_B}} = \frac{480}{\frac{100}{3} + \frac{380}{30}} = 10.4348 \text{ lb/ft}^3, \quad q_{\max} = \left\{25, \frac{10.4348(2750)}{2000}\right\} = 14.3478$$

$$s_1 = \frac{30}{10} = 3 \text{ lb/ft}^3, q_{\max} = \min\left\{25, \frac{3(2750)}{2000}\right\} = 4.125 \text{ ton}$$

$$f_1 = 100, n_1 = \frac{100}{4.125} = 24.24, w_1 = 24.24(2) = 48.48$$

$$s_2 = \frac{120}{4} = 30 \text{ lb/ft}^3, q_{\max} = \min\left\{25, \frac{30(2750)}{2000}\right\} = 25 \text{ ton}$$

$$f_2 = 380, n_2 = \frac{380}{25} = 15.2, w_2 = 15.2(2) = 30.4$$

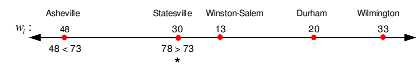


$$f_3 = 0.20 f_{\text{agg}} = 96, n_3 = \frac{96}{14.3478} = 6.69, w_3 = 6.69(2) = 13.38$$

$$f_4 = 0.30 f_{\text{agg}} = 144, n_4 = \frac{144}{14.3478} = 10.04, w_4 = 10.04(2) = 20.07$$

$$f_5 = 0.50 f_{\text{agg}} = 240, n_5 = \frac{240}{14.3478} = 16.73, w_5 = 16.73(2) = 33.45$$

$$W = 146, \quad \frac{W}{2} = 73$$



$$\text{(Montetary) Weight Losing:} \quad \Sigma w_{\text{in}} = 79 > \Sigma w_{\text{out}} = 67 \quad \left(\Sigma n_{\text{in}} = 39 > \Sigma n_{\text{out}} = 33\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$$

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Ex: FTL Location

- (Extension) Include monthly outbound frequency constraint:
  - Outbound shipments must occur at least once each month

    (provides an implicit means of including inventory costs in location decision)

$$t_{\max} = \frac{1}{12} \text{ yr/TL} \quad \Rightarrow \quad n_{\min} = \frac{1}{t_{\max}} = 12 \text{ TL/yr}$$

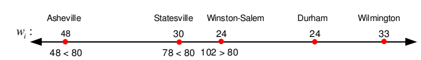
$$TC'_{FTL} = \max\{n, n_{\min}\} rd$$

$$n_3 = \max\{6.69, 12\} = 12, w_3 = 12(2) = 24$$

$$n_4 = \max\{10.04, 12\} = 12, w_4 = 12(2) = 24$$

$$n_5 = \max\{16.73, 12\} = 16.73, w_5 = 16.73(2) = 33.45$$

$$W = 160, \quad \frac{W}{2} = 80$$



$$\text{(Montetary) Weight Gaining:} \quad \Sigma w_{\text{in}} = 79 < \Sigma w_{\text{out}} = 81 \quad \left(\Sigma n_{\text{in}} = 39 < \Sigma n_{\text{out}} = 41\right)$$

$$\text{Physically Weight Unchanging (DC):} \quad \Sigma f_{\text{in}} = 480 = \Sigma f_{\text{out}} = 480$$

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Location and Transport Costs

- Monetary weights *w* used for location are, in general, a function of the location of a NF
  - Distance *d* appears in optimal TL size formula
  - TC & IC functions of location $\Rightarrow$ Need to minimize TLC instead of TC
  - FTL (since size is fixed at max payload) results in only constant weights for location $\Rightarrow$ Need to only minimize TC since IC is constant in TLC

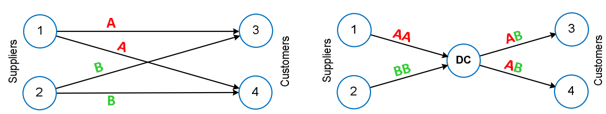
$$TLC_{TL}(\mathbf{x}) = \sum_{i=1}^{m} w_i(\mathbf{x})d_i(\mathbf{x}) + \alpha v h q_i(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_i(\mathbf{x})} r d_i(\mathbf{x}) + \alpha v h\, q_i(\mathbf{x})$$

$$= \sum_{i=1}^{m} \frac{f_i}{\sqrt{\frac{f_i r d_i(\mathbf{x})}{\alpha v h}}} r d_i(\mathbf{x}) + \alpha v h \sqrt{\frac{f_i r d_i(\mathbf{x})}{\alpha v h}} = \sum_{i=1}^{m} \sqrt{f_i r d_i(\mathbf{x})} \left(\frac{1}{\sqrt{\alpha v h}} + \sqrt{\alpha v h} \right)$$

$$TLC_{FTL}(\mathbf{x}) = \sum_{i=1}^{m} \frac{f_i}{q_{\max}} r d_i(\mathbf{x}) + \alpha v h\, q_{\max} = \sum_{i=1}^{m} w_i d_i(\mathbf{x}) + \alpha v h\, q_{\max} = TC_{FTL}(\mathbf{x}) + \text{constant}$$

Transshipment

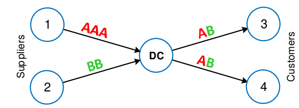
- *Direct*: P2P shipments from Suppliers to Customers



- *Transshipment*: use DC to consolidate outbound shipments
  - *Uncoordinated*: determine separately each optimal inbound and outbound shipment $\Rightarrow$ hold inventory at DC
  - *(Perfect) Cross-dock*: use single shipment interval for all inbound and outbound shipments $\Rightarrow$ no inventory at DC (non-perfect: only cross-dock a selected subset of shipments)

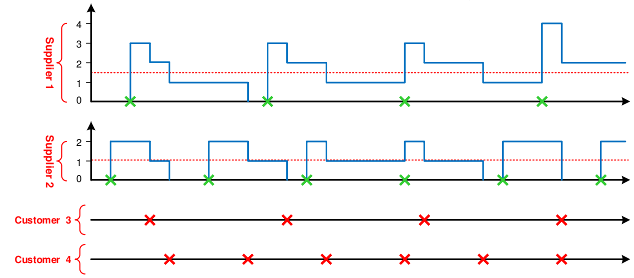
Uncoordinated Inventory

- Average in-transit inventory level at DC:



$$q_A^* = 3, \quad q_B^* = 2$$

$$\alpha = \begin{cases} \alpha_S + \frac{1}{2}, & \text{inbound} \\ 0 + \alpha_C, & \text{outbound} \end{cases}$$



$$1.55 \approx \frac{q_A^*}{2}$$

$$1.11 \approx \frac{q_B^*}{2}$$

TLC with Transshipment

- Uncoordinated: $TLC_i = TLC$ of supplier/customer i

$$q_i^* = \arg\min_q TLC_i(q)$$

$$TLC^* = \sum TLC_i\left(q_i^*\right)$$

- Cross-docking: $t = \frac{q}{f}$, shipment interval $(\Rightarrow q = ft)$

$$TLC_i(t) = \frac{c_0(t)}{t} + \alpha v h f t \quad \left(\text{cf. } TLC_i(q) = \frac{f}{q} c_0(q) + \alpha v h q\right)$$

$c_0(t) =$ independent transport charge as function of t

$$\alpha = \begin{cases} \alpha_S + 0, & \text{inbound} \\ 0 + \alpha_C, & \text{outbound} \end{cases}$$

$$t^* = \arg\min_t \sum TLC_i(t)$$

$$TLC^* = \sum TLC_i\left(t^*\right)$$

Ex: TLC Location with Transshipment

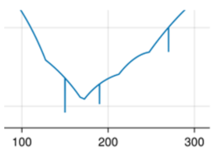
- Continuing with *Ex: FTL Location*:
  1. All transport TL (no LTL)
  2. Each carton of A valued at $300
  3. Each carton of B valued at $450
  4. Batch production, constant consumption
  5. Transshipments at the DC use either uncoordinated inventory (UC) or perfect cross-docking (XD)

$$\alpha = \begin{cases} \alpha_S + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}, & \text{inbound} \\ 0 + \alpha_C = 0 + \frac{1}{2} = \frac{1}{2}, & \text{outbound} \end{cases}$$

$$v_1 = \frac{\text{unit value A}}{\left(\frac{\text{unit weight A}}{2000}\right)} = \frac{300}{\left(\frac{30}{2000}\right)} = \$20{,}000/\text{ton}$$

- Where should the DC be located to minimize TLC?
- UC Result: Locate at 150 (Statesville) by plot inspection

Row	f	s	v	α	h	d	qmax	q°	TLC°	isLTL
	Float64	Float64	Float64	Float64	Float64	Int64	Float64	Float64	Float64	Bool
1	100.0	3.0	20000.0	0.5	0.3	100	4.125	2.58199	15491.9	false
2	380.0	30.0	7500.0	0.5	0.3	0	25.0	3.89872	4386.06	false
3	96.0	10.4348	10104.2	0.5	0.3	40	14.3478	2.25105	6823.49	false
4	144.0	10.4348	10104.2	0.5	0.3	120	14.3478	4.77519	14474.8	false
5	240.0	10.4348	10104.2	0.5	0.3	270	14.3478	9.24712	28030.3	false



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Ex: Direct vs Transhipment

- 3 different products supplied to 4 customers
- Compare:
  1. Direct shipments
  2. Uncoordinated at existing DC in Memphis
  3. Cross-docking at DC in Memphis
- Optimal DC location:
  - Denver for UC ($754K)
  - Kansas for XD ($598K)

Row	f	s	α	v	d	qmax	q°	TLC°	isLTL	t
	Float64	Int64	Float64	Int64	Float64	Float64	Float64	Float64	Bool	Float64
1	100.0	32	0.5	50000	2641.83	25.0	1.99905	1.14292e5	true	7.30154
2	15.0	3	0.5	25000	3043.26	4.125	4.125	43786.7	false	100.444
3	60.0	12	0.5	10000	2057.81	16.5	14.5131	43539.3	false	88.3485
4	175.0	32	0.5	50000	2346.59	25.0	3.10225	1.77379e5	true	6.47485
5	26.25	3	0.5	25000	2136.29	4.125	4.125	50256.0	false	57.3964
6	105.0	12	0.5	10000	906.461	16.5	12.7424	38227.2	false	44.3253
7	125.0	32	0.5	50000	1709.51	25.0	2.07247	1.17048e5	true	6.05576
8	18.75	3	0.5	25000	1867.03	4.125	4.125	37184.9	false	80.355
9	75.0	12	0.5	10000	957.05	16.5	11.0657	33197.2	false	53.8901
10	100.0	32	0.5	50000	626.061	25.0	1.2388	66432.0	true	4.52473
11	15.0	3	0.5	25000	1295.17	4.125	3.641	27307.5	false	88.6583
12	60.0	12	0.5	10000	1327.78	16.5	11.6579	34973.7	false	70.9675

Row	Method	TLC	t̄	LTL
	String	Int64	Float64	Int64
1	Direct Shipments	783623	50.7285	4
2	Uncoord Inv at DC	797779	18.6735	0
3	Cross-Docking at DC	657532	18.2625	0

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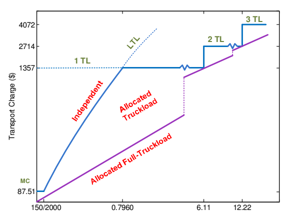
Ex: TLC Location with Transshipment

- **Problem:** Solution not realistic because both UC and XD shipments using only small portion of maximum TL payload; fixes:
  - Include LTL as a transport option (35 mi distance limit a problem)
  - Consider current XD solution as UB on TLC, use allocated FTL as LB




```
Uncooridinated         = 69,206.63 ($/yr)
Perfect Cross-Docking  = 55,539.18 ($/yr)
Allocated FTL          = 44,594.80 ($/yr)
```

$$r_{FTL} = \frac{r_{TL}}{q_{\max}}$$

$$TLC_{AllocFTL}(q^*_{TL}) = \frac{f}{q^*_{TL}}\left(q^*_{TL} r_{FTL} d\right) + \alpha v h q^*_{TL}$$

$$= f\,\frac{r_{TL}}{q_{\max}} d + \alpha v h q^*_{TL}$$

Row	f	qmax	q°	TLC°	qXD	TLC_XD	TLC_AFTL
	Float64	Float64	Float64	Float64	Float64	Float64	Float64
1	100.0	4.125	2.58199	15491.9	3.81713	5239.54	4848.48
2	380.0	25.0	3.89872	4386.06	14.5051	0.0	0.0
3	96.0	14.3478	2.25105	6823.49	3.66444	7649.74	6089.19
4	144.0	14.3478	4.77519	14474.8	5.49666	14618.3	10739.6
5	240.0	14.3478	9.24712	28030.3	9.1611	28031.6	22917.5

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TLC and Location

- TLC should include all logistics-related costs
  - ⇒ TLC can be used as sole objective for network design (incl. location)
- Facility fixed costs, two options:
  1. Use non-transport-related facility costs (mix of top-down and bottom-up) to estimate fixed costs via linear regression
  2. For DCs, might assume public warehouses to be used for all DCs ⇒ Pay only for time each unit spends in WH ⇒ No fixed cost at DC
- Transport fixed costs:
  - Costs that are independent of shipment size (e.g., $/mi vs. $/ton-mi)
    - Costs that make it worthwhile to incur the inventory cost associated with larger shipment sizes in order to spread out the fixed cost
  - Main transport fixed cost is the indivisible labor cost for a human driver
    - Why many logistics networks (e.g., Walmart, Lowes) designed for all FTL transport

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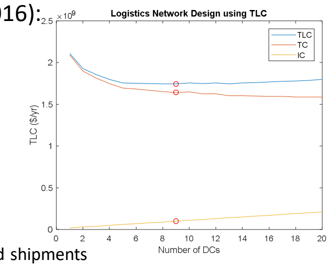
Ex: Optimal Number DCs for Lowe's

- Example of logistics network design using TLC
- Lowe's logistics network (2016):
  - Regional DCs (15)
  - Costal holding facilities
  - Appliance DCs and Flatbed DCs
  - Transloading facilities



- Modeling approach:
  - Focus only on Regional DCs
  - Mix of top-down (COGS) and bottom-up (typical load/TL parameters)
  - FTL for all inbound and outbound shipments
  - ALA used to determine TC for given number of DCs
  - $IC = \alpha vhq_{\max} \times$ (number of suppliers $\times$ number of DCs + number stores)
  - Assume uncoordinated DC inventory, no cross-docking
  - Ignoring max DC-to-store distance constraints, consolidation, etc.
- Determined 9 DCs min TLC (15 DCs $\Rightarrow$ 0.87% increase in TLC)

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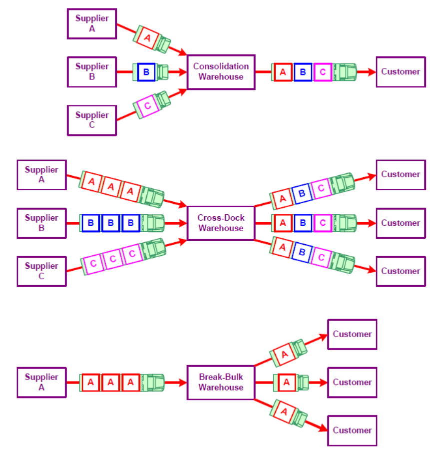
Types of Warehouses



Warehousing

- *Warehousing* are the activities involved in the design and operation of warehouses
- A *warehouse* is the point in the supply chain where raw materials, work-in-process (WIP), or finished goods are stored for varying lengths of time.
- Warehouses can be used to add value to a supply chain in two basic ways:
  1. Storage. Allows product to be available where and when its needed.
  2. Transport Economies. Allows product to be collected, sorted, and distributed efficiently.
- A *public warehouse* is a business that rents storage space to other firms on a month-to-month basis. They are often used by firms to supplement their own *private warehouses*.

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Warehouse Automation

- Historically, warehouse automation has been a craft industry, resulting highly customized, one-off, high-cost solutions
- To survive, need to
  - adapt mass-market, consumer-oriented technologies in order to realize to economies of scale
  - replace mechanical complexity with software complexity
- How much can be spent for automated equipment to replace one material handler:

$$\$45,432\left(\frac{1-1.017^{-5}}{1-1.017^{-1}}\right)=\$45,432(4.83)=\$219,692$$

  - \$45,432: median moving machine operator annual wage + benefits
  - 1.7% average real interest rate 2005-2009 (real = nominal – inflation)
  - 5-year service life with no salvage (service life for Custom Software)

KIVA Mobile-Robotic Fulfillment System

- Goods-to-man order picking and fulfillment system
- Multi-agent-based control
  - Developed by Peter Wurman, former NCSU CSC professor
- Kiva (founded 2003) now called Amazon Robotics
  - purchased by Amazon in 2012 for $775 million

Mick Mountz, Peter Wurman and Raffaello D'Andrea (left to right), creators of Kiva Systems. | Credit: National Inventors Hall of Fame

Topics

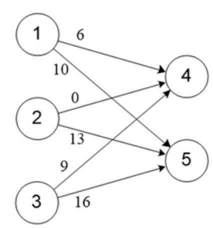
1. Introduction
2. Location
3. Transport
   - Exam 1 (take home)
4. **Networks**
5. Routing
   - Exam 2 (take home)
6. Inventory
   - Final exam (in class)

Networks 1: Assignment and Transportation Problems

- Linear assignment problem (LAP) special case of transportation problem
- Special procedures more efficient than LP were developed to solve LAP and transportation problems
  - e.g., Hungarian method
- Now usually easier to transform into LP since solvers are so
  - Special, very efficient procedures only used for shortest path problem (Dijkstra)

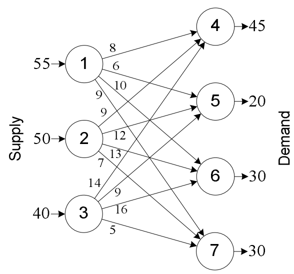
Linear Assignment Problem

- Find least cost set of arcs that connect each destination node with a source node
  - Also called matching problem on a weighted bipartite graph
  - Quadratic assignment problem is when cost of an assignment depends on the other assignments (more difficult to solve)



Transportation Problem

- Satisfy node demand from supply nodes
  - Can be used for allocation in ALA when NFs have capacity constraints
  - Min cost/distance allocation ⇔ infinite supply at each node



Trans	4	5	6	7	Supply
1	8	6	10	9	55
2	9	12	13	7	50
3	14	9	16	5	40
Demand	45	20	30	30	

Greedy Solution Procedure

- Procedure for transportation problem: *Continue to select lowest cost supply until all demand is satisfied*
  - Fast, but not always optimal for transportation problem
  - Dijkstra's shortest path and simplex method for LP are optimal greedy procedures

Trans	4	5	6	7	Supply
1	8	6	10	9	55 -20 = 35 -35 = 0
2	9	12	13	7	50 -10 = 40 -30 = 10
3	14	9	16	5	40 -30 = 10
Demand	~~45~~	~~20~~	~~30~~	~~30~~	
	~~10~~ 0	0	0	0	

$$TC_{\text{greedy}} = 5(30)+6(20)+8(35)+9(10)+13(30) = 1{,}030$$

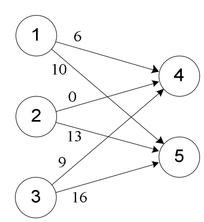
$$TC_{\text{optimal}} = 5(30)+6(20)+8(5)+9(40)+10(30) = 970$$

Networks 2: Shortest Paths and Road Networks

- Great circle distances are not accurate
  - Over shorter (< 50 mi) distances
  - In areas with waterways that restrict road travel
- Actual distance using road network more accurate
- Usually want to find the shortest travel time path
  - Not the shortest distance path

Graph Representations

- Complete bipartite directed (or digraph):
  - Suppliers to multiple DCs, single mode of transport




```
C:  1   2
-:-------
1:  6  10
2:  0  13
3:  9  16
```

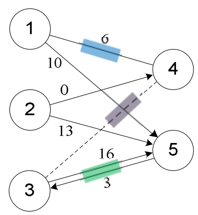
Interlevel matrix

$$W = \begin{matrix} 0 & 0 & 0 & 6 & 10 \\ 0 & 0 & 0 & \sqrt{\varepsilon} & 13 \\ 0 & 0 & 0 & 9 & 16 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Weighted adjacency matrix

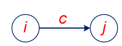
Graph Representations

- Bipartite:
  - One- or two-way connections between nodes in two groups



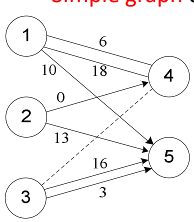

```
W =   0    0    0    6   10
      0    0    0   √ε   13
      0    0    0    0   16
      6    0    0    0    0
      0    0    3    0    0

IJC =
      4    1    6
      5    3    3
      1    4    6
      2    4    0
      1    5   10
      2    5   13
      3    5   16
```

Arc list matrix



Graph Representations

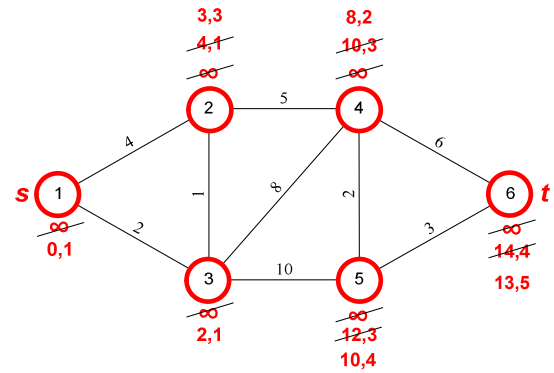
- Multigraph:
  - Multiple connections, multiple modes of transport
  - Simple graph does not have multiple connections




```
IJC = 1   -4    6
      1   -4   18
      1    5   10
      2    4    0
      2    5   13
      3    5   16
      3    5    3
no_W =
      0    0    0   24   10
      0    0    0  NaN   13
      0    0    0    0   19
     24    0    0    0    0
      0    0    0    0    0
```

Can't represent using adjacency matrix

Dijkstra Shortest Path Procedure



Path: $1 \leftarrow 3 \leftarrow 2 \leftarrow 4 \leftarrow 5 \leftarrow 6: \quad 13$

Dijkstra Shortest Path Procedure

procedure $dijkstra(\mathbf{W}, n, s)$
$S \leftarrow \{\,\}, \quad \bar{S} \leftarrow \{1,\ldots,n\}$
for $i \in \bar{S}$, $d(i) \leftarrow \infty$, **endfor**
$d(s) \leftarrow 0, \quad pred(s) \leftarrow 0$
while $|S| < n$
  $i \leftarrow \arg\min_j \{d(j) : j \in \bar{S}\}$
  $S \leftarrow S \cup i, \quad \bar{S} \leftarrow \bar{S} \setminus i$
  for $j \in \arg\{W_{i(j)} : W_{ij} \neq 0\}$
    if $d(j) > d(i) + W_{ij}$
      $d(j) \leftarrow d(i) + W_{ij}$
      $pred(j) \leftarrow i$
    endif
  endfor
endwhile
return d, pred

Procedure	Problem	Time Complexity
Simplex	LP	$O(2^m)$
Ellipsoid	LP	$O(m^4)$
Hungarian	Transportation	$O(m^3)$
Floyd-Warshall	Shortest Path with Cycles	$O(m^3)$
Dijkstra (linear min)	Shortest Path without Cycles	$O(m^2)$
Dijkstra (Fibonocci heap)	Shortest Path without Cycles	$O(n \log m)$
Number of nodes		m
Number of arcs		n

Other Shortest Path Procedures

- Dijkstra requires that all arcs have positive or negative lengths
  - It is a “label setting” algorithm since step to final solution made as each node labeled
  - Can find longest path (used, e.g., in CPM) by negating *all* arc lengths
- Networks with only *some* negative arcs require slower “label correcting” procedures that repeatedly check for optimality at all nodes or detect a negative cycle
  - Requires $O(n^3)$ via Floyd-Warshall algorithm (cf., $O(n^2)$ Dijkstra)
  - Negative arcs used in project scheduling to represent maximum lags between activities
- A* algorithm adds to Dijkstra an heuristic LB estimate of each node’s remaining distance to destination
  - Used in AI search for all types of applications (tic-tac-toe, chess)
  - In path planning applications, great circle distance from each node to destination could be used as LB estimate of remaining distance

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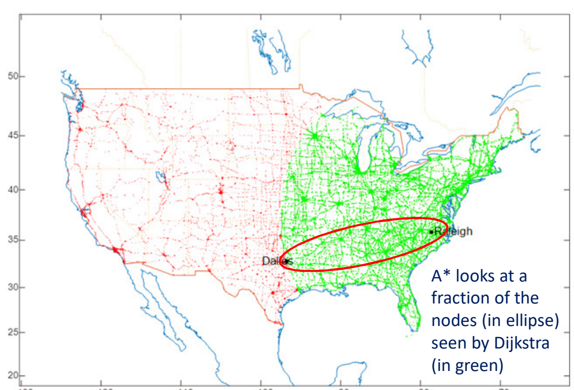
A* Path Planning Example 1

$$d_{A^*}(\text{Raleigh, Dallas}) = d_{dijk}(\text{Raleigh}, i) + d_{GC}(i, \text{Dallas}), \quad \text{for each node } i$$



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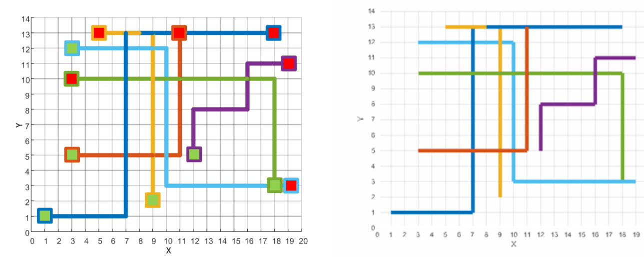
A* Path Planning Example 2

- 3-D (x,y,t) A* used for planning path of each container in a DC
- Each container assigned unique priority that determines planning sequence
  - Paths of higher-priority containers become obstacles for subsequent containers



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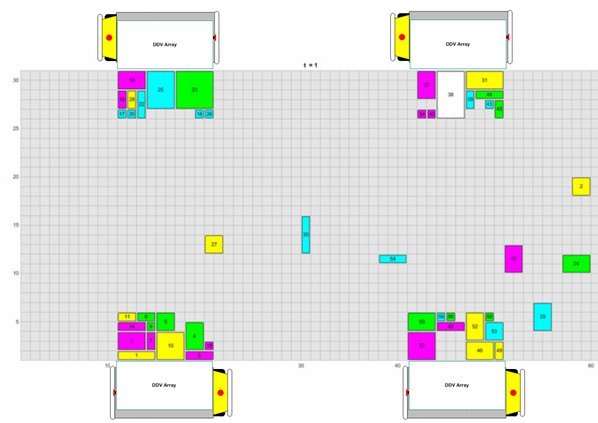
A* Path Planning Example 2



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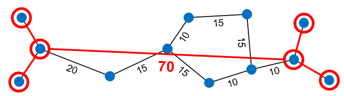
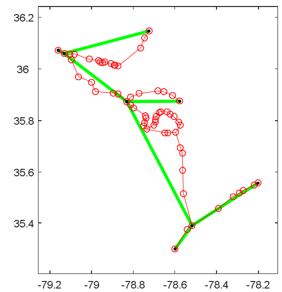
Road Network Modifications

1. **Thin**
   - Remove all degree-2 nodes from network
   - Add cost of both arcs incident to each degree-2 node
   - If results in multiple arcs between pair of nodes, keep minimum cost



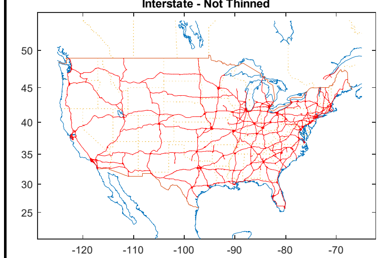
Thinned I-40 Around Raleigh



Road Network Modifications

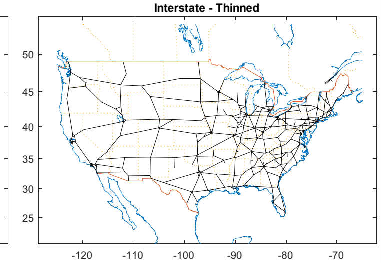
11,124 Arcs

Interstate - Not Thinned



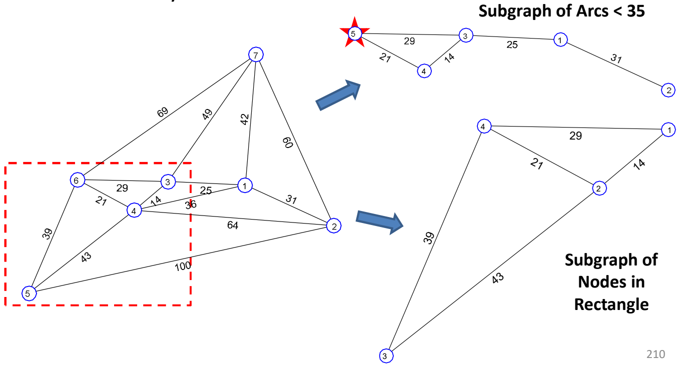
1,223 Arcs

Interstate - Thinned



Road Network Modifications

2. **Prune and Reindex**
   - Extract portion of graph with only those nodes and/or arcs that satisfy some condition

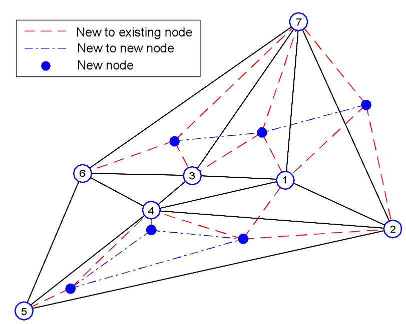


Subgraph of Arcs < 35

Subgraph of Nodes in Rectangle

Road Network Modifications

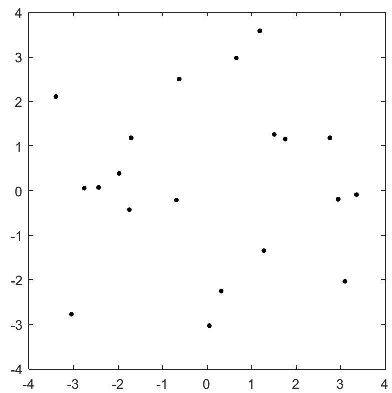
3. **Add connector**
   - Given new nodes, add arcs that connect the new nodes to the existing nodes in a graph and to each other



- Distance of connector arcs = GC distance x circuity factor (1.3)
- New node connected to 3 closest existing nodes, except if
  - Ratio of closest to 2nd and 3rd closest < threshold (0.1)
  - Distance shorter using other connector and graph

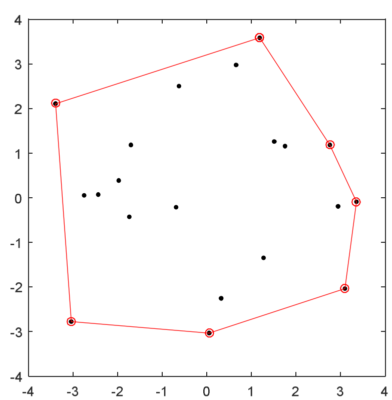
Computational Geometry

- Design and analysis of algorithms for solving geometric problems
  - Modern study started with Michael Shamos in 1975
- Facility location:
  - geometric data structures used to "simplify" solution procedures



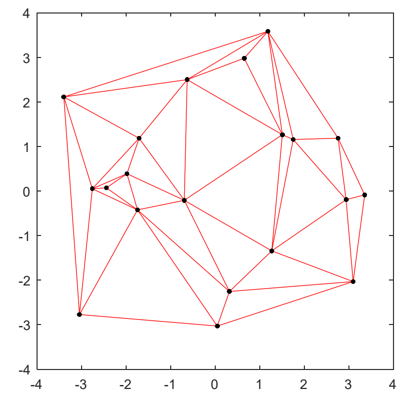
Convex Hull

- Find the points that enclose all points
  - Most important data structure
  - Calculated, via Graham's scan in $O(n \log n)$, n points



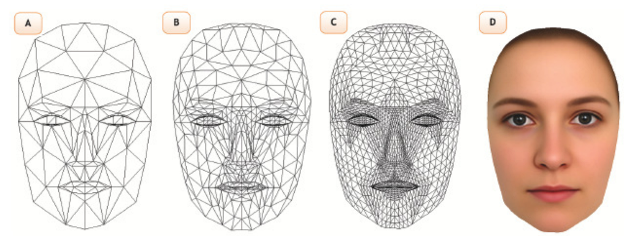
Delaunay Triangulation

- Find the triangulation of points that maximizes the minimum angle of any triangle
  - Captures proximity relationships
  - Used in 3-D animation
  - Calculated, via divide and conquer, in $O(n \log n)$, n points



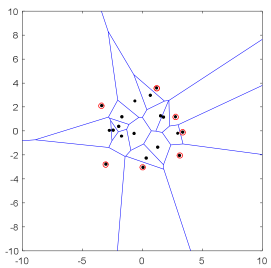
3-D Delaunay Triangulation

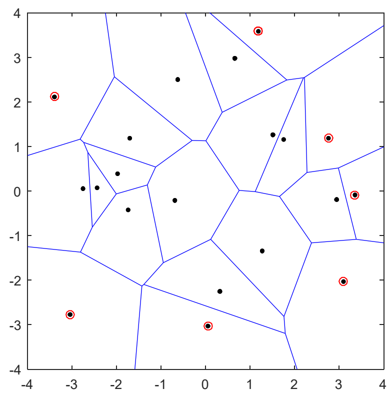
- Used in computer graphics to render synthetic images
  - Orientation of each triangle face used to determine reflection and shading relative to light sources and other objects



Voronoi Diagram

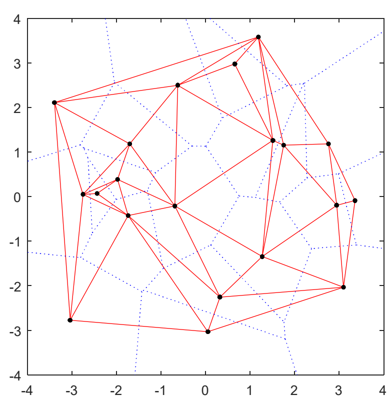
- Each region defines area closest to a point
  - Open face regions indicate points in convex hull





Delaunay-Voronoi

- Delaunay triangulation is straight-line dual of Voronoi diagram
  - Can easily convert from one to another

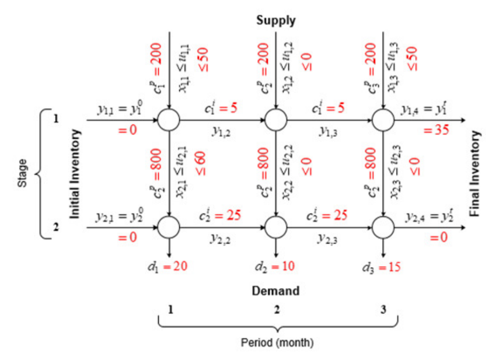


Networks 3: Production-Inventory Planning: Single Product

- Production-inventory planning models are one of the main uses of mathematical programming in industry
  - They provide a means to make complex decisions over a rolling planning horizon
  - Decisions are complex because each has impacts across multiple time periods and multiple stages in the production process
  - Models are resolved each time period, using the latest mix of firm and forecasted orders over the planning horizon

Production and Inventory: One Product

- Time-expanded network used to model multi-period production-inventory problems.



- Inventory cost at each stage based on accumulated prod (mfg and trans) expenditures:

$$h = 0.3\frac{\$}{\$\text{-yr}} = 0.3$$

$$\frac{h}{t} = \frac{0.3}{12}\frac{\$}{\$\text{-month}} = 0.025$$

$$c_m^i = \frac{h}{t}\sum_{j=1}^{m} c_j^p$$

$$c_1^i = \frac{0.3}{12}200 = 5$$

$$c_2^i = \frac{0.3}{12}(200+800)$$
$$= 25$$

Production and Inventory: One Product

$$\min \sum_{i=1}^{m}\sum_{j=1}^{t} c_i^p x_{ij} + \sum_{i=1}^{m}\sum_{j=2}^{t+1} c_i^i y_{ij}$$

subject to

Flow balance $\left\{\begin{array}{ll} x_{ij} - x_{(i+1)j} + y_{ij} - y_{i(j+1)} = 0, & i = 1,\ldots,m-1;\ j = 1,\ldots,t \\ x_{mt} + y_{mj} - y_{m(j+1)} = d_j, & j = 1,\ldots,j \end{array}\right.$

Capacity $\left\{ x_{ij} \le u_{ij}, \quad i = 1,\ldots,m;\ j = 1,\ldots,t \right.$

Initial/Final inventory $\left\{\begin{array}{ll} y_{i1} = y_i^0, & i = 1,\ldots,m \\ y_{i(t+1)} = y_i^{t+1}, & i = 1,\ldots,m \end{array}\right.$

Use var. LB & UB instead of constraints

$x, y \ge 0$ and continuous

where

m = number of production stages

c_i^p = production cost in stage i (\$/ton)

x_{ij} = production at stage i in period j (ton)

c_i^i = inventory cost for stage i (\$/ton)

y_{ij} = stage-i inventory period $j-1$ to j (ton)

t = number of periods of production

d_j = demand in period j (ton)

u_{ij} = capacity of stage i in period j (ton)

y_i^0 = initial inventory of stage i (ton)

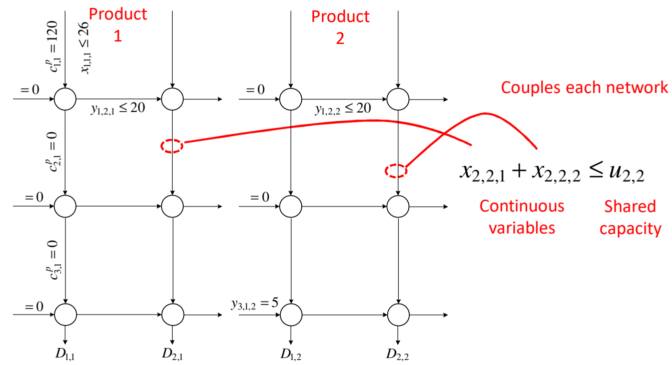
y_i^{t+1} = final inventory of stage i (ton)

Networks 4: Production-Inventory Planning: Multiple Products

- Adding constraints to a math program can never improve a solution
- Can ignore situations that would make a solution worse
  - Makes it easier to add constraints to implement a decision
  - Cf. setup constraints

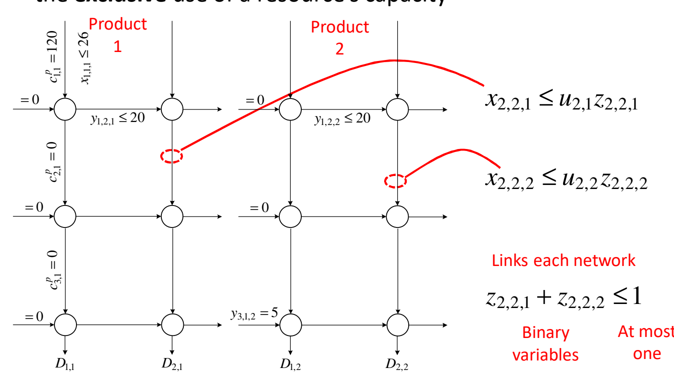
Coupling Constraints

- Otherwise separate product networks connected via sharing the **combined** use of a resource's capacity

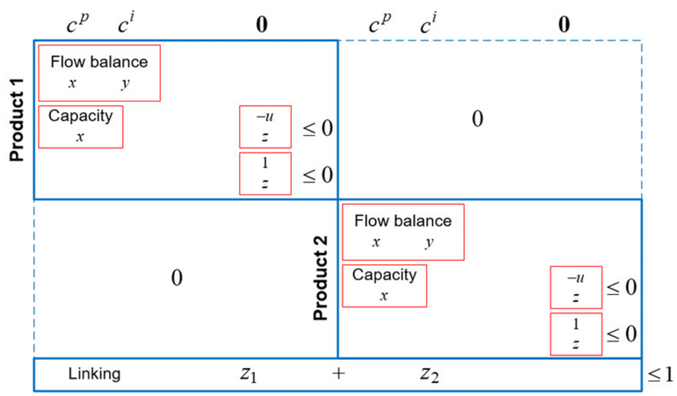


Linking Constraints

- Otherwise separate product networks connected via sharing the **exclusive** use of a resource's capacity



Multiple Products with Exclusive Shared Resources

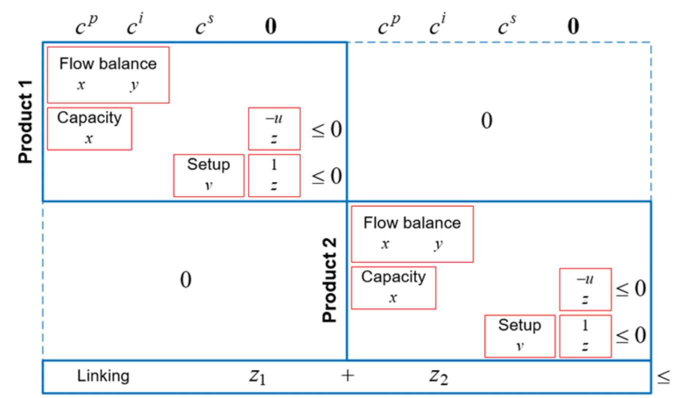


Multiple Products with Exclusive Shared Resources

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{m}\sum_{j=1}^{t}\sum_{k=1}^{g} c^p_{ik} x_{ijk} + \sum_{i=1}^{m}\sum_{j=2}^{t+1}\sum_{k=1}^{g} c^i_{ik} y_{ijk} \\
\text{subject to} \quad & -x_{ijk} + x_{(i+1)jk} - y_{ijk} + y_{i(j+1)k} = 0, && i=1,\dots,m-1; j=1,\dots,t; k=1,\dots,g \quad (a) \\
& -x_{mjk} - y_{mjk} + y_{m(j+1)k} = d_{jk}, && j=1,\dots,t; k=1,\dots,g \quad (b) \\
& x_{ijk} \le u_{ik} z_{ijk}, && i=1,\dots,m; j=1,\dots,t; k=1,\dots,g \quad (c) \\
& \sum_{k=1}^{g} z_{ijk} \le 1, && i=1,\dots,m; j=1,\dots,t \quad (d) \\
& y_{i1k} = y_i^0, && i=1,\dots,m; k=1,\dots,g \\
& y_{i(t+1)k} = y_{ik}^{t+1}, && i=1,\dots,m; k=1,\dots,g \\
& x, y \ge 0; z \in \{0,1\},
\end{aligned}
$$

m = number of production stages
t = number of periods of production
g = number of products
c^p_{ik} = production cost (dollar/ton) in stage i for product k
x_{ijk} = production (ton) at stage i in period j for product k
c^i_{ik} = inventory cost (dollar/ton) in stage i for product k
y_{ijk} = stage-i inventory (ton) from period $j-1$ to j for product k
z_{ijk} = production indicator at stage i in period j for product k
d_{jk} = demand (ton) in period j for product k
u_{ik} = production capacity (ton) of stage i in period j for product k
y^0_{ik} = initial inventory (ton) of stage i for product k
y^{t+1}_{ik} = final inventory (ton) of stage i for product k.

Multiple Products with Setup Costs



Setup Constraints

$z_{mtg} \in \{0,1\}$, production indicator

z_{mtg}	1	2	3	4	5	6	7
1	0	1	1	0	1	1	1
2	0	0	0	1	0	0	0

$v_{mtg} \in \{0,1\}$, setup indicator

v_{mtg}	1	2	3	4	5	6	7
1	0	1	0	0	1	0	0
2	0	0	0	1	0	0	0

	$-v_t$	$+\ z_t$	$-\ z_{t-1}$	≤ 0	
	0	0	0	0	
	0	0	1	−1	
Don't want (not feasible)	0	1	0	1	
	0	1	1	0	
	1	0	0	−1	Feasible, but not min cost
	1	0	1	−2	Feasible, but not min cost
Want (feasible)	1	1	0	0	
	1	1	1	−1	Feasible, but not min cost

Multiple Products with Setup Costs

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{m}\sum_{j=1}^{t}\sum_{k=1}^{g} c_{ik}^{p} x_{ijk} + \sum_{i=1}^{m}\sum_{j=2}^{t+1}\sum_{k=1}^{g} c_{ik}^{i} y_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{t}\sum_{k=1}^{g} c_{ik}^{s} v_{ijk} \\
\text{subject to} \quad & -x_{ijk} + x_{(i+1)jk} - y_{ijk} + y_{i(j+1)k} = 0, && i = 1,\dots,m-1; j = 1,\dots,t; k = 1,\dots,g \quad (a) \\
& -x_{mjk} - y_{mjk} + y_{m(j+1)k} = d_{jk}, && j = 1,\dots,t; k = 1,\dots,g \ (b) \\
& x_{ijk} \le u_{ik} z_{ijk}, && i = 1,\dots,m; j = 1,\dots,t; k = 1,\dots,g \ (c) \\
& -v_{i1k} + z_{i1k} \le z_{ik}^{0}, && i = 1,\dots,m; k = 1,\dots,g \ (d) \\
& -v_{ijk} + z_{ijk} - z_{i(j-1)k} \le 0, && i = 1,\dots,m; j = 2,\dots,t; k = 1,\dots,g \ (e) \\
& \sum_{k=1}^{g} z_{ijk} \le 1, && i = 1,\dots,m; j = 1,\dots,t \ (f) \\
& y_{i1k} = y_{i}^{0}, && i = 1,\dots,m; k = 1,\dots,g \\
& y_{i(t+1)k} = y_{ik}^{t+1}, && i = 1,\dots,m; k = 1,\dots,g \\
& x, y \ge 0; v, z \in \{0,1\},
\end{aligned}
$$

where,

m = number of production stages
t = number of periods of production
g = number of products
c_{ik}^{p} = production cost (dollar/ton) in stage i for product k
x_{ijk} = production (ton) at stage i in period j for product k
c_{ik}^{i} = inventory cost (dollar/ton) in stage i for product k
y_{ijk} = stage-i inventory (ton) from period $j-1$ to j for product k
c_{ik}^{s} = setup cost (dollar) in stage i for product k
v_{ijk} = setup indicator at stage i in period j for product k
z_{ijk} = production indicator at stage i in period j for product k
d_{jk} = demand (ton) in period j for product k
u_{ik} = production capacity (ton) of stage i in period j for product k
z_{ik}^{0} = initial setup at stage i for product k
y_{ik}^{0} = initial inventory (ton) of stage i for product k
y_{ik}^{t+1} = final inventory (ton) of stage i for product k.

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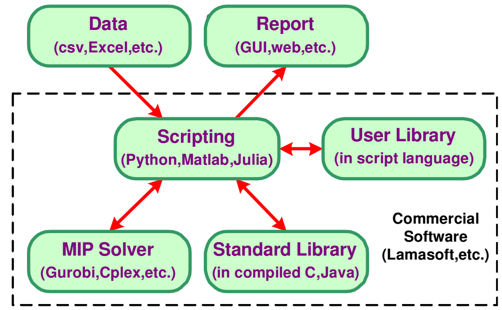
Work Flow of Logistics Software Stack



- **Flow:** *Data* → *Model* → *Solver* → *Output* → *Report*
  - reports are run on a regular period-to-period, *rolling-horizon* basis as part of normal operations management
  - model only changed when logistics network changes

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Discussion

1. Indicator variables require MILP instead of LP:
2. Lumpy resources require indicator variables to include fixed costs (setup, prod scale economies, etc)
3. Shared resources:
   - Coupling constraints used to ensures total output doesn't exceed available capacity (can still be LP)
   - Linking constraints used to ensure only one activity per period
4. Demand over planning horizon can be a mix of firm and forecasted orders from MPS/MRP/ERP
5. What is a realistic or typical number of demand periods?
   - Length should include all significant planning decisions (e.g., scheduled maintenance)
6. How is safety stock taken into consideration?

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Topics

1. Introduction
2. Location
3. Transport
   - Exam 1 (take home)
4. Networks
5. **Routing**
   - Exam 2 (take home)
6. Inventory
   - Final exam (in class)

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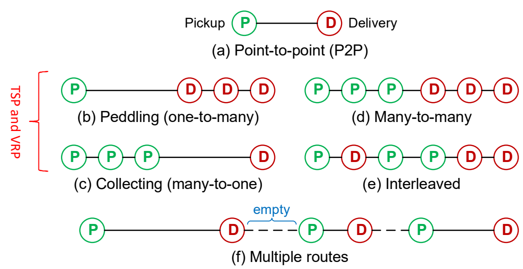
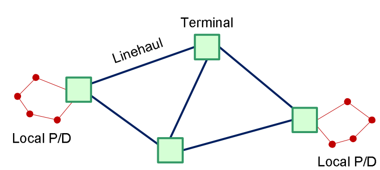
Routing 1: Traveling Salesman Problem

- Well solved problem
  - in practice and in approximation
  - not in theory

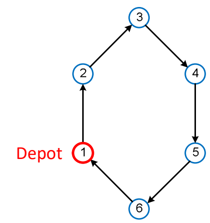
Routing Alternatives





TSP

- Problem: find connected sequence through all nodes of a graph that minimizes total arc cost
  - Termed a *tour* since returns to starting node
  - Subroutine in most vehicle routing problems
  - Node sequence can represent a route only if all pickups and/or deliveries occur at a single node (depot)

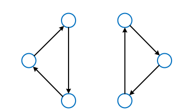


Node sequence = permutation + start node

1	2	3	4	5	6	1

$n = 6 \Rightarrow (n-1)! = 120$ possible solutions

TSP



- TSP can be solved by a mix of *construction* and *improvement* procedures
  - BIP formulation has an exponential number of constraints to eliminate subtours ($\Rightarrow$ column generation techniques)
- Asymmetric: only best-known solutions for large *n*

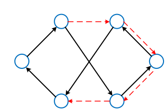
  $(n-1)! \quad n = 13 \Rightarrow \approx \frac{1}{2}$ billion solutions

- Symmetric: solved to optimal using BIP

  $c_{ij} = c_{ji} \Rightarrow \frac{(n-1)!}{2}$ solutions

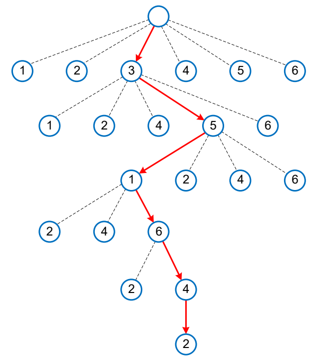
Note: $n-1$ because starting node of a route is arbitrary

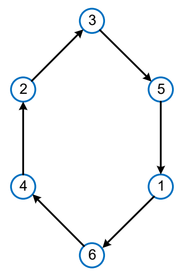
- Euclidean: arcs costs = distance between nodes



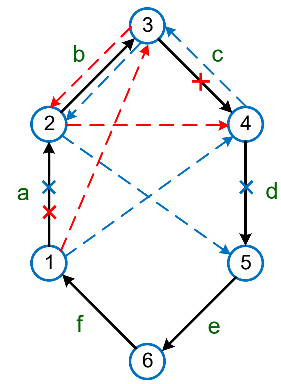
TSP Construction

- Construction easy since any permutation is feasible and can then be improved





Two-Opt Improvement

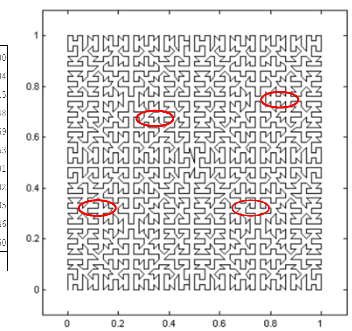


		a	b	c	d	e	f
	1	2	3	4	5	6	1
a-c	1	3	2	4	5	6	1
a-d	1	4	3	2	5	6	1
a-e	1	5	4	3	2	6	1
b-d							
b-e							
b-f							
c-e							
c-f							
d-f							

Sequences considered at end to verify local optimum: n nodes $\Rightarrow \overbrace{\sum_{j=3}^{n-1}(1)}^{\text{first arc } a} + \overbrace{\sum_{i=2}^{n-2}\sum_{j=i+2}^{n}(1)}^{\text{arcs } b \text{ to } n-2} = \dfrac{n(n-3)}{2} = 9 \text{ for } n = 6$

Spacefilling Curve

1.0	0.250	0.254	0.265	0.298	0.309	0.438	0.441	0.452	0.485	0.496	0.500
0.9	0.246	0.257	0.271	0.292	0.305	0.434	0.445	0.458	0.479	0.493	0.504
0.8	0.235	0.229	0.279	0.283	0.333	0.423	0.417	0.467	0.471 [1]	0.521	0.515
0.7	0.202	0.208	0.158	0.154 [3]	0.354	0.390	0.396	0.596	0.592	0.542	0.548
0.6	0.191	0.180	0.167	0.146	0.132	0.379	0.618	0.604	0.583	0.570	0.559
0.5	0.188	0.184	0.173	0.140	0.129	0.375	0.621	0.610	0.577	0.566	0.563
0.4	0.059	0.070	0.083	0.104	0.118	0.871	0.632	0.646	0.667	0.680	0.691
0.3	0.048	0.042	0.092	0.096	0.896	0.860	0.854	0.654	0.658	0.708	0.702
0.2	0.015	0.021 [2]	0.971	0.967	0.917	0.827	0.833	0.783 [4]	0.779	0.729	0.735
0.1	0.004	0.993	0.979	0.958	0.945	0.816	0.805	0.792	0.771	0.757	0.746
0.0	0.000	0.996	0.985	0.952	0.941	0.813	0.809	0.798	0.765	0.754	0.750
θ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0



Sequence determined by sorting position along 1-D line covering 2-D space

2: 0.021
3: 0.154
1: 0.471
4: 0.783

Bartholdi, John J., and Loren K. Platzman. "Heuristics Based on Spacefilling Curves for Combinatorial Problems in Euclidean Space." *Management Science*, vol. 34, no. 3, 1988, pp. 291–305. *JSTOR*, www.jstor.org/stable/2632046. Accessed 20 Oct. 2020.

Ex: Two-Opt Improvement

- Order in which *twoopt* considers each sequence:

1:	1	2	3	4	5	6	1	38
2:	1	3	2	4	5	6	1	39
3:	1	4	3	2	5	6	1	32
4:	1	3	4	2	5	6	1	31
5:	1	4	3	2	5	6	1	32
6:	1	2	4	3	5	6	1	31
7:	1	5	2	4	3	6	1	21
8:	1	2	5	4	3	6	1	21
9:	1	4	2	5	3	6	1	32
10:	1	3	4	2	5	6	1	31
11:	1	5	4	2	3	6	1	12
12:	1	4	5	2	3	6	1	34
13:	1	2	4	5	3	6	1	40
14:	1	3	2	4	5	6	1	39
15:	1	5	2	4	3	6	1	21
16:	1	5	3	2	4	6	1	30
17:	1	5	6	3	2	4	1	31
18:	1	5	4	3	2	6	1	13
19:	1	5	4	6	3	2	1	18
20:	1	5	4	2	6	3	1	20

Local optimal sequence

D:	1	2	3	4	5	6
1:	0	8	6	9	1	5
2:	3	0	1	5	4	2
3:	9	2	0	3	1	1
4:	8	2	1	0	10	6
5:	6	7	10	1	0	10
6:	6	2	5	2	1	0

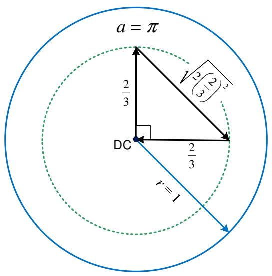
Note: Not symmetric

Sequences considered at end to verify local optimum: n nodes $\Rightarrow$

$$\sum_{j=3}^{n-1}(1) + \sum_{i=2}^{n-2}\sum_{j=i+2}^{n}(1) = \frac{n(n-3)}{2} = 9 \text{ for } n = 6$$

Expected Two-Customer Route Distance

Know: the expected distance from center of a circle, $d_a \approx \dfrac{2r}{3}$, and the expected angle between angle between two points is 90° (?); let area $a = \pi$ so that radius $r = 1$ and expected distance of an $m = 2$ customer route is $2\left(\dfrac{2}{3}\right)+\sqrt{2\left(\dfrac{2}{3}\right)^2} = \dfrac{2}{3}\left(2+\sqrt{2}\right)$.




```
span = 360; n = 1e6;
x = rand(n,1)*span; y = rand(n,1)*span;
absDiffDeg = @(a,b) min(360-mod(a-b,360), mod(a-b,360));
ang = absDiffDeg(x,y); % Abs difference between two angles
vdisp('min(ang),max(ang),mean(ang)')

 :  min(ang)  max(ang)  mean(ang)
-:-------------------------------
1:   0.0000    180.00     90.02
```

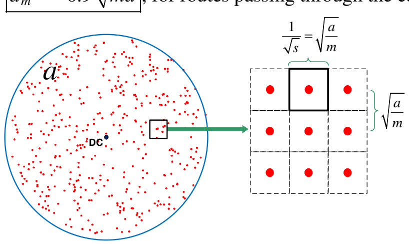
```
span = 180; n = 1e6;
x = rand(n,1)*span; y = rand(n,1)*span;
absDiffDeg = @(a,b) min(360-mod(a-b,360), mod(a-b,360));
ang = absDiffDeg(x,y); % Abs difference between two angles
vdisp('min(ang),max(ang),mean(ang)')

 :  min(ang)  max(ang)  mean(ang)
-:-------------------------------
1:   0.0002    179.64     59.98
```


Expected Route Distance

Given m customers in an area a, the density is $s = m/a$, and the expected distance between customers is $s^{-0.5} = \sqrt{a/m}$, resulting in an estimated total route distance that is proportional to $\varphi m\sqrt{a/m}$. Use known route distance for $m = 2$ to determine φ:

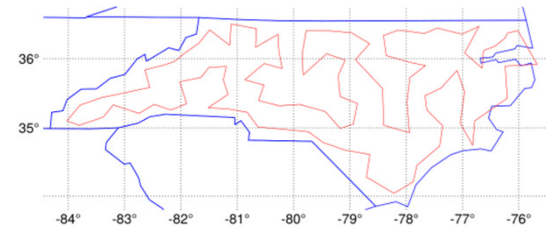
For $a = \pi$ and $m = 2$, $\varphi 2\sqrt{\dfrac{\pi}{2}} = \dfrac{2}{3}\left(2+\sqrt{2}\right) \Rightarrow \varphi = \dfrac{2\left(\sqrt{2}+1\right)}{3\sqrt{\pi}} \approx 0.9$, so that

$\boxed{\hat{d}_m^{TSP} = 0.9\sqrt{ma}}$, for routes passing through the center (DC) of a circular region.



:	m	Simulated	Estimate
1:	2	2.26	2.26
2:	5	3.79	3.57
3:	10	5.20	5.04
4:	20	7.02	7.13
5:	50	10.95	11.28
6:	100	15.50	15.95
7:	200	22.09	22.56
8:	500	35.07	35.67
9:	1,000	49.87	50.44

Ex: Tour of NC



Expected Tour Distance

```
@show a = sum(df.ALAND) + sum(df.AWATER)
@show TCᴱ = 0.9sqrt(nrow(df)*a)
@show TC°
TCᴱ/TC°
```

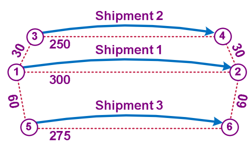
```
a = sum(df.ALAND) + sum(df.AWATER) = 53818.558999999994
TCᴱ = 0.9 * sqrt(nrow(df) * a) = 2087.8944606947925
TC° = 2141.2129756166273

0.9750989203180594
```


Routing 2:
Route-based Construction Procedures

- Two simple construction procedures, `mincostinsert` and `savings`, along with `twoopt` improvement, can be used for most routing applications
  - Can handle interleaved multi-stop routing, where each shipment has a different origin and destination

Multi-Stop Routing

- Each shipment might have a different origin and/or destination $\Rightarrow$ node/location sequence not adequate



$L = (y_1, \ldots, y_n) = (1,2,3)$ n-element shipment sequence

$R = (z_1, \ldots, z_{2n}) = (3,1,2,2,1,3)$ $2n$-element route sequence

$X = (x_1, \ldots, x_{2n}) = (5,1,3,4,2,6)$ $2n$-element location (node) sequence

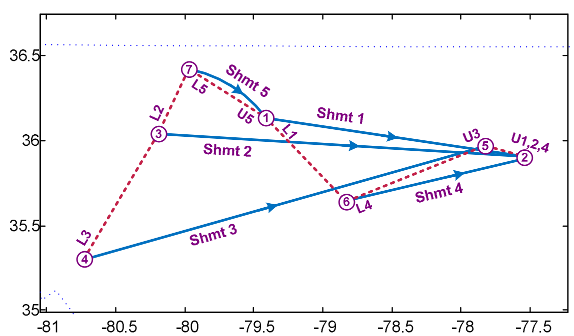
c_{ij} = cost between locations i and j

$$c(R) = \sum_{i=1}^{2n-1} c_{x_i, x_{i+1}} = 60 + 30 + 250 + 30 + 60 = 430, \quad \text{total cost of route } R$$

Route Sequencing Procedures

- Online procedure: add a shipment to an existing route as it becomes available
  - Insert and Improve: for each shipment, insert where it has the least increase in cost for route and then improve (`mincostinsert` $\rightarrow$ `twoopt`)
- Offline procedure: consider all shipments to decide order in which each added to route
  - Savings and Improve: using all shipments, determine insert ordering based on "savings," then improve final route (`savings` $\rightarrow$ `twoopt`)

5-Shipment Example



Route sequence: $R = (3,2,5,5,1,4,3,1,2,4)$

Location sequence: $X = (4,3,7,1,1,6,5,2,2,2)$

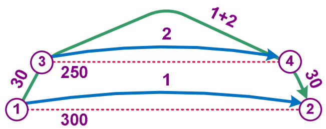
Min Cost Insert

		1			1			
1		•			•	2	2	×
2	2	•			•	2		c_2
3		•	2	2	•			c_3^*
4		•		2	•	2		c_4
5	2	•	2		•			c_5

$$\frac{n(n+3)}{2} - 1 \quad \text{evaluations}$$

		1			2		2		1	
1	3	•	3		•		•		•	
2	3	•			•	3	•		•	
3	3	•			•		•	3	•	
4	3	•			•		•		•	3
5		•	3	3	•		•		•	
6		•		3	•	3	•			
7		•		3	•		•	3		
⋮		⋮			⋮		⋮		⋮	

Pairwise Savings



$$s_{ij} = \text{pairwise savings between shipments } i \text{ and } j$$
$$= c_i + c_j - c_{ij} > 0$$
$$s_{1,2} = 300 + 250 - 310$$
$$= 240$$

Routing 3: Vehicle Routing

- Constraints turn a single TSP tour into several routes
- When a potential route violates a constraint, e.g.,
  - Total route distance
  - Total route time
  - Vehicle capacity
  - Delivery/pickup time windows
- Its cost can be set to infinity so that it is never selected

Vehicle Routing Problem

- VRP = TSP + vehicle constraints
- Constraints:
  - Capacity (weight, cube, etc.)
  - Maximum TC (HOS: 11 hr max)
  - Time windows (with/without delay at customer)
    - VRP uses absolute windows that can be checked by simple scanning
    - Project scheduling uses relative windows solved by shortest path with negative arcs
  - Maximum number of routes/vehicles (hard)
- Criteria:
  1. Number of routes/vehicles
  2. TC (time or distance)
- VRP solution can be one time or periodic
  - One time (operational) VRP minimizes TC
  - Periodic (tactical) VRP minimizes TLC (sometimes called a "milk run")

Clark-Wright (Offline) Savings Procedure

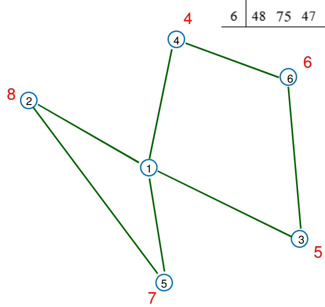
- First (1964), and still best, offline routing procedure if only have vehicle capacity constraints
- Pairs of shipments ordered in terms of their decreasing (positive) pairwise savings
- Given savings pair i-j, without exceeding capacity constraint, either:
  1. Create new route if i and j not in any existing route
  2. Add i to route only if j at beginning or end of route
  3. Combine routes only if i and j are endpoints of each route

Ex: Clark-Wright Savings Procedure

- Node 1 is depot, nodes 2-6 customers
- Customer demands 8, 3, 4, 7, 6, resp.
- Vehicle capacity is 15
- Symmetric costs

	1	2	3	4	5	6
1	0	40	48	38	33	48
2	40	0	87	46	65	75
3	48	87	0	67	41	47
4	38	46	67	0	70	34
5	33	65	41	70	0	69
6	48	75	47	34	69	0

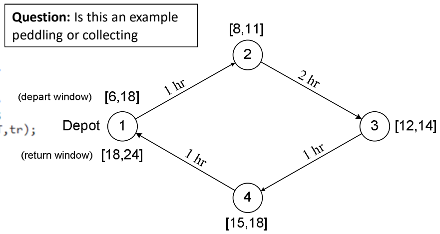
i	j	s_{ij}
2	3	$40+48-87=1$
~~2~~	~~4~~	$40+38-46=32$
$\boxed{2}$	$\boxed{5}$	8
~~2~~	~~6~~	13
~~3~~	~~4~~	19
~~3~~	~~5~~	40
$\boxed{3}$	$\boxed{6}$	49
4	5	1
$\boxed{4}$	$\boxed{6}$	52
~~5~~	~~6~~	12



Ex: VRP with Time Windows

```
T = zeros(4);
T(1,2) = 1; T(2,3) = 2;
T(3,4) = 1; T(4,1) = 1;
sh = vec2struct('b',1,'e',[2 3 4]);
sh = vec2struct(sh,'tU',0,'temin',...
   [8 12 15],'temax',[11 14 18]);
tr = struct('b',1,'e',1,'tbmin',6,...
   'tbmax',18,'temin',18,'temax',24);
[TC,~,out] = rteTC([1 2 3 1 2 3],sh,T,tr);
sdisp(sh),sdisp(out,false),TC
```

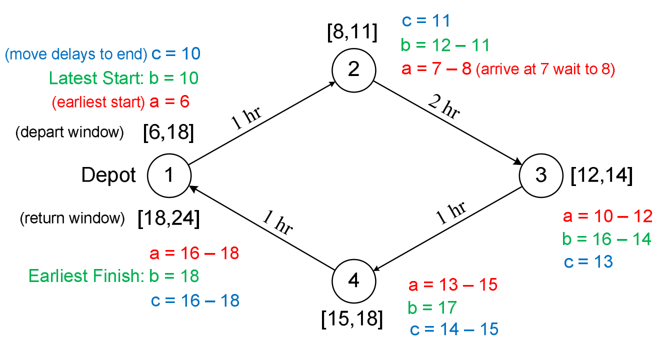
Question: Is this an example peddling or collecting




```
sh:  b  e  tU  temin  temax
--:-----------------------
 1:  1  2   0      8     11
 2:  1  3   0     12     14
 3:  1  4   0     15     18

out:  Rte  Loc  Cost  Arrive  Wait  TWmin  Start  LU  Depart  TWmax  Total
---:----------------------------------------------------------------------
  1:    0    1     0       0     0      6     10   0      10     18      0
  2:    1    1     0      10     0      6     10   0      10     18      0
  3:    2    1     0      10     0      6     10   0      10     18      0
  4:    3    1     0      10     0      6     10   0      10     18      0
  5:    1    2     1      11     0      8     11   0      11     11      1
  6:    2    3     2      13     0     12     13   0      13     14      2
  7:    3    4     1      14     1     15     15   0      15     18      2
  8:    0    1     1      16     2     18     18   0      18     24      3
TC =
```


Ex: VRP with Time Windows

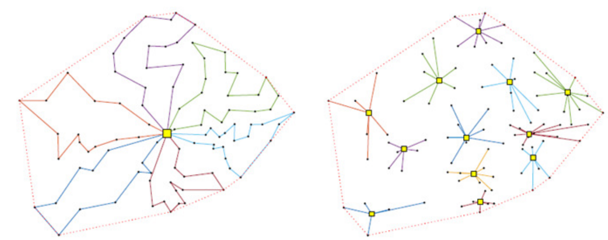
[0,24] hr; Loading/unloading time = 0; Capacity = ∞; LB = 5 hr



Earliest Finish – Latest Start = 18 – 10 = 8 hr = 5 travel + 3 delay

Tactical Routing

- Most routing procedures are for operational decisions
  - Given actual demands, determine actual route
  - For tactical decisions, can only estimate routing cost



(a) Multi-stop deliveries from a single DC. (b) Point-to-point deliveries from several DCs.

Figure 3. Multi-stop versus point-to-point delivery for the same set of customers (DC shown as yellow squares).

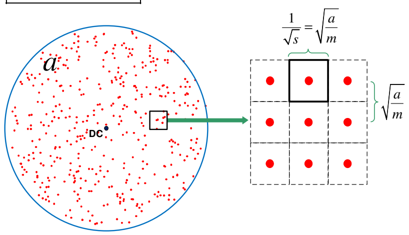
M.G. Kay, "Challenges and Opportunities Associated with Using Autonomous Vehicles and Drones for Home Delivery," MHI Whitepaper, 2020

Expected Route Distance

Given m customers in an area a, the density is $s = m/a$, and the expected distance between customers is $s^{-0.5} = \sqrt{a/m}$, resulting in an estimated total route distance that is proportional to $\varphi m \sqrt{a/m}$. Use known route distance for $m = 2$ to determine φ:

For $a = \pi$ and $m = 2$, $\varphi 2 \sqrt{\frac{\pi}{2}} = \frac{2}{3}\left(2+\sqrt{2}\right) \Rightarrow \varphi = \frac{2\left(\sqrt{2}+1\right)}{3\sqrt{\pi}} \approx 0.9$, so that

$\boxed{\hat{d}_m^{TSP} = 0.9\sqrt{ma}}$, for routes passing through the center (DC) of a circular region.



:	m	Simulated	Estimate
1:	2	2.26	2.26
2:	5	3.79	3.57
3:	10	5.20	5.04
4:	20	7.02	7.13
5:	50	10.95	11.28
6:	100	15.50	15.95
7:	200	22.09	22.56
8:	500	35.07	35.67
9:	1,000	49.87	50.44

Topics

1. Introduction
2. Location
3. Transport
   - Exam 1 (take home)
4. Networks
5. Routing
   - Exam 2 (take home)
6. **Inventory**
   - Final exam (in class)

Ex: Estimate Number of Deliveries

- Assuming that, on average, a vehicle travels at 30 mph (including stops at red lights) and, after reaching a customer, it takes two minutes to drop off a delivery. If the service area is fifty square miles, estimate the number of deliveries that the vehicle can make in eight hours assuming that it has no capacity constraints.

$$\varphi = 0.9, \quad a = 50 \text{ mi}^2, \quad u = \frac{2}{60} \text{ hr}, \quad t = 8 \text{ hr}, \quad v = 30 \text{ mph}$$

$$um + \frac{\varphi\sqrt{ma}}{v} = t \Rightarrow m = \frac{a\varphi^2 + 2tuv^2 - \sqrt{a\varphi^2\left(a\varphi^2 + 4tuv^2\right)}}{2u^2v^2} = 159.6018$$

```
dTSP = @(m,a) 0.9*sqrt(m*a);
m = fminsearch(@(m) abs(u*m + dTSP(m,a)/v - t), 1)  % m0 = 1
m =
   159.6018
```

Question: Should the estimate be 159 or 160 deliveries?
Note: Rounding to 160 (closest integer) results in total time > 8 hrs

Inventory 1: Working, Economic, and One-Time Safety Stock

- What Makes Production System Design Hard?
  1. Things not always where you want them when you want them $\Rightarrow$ Working stock
  2. Resources are lumpy $\Rightarrow$ Economic stock
  3. Things vary

     variability can be known or unknown

     uncertainty/randomness = unknown variability

     uncertainty/randomness $\Rightarrow$ Safety stock

Role/Position of Inventory

- Same units of inventory can serve multiple roles at each position in a production process

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)
- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)
- *Safety stock:* held to buffer effects of uncertainty
  - (decoupling, MRO (maintenance, repair, and operations))

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Total Logistics Cost

- *Total Logistics Cost* (TLC) includes all costs that could change as a result of a logistics-related decision

$$TLC = TC + IC + PC$$
$$TC = \text{transport cost}$$
$$IC = \text{inventory cost}$$
$$= IC_{\text{working}} + IC_{\text{economic}} + IC_{\text{safety}}$$
$$PC = \text{purchase cost}$$

Total logistics costs are any of the relevant costs associated with providing a logistics service, where a relevant cost is a cost that differs when comparing multiple alternatives and, as such, can be used in making a decision between the alternatives.

- *Economic* (*cycle*) *stock*: held to allow cheaper large shipments
- *Working* (*in-transit*) *stock*: goods in transit or awaiting transshipment
- *Safety stock*: held due to demand and transport uncertainty (e.g., shipment arriving earlier than needed "just in case")
- *Purchase cost*: can be different for different suppliers

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Determining Optimal Inventory Levels

- **Deterministic world:** min *TLC* using average demand and transport lead times
  - *Working and economic*: optimal level balances *IC* with *TC* and *PC* since ↑*IC* ⇒ ↓*TC* and ↓*PC*
  - *Safety*: optimal level = 0 ⇒ *IC* = 0
- **Stochastic (real) world:** can't just min *TLC*
  - *Working and economic*: Can still min *TLC,* using (*detrended*) averages
  - *Safety*: max Total Profit (*TP* = *TR* – *TLC*), using actual (historical or synthetic) demand and transport lead times
    - If inv level = 0 (out of stock) when a demand occurs ⇒ lost sale (0 revenue) or backorder (likely ↓ price)
    - Optimal safety stock level balances out-of-stock revenue ↓ with ↑ *IC* due to holding/disposing stock

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Inventory Cost

- *Inventory Cost* (IC) of working, economic, and safety stock can be calculated in the same general way:

$$IC = (\text{annual cost of holding one ton})(\text{average annual inventory level})$$
$$= vh\ (\$/\text{ton-yr}) \times q\ (\text{ton}) = (\$/\text{yr})$$
$$v = \text{unit value of inventory } (\$/\text{ton})$$
$$h = \text{inventory carrying rate, cost per dollar of inventory per year } (\$/\$\text{-yr} = 1/\text{yr})$$
$$q = \text{average annual inventory level (ton)}$$

- Value for v can be determined from the purchase price or production cost
- Rate h usually not known directly
  - Based on interest, warehousing, and obsolescence costs
- Determining q differs for working, economic, and safety stock

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Inventory Carrying Rate

- **Inv. Carrying Rate** (h) = interest + warehousing + obsolescence
- Interest: 5% per Total U.S. Logistics Costs
- Warehousing: 6% per Total U.S. Logistics Costs
- Obsolescence: default rate (yr) h = 0.3 $\Rightarrow h_{\text{obs}} \approx$ 0.2 (mfg product)
  - *Low* FGI cost (yr): $h = h_{\text{int}} + h_{\text{wh}} + h_{\text{obs}}$
  - *High* FGI cost (hr): $h \approx h_{\text{obs}}$, can ignore interest & warehousing
    - $(h_{\text{int}}+h_{\text{wh}})/H$ = (0.05+0.06)/2000 = 0.000055 (H = oper. hr/yr)
  - Estimate h_{obs} using "percent-reduction interval" method: given time t_h when product loses x_h-percent of its original value v, find h_{obs}

$$h_{\text{obs}} t_h v = x_h v \Rightarrow h_{\text{obs}} t_h = x_h \Rightarrow \boxed{h_{\text{obs}} = \frac{x_h}{t_h}}, \quad \text{and} \quad t_h = \frac{x_h}{h_{\text{obs}}}$$

  - Example: If a product loses 80% of its value after 2 hours 40 minutes:

$$t_h = 2 + \frac{40}{60} = 2.67 \text{ hr} \Rightarrow h = \frac{x_h}{t_h} = \frac{0.8}{2.67} = 0.3$$

  - Important: t_h should be in same time units as t_{CT}

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Working Stock

- *Working stock*: held as part of production process
  - (in-process, pipeline, in-transit, presentation)

Role		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- Was ignored for truck transport since transit time were just a few days
  - compared to weeks between shipments
- Important for comparing international transport alternatives
  - ocean (weeks) vs. air cargo (hours)

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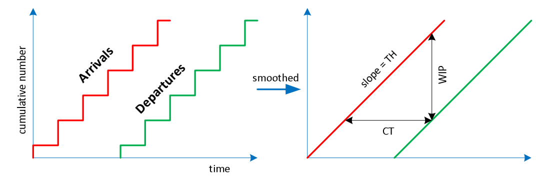
Average Working Stock = WIP



$$\textbf{Little's Law:} \quad TH(r) = \frac{WIP(q)}{CT(t)}, \quad CT = \frac{WIP}{TH}, \quad \boxed{WIP = TH \cdot CT}$$

where $TH = r = \frac{q}{t} = throughput$

= average rate at which work is produced (units per hour)

$WIP = q = work\text{-}in\text{-}process$

= average number of units of work in a production system

$CT = t = cycle\ time$

= average time each unit of work is in a production system

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Ex: Little's Law

- If it takes, on average, nine semesters for an undergraduate ISE student to graduate and 40 students, on average, graduate each semester, how many students are in the department?

$$\begin{aligned} WIP &= TH \cdot CT \\ &= 40(9) = 360 \text{ students} \end{aligned}$$

  - Note: Little's Law only works if WIP is not changing; taking the average is a good estimate of the steady state working inventory.

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Ex: In-Transit Inventory

- Currently, a factory in Los Angeles imports 12 containers per year from a supplier in Shanghai, each with 15 tons of raw material. Each ton of RM costs $7000 and loses 10% of its value after three months. Each container spends 15 days in transit, and its transport cost is $2620. What is the annual in-transit inventory cost for RM?

Shipment size: $q = 15$ ton/L, Shipment frequency: $n = 12$ L/yr

$$CT: \ t = \frac{15}{365.25} \text{ yr}, \quad TH: \ nq = 12(15) = 180 \text{ ton/yr}$$

$$WIP = TH \cdot CT: \ q_I = nqt = 12(15)\frac{15}{365.25} = 7.39 \text{ ton}$$

$$h_{\text{obs}} = \frac{x_h}{t_h} = \frac{0.1}{0.25} = 0.4, \quad h = 0.05 + 0.06 + h_{\text{obs}} = 0.51$$

$$IC_w = vhq_I = 7000(0.51)7.39 = \$26{,}390.14/\text{yr} \quad \left[\$/\text{ton}\left(1/\text{yr}\right)\text{ton} = \$/\text{yr}\right]$$

Economic Stock

- *Economic stock:* held to allow cheaper production
  - (cycle, anticipation)

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *Cycle* (economic) inventory: held to allow cheaper large shipments/orders or production batches
- *Anticipation* (economic) inventory: product that is cheaper to purchase or produced early
  – Product on sale, discounted, or discontinued
  – Production that occurs throughout the year ahead of a single peak selling period (e.g., Christmas decorations)

Ex: Supplier Selection

- Cont prev Ex: A domestic supplier has been identified with a cost of $7500 per ton. 12 TLs per year, each containing 15 tons of RM would be shipped 200 miles in one day. TL revenue per loaded mile is $2, should the domestic supplier be used?

$$\begin{aligned} TLC_1 &= TC + IC_w + PC = nc_L + vhq_I + nvq \\ &= 12(2620) + 7000(0.51)7.39 + 12(7000)15 \\ &= 31{,}440 + 26{,}390 + 1{,}260{,}000 = \$1{,}317{,}830/\text{yr} \end{aligned}$$

$$r_{TL} = \$2/\text{mi}, \quad d = 200 \text{ mi}, \quad c_L = r_{TL}d = \$400$$

$$q_I = nqt = 12(15)\frac{1}{365.25} = 0.4928 \text{ ton}$$

$$\begin{aligned} TLC_2 &= TC + IC_w + PC = nc_L + vhq_I + nvq \\ &= 12(400) + 7500(0.51)0.4928 + 12(7500)15 \\ &= 4{,}800 + 1{,}885 + 1{,}350{,}000 = \$1{,}356{,}685/\text{yr} \end{aligned}$$

Cycle Inventory

- *Cycle* (economic) inventory:
  - Inventory held between periodic shipments/orders or production batches to serve steady/constant demand
  - Average annual cycle inventory level: sum of the average level at the origin and destination

$$\begin{aligned} IC_{\text{cycle}} &= (\text{annual cost of holding one ton})(\text{average annual inventory level}) \\ &= vh \ (\$/\text{ton-yr}) \times \alpha q \ (\text{ton}) = (\$/\text{yr}) \end{aligned}$$

q = average shipment/order or production batch size (ton)

α = average inter-shipment inventory fraction at Origin and Destination

v = unit value of inventory ($/ton)

h = inventory carrying rate, cost per dollar of inventory per year ($/$-yr = 1/yr)

Note: q_I = average annual inventory level

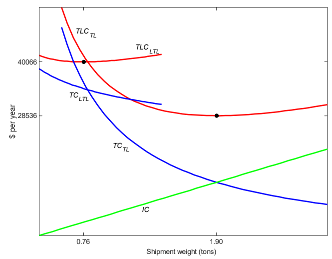
$$= \begin{cases} \alpha q, & \text{cycle (economic) inventory} \\ nqt, & \text{in-transit (working) inventory} \end{cases}$$

Optimal-Size Truck Shipments

- As shown previously (see Periodic Truck Shipments):

$$TLC_{TL}(q) = TC_{TL}(q) + IC(q) = \frac{f}{q} c_{TL}(q) + \alpha vhq = \frac{f}{q} rd + \alpha vhq$$

$$\frac{dTLC_{TL}(q)}{dq} = 0 \Rightarrow q^*_{TL} = \sqrt{\frac{f r_{TL} d}{\alpha vh}}$$



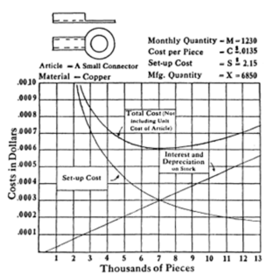
Economic Order Quantity (EOQ)

- Finds the optimal quantity (termed the EOQ) that minimizes the number of shipments/orders or production batches and the cost of holding them as inventory
- Early (1913) paper "How Many Parts to Make at Once" in *Factory, The Magazine of Management* by Ford W. Harris that introduced simple square-root formula to determine how many parts to make

$$TLC(q) = \frac{f}{q} k + \frac{1}{2} vhq, \quad \alpha = \frac{1}{2}$$

$$EOQ: \; q^* = \sqrt{\frac{2fk}{vh}}$$

$$k = \begin{cases} \text{fixed production cost} \\ \text{setup cost} \\ \text{transport cost} \\ \text{ordering cost} \end{cases}$$



Safety Stock

- *Safety stock:* held to buffer effects of uncertainty

		Position		
		Raw Material	Work in Process	Finished Goods
Role	Working Stock			
	Economic Stock			
	Safety Stock			

- *One-time* safety stock: Uncertain demand and not able to carry inventory
  - Unmet demand is a lost sale
  - Excess product is disposed of
- *Periodic* safety stock: Uncertain demand and/or transport lead time, and able to carry inventory
  - Unmet demand is either a lost sale or backordered
  - Excess product is carried over to the next period

Inventory 2: Periodic Safety Stock

- *One-time* safety stock: Uncertain demand and not able to carry inventory
  - Unmet demand is a lost sale
  - Excess product is disposed of
  - Optimal policy: Tradeoff between lost profit and disposal cost
- *Periodic* safety stock: Uncertain demand and/or replenishment lead time, and able to carry inventory
  - Unmet demand is either a lost sale or backordered
  - Excess product is carried over to the next period
  - Optimal policy: Tradeoff with lost-profit or backorder-cost, and inventory carrying cost

Squared Coefficient of Variation

- Provides a normalized measure used to estimate of variance of a process (demand, production, etc.)

$$c = \frac{\sigma}{t} = \text{coefficient of variation (CV)}$$

$$c^2 = \frac{\sigma^2}{t^2} = \text{squared coefficient of variation (SCV)}$$

$$= \begin{cases} 0, & \text{deterministic/exactly (best case, } LB\text{)} \\ < 0.75 & \text{low variability} \\ \geq 0.75, < 1.33, & \text{moderate variability} \\ 1, & \text{Poisson} \Leftrightarrow \text{totally random (practical worse case, } UB\text{)} \\ \geq 1.33, & \text{high variability (bad control)} \end{cases}$$

σ = standard deviation of process

t = mean of process

σ^2 = variance of process

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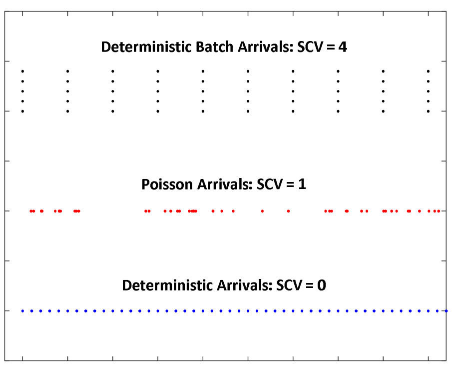
Low, Moderate and High SCVs

- All arrivals have same rate of 10 per hour



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Base Stock with Lost Sales

- Pure safety stock
- Seller makes one decision:
  1. Maximum finished goods inventory level
- Control logic for seller:
  – Start with inventory at max level
  – Order replacement after each (unit) customer sale
- Customer fulfilment process:
  – If demand and inventory level > 0, make sale; otherwise, lost sale
- Performance measures:
  1. Out-of-stock percentage
  2. Average inventory level

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Optimal Base-Stock Policy

$$\max_{q_{\max}} TP = (p - c)(1 - \pi_0) r_a - ch\bar{q}$$

where
$q_{\max}$ = maximum inventory level
p = unit sales price
c = unit cost
π_0 = probability out of stock
r_a = demand arrival rate
h = inventory carrying rate
$\bar{q}$ = average inventory level

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Order Point with Lost Sales

- Safety + cycle stock
- Seller makes two decisions:
  1. Maximum finished goods inventory level
  2. Order point (minimum inventory level)
- Control logic for seller:
  - Start with inventory at max level
  - Order up to max level when *position* falls below min level
- Customer fulfilment process:
  - If demand and inventory level > 0, make sale; otherwise, lost sale
- Performance measures:
  1. Out-of-stock percentage
  2. Average inventory level
  3. Average number of orders

Optimal Order-Point Policy

$$\max_{q_{\max},\, q_{\min}} TP = (p-c)(1-\pi_0)r_a - ch\bar{q} - c_o\bar{n}_o$$

where
$q_{\max}$ = maximum inventory level
$q_{\min}$ = order point (minimum inventory level)
p = unit sales price
c = unit cost
π_0 = probability out of stock
r_a = demand arrival rate
h = inventory carrying rate
$\bar{q}$ = average inventory level
c_o = fixed cost per order
$\bar{n}_o$ = average number of orders

Inventory 3:
Multi-Echelon Inventory Systems

- Last Level:
  - Only the last level (or echelon) of a multi-echelon system hold FG inventory and is customer facing
  - Stockouts at this level impact revenue through lost sales and/or backorder costs
- Other Levels:
  - RM and WIP inventory held to support last level
  - Stockouts at these levels always backordered, no notion of lost sales

Two-Echelon Supply Chain